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H.J.J. TE RIELE
UNITARY ALIQUOT SEQUENCES

RA

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Unitary aliquot sequences

by

Herman J.J. te Riele.

0. Summary

After a brief survey of the history of aliquot sequences, the results of the computation of all unitary aliquot sequences with starting value ≤ 40000 are reported. There were no sequences, the terms of which became too large for (our) computational means.

Unitary sociable groups of orders 1, 2, 3, 5 and 14 were found as "endpoints" of some sequences. Many directed graphs of periodic unitary aliquot sequences are given, and two long terminating unitary aliquot sequences - one with 1154 terms, one with 879 terms - are described. Furthermore, the existence of arbitrary long unitary aliquot sequences is proved, and the average value of the function $s^*(n)/n$ is computed. Finally, two theorems are given, which deal with the construction of sociable groups from other, given, sociable groups. Application of these two theorems gives more than 600 new unitary amicable number pairs, which will be published in a subsequent report, and five new unitary sociable groups of order 4.

1. Introduction

In this report n is natural number. A divisor d of n is called unitary if $(d, n/d) = 1$. With $\sigma(n)$ we denote the sum of the divisors of n , $\sigma^*(n)$ is the sum of the unitary divisors of n , $s(n)$ the sum of the aliquot divisors of n ($s(n) = \sigma(n) - n$), and $s^*(n)$ the sum of the unitary aliquot divisors of n ($s^*(n) = \sigma^*(n) - n$).

If the prime factorization of n is given by $n = q_1^{\alpha_1} \cdot q_2^{\alpha_2} \cdots q_r^{\alpha_r}$, q_1, q_2, \dots, q_r being distinct primes and $\alpha_i \geq 1$ ($i=1, 2, \dots, r$), then $s(n)$ and $s^*(n)$ can be computed by

$$s(n) = \prod_{i=1}^r (1 + q_i + \dots + q_i^{\alpha_i}) - n, \text{ and}$$

$$s^*(n) = \prod_{i=1}^r (1 + q_i^{\alpha_i}) - n.$$

An aliquot sequence (AS) of n is the sequence (n_i) , defined by

$$n_0 = n, n_1 = s(n_0), \dots, n_i = s(n_{i-1}), \dots$$

Replacing the s -function by the s^* -function, we call the resulting sequence a unitary aliquot sequence (UAS) of n . A usual way to indicate the i -th term of an AS with starting value n_0 is $n_0 : i$, where n_0 is counted as the zero-term. For instance: $276 : 362$ means the 362nd term of the AS with starting value 276. We shall adopt this notation for the indication of terms of unitary aliquot sequences.

A k -tuple of distinct numbers $(n_0, n_1, \dots, n_{k-1})$ with

$$n_i = s(n_{i-1}) \quad (i=1, 2, \dots, k-1), \quad s(n_{k-1}) = n_0, \quad k \geq 1,$$

is called a sociable group (SG) of order k ; well-known special cases are perfect numbers ($k=1$) and pairs of amicable numbers ($k=2$). Again, writing s^* instead of s , the resulting k -tuple is called a unitary sociable group (USG) of order k . Special cases for $k = 1$ and $k = 2$ are called unitary perfect numbers and pairs of unitary amicable numbers.

In this report the behaviour of unitary aliquot sequences is studied, in view of that of (ordinary) aliquot sequences.

2. Aliquot sequences, a brief survey

The behaviour of AS-s has been investigated by Alanen [1], Catalan [3], Cohen [4], Dickson [6], Guy and Selfridge [9], Lehmer [13], Paxson [15], Poulet [16], te Riele [17] and many others.

AS-s can be divided into 3 classes:

- (i) A certain term of the AS is a prime number, the next term is 1 and subsequent terms are uninteresting. Such AS-s are called terminating.
- (ii) The AS (n_i) is periodic - ultimately (but not initially) or purely -, i.e. there exists an index k such that the t -tuple of distinct terms $(n_k, n_{k+1}, \dots, n_{k+t-1})$ is a sociable group of order t .
- (iii) The terms of the AS become too large for practical computation.

The number l such that $n_0 : l = 1$ is called the length of the AS of n_0 .

In case of periodic AS-s, the least number m for which $n_0 : m$ is a member of a sociable group, is called the preperiod l' ; we call $n_0 : l'$ the endpoint of the periodic AS.

Catalan and Dickson conjectured that every AS belongs to class (i) or (ii), in other words, that every AS is bounded. This conjecture has been verified for many starting values n_0 , first by hand, later on by use of computers. The starting value $n_0 = 276$ is the smallest one for which the behaviour of the AS is unknown. Lehmer [13] has computed $276 : 362$, a 32-digit number. Guy and Selfridge [9] and Alanan [1] give many other AS-s with unknown behaviour. Table 1 shows how many AS-s with starting value $n_0 \leq 40000$ belong to classes (i), (ii) and (iii).

TABLE 1

	terminating	periodic			unknown
		t = 1	t = 2	t > 2	
$n_0 \leq 10000$	8994	93	121	10	782
(G&S) %	89.9	.9	1.2	.1	7.8
$10000 < n_0 \leq 40000$	24704	232	374	44	4646
(A) %	82.3	.8	1.2	.1	15.5

Remark: Guy and Selfridge place an AS in class (iii) (unkown) if some term exceeds 10^{18} , while Alanan took 10^{10} for this bound.

The existence of very long, monotonically increasing AS-s has been shown by te Riele [17]; H.W. Lenstra Jr. [14], then proved that it is possible to construct AS-s with m monotonically increasing terms, for any given natural number m .

An old problem is the determination of sociable groups of order k . The case $k = 1$ (perfect numbers) is well-known, and, recently, B. Tuckerman [20] has found the 24th perfect number. Pairs of amicable numbers ($k=2$) have also been extensively investigated. A nearly complete historical survey on this subject, with an extensive bibliography, has been given by Lee [12]; Lee presents a list of more than 1000 amicable number pairs. Sociable groups of

order $k > 2$ are scarce. Poulet [16] discovered one of order 5 and one of order 28. Recently, Borho [2] gave one, Cohen [4] eight and David [5] four sociable groups of order 4.

3. Unitary aliquot sequences

Since

$$s^*(n) \leq s(n),$$

with equality if and only if n is squarefree, it is probable that UAS-s are easier to compute than AS-s.

We define the same 3 classes for UAS-s as we did for AS-s in section 2. A computation of UAS-s for all starting values $n_0 \leq 40000$ revealed that all these sequences are bounded, i.e. all sequences belong to class (i) or (ii). Nevertheless, very long UAS-s were found and, in fact, the existence of arbitrary long UAS-s can be proved (along the same lines as H.W. Lenstra, Jr. proved this for AS-s); this will be done in section 4. Hence, the conjecture that every UAS is bounded, is as uncertain as the Catalan-Dickson conjecture for AS-s.

The computation produced some new unitary sociable groups of order 2, 3, 5 and 14. (In section 5, moreover, five new USG-s of order 4 will be given). Many results of the computations are collected in tables 2, 3, 4, 5, 6 and 7.

Table 2 shows the distribution of UAS-s among classes (i) and (ii), for all starting values $n_0 \leq 40000$.

TABLE 2

	terminating	periodic			unknown
		t = 1	t = 2	t > 2	
frequency	35701	728	966	2605	0
%	89.2	1.8	2.4	6.5	0

Table 3 gives all unitary sociable groups, into which lead some UAS-s with starting value $n_0 \leq 40000$.

