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CONTROLLABILITY DISTRIBUTIONS FOR NONLINEAR CONTROL SYSTEMS

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Controllability distributions for nonlinear control systems

by

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ABSTRACT

The purpose of this paper is to relate the notion of controlled invariance to a notion of controllability for nonlinear control systems. By means of the 'linear' ideas of a subsystem and a quotientsystem we derive a very structured description of nonlinear systems.

KEY WORDS & PHRASES: *nonlinear systems, controlled invariance, controllability, subsystem, quotientsystem*

1. INTRODUCTION

Basic to the solution of various synthesis problems in linear systems theory is the notion of (A,B) invariance or controlled invariance (cf.[11]). A special class of controlled invariant subspaces, the so called controllability subspaces, play an important role in the structure-analysis of linear systems (cf.[11]). For example stabilization, decomposition and noninteracting control can be investigated by means of controllability subspaces.

First we will briefly sketch the linear situation. Let

$$\overset{\circ}{x} = Ax + Bu \qquad x \in \mathcal{X} := \mathbb{R}^n$$
$$u \in \mathcal{U} := \mathbb{R}^m$$

and A,B matrices of appropriate dimension.

A subspace $\mathcal{V} \subset \mathcal{X}$ is called (A,B) invariant if $A\mathcal{V} \subset \mathcal{V} + \mathcal{B}$ ($\mathcal{B}$:= Im B), which is equivalent with the existence of a feedback $F : \mathcal{X} \to \mathcal{U}$ such that

$$A_F\mathcal{V} \subset \mathcal{V} \qquad (A_F := A + BF).$$

A subspace $\mathcal{V} \subset \mathcal{X}$ is said to be a controllability subspace if $F : \mathcal{X} \to \mathcal{U}$ such that

$$\mathcal{V} = \langle A_F | \mathcal{B} \cap \mathcal{V}\rangle := \mathcal{B} \cap \mathcal{V} + A_F(\mathcal{B} \cap \mathcal{V}) + \ldots\ldots + A_F^{n-1}(\mathcal{B} \cap \mathcal{V}).$$

There is a direct relation between controllability subspaces and the set reachable from $x(0) = \underline{0}$, namely the set reachable from $\underline{0}$ is the smallest (A,B)-invariant subspace which contains $\mathcal{B}$, and this is exactly the controllability subspace $\langle A|\mathcal{B}\rangle$. In a similar way an arbitrary controllability subspace $\mathcal{V}$ is the set reachable from $\underline{0}$ of a 'subsystem' ([11]) $\overset{\circ}{x} = A_Fx + BGu$ for a matrix $G : \mathcal{U} \to \mathcal{U}$, i.e.

$$\mathcal{V} = \langle A_F | \text{Im } BG\rangle.$$

Furthermore we note that the original system $\overset{\circ}{x} = Ax + Bu$ reduces to a new linear system $\overset{\circ}{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u}$ on the factorspace (quotientspace) $\mathcal{X}(\text{mod } \mathcal{V})$ where $\bar{A}$ and $\bar{B}$ are the induced maps in $\mathcal{X}(\text{mod } \mathcal{V})$ by A + BF resp. B.

Recently several people introduced the notion of nonlinear controlled invariance for systems of the form $\overset{\circ}{x} = A(x) + B(x)u$ (cf.[2,3]), which has been elaborated in [4,5,6,7]. The results of [4] and [6] seem to be very conclusive for the problem of nonlinear (A,B)-invariance. With the aid of this notion of controlled invariance we will set up a similar theory for nonlinear controllability distributions. Indeed in the same way as in the linear theory we will derive controllability (in the sense of SUSSMANN and JURDJEVIC [10]) by means of the smallest controlled invariant distribution which contains the inputs. We will show that controllability distributions directly lead to the study of subsystems and quotient systems.

The outline of the paper is as follows. Section 2 contains preliminaries on notation and nonlinear controlled invariance. In section 3 we briefly discuss the notion of controllability, while in section 4 we will introduce controllability distributions by means of degenerate controlled invariance. Finally we terminate with a discussion of the results in section 5.

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2. PRELIMINARIES

We assume that the reader is familiar with the basic notions of differential geometry (cf.[9]). Throughout this paper all our objects like manifolds, maps etc. are C^∞. We call Δ an affine distribution on a manifold M if Δ in every $x \in M$ is given by an affine subspace $\Delta(x) \subset T_xM$ (in a smooth way). Given two (affine) distributions D_1 and D_2, then we define the distribution

$$[D_1,D_2] = \{[X,Y] \mid X \in D_1,\ Y \in D_2\},$$

where [,] is the Lie-bracket.

