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GRAPH TERMINOLOGY AND ELEMENTARY ANALYSES FOR THE SOCIAL SCIENCES

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Graph terminology and elementary analyses for the social sciences *)

by

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ABSTRACT

A number of concepts from the theory of graphs are introduced. A number of elementary analyses can be used to obtain some insight into the structure of a graph and to identify its central and peripheral vertices. These analyses have been used in the study of interlocking directorates and joint-venture relations.

KEY WORDS & PHRASES: *graphs, social sciences*

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*) This paper is a slightly modified translation of Chapter 4 of: H.M. Helmers, R.J. Mokken, R.C. Plijter, F.N. Stokman, in collaboration with Jac.M. Anthonisse: *Graven naar macht* (Van Gennep, Amsterdam, 1975). This paper is not for review; it is meant for publication elsewhere.

0. INTRODUCTION

This paper contains definitions of a number of concepts from the theory of graphs and descriptions of some methods of generating and analyzing graphs. Theorems from the theory of graphs are not discussed; these can be found in the books by HARARY [3] and WILSON [5].

MOKKEN & STOKMAN [4] is an application of the analysis of graphs as a tool in the study of interlocking directorates.

1. GRAPHS AND NETWORKS

A *graph* is an abstract object, composed of two types of elements called *points* and *lines* respectively, where each line is *incident* with one or with two points. Each point that is incident with a line is called an *end point* of that line.

Consider, as an example, the graph composed of the vertices u,v,w,x,y,z and the lines a,b,c,d,e,f,g,h with the incidence relations as defined by table 1.

line	incident with
a	v,y
b	x
c	w,v
d	u,x
e	w
f	y,w
g	x
h	u,x

TABLE 1: Incidence Relations

The incidence relations can also be described by a point-line incidence matrix, each row of the matrix corresponding to a point of the graph, each column corresponding to a line; a matrix element has the value 1 (or 0) if the elements of the graph corresponding to the element of the matrix are (or are not) incident with each other. Table 2 contains the point-line incidence matrix of the above example.

	a	b	c	d	e	f	g	h
u	0	0	0	1	0	0	0	1
v	1	0	1	0	0	0	0	0
w	0	0	1	0	1	1	0	0
x	0	1	0	1	0	0	1	1
y	1	0	0	0	0	1	0	0
z	0	0	0	0	0	0	0	0

TABLE 2: Point-Line Incidence Matrix

From both table 1 and table 2 it can be seen directly that lines b, e and g are incident with only a single point. Such lines are called *loops*; line b is a loop on point x, line e is a loop on point w and line g is another loop on point x.

Lines which are not loops, and thus incident with two points, constitute a direct connection between those two points; such points are called *adjacent* to each other. Line a is incident with points v and y, so point v is adjacent to point y and point y is adjacent to point v. Due to line f point w also is adjacent to point y. If a point x is adjacent to a point y, then y is called a *neighbour* of x. The set of neighbours of x is the *neighbourhood* of x.

A point without adjacency relations is an *isolated* point; in the example, point z is isolated.

Both lines d and h are incident with the points u and x; such lines are *parallel* with each other. The number of parallel lines constituting a direct connection between two points is the *multiplicity* of the connection. Instead of multiplicity of the connection the term multiplicity of a line is also used.

The term *multi-graph* is often used to indicate that multiple lines are present, or are not excluded

A survey of the multiplicities of the connections can be given with the help of the *multiplicity matrix*, that is a square matrix, each row and each column corresponding to a point of the graph. A matrix element on the main diagonal corresponds to one point of the graph; the value of that element is the number of loops on that point. Other matrix elements correspond with two points of the graph; the value of such elements is the number of lines which are incident with both points. Table 3 is the multiplicity matrix of the above example.

	u	v	w	x	y	z
u	0	0	0	2	0	0
v	0	0	1	0	1	0
w	0	1	1	0	1	0
x	2	0	0	2	0	0
y	0	1	1	0	0	0
z	0	0	0	0	0	0

TABLE 3: Multiplicity Matrix

The adjacency relations can be summarized in another square matrix where each row and each column corresponds to a point of the graph. The value of a matrix element is 1 (or 0) if the corresponding points are (or are not) adjacent to each other. The elements on the main diagonal are 0 as no point is adjacent to itself. Table 4 gives an example of an *adjacency matrix*.

	u	v	w	x	y	z
u	0	0	0	1	0	0
v	0	0	1	0	1	0
w	0	1	0	0	1	0
x	1	0	0	0	0	0
y	0	1	1	0	0	0
z	0	0	0	0	0	0

TABLE 4: Adjacency Matrix

Each of the tables 1,2,3, and 4 can be interpreted as a representation of the same graph. With table 1, however, it should be noted that the graph contains an isolated point, not listed in the table.