TABLE 3

order	unitary sociable groups
1	(6) (60) (90)
2	(114,126) (1140,1260) (18018,22302) (32130,40446) (56430,64530) (1080150,1291050)
3	(30,42,54)
5	(1482,1878,1890,2142,2178)
14	(2418,2958,3522,3534,4146,4158,3906,3774,4434,4446,3954,3966, 3978,3582) (24180,29580,35220,35340,41460,41580,39060,37740,44340,44460, 39540,39660,39780,35820) (35238,45402,65190,98106,101478,117258,117270,117450,74430, 74610,74790,65322,49878,38682)

Table 2 shows that 4299 UAS-s are periodic. The starting values of these UAS-s, their preperiods and their endpoints are given in table 7 (p. 14 - 31). For instance: $1702 : 7 = 30$ means that the 7th term of the UAS with starting value $n_0 = 1702$ (counting 1702 as the zero-term) equals 30 (a member of the unitary sociable group (30,42,54)). If a starting value itself is a member of a USG (then the UAS is purely periodic), this is indicated in the table as (for instance) $30 = 30$. 44 UAS-s are purely periodic and 4255 ultimately periodic.

Table 4 tells how often a member of a USG is endpoint of the periodic UAS-s with starting value $n_0 \leq 40000$.

TABLE 4

frequency	endpoint	member of un. sociable group of order	frequency	endpoint	member of un. sociable group of order
1	6	1	81	2418	14
93	60	1	12	2958	14
634	90	1	3	3522	14
			2	3534	14
231	114	2	3	4146	14
204	126	2	4	4158	14
			1	3906	14
20	1140	2	3	3774	14
1	1260	2	2	4434	14
			3	4446	14
14	18018	2	1	3954	14
1	22302	2	6	3966	14
			1	3978	14
1	32130	2	1	3582	14
462	56430	2	18	24180	14
			3	29580	14
32	1291050	2	1	35220	14
			1	35340	14
941	30	3	1	39060	14
612	42	3	1	37740	14
327	54	3	1	39540	14
			2	39660	14
1	1482	5	1	39780	14
15	1878	5	1	35820	14
66	1890	5			
9	2142	5	2	35238	14
477	2178	5	2	38682	14

Table 5 (p. 8 - 12) displays some directed graphs of the periodic UAS-s which lead into the following USG-s:

(60) (1140,1260) (18018,22302) (32130,40446) (1080150,1291050), three of order 14 (see TABLE 3) and (1482,1878,1890,2142,2178). All numbers ≤ 40000 which lead into one of these USG-s appear in TABLE 5, except for those which lead into 2178 (a member of the USG of order 5). Underlined numbers indicate the distance of the terms from a member of the USG, i.e. the length of the preperiod. Use is made of Dickson's [6] notation, for instance:

$$\begin{array}{c} 5736 | \\ 37564 | 9396 | 2904 \end{array}$$

means that $s^*(37564) = 9396$, and $s^*(9396) = s^*(5736) = 2904$. Sometimes, another notation is used, e.g. for the USG (1080150,1291050). In this case, the table should be read from above downwards, following the arrows where they occur.

Table 6 (p. 13) gives some information about very long UAS-s. Only two sequences were found with maximum term $> 10^9$, both were terminating. Table 6 gives (partly) the directed graph of these two sequences and, extracted from this graph, a list of all starting values $n_0 \leq 40000$ (with the length of the sequence) which are part of this graph.

Unitary sociable groups of order k have been studied by Subbarao and Warren [19], Wall [21,22] ($k=1$) and by Hagis, Jr. [10] ($k=2$). Five unitary perfect numbers are known; three of them, 6, 60 and 90 occur in table 4 as endpoints of some periodic UAS-s. Hagis, Jr. gives 108 pairs of unitary amicable numbers; all pairs of table 3, except for (1080150,1291050), appear in his list.

In section 5 we shall prove two theorems which deal with the construction of (unitary) sociable groups from other given (unitary) sociable groups. An application of these two theorems yields many new unitary amicable number pairs and five new unitary sociable groups of order 4.

TABLE 5

SOME DIRECTED GRAPHS OF PERIODIC UNITARY ALIQUOT SEQUENCES

<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>0</u>
		5736				
	37564	9396	2904			
22704		6744				
26328	13200	8016	3408			
38556	20484					
	32044					
	35452	8868				
		10868				
		13748				
		16148	5932			
	37332	18468				
35508	29292	19548				
		23708				
		34568				
		39384				
		39724	9936			
			23536	1488		
		5270				
23966	13618					
23426	17398	8702				
	10318	9266	5098			
			13432	2552		
			17248			
		36680	20344		688	
	27838					
	30718	15362				
		31520	7684	2576		
			5458			
		20840	7348	2732		
			5544	3096		
		33216	12024			
			17064			
	27446	13726	6866			
	24838	15842	7924			
	28150					
		31676				
		17316	9284			
	31958	18562				
39370	34358			3436	864	
	36804	24556				
		27036				
		34316	10564			
		35836				
		37436				
		38236				
			11124			
			11524			
		31636	11564			
		39672	14328	3672		

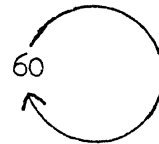


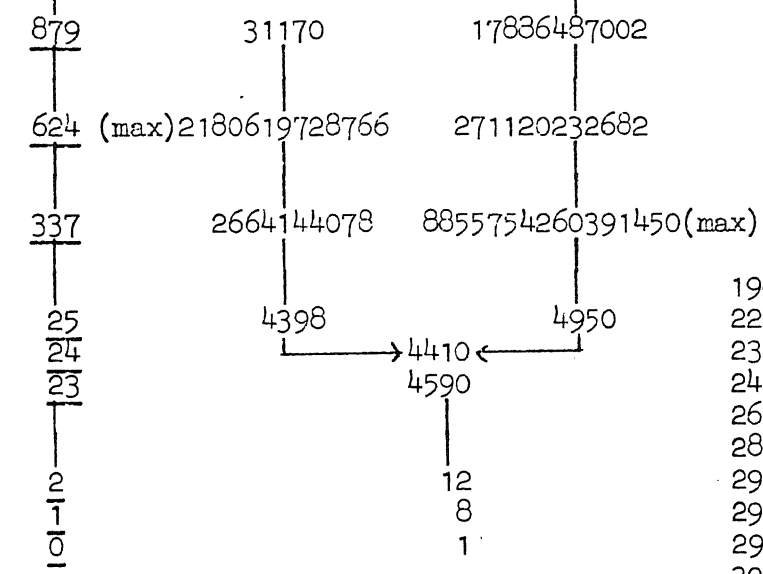
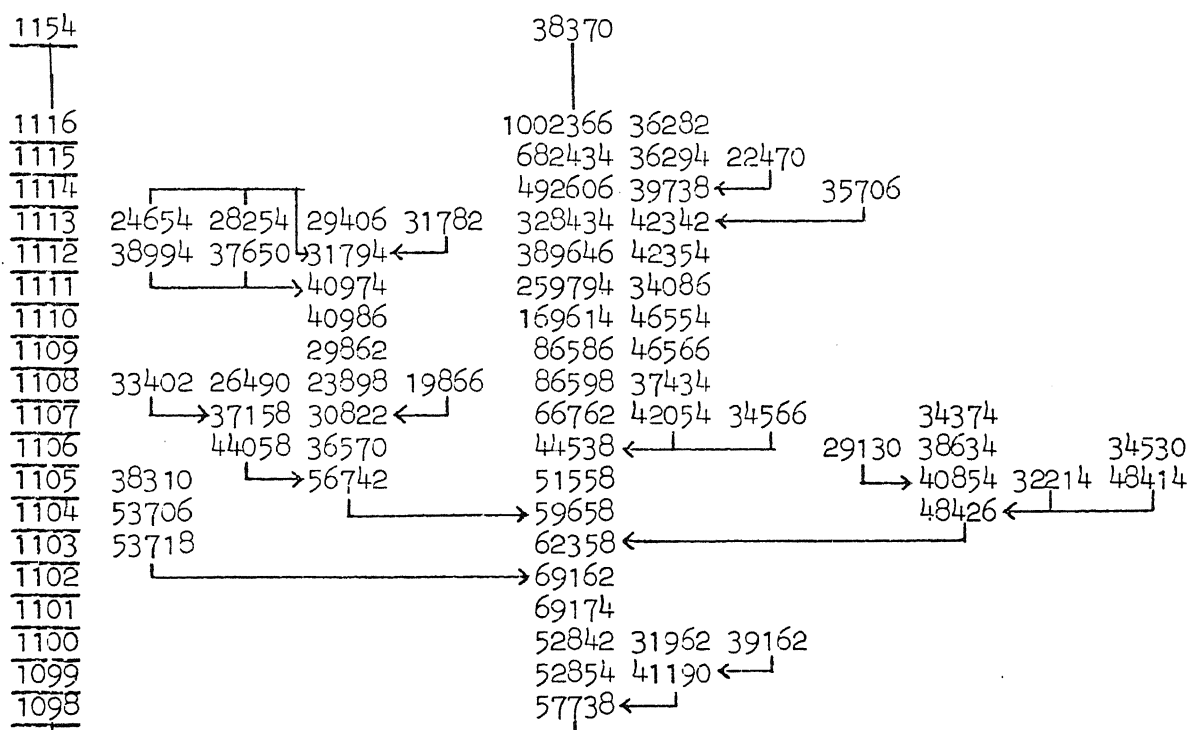
TABLE 5 (cont.)

