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TECHNICAL ASPECTS OF THE ITERATIVE SOLUTION OF THE AUTOMOBILE INSURANCE PROBLEM

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Technical aspects of the iterative solution of the automobile insurance problem

by

P.J. Weeda

ABSTRACT

A specific version of the automobile insurance problem is solved by using the iteration cycle of the generalized Markov programming method developed by DE LEVE. The emphasis is on the way the functional equation, appearing in the strategy evaluation part of the method, is solved numerically. The computer output of the course of the iteration of two numerical examples is presented.

KEY WORDS & PHRASES: *Automobile insurance problem, generalized Markov programming method, numerical solution.*

1. INTRODUCTION AND PROBLEM FORMULATION

In DE LEVE & WEEDA [5] and DE LEVE, TIJMS & WEEDA [4] it has been shown how a specific version of the automobile insurance problem can be solved by a direct approach to an optimal claiming strategy. This method leads to a functional equation, which possesses an explicit solution in case the damage distribution is negative exponential. However for other distributions the functional equation has to be solved numerically. In these cases preference has been given to an iterative approach to an optimal strategy by using the iteration scheme of the generalized Markov programming method of DE LEVE [2]. The technical aspects of the iterative approach have never been published thus far. This report is to fill up this gap and to satisfy recent interest in the subject.

The problem is to determine an optimal claiming strategy for a policy holder of an accident insurance under the following conditions. The insurance runs for 1 year. The premium due for the first year amounts to E_0. If no claim has been filed during i successive years, $i = 1,2,\dots,N-1$ with N a finite natural number, the premium is reduced to E_i. If in N or more successive years no claim is filed, the premium due amounts to E_{N-1}. If a claim is filed in any year, then the premium due for the next year amounts to E_0 again. The number of accidents is assumed to be Poisson distributed with a mean of λ per year. It is further assumed that the damages caused by the accidents are mutually independent random variables which have a common distribution function $F(s)$ with finite mean and variance. Furthermore, the damages are assumed to be stochastically independent of the Poisson process which generates the accidents. The purpose is to find a claiming strategy that minimizes the expected average costs per year in the long run.

2. REVIEW OF THE ITERATIVE SOLUTION

The state space of the problem consists of

(1) N points E_i, $i = 0,1,\dots,N-1$. In these states the corresponding premium has to be paid; damages are no longer covered by insurance.

(2) A 3-dimensional Eucledian subspace (t,s,u) with $11 \le t < 1N$, $s \ge 0$ and $u \ge 0$. The state of the system is a point of this space as soon as at least one claim has been filed in the current premium year. The variable t denotes time. We have $1i \le t < 1i + 1$,*) $i = 1,\dots,N$ if the premium due at the beginning of the current premium year has been E_{i-1} if $i \le N-1$ and E_{N-1} if $i = N$. The variable s denotes the extent of the last damage. The variable u denotes the time elapsed in the current premium year since the first claim in that year and equals zero if no claim has been filed.

(3) A 1-dimensional interval $21 \le t < 2N + 1$. Each value of t in the subinterval $2i \le t < 2i + 1$, $i = 1,2,\dots,N$ represents the state of the system if no claim has been filed in the current premium year.

In each state the decisionmaker has at most one intervention and at most one null decision at his disposal. An intervention results in a deterministic change of the state of the system in time zero, while the null decision leaves the state of the system unaltered. In the states E_i, $i = 0,\dots$ $\dots,N-1$ only the intervention "pay the premium" is feasible which transforms the system to state $t = 2i$. In the states $t = 2i + \tau$, $0 < \tau < 1$ the state of the system becomes $(1i+\tau,s,0)$ if an accident occurs with damage s. If the claim is filed (null decision!) the system remains in the 3-dimensional subspace. If the claim is not filed (intervention!), the state of the system becomes $t = 2i + \tau$ again. If no claim is filed during the year represented by the interval $2i \le t < 2i + 1$, the state of the system becomes E_i if $i \le N - 1$ and E_{N-1} if $i = N$. If at least one damage has been claimed, the state of the system at the end of the premium year becomes E_0.