A distribution D is said to be involutive, if D is closed under taking Lie-brackets of vector fields in D. The involutive closure of a distribution D, i.e. the closure of D by taking all iterated Lie-brackets of vector fields in D, is denoted by $\bar{D}$. (So we have for an involutive distribution

D that $D = \overline{D}$). For each $x \in M, I(D,x)$ will denote the maximal integral submanifold of an involutive distribution D through x (The existence of $I(D,x)$ is guaranteed by Frobenius' theorem), where we assume, to avoid technical difficulties, that D has fixed dimension.
For $X,Y \in V(M)$, i.e. smooth vectorfields on M, we define $ad_X^0 Y = Y$, $ad_X Y = [X,Y]$ and

$$ad_X^k Y = [X, ad_X^{k-1} Y] \quad k = 1,2,\ldots .$$

In this paper we shall consider systems of the form (locally)

$$(2.1) \qquad \overset{\circ}{x}(t) = A(x(t)) + \sum_{i=1}^{m} u_i(t) B_i(x(t)),$$

where $x \in M$, $A, B_1, \ldots, B_m \in V(M)$.
We will give a slightly different definition for the control system (2.1), which turns out to be very useful in the sequel.

DEFINITION 2.1. (cf.[5,6,7]). A C^∞ m-dimensional *affine control system* on M will be a smooth manifold M together with a m-dimensional affine distribution Δ on M.

REMARKS.

(i) It will be clear that locally we can identify Δ with (2.1). The advantage of this definition, besides its compactness, is that it gives a feedback invariant definition of a control system.

(ii) An affine control system Δ induces a m-dimensional distribution $\Delta_0 := \Delta - \Delta = \{X-Y \mid X,Y \in \Delta\}$, or if $\Delta(x) = A(x) + \mathrm{Span}\{B_1(x),\ldots,B_m(x)\}$ then $\Delta_o(x) = \mathrm{Span}\{B_1(x),\ldots,B_m(x)\}$, i.e. the directions in which we can steer.

Next we will briefly describe the results on controlled invariance (cf.[2,3,4,5,6]).

DEFINITION 2.2. An involutive distribution D on M is (locally) *controlled invariant* for an affine control system Δ on M if (locally) there exist $\tilde{A}, \tilde{B}_1, \ldots \tilde{B}_m \in V(M)$ such that

$$\Delta(x) = \tilde{A}(x) + \mathrm{Span}\{\tilde{B}_1(x),\dots,\tilde{B}_m(x)\} \text{ and}$$

$$[A,D] \subset D$$

$$[\tilde{B}_i,D] \subset D \qquad i = 1,\dots,m.$$

Whether or not a distribution D is controlled invariant can directly be checked in terms of the control system Δ.

THEOREM 2.3. *An involutive distribution* D *is controlled invariant iff* $[\Delta,D] \subset D + \Delta_0$.

REMARK. To avoid technical difficulties we suppose that the distribution D as well as $D \cap \Delta_0$ have fixed dimension.

3. CONTROLLED INVARIANCE AND CONTROLLABILITY

In this section we want to describe the notion of controllability of an affine system as it has been given by SUSSMANN & JURJEVIC [10]. (In the literature there exist several notions of controllability, reachability, accessibility, e.g. [1,10]) The notion as introduced in [10] perfectly fits in the framework of controlled invariance, although also the notion of (local) weak controllability as introduced by HERMANN & KRENER [1] can be related to controlled invariance (See remark (i) after theorem 3.5). In what follows we will use the next assumptions:

Assumption 1: M is a compact manifold;

Assumption 2: All distributions we consider have fixed dimension.

The *reachable set* from $x_0 \in M$ at time t, $R_t(x_0)$, is given by

$$R_t(x_0) = \{X^1_{t_1} \circ \dots \circ X^k_{t_k}(x_0) \mid X^1,\dots,X^k \in \Delta,\ t_i > 0,\ \sum_{i=1}^{k} t_i = t\}$$

where $X_t(x_0)$ is the time t integral of the vector field X starting at t = 0 in x_0.

