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# Note on a Previous paper on Fermat's last theorem.

(Nieuw Archief voor Wiskunde, 3e Serie, 2 (1954), p 40-41).

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# NOTE ON A PREVIOUS PAPER ON FERMAT'S LAST THEOREM

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6. In a previous paper [1]) a lower bound for $z$ satisfying

$$x^p + y^p = z^p \quad (x, y, z \text{ positive integers}; \; p \nmid xyz; \; x < y < z)$$

was given. It here may be remarked that from the derived results easily lower bounds also for $x$ and $y$ can be found.

From the formulae

$$2x = a^p - b^p + c^p, \; 2y = -a^p + b^p + c^p \tag{2.6}$$

one gets, using $0 < a < b < c$ and

$$c > \frac{p^3}{3 + \log 2p} - p, \tag{4.2}$$

the result $2y > c^p$, hence

$$y > \tfrac{1}{2}c^p, \; c > \frac{p^3}{3 + \log 2p} - p. \tag{6.1}$$

Further one deduces from (2.6)

$$2x = a^p - b^p + c^p \geq a^p + p(c - b)b^{p-1} \geq a^p + pb^{p-1} > pb^{p-1}. \tag{6.2}$$

Using $b = \beta c$ and the formulae (4.2) and

$$\beta > 1 - \frac{\log 2pe}{p}, \tag{3.4}$$

one gets

1) A remark on Fermat's last theorem, H. J. A. Duparc and A. van Wijngaarden, Nieuw Archief voor Wiskunde (3) I, 123—128 (1953).

$$b > \left(1 - \frac{\log 2pe}{p}\right)\left(\frac{p^3}{3 + \log 2p} - p\right) =$$

$$= \left(1 - \frac{\log 2pe}{p}\right)\left(\frac{1}{\log 2pe^3} - \frac{1}{p^2}\right)p^3. \qquad (6.3)$$

Now

$$\left(1 - \frac{\log 2pe}{p}\right)^{p-1} = e^{(p-1)\log\left(1-\frac{\log 2pe}{p}\right)}$$

$$> e^{-(p-1)\left(\frac{\log 2pe}{p}+\frac{\log^2 2pe}{p^2}+\dots\right)}$$

$$= e^{-(p-1)\frac{\log 2pe}{p}\frac{p}{p-\log 2pe}} = \left(\frac{1}{2pe}\right)^{\frac{p-1}{p-\log 2pe}}$$

and

$$\left(\frac{1}{\log 2pe^3} - \frac{1}{p^2}\right)^{p-1} > \left(\frac{1}{\log 2pe^3}\right)^{p-1}\left(1 - \frac{\log 2pe^3}{p^2}\right)^{p}$$

$$> \left(\frac{1}{\log 2pe^3}\right)^{p-1}\left(1 - \frac{\log 2pe^3}{p}\right).$$

Consequently from (6.2) and (6.3) one obtains

$$x > \tfrac{1}{2}\left(\frac{1}{2pe}\right)^{\frac{p-1}{p-\log 2pe}}\left(\frac{1}{\log 2pe^3}\right)^{p-1}\left(1 - \frac{\log 2pe^3}{p}\right)p^{3p-2}. \qquad (6.4)$$

From the bound $p \geqq 253747889$ one then gets the same numerical lower bound for $x$, $y$ and $z$, viz.

$$z > y > x > 10^{6\times 10^9}. \qquad (6.5)$$

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