In table 1 and in table 2 points and lines are identified by a name. Tables 3 and 4 contain names for the points but not for the lines. The lines are identified by their end points: line (v,w) is a direct connection between points w and v.

If the lines are identified by their end points, then the representation of a graph can consist of two lists:
a list containing the names of the points, and
a list of pairs of points, where each pair corresponds with a line.

This is the form most commonly used to represent graphs in practical applications; in table 5 the example is represented in this form. (cf. ANTHONISSE [1].)

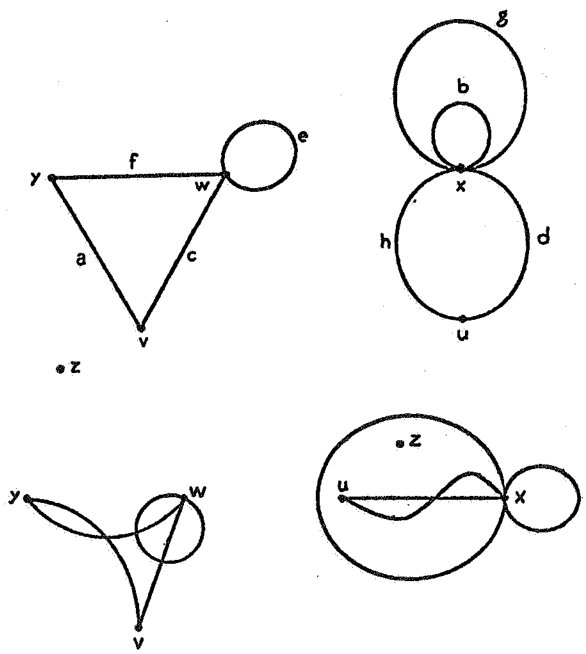
points:	u, v, w, x, y, z.
lines:	(v,y), (x,x), (w,v), (u,x), (w,w), (y,w), (x,x), (u,x).

TABLE 5: List Representation of a Graph

Graphs also can be described with the help of a picture. Each point of the graph corresponds to a point or dot in the picture. Each line corresponds to a, not necessarily straight, curve connecting two points with each other or a point with itself.

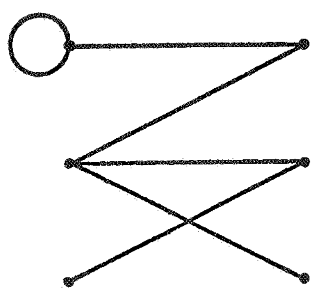
While drawing the picture care should be taken that a curve passes only through those points it is incident with. Crosspoints of curves should not be confused with points of the graph. Figure 1 contains two pictorial representations of the example.

Figure 1: Pictorial representations of a graph



A graph is called a *bipartite graph* if its set of points has been partitioned into two disjoint, non-empty subsets in such a way that there are no adjacency relations within the two sets. Thus all neighbours of a point belong to the subset not containing that point.

Figure 2: Bipartite graph



The data for research on interlocking directorates can be represented as a bipartite graph. The points in one subset correspond to persons, the points in the other subset correspond to companies. A company and an individual are connected by a line if that individual sits on the board of that company.

Bipartite graphs also are used to analyze relations between different classes of entities, e.g. the relations between financial enterprises and industrial firms. In such cases the relations within the two subsets are disregarded.

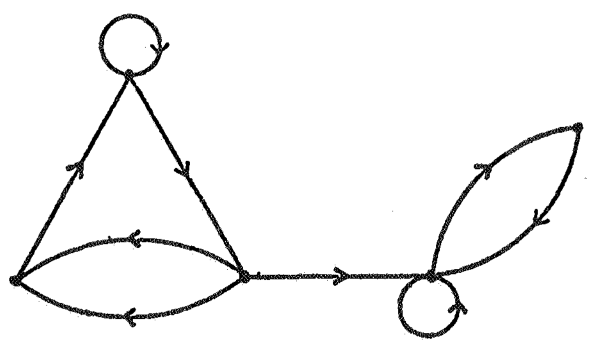
A graph is *complete* if all adjacency relations exist. Thus the complete graph on n points contains $\frac{1}{2}n(n-1)$ adjacency relations, as each point is connected with each other point. The complete bipartite graph, with q and r points in the two subsets respectively, contains qr lines: each point is adjacent with each point in the other subset.