For a particular strategy z, the set of intervention states, denoted by A_z, is given by

$$A_z = \{(t,s,u):\ 11 \le t < 1N + 1,\ 0 \le s \le s_z(i,t),\ u = 0\} \cup$$
$$\cup \{E_i:\ i = 0,1,2,\dots,N-1\}$$

where $s_z(i,t)$ denotes the boundary of the set of states in which the dam-

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*) The notation $1i$, $i = 1,\dots,N$ means $11,12,\dots,19$ if $N = 9$; $101,102,\dots,110$ if $N = 10$, etc.

age is *not* claimed. Note that state E_0 is accessible from each other state in the Markov process in A_z. Hence this process has only one simple ergodic set and therefore the expected average costs per year for a strategy z are independent of the state.

We define

$$(1) \qquad v(E_0;z) = 0$$

to obtain a unique solution in the function $v(S;z)$. Further we have

$$(2) \qquad v((t,s,u);z) = \begin{cases} 0 & \text{for } (t,s,u) \notin A_z \\ \max[s-a_0,0] + v(t+10;z) & \text{for } (t,s,u) \in A_z \end{cases}$$

$$(3) \qquad \lim_{\tau \uparrow 1} v(2i+\tau;z) = v(E_i;z) \qquad \text{for } i = 1,2,\dots,N-1$$

and

$$(4) \qquad \lim_{\tau \uparrow 1} v(2i+\tau;z) = \lim_{\tau \uparrow 1} v(2i+\tau+1;z) = v(E_{N-1};z) \quad \text{for } i = N-1.$$

Relation (4) follows from the fact that after n years of claim-free driving, the premium due amounts to E_{N-1} if and only if $n \geq N-1$. For $v(E_{i-1};z)$ we have

$$(5) \qquad v(E_{i-1};z) = E_{i-1} + \lambda k(a_0) - y(z) + v(2i;z) \quad \text{for } i = 1,2,\dots,N$$

where $k(a_0)$ is the expected cost of the policy holder per accident if he claims whenever $s > a_0$

$$(6) \qquad k(a_0) = \int_0^{a_0} s\, dF(s) + a_0(1 - F(a_0)).$$

Finally we have for $v(2i+\tau;z)$ $i = 1,2,\dots,N$

$$(7) \qquad v(2i+\tau;z) =$$

$$v(E_i;z) \int_{1-\tau}^{\infty} \lambda e^{-\lambda t} dt + v(E_0;z) \int_0^{1-\tau} \lambda e^{-\lambda t} dt \int_{s_z(1i+\tau+t)}^{\infty} d\,F(u) +$$

$$\int_0^{1-\tau} \lambda e^{-\lambda t} dt \int_0^{s_z(1i+\tau+t)} v((1i+\tau+t,u,0);z)\, d\,F(u).$$

Substitution of (1), (2) and (3) followed by differentiation with respect to τ yields

$$\text{(8)} \qquad \frac{d\, v(2i+\tau;z)}{d\tau} = \lambda v(2i+\tau;z) \int_{s_z(1i+\tau)}^{\infty} d\,F(u) + - \lambda \int_{a_0}^{s_z(1i+\tau)} (u-a_0)\, d\,F(u).$$

The policy improvement and cutting operation of generalized Markov programming applied to this problem combine, as proved in [3], to the computation of the boundary $s_{z_{n+1}}(1i+\tau)$ of the next strategy z_{n+1} by the following identity

$$\text{(9)} \qquad s_{z_{n+1}}(1i+\tau) = a_0 - v(2i+\tau;z_n).$$

For a more detailed treatment the reader is referred to [3], [4] or [5].