<u>REMARK</u>. Here we used the fact that M is a compact manifold. If M is not compact then $X_t(x_o)$ is possibly not defined for all t. In fact we can work without assumption 1 by setting

$$R_t(x_0) = \{X^1_{t_1} \circ \dots \circ X^k_{t_k}(x_0) \mid X^1, \dots, X^k \in \Delta, \; t_i > 0, \; \sum_{i=1}^{k} t_i = t \text{ and } X^1_{t_1} \circ \dots \circ X^k_{t_k}(x_0) \text{ is well-defined}\}.$$

It is known that the reachable set is related to the following distribution, called the *derived* distribution,

$$\bar{L}_0 = \text{involutive closure of } \{ad^k_A B, \; k \in \mathbb{N}, \; A \in \Delta, \; B \in \Delta_0\}$$

(by assumption 2 has fixed dimension).
Then we have

<u>LEMMA 3.1</u>. *Let* X,Y $\in \Delta$, *then for all* $x \in H$, $t \in \mathbb{R}$,

$$X_t(I(\bar{L}_0,x)) = Y_t(I(\bar{L}_0,x)).$$

<u>PROOF</u>. See [10], but for affine control systems the proof can be simplified in the following way:
Let X,Y $\in \Delta$, then Y = X + B for B $\in \Delta_0$ for $p \in I(\bar{L}_0,x)$ we have

$$Y_t(p) = (X + B)_t(p) = \lim_{n\to\infty} \left(X_{\frac{t}{n}} \circ B_{\frac{t}{n}} \right)^n (p) \in X_i(I(\bar{L}_0,x)).$$

Therefore

$$Y_t(I(\bar{L}_0,x)) \subset X_t(I(\bar{L}_0,x))$$

and by the same argument the converse also holds. □

This result motivates the following definition.

DEFINITION 3.2. Let $X \in \Delta$, then we define

$$I_t(\bar{L}_0,x) := I(\bar{L}_0,X_t(x_0)) = X_t(I(\bar{L}_0,x))$$

(by 3.1 this is independent of $X \in \Delta$).

In [10] the following crucial result is proven:

THEOREM 3.3. *Let* Δ *be an affine controlsystem on* M *with derived distribution* $\bar{L}_0$, *then for all* $x \in M$, *for all* $t \in \mathbb{R}$ $R_t(x) \subset I_t(\bar{L}_0,x)$ *and with respect to the topology of* $I_t(\bar{L}_0,x)$, $R_t(x)$ *is contained in the closure of its interior.*

REMARKS.

(i) This result merely states that, except for its boundary, $R_t(x)$ is submanifold of $I_t(\bar{L}_0,x)$ with $\dim R_t(x) = \dim I_t(\bar{L}_0,x)$.

(ii) For the smooth counterpart of the analytic framework of [10] we need the fixed dimension-assumption.

Based on this result we define:

DEFINITION 3.4. Let Δ be an affine *control system on* M. Then we will call $\bar{L}_0 = \overline{\{ad_A^k B,\ k \in \mathbb{N},\ A \in \Delta,\ B \in \Delta_0\}}$ the *accessibility distribution* of Δ.

Now we are able to show that the accessibility disbtribution $\bar{L}_0$ plays the same role as in the linear case (see the Introduction, also [8]).

THEOREM 3.5. *Let* Δ *be an affine control system on* M. *The smallest controlled invariant distribution* D *on* M *which contains* Δ_0 *equals the accessibility distribution* $\bar{L}_0$.

PROOF. Let as before $\Delta_0 = \Delta - \Delta$ and define $\Delta_k = [\Delta,\Delta_{k-1}]$ $k = 1,2,\ldots$ (see [5]). Let $D := \lim_{k\to\infty} \Delta_k = \Delta_{n-2}$.

By using the Jacobi-identity ot follows that D is involutive. Furthermore, it is easy to see that $D = \bar{L}_0$. The algorithm given here produces the smallest controlled invariant distribution which contains Δ_0. □

REMARKS.

(i) In a similar way (see [3]) we get the (local weak controllability distribution of [1] as the smallest controlled invariant distribution which contains Δ, namely you start with the smallest distribution which contains the affine distribution Δ. In this case it is also allowed to travel backwards in time.

(ii) Suppose that $N := M(\text{mod } \bar{L}_0)$ is a manifold then the affine control system Δ on M reduces to a unique vectorfield on N; the system reduces to an autonomous system on N (no inputs). See the definition of a quotientsystem in [7].

4. DEGENERATE CONTROLLED INVARIANCE AND CONTROLLABILITY DISTRIBUTIONS

Instead of the notion of controlled invariance as given in 2.2, also called controlled invariance *with full control* [3], there is also a definition of controlled invariance *with partial control* [3] or what we shall call, following [7], *degenerate controlled invariance*. The difference is that for controlled invariance we have that there is a basis $\{\tilde{B}_1,\dots,\tilde{B}_m\}$ for Δ_0 such that the distribution D satisfies $[D,\tilde{B}_i] \subset D$ $i = 1,\dots,m$, where D is the controlled invariant distribution, while for degenerate controlled invariance there is only a subbasis $\{\tilde{B}_1,\dots,\tilde{B}_k\}$ $k < m$, such that $[D,\tilde{B}_i] \subset D$ $i = 1,\dots,k$. We will formalize this in the following way.