A *directed graph* is a graph together with an *orientation* of each line. A line is oriented from one of the points it is incident with towards the other point. A line together with each orientation is called an *arc*. A line incident with points x and y, oriented from x to y then is an arc from x to y; x is the *tail* of the arc, y is the *head* of the arc.

The lines in a non-directed graph define symmetric relations between the points. The arcs in a directed graph define non-symmetric relations; only if the graph contains both an arc from x to y and an arc from y to x can the relation between x and y be called a symmetric or reciprocal one.

Directed graphs occur in the study of joint ventures. The points of the graph correspond with companies; there is an arc from x to y if company x is one of the parent companies of y.

Figure 3: Directed graph



With the help of directed and non-directed graphs the existence of relations between entities can be described. In many cases it is important to include the type or intensity of the relations. Then *information* is associated with the lines (or arcs) of the graph. Information also can be associated with the points of the graph. The kind of information associated with the elements of the graph is completely determined by the kind of application using the graphs as models.

A graph, together with the information associated with its elements, is called a *network*.

The bipartite graph describing the relations between individuals and companies becomes a network by assigning to each line the name of the function of that individual in that company. The directed graph mentioned above becomes a network by associating with the arc from x to y the percentage of x's participation in y.

2. GRAPH GENERATORS

Before a graph can be analyzed it must be generated. To generate a graph, data are required, and a rule describing the relation between data and graph. If the data are available in the form of a graph or network, then the definition of the new graph can be given in graph-theoretical terminology. The new graph is derived from the structure of the original one and from the information associated with its elements. Three processes to derive a graph from another graph are: selection, aggregation and induction.

A graph is obtained by *selection* if its elements are selected from the set of elements of the original graph. The new graph consists of those points and lines that satisfy the selection criteria. No line can be selected unless its end points belong to the set of selected points.

A *subgraph* consists of a subset of the points of the original graph and all lines of the original graph having both end points in the subset. Thus a subgraph can be obtained by removing from a graph a subset of its points and all lines incident with this subset.

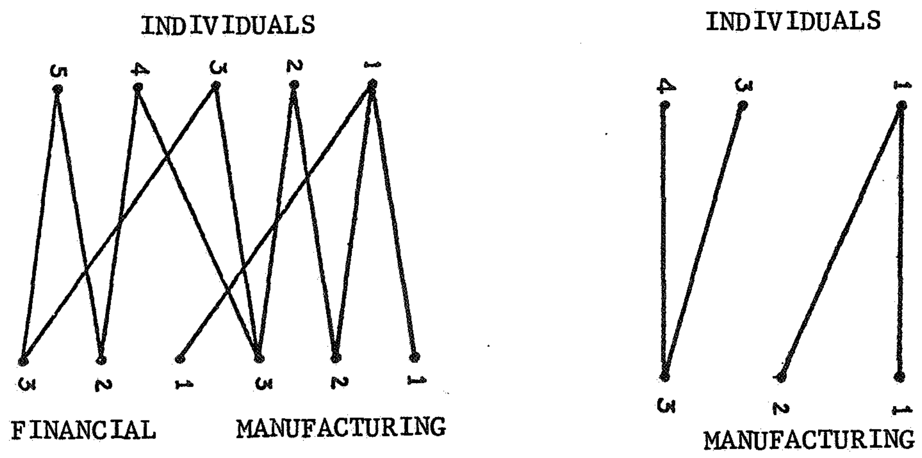
A *partial graph* consists of all points of the original graph and a subset of its lines.

A *partial subgraph* is a partial graph of a subgraph of the original graph, and can thus be obtained by removing lines from a subgraph.

A *clique* is a maximal complete subgraph, i.e. all points in the clique are adjacent to each other (complete), each point outside the clique is non-adjacent to at least one point in the clique (maximal).

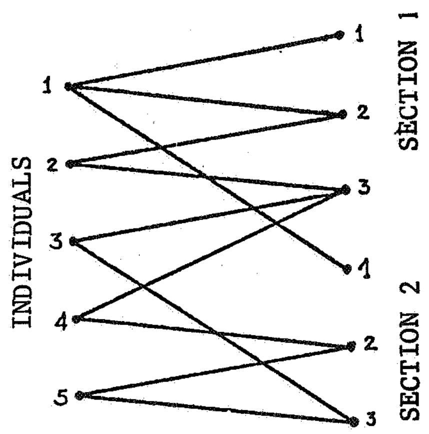
Figure 4 contains a graph and a partial subgraph. The new graph describes the relations between the manufacturing companies and those individuals related to both a financial and a manufacturing company.