<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>0</u>
					24716	7204	1806	
					25596			
	24916	6234				28624		
		24964	6246			30928	1950	
29476	7374	7386			1902			
	11034		7398	4194	2826			
		29572						
	24246	13554	7614					
	22988					1914		
	32108	9892	2478					
	35708							
	39548			3282				
	4854	4866	4878		3294			
38882	19444		5766	5778				
			8622					
	23824	1506	1518					
		17948		1938	2382			
		25868	7732					
		27548		9508				
		30908						
			3330	3510			2406	
	27692	14548	3642	3654				
			5262	5274	3546			
		37348	9342					
				14164				
				31004				
				33004				
				34364	9556			
				34444				
				38204				
					38032			
				26158				
				30118	19202			
				34150		9604		
					22316			
						38224		
							38416	

2418(see p11)

TABLE 6

TWO LONG (TERMINATING) UNITARY ALIQUOT SEQUENCES



- 19866:1108=1
- 22470:1115=1
- 23898:1108=1
- 24654:1113=1
- 26490:1108=1
- 28254:1113=1
- 29130:1106=1
- 29406:1113=1
- 29862:1109=1
- 30822:1107=1
- 31170:879=1
- 31782:1113=1
- 31794:1112=1
- 31962:1100=1
- 32214:1105=1
- 33402:1108=1
- 34086:1111=1
- 34374:1107=1
- 34530:1106=1
- 34566:1107=1
- 35706:1114=1
- 36282:1116=1
- 36294:1115=1
- 36570:1106=1
- 37434:1108=1
- 37650:1112=1
- 38310:1105=1
- 38370:1154=1
- 38634:1106=1
- 38994:1112=1
- 39162:1100=1
- 39738:1114=1

TABLE 7 PERIODIC UNITARY ALIQUOT SEQUENCES
 starting values (≤ 40000), preperiods and endpoints

6	=	6	740:	2=	42	1380:	3=	1140	1946:	8=	30
30	=	30	756:	3=	54	1382:	6=	30	1948:	5=	42
42	=	42	758:	4=	30	1392:	2=	90	1950:	1=	2418
54	=	54	764:	2=	54	1408:	3=	30	1956:	4=	30
60	=	60	766:	3=	54	1414:	7=	30	1960:	3=	42
66:	2=	90	770:	6=	90	1420:	3=	42	2020:	4=	114
78:	1=	90	780:	3=	42	1436:	3=	54	2028:	5=	42
90	=	90	784:	3=	90	1438:	4=	54	2058:	5=	1390
100:	1=	30	792:	2=	42	1440:	6=	30	2062:	7=	30
102:	1=	114	794:	2=	42	1482	=	1482	2066:	3=	126
114	=	114	812:	3=	114	1488:	2=	60	2068:	4=	114
126	=	126	820:	3=	30	1498:	5=	54	2070:	4=	1890
140:	2=	30	832:	2=	90	1500:	2=	1140	2088:	3=	42
148:	1=	42	836:	3=	54	1506:	6=	2418	2092:	3=	42
194:	2=	30	864:	1=	60	1508:	2=	54	2096:	2=	42
196:	1=	54	866:	2=	114	1518:	5=	2418	2128:	2=	54
208:	1=	30	870:	2=	1878	1522:	3=	54	2136:	4=	42
220:	3=	30	900:	2=	42	1524:	3=	126	2142	=	2142
238:	3=	30	932:	4=	30	1530:	2=	1890	2144:	2=	30
244:	3=	90	938:	6=	30	1532:	3=	114	2154:	2=	2178
252:	2=	42	956:	4=	90	1536:	4=	54	2166:	1=	2178
274:	3=	30	958:	5=	90	1552:	1=	114	2178	=	2178
288:	1=	42	960:	4=	30	1580:	4=	30	2182:	5=	54
292:	2=	90	970:	3=	42	1592:	2=	30	2208:	5=	30
300:	4=	30	976:	2=	90	1600:	1=	90	2218:	3=	42
336:	2=	30	984:	3=	42	1618:	4=	114	2226:	1=	2958
348:	3=	42	1020:	1=	1140	1644:	3=	126	2232:	2=	90
350:	4=	30	1034:	6=	30	1658:	3=	90	2238:	4=	1890
364:	2=	54	1036:	2=	126	1666:	7=	30	2250:	3=	1890
374:	4=	30	1040:	3=	114	1668:	3=	42	2258:	3=	42
380:	4=	30	1048:	3=	30	1670:	5=	90	2264:	3=	90
382:	3=	30	1056:	3=	42	1686:	3=	1890	2294:	5=	90
386:	2=	54	1076:	4=	30	1688:	4=	30	2300:	4=	30
388:	2=	114	1078:	4=	54	1692:	6=	42	2320:	3=	42
400:	1=	42	1094:	4=	54	1698:	2=	1890	2324:	3=	126
420:	6=	30	1104:	3=	42	1702:	7=	30	2348:	2=	54
436:	1=	114	1112:	2=	42	1710:	1=	1890	2352:	4=	30
440:	2=	30	1116:	2=	126	1716:	4=	126	2368:	2=	114
476:	4=	90	1128:	4=	30	1724:	2=	114	2376:	2=	90
482:	4=	90	1132:	2=	42	1726:	3=	114	2382:	3=	2418
484:	1=	126	1140	=	1140	1744:	1=	126	2386:	3=	126
492:	4=	42	1150:	4=	54	1786:	5=	54	2394:	2=	2418
516:	3=	54	1168:	1=	90	1794:	5=	1890	2406:	1=	2418
528:	2=	42	1188:	5=	42	1800:	6=	30	2410:	9=	30
540:	5=	30	1196:	2=	126	1806:	1=	2418	2418	=	2418
542:	4=	30	1222:	3=	42	1832:	4=	30	2424:	5=	30
550:	3=	54	1230:	6=	1890	1858:	5=	30	2428:	3=	42
570:	3=	1878	1232:	2=	42	1866:	1=	1878	2432:	2=	42
578:	3=	90	1248:	4=	30	1878	=	1878	2448:	3=	42
592:	1=	54	1258:	3=	42	1890	=	1890	2458:	3=	42
600:	3=	30	1260	=	1260	1896:	4=	42	2478:	5=	2418
612:	2=	42	1270:	7=	30	1902:	3=	2418	2480:	4=	90
648:	1=	90	1288:	3=	30	1906:	5=	90	2486:	5=	114
660:	4=	42	1290:	1=	1878	1908:	3=	42	2502:	3=	1890
680:	3=	90	1324:	3=	30	1914:	2=	2418	2520:	7=	30
688:	1=	60	1326:	3=	1890	1916:	2=	126	2544:	5=	30
694:	5=	30	1328:	2=	30	1932:	4=	42	2552:	2=	60
708:	5=	42	1354:	4=	90	1934:	4=	42	2572:	2=	90
720:	5=	30	1360:	5=	90	1938:	4=	2418	2576:	2=	60
722:	3=	54	1372:	4=	42	1944:	3=	42	2592:	1=	114