3. COMPUTATIONAL DETAILS OF THE ITERATIVE SOLUTION

The differential equation (8) is linear and of the first order. Introducing the notation

$$v_i(\tau) \overset{\text{def}}{=} v(2i+\tau;z_n)$$

$$P_i(\tau) \overset{\text{def}}{=} \lambda \int_{s_{z_n}(1i+\tau)}^{\infty} d\,F(y)$$

$$Q_i(\tau) \overset{\text{def}}{=} \lambda \int_{a_0}^{s_{z_n}(1i+\tau)} (u-a_0)\, d\,F(u)$$

its general solution is

$$\text{(10)} \qquad v_i(\tau) = e^{\int P_i(\tau)d\tau} \left[C_i - \int Q_i(\tau)\, e^{-\int P_i(\tau)d\tau} d\tau \right]$$

where C_i is an integration constant. If we introduce the additional notation

$$A_i(\tau) \overset{\text{def}}{=} \int P_i(\tau)d\tau$$

$$B_i(\tau) \overset{\text{def}}{=} \int Q_i(\tau)\, e^{-A_i(\tau)}\, d\tau$$

then (10) is equivalent to

$$\text{(11)} \qquad v_i(\tau) = e^{A_i(\tau)} [C_i - B_i(\tau)].$$

LEMMA. *For* $\tau \geq 0$ (10) *is equivalent to*

$$\text{(12)} \qquad v_i(\tau) = e^{\int_0^\tau P_i(x)dx} \left[v_i(0) - \int_0^\tau Q_i(x)\, e^{-\int_0^x P_i(y)dy} dx \right].$$

PROOF. We have for $\tau \geq 0$

$$\int_0^\tau P_i(x)dx = A_i(\tau) - A_i(0)$$

and

$$\int_0^\tau Q_i(x)\, e^{-\int_0^x P_i(y)dy} dx = \int_0^\tau Q_i(x)\, e^{-A_i(x)+A_i(0)} dx =$$

$$= e^{A_i(0)} [B_i(\tau) - B_i(0)].$$

Hence if we substitute in (11) the relations

$$A_i(\tau) = A_i(0) + \int_0^\tau P_i(x)dx$$

and

$$B_i(\tau) = e^{-A_i(0)} \int_0^\tau Q_i(x)\, e^{-\int_0^x P_i(y)dy} dx + B_i(0)$$

we obtain that for $\tau \geq 0$ (11) and also (10) are equivalent to

$$v_i(\tau) = e^{\int_0^\tau P_i(x)dx}\left[e^{A_i(0)}\{C_i - B_i(0)\} - \int_0^\tau Q_i(x)\, e^{-\int_0^x P_i(y)dy} dx\right] =$$

$$= e^{\int_0^\tau P_i(x)dx}\left[D_i - \int_0^\tau Q_i(x)\, e^{-\int_0^x P_i(y)dy} dx\right]$$

with $D_i = e^{A_i(0)}\{C_i - B_i(0)\}$. Substitution of $\tau = 0$ yields $v_i(0) = D_i$ implying the assertion. □

The numerical solution is obtained by numerical integration of the definite integrals

$$\int_0^\tau \lambda\, dx \int_{s_{z_n}(li+x)}^\infty d\, F(y)$$

and

$$\int_0^\tau \left\{e^{-\int_0^x \lambda dy \int_{s_{z_n}(li+y)}^\infty d\, F(w)}\ \lambda \int_{a_0}^{s_{z_n}(li+x)} (u-a_0)\ d\, F(u)\right\} dx.$$

If the density function of the damage distribution is explicitly known, the inner integrals are computed by the ALGOL 60-procedure QADRAT described in [6]*). The interval $0 \le \tau \le 1$ is discretized to the equidistant points jh, $j = 0,1,\ldots,J$ with $hJ = 1$. By means of QADRAT the array elements $P[i,j] \overset{def}{=} P_i(jh)$ and $Q[i,j] \overset{def}{=} Q_i(jh)$ are computed for $j = 0,1,\ldots,J$ and $i = 1,\ldots,N$ using the formulas

$$P[i,j] = \lambda(1 - \int_0^{s[i,j]} d\, F(y))$$

and

$$Q[i,j] = \lambda \int_{a_0}^{s[i,j]} (u-a_0)\ d\, F(u)$$

*) In the earlier version the ALGOL 60-procedure QAD has been used.