DEFINITION 4.1. Let Δ be an m-dimensional affine control system on M. Let $k < m$. An k-dimensional affine control system $\tilde{\Delta}$ on M is a *subsystem* of Δ if $\tilde{\Delta} \subset \Delta$.

REMARKS.

(i) Suppose Δ is given by $\Delta(x) = A(x) + \text{Span}\{B_1(x),\dots B_m(x)\}$ for smooth vectorfields $A,B_1,\dots,B_m$ then a k-dimensional subsystem $\tilde{\Delta}$ can be written by

$$\tilde{\Delta}(x) = \tilde{A}(X) + \text{Span}\{\tilde{B}_1(x),\dots,\tilde{B}_k(x)\}$$

where

$$\tilde{A}(x) = A(x) + \sum_{j=1}^{m} \alpha_j(x) B_j(x)$$

$$\tilde{B}_i(x) = \sum_{j=1}^{m} \beta_j^i(x) B_j(x), \quad i = 1,\dots,k,$$

and the matrix $(\beta_j^i(x))_{ij}$ has rank k.
This means that we get the subsystem

$$\overset{\circ}{x}(t) = \tilde{A}(x(t)) + \sum_{i=1}^{k} v_i(t) B_i(x(t))$$

from the system

$$\overset{\circ}{x}(t) = A(x(t)) + \sum_{i=1}^{m} u_i(t) B_i(x(t))$$

by applying a state feedback and a change of the inputfields and then setting some of the new inputs equal to zero. The new input vectorfields belong to the distribution $\tilde{\Delta}_0 = \tilde{\Delta} - \tilde{\Delta}$. Note that $\tilde{\Delta}_0 \subset \Delta_0$.

(ii) In the 'category' of linear systems (see the introduction) we end up with the following. Δ is given by system matrices (A,B) and then a subsystem $\tilde{\Delta}$ is given by matrices (A + BF, BG) for an arbitrary feedback matrix F and arbitrary G.

<u>DEFINITION 4.2</u>. Let Δ be an m-dimensional control system on M. An involutive distribution D is said to be *degenerate controlled invariant* (of dim k) if there exists a k-dimensional subsystem $\tilde{\Delta} \subset \Delta$ such that $[\tilde{\Delta}, D] \subset D + \tilde{\Delta}_0$.

From this definition we see that a degenerate controlled invariant distribution D for Δ is a controlled invariant distribution for the subsystem $\tilde{\Delta}$. Under the usual regularity conditions - D and $D \cap \Delta_0$ have fixed dimension - we know by theorem 2.3 that this is equivalent with:
there exist $\tilde{A}, \tilde{B}, \dots, \tilde{B}_k \in V(M)$ such that

$$\tilde{\Delta}(x) = \tilde{A}(x) + \mathrm{Span}\{\tilde{B}_1(x), \dots, \tilde{B}_k(x)\} \text{ and}$$

$$[\tilde{A}, D] \subset D$$

$$[\tilde{B}_i, D] \subset D \qquad i = 1,\dots,k.$$

REMARKS.

(i) Here we have in terms of [6] degenerate input-insensitivity with respect to the basis $\{\tilde{B}_i,\ldots,\tilde{B}_k\}$ of $\tilde{\Delta}_0$. It seems to be a very hard problem to find out whether or not an involutive distribution D is degenerate controlled invariant for a control system Δ. In practice this could be a very interesting problem; if we are only concerned with closed loop controlled invariance ([6]) - i.e. there exists a vectorfield $\tilde{A} \subset \Delta$ with $[\tilde{A},D] \subset D$ - then degenerate controlled invariance will play an important role. On the other hand for a given subsystem $\tilde{\Delta}$ of Δ controlled invariance of a distribution D is easily verified by theorem 2.3 and so we can immediately see that if D is controlled invariant for $\tilde{\Delta}$ then D is degenerated controlled invariant for Δ.

(ii) For linear systems degenerate controlled invariance automatically implies full controlled invariance (cf.[2,7]).

Now we are able to give the definition of controllability distributions for an affine control system Δ (motivated by the structure of linear systems).

DEFINITION 4.3. An involutive distribution D on M is a *controllability distribution* of an affine control system Δ if there is a subsystem $\tilde{\Delta} \subset \Delta$ such that D is the accessibility distribution of $\tilde{\Delta}$.

REMARKS.