TABLE 7 (cont.)

2626:	4=	90	3124:	3=	126	3774	=	3774	4390:	6=	90
2642:	4=	30	3136:	1=	114	3790:	7=	54	4392:	6=	42
2664:	4=	54	3140:	2=	90	3806:	4=	42	4420:	3=	90
2668:	5=	30	3148:	3=	42	3812:	6=	90	4422:	34=	56430
2674:	5=	42	3168:	3=	42	3826:	3=	126	4434	=	4434
2684:	3=	126	3172:	2=	90	3860:	4=	42	4438:	3=	90
2688:	7=	30	3178:	6=	90	3862:	5=	42	4446	=	4446
2712:	3=	90	3194:	2=	90	3884:	3=	90	4454:	6=	42
2732:	2=	60	3216:	4=	30	3904:	1=	126	4464:	3=	90
2738:	5=	42	3222:	1=	2178	3906	=	3906	4476:	5=	54
2750:	6=	54	3234:	1=	3966	3926:	4=	42	4492:	5=	30
2760:	6=	30	3240:	6=	42	3934:	7=	54	4494:	25=	2178
2772:	6=	42	3248:	3=	90	3954	=	3954	4500:	7=	30
2774:	8=	30	3268:	3=	42	3956:	4=	30	4506:	8=	1890
2780:	4=	42	3276:	4=	126	3960:	8=	30	4518:	7=	1890
2802:	4=	2178	3282:	4=	2418	3966	=	3966	4528:	5=	30
2806:	4=	90	3294:	3=	2418	3970:	3=	90	4540:	5=	30
2808:	6=	30	3308:	3=	90	3972:	6=	30	4552:	4=	90
2814:	3=	2178	3326:	8=	30	3978	=	3978	4560:	5=	30
2826:	3=	2418	3330:	5=	2418	3980:	5=	114	4580:	4=	42
2832:	5=	30	3332:	2=	90	3996:	4=	30	4594:	5=	30
2834:	6=	54	3334:	6=	90	4028:	5=	42	4632:	5=	30
2840:	4=	30	3348:	3=	42	4030:	6=	114	4634:	7=	90
2842:	4=	90	3360:	6=	30	4032:	2=	90	4652:	2=	90
2864:	2=	54	3398:	8=	30	4034:	5=	114	4664:	2=	90
2866:	4=	54	3400:	4=	114	4042:	6=	90	4680:	3=	1140
2868:	5=	42	3408:	3=	60	4060:	3=	90	4704:	5=	42
2870:	7=	90	3416:	4=	30	4094:	4=	126	4712:	4=	30
2880:	2=	1140	3424:	3=	30	4100:	6=	90	4726:	7=	54
2892:	6=	42	3436:	2=	60	4112:	4=	30	4732:	5=	114
2904:	3=	60	3440:	4=	30	4124:	3=	126	4738:	7=	54
2910:	1=	4146	3442:	3=	114	4126:	4=	126	4752:	5=	30
2912:	4=	90	3444:	5=	126	4136:	4=	30	4760:	4=	54
2920:	5=	30	3446:	4=	114	4142:	4=	42	4766:	4=	126
2924:	3=	126	3448:	3=	30	4146	=	4146	4776:	6=	30
2926:	7=	54	3476:	4=	30	4158	=	4158	4800:	4=	42
2928:	4=	30	3480:	6=	54	4178:	4=	42	4810:	5=	126
2946:	1=	2958	3482:	2=	126	4194:	4=	2418	4816:	2=	90
2948:	3=	42	3492:	4=	30	4204:	4=	42	4836:	4=	126
2958	=	2958	3510:	4=	2418	4214:	4=	126	4848:	4=	42
2976:	5=	30	3522	=	3522	4246:	6=	42	4854:	7=	2418
2984:	4=	30	3530:	5=	90	4260:	9=	30	4866:	6=	2418
2992:	4=	90	3534	=	3534	4280:	2=	114	4868:	4=	42
2996:	4=	30	3546:	3=	2418	4282:	3=	30	4878:	5=	2418
3000:	5=	54	3550:	4=	126	4292:	4=	30	4900:	2=	90
3004:	4=	54	3566:	6=	54	4300:	4=	42	4914:	26=	2178
3010:	9=	30	3582	=	3582	4308:	7=	42	4944:	3=	54
3012:	6=	42	3600:	4=	30	4324:	4=	54	4954:	5=	90
3016:	3=	54	3630:	29=	2178	4328:	4=	54	4966:	3=	114
3024:	4=	90	3632:	4=	90	4330:	3=	126	4980:	11=	30
3032:	3=	114	3642:	5=	2418	4336:	2=	42	4992:	3=	90
3042:	6=	1890	3648:	2=	114	4344:	6=	30	5010:	24=	2178
3044:	4=	54	3654:	4=	2418	4346:	4=	42	5030:	7=	90
3050:	6=	54	3672:	2=	60	4350:	25=	2178	5040:	3=	114
3056:	2=	30	3696:	6=	30	4352:	4=	30	5076:	4=	126
3068:	3=	42	3702:	3=	2178	4356:	2=	126	5080:	5=	30
3074:	6=	54	3714:	2=	2178	4358:	6=	54	5098:	3=	60
3096:	2=	60	3726:	1=	2178	4360:	5=	30	5100:	10=	30
3098:	2=	114	3736:	5=	90	4368:	4=	90	5122:	3=	90
3120:	2=	114	3770:	8=	54	4380:	8=	30	5132:	4=	30

TABLE 7 (cont.)