with $s[i,j] \stackrel{def}{=} s_{z_n}(li+jh)$. The integrals $\int_0^x P_i(x)dx$ and $\int_0^\tau Q_i(x)e^{-\int_0^x P_i(y)dy}dx$ are computed by using the Newtonian integration formulas, see [1] p.140, (for brevity we discuss the procedure only for the first integral)

$$\text{(13)} \qquad \int_{jh}^{jh+h} P_i(x)\,dx = $$

$$= h\left(1 + \frac{1}{2}\Delta^1 - \frac{1}{12}\Delta^2 + \frac{1}{24}\Delta^3 - \frac{19}{720}\Delta^4 + \frac{3}{160}\Delta^5 + \ldots\right) P_i(jh)$$

and

$$\text{(14)} \qquad \int_{jh-h}^{jh} P_i(x)\,dx = $$

$$= h\left(1 - \frac{1}{2}\nabla^1 - \frac{1}{12}\nabla^2 - \frac{1}{24}\nabla^3 - \frac{19}{720}\nabla^4 - \frac{3}{160}\nabla^5 + \ldots\right) P_i(jh).$$

In formula (13) Δ^r, $r = 0,1,2,\ldots$ denotes the r^{th} order forward difference operator defined recursively by

$$\text{(15)} \qquad \begin{cases} \Delta^0 P_i(jh) \stackrel{def}{=} P[i,j] \\ \Delta^1 P_i(jh) \stackrel{def}{=} P[i,j+1] - P[i,j] \\ \Delta^{r+1} P_i(jh) \stackrel{def}{=} \Delta^r P_i(jh+h) - \Delta^r P_i(jh) \end{cases}$$

for $r = 0,1,2,\ldots$. In formula (14) ∇^r, $r = 0,1,2,\ldots$ denotes the r^{th} order backward difference operator defined recursively by

$$\text{(16)} \qquad \begin{cases} \nabla^0 P_i(jh) \stackrel{def}{=} P[i,j] \\ \nabla^1 P_i(jh) \stackrel{def}{=} P[i,j] - P[i,j-1] \\ \nabla^{r+1} P_i(jh) \stackrel{def}{=} \nabla^r P_i(jh) - \nabla^r P_i(jh-h). \end{cases}$$

The integral $\int_0^{jh+h} P_i(x)dx$ is computed recursively by

$$\int_0^{jh+h} P_i(x)\,dx = \int_0^{jh} P_i(x)\,dx + \int_{jh}^{jh+h} P_i(x)\,dx.$$

In the ALGOL 60 program both formulas (13) and (14) are truncated after the 5^{th} order term. Because the use of the formulas would imply the calculation of the differences at each integration step, the formulas are used in a form which uses the function values rather than their differences. The coefficients of the function values in this form are computed in the program once and for all. Because the use of the formula (13) involves function values outside the interval $2i \le \tau < 2i + \tau$, formula (13) is used in the points $j = 0,1,\ldots,[J/2]^-$ and formula (14) in the points $j = [J/2]^+,\ldots,J$ where the notation $[\cdot]^-$ represents the largest integer smaller than the argument and $[\cdot]^+ \overset{def}{=} [\cdot]^- + 1$.

Denoting the first 6 coefficients in (13) by α_r, $r = 0,1,\ldots,5$ and using the relations

$$\Delta^r P_i(jh) = \sum_{n=0}^{r} \binom{r}{n} (-1)^{r+n} P[i,j+n] \tag{17}$$

and

$$\nabla^r P_i(jh) = \sum_{n=0}^{r} \binom{r}{n} (-1)^{n} P[i,j-n] \tag{18}$$

which are implied by (15) and (16) respectively, we obtain

$$\int_{jh}^{jh+h} P_i(x)\,dx \approx \sum_{r=0}^{5} \alpha_r \sum_{n=0}^{r} \binom{r}{n} (-1)^{r+n} P[i,j+n] \tag{19}$$

and

$$\int_{jh-h}^{jh} P_i(x)\,dx \approx \sum_{r=0}^{5} (-1)^r \alpha_r \sum_{n=0}^{r} \binom{r}{n}(-1)^{n} P[i,j-n]. \tag{20}$$

Both formulas (19) and (20) use the function values explicitly. Note that in this form they have the same coefficients as should be so by symmetry.