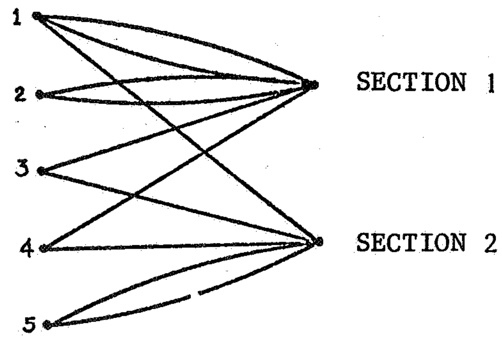
Figure 4: Selection



A new graph is obtained by *aggregation of points* if a subset of points of the original graph is replaced by a single new point. Lines having both end points in the subset, in particular loops within the subset, become loops on the new point. Each line connecting a point inside the subset with a point outside the subset becomes a line connecting the new point with the point outside the subset Figure 5 contains a description of relations between companies and individuals. By aggregation a graph describing relations between individuals and sectors of the economy is obtained.

Figure 5: Aggregation of points





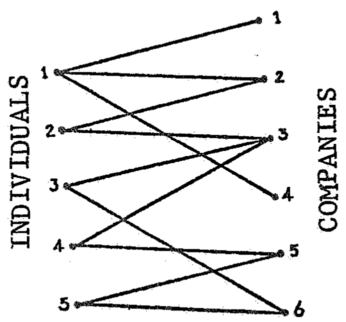
The example of *aggregation of lines* is the replacement of paralleled lines by a single line.

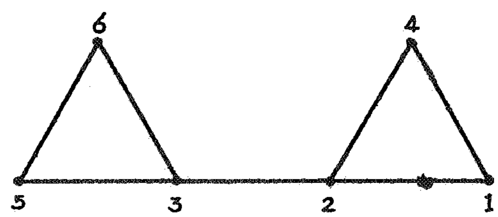
A graph is obtained by *induction* if two points in the new graph are adjacent if they have a common neighbour in the original graph.

The points of the induced graph are a subset of the points of the original graph. The induction process can be restricted to common neighbours satisfying certain criteria, which might concern both the information associated with the points and that associated with the lines.

Figure 6 again contains relations between companies and individuals. By induction a graph of relations between the companies is obtained. Each individual induces a line for each pair of companies he is related to; thus the number of lines between two companies is equal to the number of individuals related to both companies.