5134:	7=	54	5736:	4=	60	6396:	6=	42	6994:	5=	42
5136:	6=	30	5766:	5=	2418	6398:	6=	30	7008:	7=	30
5142:	29=	2178	5770:	8=	90	6408:	7=	42	7010:	4=	90
5154:	28=	2178	5772:	5=	42	6432:	6=	30	7030:	9=	30
5160:	7=	30	5778:	4=	2418	6448:	2=	90	7054:	6=	90
5166:	27=	2178	5780:	6=	30	6452:	5=	114	7062:	21=	2178
5170:	11=	30	5784:	5=	30	6454:	8=	90	7068:	6=	54
5192:	4=	30	5818:	5=	90	6488:	4=	30	7086:	23=	2178
5198:	10=	30	5830:	7=	30	6500:	4=	42	7092:	7=	30
5206:	7=	54	5834:	6=	30	6522:	4=	2178	7094:	5=	126
5216:	2=	54	5862:	25=	2178	6528:	7=	30	7098:	22=	2178
5244:	3=	126	5866:	5=	126	6534:	3=	2178	7156:	6=	1890
5262:	5=	2418	5874:	24=	2178	6572:	5=	114	7172:	6=	30
5270:	4=	60	5908:	3=	90	6576:	7=	30	7190:	9=	90
5274:	4=	2418	5932:	3=	60	6584:	3=	90	7194:	24=	2178
5276:	4=	30	5936:	4=	30	6588:	4=	42	7196:	4=	126
5284:	4=	1890	5952:	3=	114	6600:	6=	30	7200:	4=	1140
5296:	4=	42	5972:	6=	54	6606:	1=	4434	7204:	2=	2418
5312:	2=	42	5980:	7=	90	6608:	2=	114	7224:	8=	30
5328:	3=	42	5990:	6=	126	6616:	4=	54	7228:	3=	90
5338:	3=	90	6002:	5=	54	6646:	9=	30	7270:	7=	30
5342:	6=	42	6020:	6=	30	6648:	7=	30	7272:	4=	42
5350:	7=	54	6056:	4=	54	6650:	8=	30	7296:	5=	90
5352:	4=	90	6058:	9=	54	6652:	4=	42	7298:	7=	90
5364:	5=	42	6060:	9=	90	6658:	3=	90	7300:	4=	42
5368:	3=	30	6078:	34=	56430	6662:	7=	90	7348:	3=	60
5370:	33=	56430	6082:	5=	54	6664:	4=	54	7372:	4=	42
5390:	6=	42	6088:	7=	90	6684:	6=	54	7374:	7=	2418
5410:	5=	42	6090:	33=	56430	6688:	3=	42	7384:	5=	30
5418:	30=	2178	6100:	4=	42	6702:	10=	1890	7386:	6=	2418
5430:	27=	2178	6106:	9=	30	6704:	2=	114	7398:	5=	2418
5440:	5=	30	6120:	5=	30	6714:	9=	1890	7416:	4=	42
5448:	7=	30	6124:	5=	54	6724:	4=	1890	7426:	5=	126
5458:	3=	60	6128:	2=	42	6726:	27=	2178	7432:	7=	30
5468:	5=	42	6160:	5=	90	6740:	5=	114	7440:	5=	30
5472:	5=	30	6168:	3=	114	6744:	4=	60	7444:	2=	1878
5494:	7=	54	6170:	6=	90	6752:	4=	90	7448:	2=	114
5514:	4=	2178	6174:	1=	4146	6766:	6=	114	7458:	23=	56430
5522:	5=	126	6180:	8=	90	6768:	3=	90	7462:	9=	30
5526:	3=	2178	6182:	4=	90	6772:	3=	1890	7466:	6=	90
5530:	7=	126	6192:	4=	30	6788:	8=	30	7472:	2=	126
5544:	3=	60	6200:	4=	30	6812:	4=	42	7492:	1=	1878
5552:	3=	54	6202:	7=	42	6816:	8=	30	7504:	2=	126
5554:	5=	42	6220:	3=	90	6826:	5=	30	7518:	7=	2178
5568:	3=	90	6232:	3=	30	6832:	2=	90	7524:	6=	54
5572:	4=	42	6234:	6=	2418	6840:	9=	30	7532:	4=	42
5588:	4=	42	6240:	5=	42	6842:	7=	90	7534:	9=	54
5592:	6=	30	6242:	4=	126	6856:	3=	114	7562:	4=	90
5606:	5=	90	6246:	5=	2418	6864:	6=	30	7568:	4=	30
5614:	6=	114	6262:	6=	90	6866:	3=	60	7576:	5=	90
5616:	4=	30	6270:	44=	56430	6870:	204=	56430	7584:	6=	30
5620:	5=	30	6274:	3=	90	6878:	7=	90	7590:	32=	56430
5626:	3=	90	6276:	5=	42	6880:	5=	30	7596:	5=	54
5636:	8=	30	6288:	8=	30	6884:	4=	114	7598:	4=	30
5644:	3=	126	6300:	7=	90	6902:	10=	54	7604:	6=	90
5656:	5=	30	6308:	4=	42	6908:	3=	90	7606:	5=	42
5674:	5=	30	6338:	3=	90	6950:	4=	90	7610:	10=	30
5722:	3=	54	6372:	6=	42	6952:	5=	30	7614:	5=	2418
5732:	5=	54	6380:	4=	90	6968:	2=	90	7616:	2=	126
5734:	3=	90	6382:	3=	90	6992:	2=	90	7646:	4=	126

TABLE 7 (cont.)

7658:	8=	54	8234:	8=	54	8664:	5=	90	9296:	3=	54
7660:	5=	42	8240:	3=	114	8670:	36=	56430	9298:	3=	90
7662:	27=	2178	8248:	4=	114	8676:	4=	30	9328:	5=	30
7672:	4=	90	8252:	5=	114	8698:	5=	30	9336:	6=	42
7674:	26=	2178	8262:	27=	2178	8702:	4=	60	9342:	5=	2418
7680:	6=	30	8266:	5=	30	8714:	6=	30	9384:	4=	114
7684:	3=	60	8278:	5=	42	8732:	6=	30	9386:	6=	90
7686:	25=	2178	8280:	4=	114	8742:204=	56430		9388:	5=	30
7688:	4=	42	8288:	2=	126	8750:	4=	90	9390:	21=	2178
7690:	7=	90	8290:	9=	30	8754:	26=	2178	9396:	4=	60
7712:	4=	30	8294:	6=	30	8760:	9=	30	9400:	5=	30
7718:	6=	30	8308:	3=	90	8766:	25=	2178	9430:	7=	30
7728:	4=	42	8312:	4=	30	8772:	7=	54	9432:	4=	42
7730:	8=	42	8332:	4=	42	8778:	29=	56430	9446:	8=	54
7732:	5=	2418	8338:	7=	42	8780:	4=	90	9452:	4=	42
7736:	3=	90	8342:	6=	30	8790:	23=	2178	9458:	6=	114
7750:	5=	42	8348:	4=	42	8798:	6=	126	9460:	5=	90
7762:	4=	90	8362:	8=	90	8806:	11=	30	9476:	5=	54
7770:	21=	56430	8366:	6=	30	8812:	6=	30	9484:	3=	90
7772:	4=	42	8368:	6=	30	8820:	9=	90	9496:	3=	126
7786:	8=	90	8384:	2=	54	8824:	3=	42	9508:	4=	2418
7804:	5=	30	8402:	5=	42	8852:	4=	42	9524:	4=	126
7828:	3=	90	8422:	5=	126	8862:	15=	2178	9528:	5=	42
7836:	4=	126	8424:	4=	42	8868:	4=	60	9534:	10=	2178
7846:	5=	42	8428:	3=	90	8884:	2=	2958	9554:	6=	30
7862:	8=	54	8432:	6=	90	8888:	3=	54	9556:	3=	2418
7866:	4=	2178	8450:	6=	126	8902:	7=	42	9560:	5=	114
7876:	4=	126	8454:206=	56430		8904:	8=	30	9564:	7=	42
7912:	3=	30	8466:205=	56430		8918:	8=	126	9570:	20=	2178
7924:	3=	60	8470:	6=	90	8932:	6=	42	9604:	2=	2418
7934:	4=	90	8474:	4=	114	8954:	6=	90	9624:	5=	42
7936:	2=	42	8476:	5=	54	8958:	22=	56430	9626:	3=	90
7960:	5=	30	8484:	5=	126	8968:	5=	30	9652:	4=	42
7968:	3=	114	8486:	7=	42	8970:	21=	56430	9658:	5=	90
7982:	6=	90	8490:	20=	2178	8978:	6=	30	9676:	4=	126
7984:	4=	54	8492:	4=	42	8984:	3=	42	9678:204=	56430	
7990:	5=	90	8500:	5=	30	8990:	10=	30	9688:	5=	30
7994:	4=	90	8524:	5=	42	9002:	9=	90	9690:203=	56430	
8004:	7=	42	8530:	8=	90	9010:	8=	42	9692:	4=	42
8010:	20=	2178	8532:	6=	30	9020:	5=	42	9704:	4=	42
8016:	4=	60	8548:	1=	2142	9030:	25=	2178	9710:	9=	90
8028:	3=	90	8552:	5=	54	9066:	24=	2178	9714:	7=	2178
8040:	8=	30	8568:	7=	42	9074:	4=	90	9716:	5=	42
8054:	7=	114	8576:	2=	54	9078:	23=	2178	9726:	6=	2178
8058:	22=	2178	8578:	5=	30	9098:	5=	90	9730:	9=	90
8070:	56=	1291050	8580:	7=	42	9126:	29=	2178	9738:	5=	2178
8080:	4=	126	8592:	3=	114	9144:	3=	90	9740:	3=	90
8082:	31=	2178	8594:	5=	42	9182:	6=	30	9744:	8=	30
8096:	4=	30	8598:	21=	2178	9192:	6=	30	9762:	6=	2178
8112:	4=	30	8602:	5=	90	9196:	5=	54	9768:	8=	30
8114:	4=	90	8606:	4=	90	9200:	4=	30	9774:	5=	2178
8130:	23=	56430	8610:	20=	2178	9204:	6=	54	9784:	3=	42
8132:	6=	30	8616:	7=	30	9216:	7=	30	9792:	4=	42
8148:	5=	42	8622:	5=	2418	9222:	21=	2178	9796:	5=	54
8182:	5=	126	8628:	6=	42	9240:	7=	30	9816:	4=	54
8190:	19=	2178	8634:	24=	2178	9252:	3=	114	9822:	28=	56430
8202:	7=	2178	8644:	2=	2178	9266:	4=	60	9834:	27=	56430
8212:	6=	1890	8646:	23=	2178	9284:	3=	60	9840:	6=	90
8214:	6=	2178	8654:	4=	126	9288:	7=	30	9852:	5=	42
8226:	5=	2178	8660:	6=	30	9292:	4=	42	9854:	10=	30