In the numerical solution of the differential equation the constants $v_i(0)$, $i = 1,2,\ldots,N$ are still unknown. If we introduce the notation

$$G_i(\tau) \overset{def}{=} e^{\int_0^\tau P_i(x)dx}$$

$$H_i(\tau) \overset{def}{=} G_i(\tau) \int_0^\tau Q_i(x)\, e^{-\int_0^x P_i(y)dy} dx$$

then we have the following relations in $v_i(0)$ and $v_i(1)$

(21) $$v_i(1) = G_i(1)\, v_i(0) - H_i(1) \qquad \text{for } i = 1,2,\dots,N$$

in which $G_i(1)$ and $H_i(1)$, $i = 1,\dots,N$ are known after the numerical integration procedure described above. By equations (3) and (5) we have

(22) $$v_i(1) = E_i + \lambda k(a_0) - y(z) + v_{i+1}(0) \qquad \text{for } i = 1,\dots,N-1$$

by (4) we have

(23) $$v_N(1) = v_{N-1}(1)$$

and by (1) and (5) we have

(24) $$0 = E_0 + \lambda k(a_0) - y(z) + v_1(0).$$

The 2N + 1 linear equations (21)...(24) in the 2N + 1 unknowns $v_i(0)$, $v_i(1)$, $i = 1,2,\dots,N$ and $y(z)$ can be solved by one of the general numerical procedures available for this purpose. Because of the special structure a faster method has been developed, which is specified below.

From (24) and by substitution of (21) into (24) we obtain

(25) $$v_1(0) = y(z) - E_0 - \lambda k(a_0)$$

and

(26) $$v_{i+1}(0) = y(z) - E_i - \lambda k(a_0) + G_i(1)v_i(0) - H_i(1)$$

for $i = 1,2,\dots,N-1$. Further, it will be convenient to define $v_{N+1}(0)$ by

(27) $$v_{N+1}(0) = y(z) - E_{N-1} - \lambda k(a_0) + G_N(1)v_N(0) - H_N(1).$$

To develop the computational procedure for $y(z)$ and $v_n(0)$ we define with $H_0(1) \overset{\text{def}}{=} 0$ and $G_0(1) \overset{\text{def}}{=} 0$

$$(28)\qquad r_n \overset{\text{def}}{=} \begin{cases} 0 & \text{for } n = 0 \\ -E_{n-1} - k(a_0) - H_{n-1}(1) & \text{for } n = 1,\dots,N \\ -E_{N-1} - k(a_0) - H_N(1) & \text{for } n = N+1 \end{cases}$$

$$(29)\qquad t_n \overset{\text{def}}{=} \begin{cases} 0 & \text{for } n = 0 \\ G_{n-1}(1)t_{n-1} + r_n & \text{for } n = 1,\dots,N+1 \end{cases}$$

and

$$(30)\qquad u_n \overset{\text{def}}{=} \begin{cases} 0 & \text{for } n = 0 \\ G_{n-1}(1)u_{n-1} + 1 & \text{for } n = 1,\dots,N+1 \end{cases}$$

<u>LEMMA 2</u>. *The definitions* (28)...(39) *and the relations* (25)...(27) *imply*

$$(31)\qquad v_n(0) = u_n y(z) + t_n \qquad\qquad n = 1,2,\dots,N+1$$

and if $u_{N+1} \neq u_N$

$$(32)\qquad y(z) = \frac{t_{N+1} - t_N}{u_{N+1} - u_N}\,.$$

<u>PROOF</u>. By induction we prove (31). From (25), (28)...(30) we have

$$v_1(0) = y(z) + r_1 = u_1 y(z) + t_1 .$$

Suppose $v_n(0) = u_n y(z) + t_n$, then by (28)...(30) we have from (26)

$$\begin{aligned} v_{n+1}(0) &= y(z) - E_n - \lambda k(a_0) + G_n(1)v_n(0) - H_n(1) \\ &= y(z) - E_n - \lambda k(a_0) + G_n(1)(u_n y(z) + t_n) - H_n(1) \\ &= (G_n(1)u_n + 1)y(z) + G_n(1)t_n + r_{n+1} \\ &= u_{n+1}y(z) + t_{n+1} . \end{aligned}$$