Figure 6: Induction



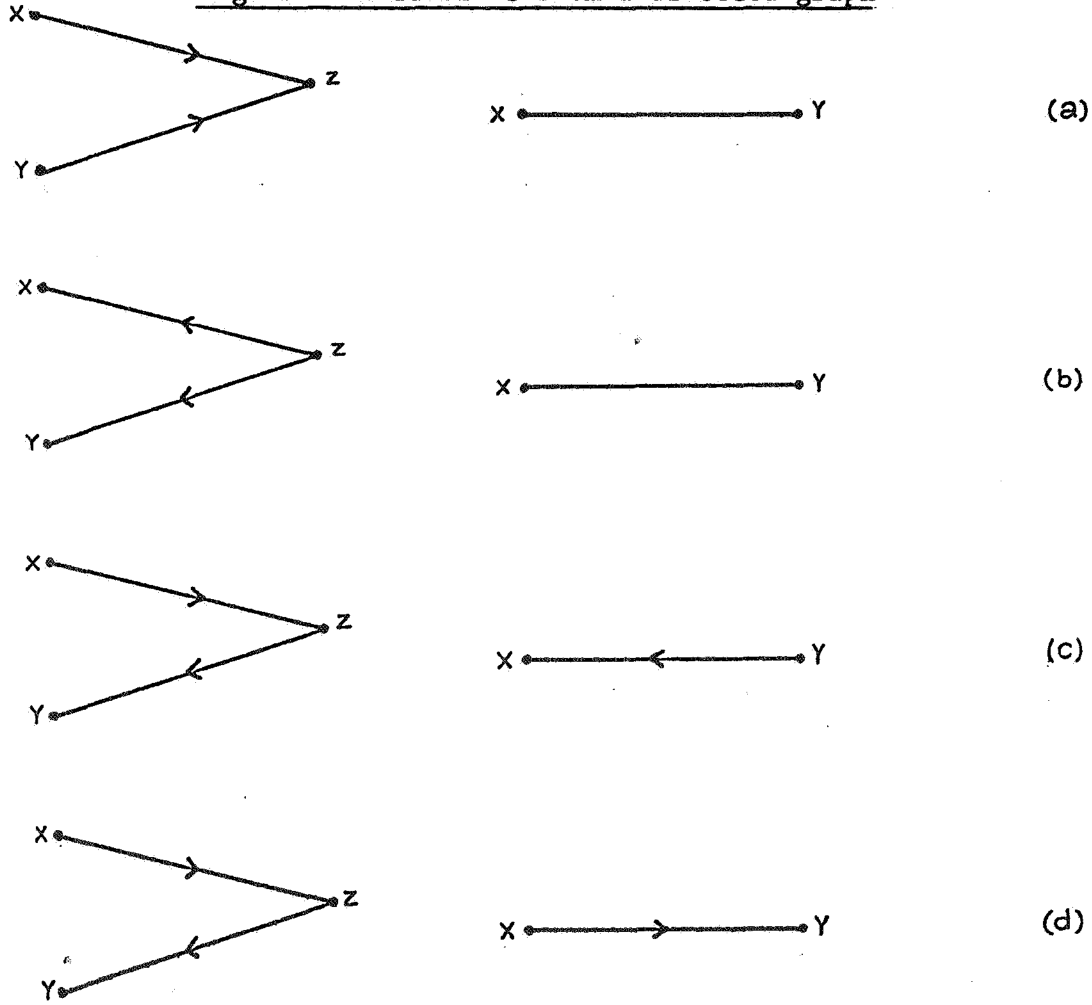


From this graph an interesting sequence of partial graphs can be generated. Each graph in this sequence contains all vertices, but only those lines having multiplicity $\geq m$, for $m = 1,2,\ldots$.

If the induction process is applied to a graph of interlocking directorates (i.e. there is a line between two companies if their directorates interlock), then the number of lines between two companies in the new graph equals their number of common neighbours in the original graph. In a common neighbour (representatives of) the two companies meet each other; the number of common neighbours is a proximity measure for pairs of companies.

Induction also can be performed on directed graphs; some possibilities are indicated in Figure 7. Construction (a) can be used to obtain a graph of partnership relations from a graph of participation relations. There is a partnership relation between x and y for each point z with the property that both x and y participate in z.

Figure 7: Inductions from a directed graph



3. ELEMENTARY ANALYSIS OF GRAPHS

In all applications of graphs the analyses to be performed should be determined by the field of application. The problem to be solved should be translated from the terminology of the field of application into graph-theoretical terms. The information obtained by the analysis should be translated back into the original terminology.

In this section a number of analyses is described, mainly with the help of examples. Only non-directed graphs are considered; problems of computational efficiency are not discussed. Details on these and other analyses can be found in ANTHONISSE & LAGEWEG [2].

The majority of the analyses to be described here are used to obtain some insight into the structure of a graph and to identify its central and peripheral points. By application of these elementary analyses to (partial) subgraphs the relations within and between sections of the economy or other subsets of points can be studied. Using a number of coefficients, the networks and the (dis)similarities between them can be described and interpreted.

Adjacency and Connectivity

The most trivial information about a graph is undoubtedly its number of points and its number of lines. If it is known a priori that the graph contains neither loops nor multiple lines, then the number of lines equals the number of adjacency relations. If loops or multiple lines might be present, then the number of loops, the number of adjacency relations and their multiplicities can be determined. The graph in figure 8 contains 6 points and 15 lines.

Figure 8: A graph

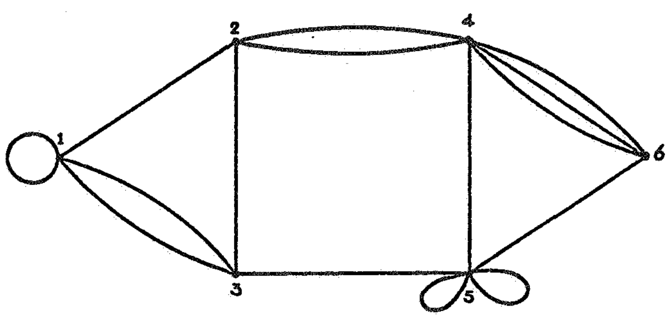


Table 6 contains a survey of the multiplicities.

loops		other lines	
mult.	frequency	mult.	frequency
1	1	1	5
2	1	2	2
		3	1
	2		8

TABLE 6: Multiplicity of Loops and Lines

The *coefficient of adjacency* (or *density*) of a graph is its number of adjacency relations, expressed as a fraction of the number of possible adjacency relations. If a graph G has p points and a adjacency relations, then the value of this coefficient is

$$\mathrm{adj}(G) = \frac{a}{\frac{1}{2}p(p-1)}$$

and can be interpreted as the probability that two points, chosen at random, are adjacent.