TABLE 7 (cont.)

9882:	34=	56430	10508:	3=	90	11056:	6=	42	11524:	3=	60
9892:	6=	2418	10512:	5=	114	11064:	4=	90	11526:	30=	56430
9902:	6=	90	10514:	10=	54	11068:	7=	42	11548:	7=	42
9916:	5=	54	10516:	4=	90	11072:	4=	30	11560:	7=	90
9918:	32=	2178	10520:	6=	90	11078:	8=	42	11564:	3=	60
9930:	20=	2178	10528:	3=	30	11086:	4=	90	11578:	7=	30
9936:	3=	60	10540:	6=	114	11092:	4=	90	11580:	6=	42
9944:	3=	114	10546:	5=	30	11098:	5=	90	11628:	7=	42
9948:	5=	42	10552:	3=	30	11100:	7=	30	11640:	6=	42
9958:	7=	90	10562:	5=	1890	11102:	10=	90	11644:	5=	30
9964:	5=	30	10564:	3=	60	11104:	5=	30	11654:	8=	30
9970:	5=	90	10572:	7=	54	11112:	7=	30	11670:	202=	56430
9982:	7=	126	10574:	4=	90	11124:	3=	60	11682:	33=	2178
9992:	4=	42	10586:	4=	90	11128:	5=	90	11700:	5=	42
9996:	8=	42	10592:	3=	54	11136:	7=	30	11710:	7=	90
10002:	12=	1890	10602:	22=	2178	11138:	5=	42	11724:	5=	126
10004:	4=	54	10608:	8=	30	11142:	24=	56430	11730:	28=	2178
10008:	2=	114	10618:	3=	42	11158:	5=	90	11736:	5=	90
10010:	8=	30	10632:	5=	90	11160:	6=	30	11742:	21=	2178
10014:	11=	1890	10636:	5=	54	11168:	4=	30	11744:	2=	42
10026:	10=	1890	10650:	23=	56430	11170:	7=	90	11748:	6=	42
10028:	3=	90	10652:	6=	30	11180:	5=	42	11754:	5=	2178
10032:	9=	30	10678:	5=	126	11188:	5=	2178	11766:	26=	56430
10034:	4=	90	10680:	10=	30	11190:	32=	56430	11790:	19=	2178
10058:	8=	54	10682:	6=	90	11192:	4=	30	11798:	6=	42
10096:	2=	90	10694:	8=	54	11200:	4=	42	11812:	1=	2958
10112:	2=	30	10704:	6=	30	11204:	5=	90	11814:	22=	56430
10144:	5=	30	10706:	6=	90	11212:	7=	30	11816:	4=	30
10146:	23=	56430	10716:	6=	126	11230:	10=	90	11818:	11=	54
10152:	4=	42	10728:	7=	42	11236:	4=	2178	11824:	4=	54
10154:	6=	30	10732:	8=	30	11246:	4=	90	11868:	4=	114
10156:	6=	30	10758:	26=	56430	11252:	4=	30	11884:	6=	30
10200:	8=	30	10774:	7=	42	11264:	3=	126	11890:	7=	30
10212:	4=	90	10790:	6=	30	11282:	4=	126	11902:	11=	30
10218:	20=	2178	10794:	20=	2178	11288:	4=	42	11912:	6=	54
10232:	4=	30	10812:	7=	42	11296:	3=	54	11946:	29=	56430
10238:	4=	90	10814:	6=	42	11298:	9=	2178	11958:	19=	2178
10250:	4=	90	10820:	6=	30	11306:	6=	30	11966:	4=	90
10286:	6=	30	10828:	4=	90	11326:	5=	90	11970:	18=	2178
10288:	5=	42	10832:	6=	30	11348:	5=	90	11974:	7=	126
10292:	4=	42	10846:	6=	42	11362:	7=	126	11980:	7=	30
10302:	29=	2178	10854:	25=	2178	11364:	6=	54	11992:	3=	54
10310:	6=	30	10868:	4=	60	11370:	55=	1291050	11996:	5=	54
10318:	5=	60	10870:	7=	30	11398:	5=	126	11998:	6=	42
10334:	12=	30	10872:	7=	30	11416:	4=	54	12000:	6=	30
10360:	5=	54	10874:	6=	30	11424:	7=	30	12010:	4=	90
10362:	22=	2178	10900:	5=	114	11432:	5=	54	12018:	43=	56430
10378:	5=	30	10914:	22=	2178	11442:	23=	56430	12024:	3=	60
10382:	6=	90	10920:	10=	30	11444:	5=	54	12030:	42=	56430
10416:	3=	90	10956:	7=	54	11454:	22=	56430	12036:	8=	42
10422:	25=	2178	10984:	7=	30	11456:	4=	90	12040:	3=	90
10428:	8=	54	11008:	5=	30	11460:	7=	42	12044:	4=	54
10430:	8=	90	11010:	43=	56430	11466:	11=	2178	12050:	6=	30
10434:	23=	56430	11022:	20=	56430	11472:	5=	42	12054:	32=	56430
10472:	6=	30	11028:	5=	42	11478:	15=	2178	12076:	5=	90
10480:	5=	30	11034:	7=	2418	11480:	5=	54	12080:	4=	30
10484:	5=	90	11038:	6=	126	11490:	14=	2178	12082:	5=	126
10492:	4=	42	11040:	4=	114	11494:	9=	54	12106:	5=	54
10504:	3=	54	11050:	5=	90	11496:	6=	30	12126:	21=	2178
10506:	20=	2178	11054:	8=	126	11508:	8=	54	12140:	5=	42

TABLE 7 (cont.)