Because (23) implies $v_{N+1}(0) = v_N(0)$ we have

$$u_{N+1}y(z) + t_{N+1} = u_N y(z) + t_N$$

which is equivalent to (32) provided that $u_{N+1} \neq u_N$. □

Definitions (28)...(30) and lemma 2 imply an obvious procedure to compute y(z) and $v_n(0)$, n = 1,...,N. Thereafter the function values of v(2i+τ;z) are easily obtained using (12). After that (9) is used to find the boundary $s_{z_{n+1}}(1i+\tau)$ of the set of intervention states of strategy z_{n+1}. The iteration is stopped as soon as the absolute value of the difference between the y(z) of two successive strategies is smaller than a prespecified small positive number.

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6. TWO EXAMPLES OF THE COURSE OF THE ITERATION

The computer output of two of the five examples given in [4] is presented in the succeeding pages. The numerical data are:

EXAMPLE 1

$E_0 = 1.6$; $E_1 = 1.4$; $E_2 = 1.2$; $E_3 = 1.1$

$a_0 = 0.4$

$\lambda = 2$

damage distribution: negative exponential with expectation 1.

initial strategy: $s_{z_1}(1i+\tau) = .45$, $i = 1,2,3,4$, $0 \leq \tau \leq 1$.

EXAMPLE 2

Same data as example 1 except the damage distribution being log normal with expectation 1 and coefficient of variation 1/3.

The total CPU-time for both iterations together has been 22.2 sec. on the CDC CYBER 73.

ITERATIESTAP 1

Y(Z)= 2.18652051

RAND NIEUWE INTERVENTIEVERZAMELING

0.47283940	0.53655191	0.56450210	0.56450210
0.48109308	0.55194626	0.58302900	0.58302900
0.49032416	0.56911830	0.60368468	0.60368468
0.50058982	0.58821492	0.62665538	0.62665538
0.51200602	0.60945182	0.65220055	0.65220055
0.52470170	0.63306887	0.68060872	0.68060872
0.53882028	0.65933284	0.71220078	0.71220078
0.55452120	0.68854038	0.74733356	0.74733356
0.57198184	0.72102139	0.78640390	0.78640390
0.59139939	0.75714275	0.82985310	0.82985310
0.61299320	0.79731247	0.87817192	0.87817192
0.63700715	0.84198426	0.93190614	0.93190614
0.66371251	0.89166270	0.99166270	0.99166270

ITERATIESTAP 2

Y(Z)= 2.17730913

RAND NIEUWE INTERVENTIEVERZAMELING

0.48205078	0.56458483	0.60303687	0.60303687
0.49121097	0.58203339	0.62407361	0.62407361
0.50146404	0.60155397	0.64760011	0.64760011
0.51280663	0.62294631	0.67327853	0.67327853
0.52534025	0.64634383	0.70124199	0.70124199
0.53917273	0.67188047	0.73162002	0.73162002
0.55441794	0.69968882	0.76453630	0.76453630
0.57119533	0.72989818	0.80010641	0.80010641
0.58962925	0.76263238	0.83843554	0.83843554
0.60984811	0.79800763	0.87961654	0.87961654
0.63198322	0.83613045	0.92372838	0.92372838
0.65616753	0.87709589	0.97083518	0.97083518
0.68253405	0.92098609	1.02098609	1.02098609

ITERATIESTAP 3

Y(Z)= 2.17701085

RAND NIEUWE INTERVENTIEVERZAMELING

0.48234906	0.56551572	0.60433741	0.60433741
0.49150793	0.58285885	0.62517020	0.62517020
0.50178664	0.60241100	0.64871228	0.64871228
0.51315604	0.62382987	0.67439574	0.67439574
0.52571751	0.64724834	0.70235356	0.70235356
0.53957875	0.67280023	0.73271600	0.73271600
0.55485352	0.70061859	0.76560859	0.76560859
0.57166118	0.73083390	0.80114998	0.80114998
0.59012607	0.76357204	0.83944976	0.83944976
0.61037670	0.79895215	0.88060626	0.88060626
0.63254468	0.83708439	0.92470448	0.92470448
0.65676342	0.87806784	0.97181419	0.97181419