As the graph in figure 8 has 6 points and 8 adjacencies its coefficient of adjacency is 8/15 = 0.53.

For bipartite graphs the coefficient can be defined as

$$\mathrm{adj}(B) = \frac{a}{qr},$$

where q and r are the numbers of points in the two defining subsets. For the bipartite graph in figure 6 this leads to adj(B) = 0.37.

If two points in a graph are not adjacent, thus not directly connected by a line, then an indirect connection (e.g. by a common neighbour) might exist. A *path* between point x and point y of a graph consists of a sequence of lines l_i and points z_i

$$x = z_0,\ l_1,\ z_1, \ldots, z_{k-1},\ l_k,\ z_k = y$$

where l_i is incident with z_{i-1} and z_i, for i = 1,2,...,k.

The sequence w,f,y,a,v is a path between points w and v of the graph in figure 1.

The sequence 5,(5,3),3,(3,2),2,(2,1),1,(1,1),1 is a path between points 1 and 5 of the graph in figure 8.

Often it is sufficient to describe a path by its sub-sequence of points e.g. w,y,v or 5,3,2,1,1.

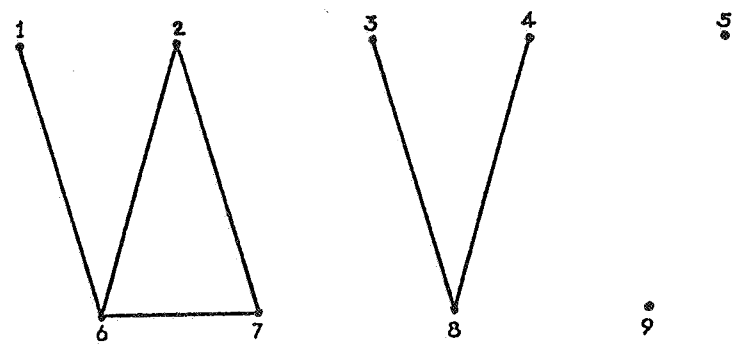
The path then consists of the points, in that order, and, for each pair of subsequent points, one of the lines connecting the two points.

A graph is a *connected* one if there is a path for each pair of points.

If a graph is not connected, then it consists of a number of connected *components*: two points belong to the same component if there is a path connecting them; if two points are not connected by a path, then they belong to different components.

Components can be analyzed as separate (sub)graphs. The graph in figure 9 consists of 4 components, the sizes of the components are 4,3,1,1.

Figure 9: A graph



The *coefficient of connectivity* of a graph is the number of pairs of points connected by a path, expressed as a fraction of the total number of pairs. This coefficient can be interpreted as the probability that two points, chosen at random, belong to the same component. If the graph G consists of p points and c components, and if the i-th component contains p_i points, then the coefficient is

$$\text{conn}\ (G) = \frac{1}{2p(p-1)} \sum_{i=1}^{c} \tfrac{1}{2}\, p_i(p_i-1)\ .$$

For the graph in figure 9 this leads to conn (G) = 0.25.

Bipartite graphs do not require a separate formula as points in the same defining subset can be connected by a path.

Apart from the connectivity of a graph the connectivity within a subset and between two disjoint subsets of points can be defined. These coefficients should be distinguished from the connectivity of the corresponding 2 subgraphs or bipartite subgraphs as the paths may contain points which do not belong to the defining subsets.

If subset Q contains q points, and the i-th component of the graph contains q_i points of Q, then the connectivity within Q is

$$\text{conn}_Q\ (G) = \frac{1}{\tfrac{1}{2}q(q-1)} \sum_{i=1}^{c} \tfrac{1}{2}\, q_i(q_i-1).$$

If the two disjoint subsets Q and R contain q and r points respectively, and the i-th component of the graph contains q_i and r_i points of Q and R respectively, then the connectivity between Q and R is

$$\text{conn}_{QR}\ (G) = \frac{1}{qr} \sum_{i=1}^{c} q_i r_i .$$

Define, for the graph in figure 9, Q = {1,2,1,3,5} and R = {6,7,8,9}, thus q=5 and r=4. It follows from table 7 that $\text{conn}_Q(G)$ = 0.20, $\text{conn}_R(G)$ = 0.17 and $\text{conn}_{QR}(G)$ = 0.30.

p_i	q_i	r_i	$\tfrac{1}{2}q_i(q_i-1)$	$\tfrac{1}{2}r_i(r_i-1)$	$q_i r_i$
4	2	2	1	1	4
3	2	1	1	0	2
1	1	0	0	0	0
1	0	1	0	0	0
9	5	4	2	1	6

TABLE 7: Computations for Connectivities

Centrality

There are several methods to compute a measure of centrality for each point of a graph. Points with a maximum centrality are the central points, points with a minimum centrality are the peripheral points. The concept of centrality to be used depends upon the application.