(i) It automatically follows that the accessibility distribution of Δ is a controllability distribution of Δ.

(ii) Applying this procedure to linear systems we precisely get the controllability subspaces (cf. [11]).

As a direct consequence of section 3 we have:

THEOREM 4.4. *Let* D *be a controllability distribution for an affine control system* Δ *on* M. *Then* D *is a degenerate controlled invariant distribution for* Δ, *i.e. there exists a subsystem* $\tilde{\Delta}$ *such that* $[\tilde{\Delta},D] \subset D + \tilde{\Delta}_0$. *Moreover* D *is the smallest controlled invariant distribution for* $\tilde{\Delta}$ *which contains* $\tilde{\Delta}_0$.

As a drawback of the notion of controllability distribution we note that in general it is not true that the involutive sum $\overline{D_1 + D_2}$ of two controllability distributions is again a controllability distribution. (As we know this is true for linear systems.) Although this set up probably will play an important role in the analysis of the structure of nonlinear control systems, we will introduce here a special class of controllability distributions which is far more structured.

DEFINITION 4.5. Let Δ be an affine control system on M. An involutive distribution D on M is called a *regular controllability distribution* if D is a controllability distribution for Δ and D is a controlled invariant distribution for Δ. This means that there exist a subsystem $\tilde{\Delta}$ of Δ such that D is the smallest distribution on M such that:

$$D \supset \tilde{\Delta}_0$$

$$[\tilde{\Delta},D] \subset D \text{ and}$$

$$[\Delta,D] \subset D + \Delta_0.$$

By using theorem 2.3 we see that there exist $\tilde{A},\tilde{B}_1,\ldots,\tilde{B}_m \in V(M)$ such that

$$\Delta(x) = \tilde{A}(x) + \mathrm{Span}\{\tilde{B}_1,\ldots,\tilde{B}_m\}$$

$$\tilde{\Delta}(x) = A(x) + \mathrm{Span}\{\tilde{B}_1,\ldots,\tilde{B}_k\}$$

$$D = \overline{\{\mathrm{ad}^j_{\tilde{A}}\, B_i \mid j \in \mathbb{N},\ i = 1,\ldots,k\}}$$

$$[D,\tilde{B}_i] \subset D \qquad i = 1,\ldots,m,$$

and automatically

$$[D,\tilde{A}] \subset D.$$

For the quotientspace N := M(mod D) - we assume for a moment that this is a manifold - this leads to the following appealing representation:
There exist

$$\bar{A},\bar{B}_{k+1},\ldots,\bar{B}_m \in V(N)$$

such that if

$$\overset{\circ}{x}(t) = \tilde{A}(x(t)) + \sum_{i=1}^{m} u_i(t)\tilde{B}_i(x(t))$$

and

$$\bar{x}(t) = x(t) \bmod D$$

then

$$\overset{\circ}{\bar{x}}(t) = \bar{A}(\bar{x}(t)) + \sum_{i=k+1}^{m} u_i(t)\, \bar{B}_i(\bar{x}(t)).$$

<u>REMARK</u>. Some of the $\tilde{B}_i$ $i = k+1,\ldots,m$ can be annihilated by D, i.e. $\tilde{B}_i \in D$ then the corresponding $\bar{B}_i$ is the zero section from $N \to TN$ (so on N the input u_i has no effect).

We conclude this section with two nice 'linear' theorems.

<u>THEOREM 4.6</u>. *Let* D_1, D_2 *be regular controllability distributions for* Δ. *Then* $D_1 + D_2$ *is a regular controllability distribution for* Δ.

<u>PROOF</u>. This follows from the fact that the sum of two controlled invariant distributions is again controlled invariant (cf.[2,3]). The corresponding subsystem has as its 'input' space the sum of the 'input' spaces of the controllability distributions D_1 and D_2. □

<u>THEOREM 4.7</u>. *Let* Δ *be an affine control system on* M *and let* K *be an involutive distribution on* M. *Then there exist a unique maximal regular controllability distribution on* M *which is contained in* K.

<u>PROOF</u>. This follows from theorem 4.6 and [2,3,6]. □

5. DISCUSSION

The results of this paper directly relate the concept of controlled invariance to the results on controllability of nonlinear systems. Probably the whole set up given here, can be extended, by using [7], to nonlinear systems of the form $\overset{\circ}{x} = f(x,u)$. The concept of degenerate controlled invariance as well as controllability distributions are probably of some practical interest. The regular controllability distributions play a very nice role in the study of nonlinear control systems and can probably be used for developing nonlinear analogues of linear systems theory (e.g. Noninteracting control [11], which also is studied without the notion of controllability distribution in [3]).

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