12144:	6=	30	12694:	5=	90	13176:	4=	42	13858:	5=	90
12148:	7=	1890	12702:	21=	2178	13178:	6=	126	13894:	5=	90
12158:	6=	54	12708:	4=	90	13192:	4=	126	13898:	6=	30
12166:	7=	30	12726:	6=	2178	13200:	5=	60	13900:	5=	42
12172:	6=	42	12728:	4=	42	13208:	6=	30	13902:	19=	2178
12184:	4=	114	12738:	21=	56430	13210:	5=	90	13906:	9=	54
12188:	6=	30	12744:	3=	90	13218:	20=	2178	13920:	7=	90
12190:	6=	42	12756:	6=	42	13222:	7=	126	13938:	20=	2178
12192:	6=	42	12758:	4=	90	13230:	19=	2178	13948:	5=	30
12210:	35=	56430	12780:	10=	90	13244:	5=	126	13950:	44=	56430
12222:	10=	2178	12790:	5=	90	13248:	5=	30	13952:	4=	30
12238:	8=	90	12804:	7=	126	13256:	8=	30	13962:	39=	56430
12240:	6=	30	12824:	6=	90	13272:	9=	30	13974:	19=	2178
12242:	6=	54	12846:	26=	56430	13286:	8=	30	13982:	6=	42
12250:	9=	30	12858:	25=	56430	13316:	7=	90	13992:	7=	42
12260:	6=	90	12864:	3=	90	13318:	8=	90	14000:	2=	114
12280:	5=	30	12868:	2=	2178	13320:	5=	1140	14008:	5=	30
12288:	7=	90	12870:	24=	56430	13322:	5=	54	14052:	6=	30
12294:	6=	2178	12872:	5=	114	13342:	7=	30	14054:	10=	30
12312:	4=	42	12878:	6=	42	13344:	7=	30	14066:	6=	30
12320:	4=	42	12880:	3=	114	13366:	8=	90	14068:	1=	3522
12322:	9=	30	12898:	6=	114	13372:	4=	42	14070:	81=	56430
12340:	5=	30	12902:	9=	90	13402:	3=	114	14076:	7=	54
12344:	2=	114	12908:	5=	42	13416:	10=	30	14082:	6=	2178
12354:	9=	2178	12912:	8=	30	13430:	10=	30	14094:	5=	2178
12378:	22=	2178	12916:	2=	3966	13432:	3=	60	14102:	9=	42
12380:	4=	90	12924:	5=	126	13450:	11=	30	14106:	21=	56430
12384:	5=	42	12936:	6=	90	13474:	6=	114	14116:	1=	3534
12390:	21=	2178	12940:	5=	42	13478:	8=	90	14118:	20=	56430
12394:	5=	30	12948:	8=	54	13494:	32=	56430	14122:	8=	114
12398:	8=	42	12960:	5=	126	13498:	5=	90	14128:	3=	42
12400:	2=	126	12984:	8=	30	13526:	7=	114	14136:	9=	30
12414:	21=	2178	12988:	5=	30	13530:	30=	2178	14142:	9=	2178
12426:	20=	2178	12990:	36=	56430	13544:	8=	30	14154:	8=	2178
12432:	4=	114	12998:	6=	42	13554:	6=	2418	14164:	4=	2418
12434:	4=	90	13026:	21=	56430	13560:	5=	90	14174:	6=	30
12458:	4=	30	13032:	4=	42	13566:	8=	2178	14176:	5=	90
12468:	5=	42	13034:	6=	42	13596:	7=	54	14178:	9=	2178
12474:	25=	56430	13038:	10=	2178	13608:	9=	30	14182:	7=	30
12476:	4=	126	13046:	8=	42	13612:	6=	42	14210:	6=	90
12490:	9=	30	13056:	8=	30	13618:	5=	60	14214:	32=	56430
12504:	9=	30	13066:	6=	126	13646:	6=	30	14216:	6=	54
12512:	2=	126	13074:	28=	2178	13648:	3=	1878	14248:	3=	90
12542:	4=	90	13076:	4=	126	13656:	7=	30	14250:	55=	1291050
12568:	5=	30	13080:	11=	30	13658:	3=	90	14260:	5=	90
12570:	14=	2178	13082:	8=	30	13668:	8=	42	14262:	28=	56430
12576:	5=	42	13084:	5=	126	13708:	5=	30	14272:	2=	42
12600:	6=	30	13086:	27=	2178	13716:	5=	42	14274:	27=	56430
12604:	5=	30	13090:	10=	42	13726:	4=	60	14276:	5=	42
12612:	4=	90	13098:	29=	56430	13748:	4=	60	14314:	5=	114
12624:	4=	42	13100:	4=	90	13760:	5=	114	14316:	8=	42
12628:	5=	42	13104:	5=	30	13762:	11=	30	14322:	12=	2178
12652:	4=	42	13124:	3=	42	13782:	44=	56430	14326:	7=	30
12654:	24=	56430	13128:	7=	30	13790:	8=	30	14328:	3=	60
12660:	11=	90	13138:	6=	114	13794:	43=	56430	14342:	8=	90
12664:	3=	30	13146:	31=	56430	13798:	11=	54	14348:	6=	114
12668:	3=	90	13158:	23=	2178	13800:	6=	90	14360:	6=	30
12672:	7=	30	13160:	6=	90	13816:	4=	90	14368:	5=	90
12680:	6=	30	13162:	4=	90	13818:	201=	56430	14374:	10=	90
12688:	5=	114	13170:	19=	56430	13824:	6=	30	14376:	9=	30

TABLE 7 (cont.)