0,68316666	0,92198835	1,02198835	1,02198835

ITERATIESTAP 4

Y(Z)= 2,17700797

RAND NIEUWE INTERVENTIEVERZAMELING

0,48235194	0,56552468	0,60435003	0,60435003
0,49151015	0,58286310	0,62517477	0,62517477
0,50178909	0,60241562	0,64871720	0,64871720
0,51315874	0,62383489	0,67440105	0,67440105
0,52572048	0,64725379	0,70235928	0,70235928
0,53958202	0,67280613	0,73272215	0,73272215
0,55485711	0,70062496	0,76561520	0,76561520
0,57166512	0,73084078	0,80115707	0,80115707
0,59013038	0,76357946	0,83945735	0,83945735
0,61038142	0,79896011	0,88061437	0,88061437
0,63254982	0,83709293	0,92471312	0,92471312
0,65676902	0,87807697	0,97182337	0,97182337
0,68317273	0,92199809	1,02199809	1,02199809

ITERATIESTAP 1

Y(Z)= 2,36809021

RAND NIEUWE INTERVENTIEVERZAMELING

0,43162110	0,46121980	0,47506123	0,47506123
0,43730793	0,47220918	0,48853027	0,48853027
0,44402690	0,48518064	0,50442561	0,50442561
0,45194957	0,50047591	0,52316858	0,52316858
0,46129157	0,51851129	0,54526931	0,54526931
0,47230716	0,53977767	0,57132933	0,57132933
0,48529618	0,56485389	0,60205795	0,60205795
0,50061214	0,59442245	0,63829154	0,63829154
0,51867193	0,62928815	0,68101630	0,68101630
0,53996710	0,67039997	0,73139511	0,73139511
0,56507724	0,71887688	0,79079917	0,79079917
0,59468582	0,77603833	0,86084534	0,86084534
0,62959870	0,84344013	0,94344013	0,94344013

ITERATIESTAP 2

Y(Z)= 2,36718933

RAND NIEUWE INTERVENTIEVERZAMELING

0,43252198	0,46599426	0,48347069	0,48347069
0,43837553	0,47782118	0,49838814	0,49838814
0,44528454	0,49177409	0,51597920	0,51597920
0,45343243	0,50819731	0,53665523	0,53665523
0,46303881	0,52750958	0,56091995	0,56091995
0,47436075	0,55018854	0,58933457	0,58933457
0,48769831	0,57677059	0,62250686	0,62250686
0,50340016	0,60784413	0,66106716	0,66106716
0,52186895	0,64403103	0,70562260	0,70562260
0,54356530	0,68595144	0,75668961	0,75668961
0,56900882	0,73416642	0,81460542	0,81460542
0,59877334	0,78909929	0,87944030	0,87944030
0,63347228	0,85094871	0,95094871	0,95094871

ITERATIESTAP 3

Y(Z)= 2,36718101

RAND NIEUWE INTERVENTIEVERZAMELING

0,43253029	0,46605232	0,48361078	0,48361078
0,43838503	0,47788413	0,49853796	0,49853796
0,44529572	0,49184725	0,51615176	0,51615176
0,45344558	0,50828184	0,53685197	0,53685197
0,46305426	0,52760641	0,56114100	0,56114100
0,47437888	0,55029813	0,58957764	0,58957764
0,48771953	0,57689246	0,62276592	0,62276592
0,50342493	0,60797643	0,66133101	0,66133101
0,52189775	0,64417002	0,70587484	0,70587484
0,54359863	0,68609174	0,75691171	0,75691171
0,56904715	0,73430274	0,81478506	0,81478506
0,59881712	0,78923017	0,87958293	0,87958293

0,63352203	0,85108049	0,95108049	0,95108049