The *distance* between two points is defined if both points belong to the same component. The distance is the minimal number of lines that is necessary to form a path between x and y.

Thus the distance between two adjacent points is 1, the distance between two non-adjacent points with a common neighbour is 2.

The *distance matrix* is a square matrix, each row and each column corresponding to a point of the graph. The value of the elements on the main diagonal is 0, the value of the other elements is the distance between the corresponding elements of the graph.

The maximal value occurring in the distance matrix is the *diameter* of the graph. For a graph on p points the diameter is at most p-1, as each path of length k contains k-1 intermediate points. Table 8 gives the distances of the graph in figure 8.

	1	2	3	4	5	6
1	0	1	1	2	2	3
2	1	0	1	1	2	2
3	1	1	0	2	1	2
4	2	1	2	0	1	1
5	2	2	1	1	0	1
6	3	2	2	1	1	0

TABLE 8: Distance Matrix

A frequency table of the distances, as in table 9, is often a useful survey of the distances, and can be used to compute the average distance in the graph.

distance	frequency	fraction
1	16	0,53
2	12	0,40
3	2	0,07
	30	1,00

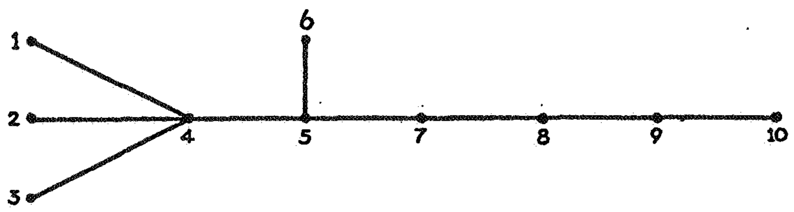
TABLE 9: Frequencies of Distances

The values in a row of the distance matrix are the distances from the point corresponding to that row to the other points. The maximum value in the row is the *excentricity* of that point. For each point the *average* and the *median* of the distances to the other points are easily determined.

The points can be sorted according to their excentricity, average or median distance. Points with a minimal value for their excentricity, average distance or median distance can be considered to be *central* points of the graph. Each of these coefficients can be used as a *measure* of centrality.

Table 10 contains, for each point of the graph in figure 10, a survey of the number of points at distance 1,2,... . With the help of these surveys the three centrality measures are easily found. Table 11 contains, for each measure, the points in non-increasing order of centrality.

Figure 10: A graph



Point	Number of points at distance 1	2	3	4	5	6	Average	Median	Excentricity
1	1	3	2	1	1	1	3.11	3	6
2	1	3	2	1	1	1	3.11	3	6
3	1	3	2	1	1	1	3.11	3	6
4	4	2	1	1	1		2.22	2	5
5	3	4	1	1			2.00	2	4
6	1	2	4	1	1		2.89	3	5
7	2	3	4				2.22	2	3
8	2	2	2	3			2.67	3	4
9	2	1	1	2	3		3.33	4	5
10	1	1	1	1	2	3	4.22	5	6

TABLE 10: Computations for Centrality Measures

measure	order
average	5, (7, 4), 8, 6, (1, 2, 3), 9, 10
median	(4, 5, 7), (1, 2, 3, 6, 8), 9, 10
excentricity	7, (8, 5), (4, 6, 9),(1, 2, 3, 10)

TABLE 11: Points in Order of Centrality

Another measure of centrality is the number of points in a *vicinity* of each point. A vicinity consists of the points within a fixed distance (d) of that point. For d=1 the vicinity of x consists of the neighbours of x. See table 12 for the sizes of three vicinities of the points in the graph in figure 10.

Point	Number of points in vicinity d=1	d=2	d=3
1	1	4	6
2	1	4	6
3	1	4	6
4	4	6	7
5	3	7	8
6	1	3	7
7	2	5	9
8	2	4	6
9	2	3	4
10	1	2	3

TABLE 12: Vicinity of Points

Table 12 leads to three new orderings of the points; these are given in table 13.

Vicinity	Order
d=1	4, 5, (7, 8, 9), (1, 2, 3, 6, 10)
d=2	5, 4, 7, (1, 2, 3, 8),(6, 9), 10
d=3	7, 5, (4, 6),(1, 2, 3, 8), 9, 10

TABLE 13: Points in Order of Vicinity

In the centrality measures mentioned above the centrality of a point is indeed measured, i.e. given as a real number, and then sorted according to the value of the coefficient. In the next concept of centrality no coefficient is computed, but the points are just sorted in *lexicographical* order of the size of their vicinities. For the points of the graph in figure 10 this leads, with the help of table 12, to the order

4, 5, 7, 8, 9, (1, 2, 3), 6, 10.

Distance

In the above the distance between two adjacent points was defined to be 1. In many applications the information associated with a line contains a measure of the distance between or proximity of the two points. If this measure on the line (x,y) can be interpreted as the *cost* of travelling between x and y, then the distance between x and y, measured in the appropriate currency, can differ from the distance between u and v connected by the line (u,v). In this case the length of a path should be defined as the sum of the lengths of its constituent lines. The distance between x and y again is the minimal length of the paths between x and y. The distance between two adjacent points can be smaller than the length of the line connecting them.

If each adjacency corresponds to a channel of communication, with a *capacity* defined for each channel, then the capacity of a path can be defined as the minimal capacity occurring in the path. The maximum of the capacities of the paths connecting x and y can then serve as a measure for the proximity of x and y. It is ascertained here that two points use a single path to communicate and that the number of lines in the path is irrelevant.

Rush, Covers, Strong Cliques

If a graph serves as a model of a communication network, then it is assumed that the contacts between the points are realized using the direct and indirect connections. In each element, point or line, of the graph the amount of throughgoing traffic can be measured and interpreted as the importance of that element for maintaining the contacts.

The *rush* in an element is the number of contacts using that element, expressed as a fraction of all contacts. It is assumed that the contacts between x and y are realized using all paths of minimal length connecting x and y; the contact is equally distributed over these paths.

The graph in figure 11 has 3 paths of minimal length between points 1 and 5. Table 14 gives the rush in each point due to a contact between 1 and 5, and also the rush due to a contact between each pair of points.

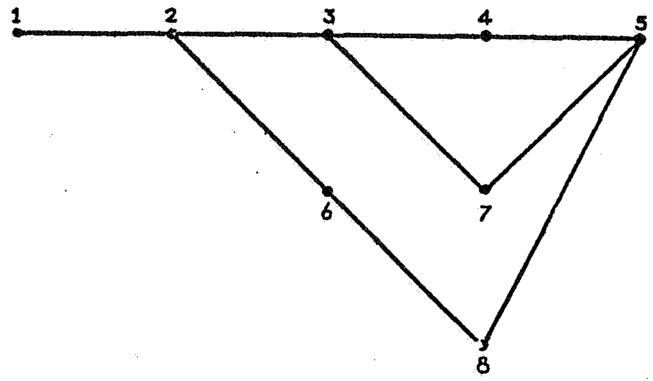


Figure 11: A graph

Point	rush (1,5)	rush (complete)
1	0.00	0.00
2	1.00	0.30
3	0.67	0.26
4	0.33	0.06
5	0.00	0.15
6	0.33	0.09
7	0.33	0.06
8	0.33	0.08

TABLE 14: Rush

The bipartite graph describing relations between individuals and companies can be interpreted as a communication network. Contacts between companies are effectuated through the individuals. These contacts generate a rush in the points corresponding to persons, and the persons can be sorted according to their rush.

Two other analyses on the above bipartite graph will be sketched.

The bipartite graph consists of two defining subsets of points, I (individuals) and C (companies) respectively. A subset S of I *covers* C if each point in C has at least 1 neighbour in S. A *minimal* cover is a cover of minimal cardinality, i.e. with a minimal number of points in S.

In the terminology of individuals and companies a minimal cover is a smallest set of persons containing at least one representative of each company. As a group these persons have all information concerning all companies. The size of a minimal cover divided by the number of companies could be used to compare the concentration of information in the sectors of the economy or to study its development in successive years.

Covers can also be defined for graphs other than bipartite graphs. In the graph describing interlocking-directorates relations between firms a minimal set of firms can be found such that these firms as a group have direct access to all firms.

As described above, the bipartite graph can be used to obtain, by induction, an interlocking-directorate graph on the firms. This latter graph contains cliques, which may arise in several ways, as figure 6 shows.

The relations between companies 1, 2 and 4 are induced by one single person, the relations between 3, 5 and 6 by three different persons. Thus different types of cliques can be distinguished in the induced graph. A *strong clique* is a subset of at least 3 firms such that there are at least 2 persons each of which is related to each of the firms. The strong cliques can be found by determining the cliques in the bipartite graph.

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