14386:	5=	126	14942:	10=	30	15520:	4=	90	16110:	38=	56430
14396:	5=	42	14946:	14=	2178	15574:	4=	90	16148:	4=	60
14410:	10=	42	14978:	2=	1878	15582:	19=	2178	16152:	5=	30
14420:	7=	114	14982:	200=	56430	15590:	10=	30	16158:	13=	2178
14430:	25=	2178	14986:	8=	114	15594:	28=	2178	16168:	5=	30
14434:	13=	30	14990:	5=	90	15596:	5=	1890	16170:	12=	2178
14472:	5=	54	15010:	9=	30	15602:	6=	30	16186:	5=	30
14482:	7=	90	15056:	6=	90	15606:	27=	2178	16188:	5=	90
14496:	4=	90	15058:	5=	42	15608:	4=	42	16222:	5=	90
14502:	34=	56430	15060:	6=	24180	15612:	9=	54	16230:	202=	56430
14514:	33=	56430	15070:	6=	42	15618:	21=	56430	16240:	4=	114
14528:	3=	90	15072:	6=	30	15624:	5=	42	16258:	7=	90
14540:	5=	42	15076:	1=	3774	15626:	6=	90	16266:	39=	56430
14548:	6=	2418	15078:	27=	2178	15640:	5=	42	16268:	6=	114
14554:	8=	42	15090:	37=	56430	15642:	24=	2178	16270:	7=	42
14560:	3=	126	15096:	6=	42	15662:	9=	90	16278:	38=	56430
14562:	6=	2178	15110:	6=	54	15688:	5=	42	16282:	9=	30
14572:	3=	114	15114:	2=	18018	15690:	18=	2178	16288:	5=	30
14578:	5=	90	15118:	5=	90	15696:	5=	54	16290:	37=	56430
14584:	5=	30	15120:	5=	42	15704:	4=	30	16304:	3=	126
14586:	201=	56430	15142:	5=	90	15716:	8=	54	16314:	24=	2178
14594:	5=	42	15146:	6=	90	15726:	32=	56430	16326:	23=	2178
14598:	7=	2178	15180:	5=	24180	15738:	31=	56430	16348:	6=	114
14610:	45=	56430	15184:	4=	42	15746:	5=	126	16350:	19=	2178
14622:	8=	2178	15186:	21=	56430	15748:	6=	114	16358:	6=	126
14634:	7=	2178	15198:	20=	56430	15756:	8=	126	16360:	7=	30
14652:	6=	42	15200:	4=	42	15768:	3=	1140	16368:	9=	30
14664:	6=	42	15202:	10=	90	15774:	24=	56430	16402:	7=	126
14668:	6=	114	15206:	6=	42	15778:	11=	30	16404:	8=	54
14674:	5=	90	15214:	11=	30	15800:	6=	30	16408:	5=	54
14680:	5=	30	15216:	8=	42	15824:	3=	54	16410:	201=	56430
14686:	11=	54	15222:	20=	56430	15828:	8=	54	16418:	7=	1890
14688:	4=	42	15248:	4=	42	15834:	18=	2178	16420:	4=	114
14696:	4=	30	15252:	8=	42	15842:	4=	60	16424:	8=	30
14722:	7=	30	15256:	3=	126	15844:	3=	54	16426:	9=	126
14728:	3=	114	15284:	4=	126	15850:	7=	90	16430:	6=	90
14738:	5=	42	15286:	5=	126	15858:	23=	2178	16446:	20=	56430
14740:	4=	90	15308:	6=	30	15864:	4=	114	16452:	3=	90
14748:	6=	42	15322:	7=	30	15866:	3=	42	16456:	4=	90
14758:	5=	90	15330:	26=	2178	15868:	7=	30	16458:	19=	56430
14764:	7=	30	15336:	7=	30	15870:	200=	56430	16462:	9=	54
14768:	3=	114	15338:	5=	90	15886:	6=	42	16468:	5=	30
14778:	35=	56430	15344:	4=	30	15888:	4=	42	16470:	36=	56430
14788:	4=	2178	15356:	4=	126	15912:	4=	90	16474:	4=	114
14790:	13=	2178	15360:	8=	30	15924:	6=	54	16480:	5=	30
14792:	6=	30	15362:	4=	60	15946:	11=	30	16496:	4=	30
14806:	7=	114	15370:	9=	30	15978:	55=	1291050	16510:	6=	126
14808:	6=	30	15374:	8=	90	15990:	54=	1291050	16512:	5=	30
14824:	5=	30	15384:	5=	42	16014:	30=	56430	16520:	6=	30
14826:	19=	56430	15394:	7=	30	16016:	3=	90	16526:	6=	30
14836:	3=	2178	15438:	19=	2178	16046:	6=	30	16536:	11=	30
14844:	6=	54	15442:	9=	126	16062:	8=	2178	16554:	2=	18018
14848:	5=	30	15450:	31=	56430	16066:	7=	90	16566:	19=	2178
14884:	2=	2178	15454:	9=	42	16074:	7=	2178	16572:	8=	42
14892:	7=	42	15472:	4=	42	16086:	18=	56430	16574:	10=	30
14904:	7=	30	15474:	43=	56430	16088:	5=	114	16582:	7=	30
14918:	10=	30	15480:	5=	1140	16098:	39=	56430	16598:	9=	42
14920:	5=	30	15486:	42=	56430	16102:	6=	42	16600:	3=	30
14928:	9=	30	15498:	7=	2178	16104:	11=	30	16604:	7=	1890
14938:	9=	30	15518:	5=	90	16108:	3=	90	16606:	7=	114

TABLE 7 (cont.)

16610:	6=	90	17178:	8=	2178	17606:	12=	30	18124:	9=	30
16612:	1=	4158	17184:	8=	30	17610:	201=	56430	18126:	28=	56430
16624:	4=	42	17186:	4=	42	17622:	36=	56430	18166:	9=	54
16626:	23=	56430	17190:	24=	56430	17642:	4=	42	18188:	5=	90
16630:	6=	54	17198:	6=	90	17664:	8=	30	18204:	6=	42
16642:	7=	90	17202:	19=	56430	17670:	13=	2178	18210:	27=	2178
16650:	37=	56430	17208:	7=	42	17672:	4=	42	18214:	7=	42
16656:	8=	30	17238:	27=	2178	17696:	4=	30	18236:	5=	1890
16658:	5=	42	17240:	8=	90	17698:	5=	42	18240:	6=	126
16664:	4=	42	17248:	3=	60	17706:	19=	56430	18246:	36=	56430
16676:	6=	54	17256:	6=	90	17710:	8=	30	18258:	35=	56430
16680:	6=	90	17260:	7=	30	17736:	9=	30	18260:	8=	30
16696:	3=	42	17262:	24=	2178	17746:	13=	30	18270:	39=	56430
16714:	7=	90	17270:	8=	90	17748:	4=	114	18280:	6=	114
16716:	5=	126	17274:	16=	2178	17764:	1=	4446	18284:	5=	126
16726:	7=	30	17276:	3=	1878	17772:	5=	114	18290:	8=	42
16732:	5=	42	17280:	7=	42	17776:	4=	114	18294:	7=	2178
16744:	3=	114	17282:	3=	2178	17780:	6=	42	18306:	6=	2178
16750:	7=	90	17284:	3=	54	17782:	11=	54	18336:	8=	30
16770:	17=	56430	17286:	15=	2178	17784:	5=	42	18348:	9=	42
16772:	4=	90	17310:	89=	56430	17796:	7=	30	18352:	4=	42
16788:	8=	30	17314:	9=	126	17798:	8=	126	18384:	5=	42
16806:	19=	2178	17316:	4=	60	17810:	6=	90	18394:	7=	30
16812:	5=	42	17318:	6=	30	17820:	6=	42	18396:	6=	90
16818:	18=	2178	17342:	7=	114	17850:	199=	56430	18416:	2=	90
16830:	17=	2178	17344:	3=	30	17858:	7=	42	18420:	13=	90
16836:	6=	126	17356:	7=	30	17862:	18=	56430	18426:	13=	56430
16838:	6=	126	17370:	23=	56430	17870:	6=	114	18432:	6=	1890
16848:	6=	30	17374:	6=	42	17874:	12=	1890	18468:	4=	60
16858:	7=	50	17376:	8=	30	17902:	7=	90	18474:	4=	18018
16864:	3=	30	17390:	10=	126	17914:	4=	90	18486:	3=	18018
16874:	9=	90	17392:	4=	42	17920:	3=	114	18488:	4=	42
16894:	7=	126	17398:	5=	60	17928:	7=	42	18498:	19=	56430
16896:	5=	42	17400:	11=	30	17930:	6=	90	18500:	6=	30
16914:	41=	56430	17412:	8=	42	17948:	6=	2418	18506:	8=	42
16926:	40=	56430	17436:	6=	30	17956:	26=	2178	18510:	18=	56430
16934:	7=	90	17448:	10=	30	17966:	6=	90	18534:	14=	56430
16936:	5=	54	17452:	5=	90	17968:	1=	1140	18536:	4=	30
16940:	6=	30	17464:	3=	30	17970:	18=	2178	18540:	12=	90
16950:	15=	2178	17478:	34=	2178	17976:	8=	30	18546:	13=	56430
16960:	7=	90	17490:	29=	2178	17980:	7=	30	18550:	7=	90
16964:	7=	42	17494:	5=	90	17982:	28=	56430	18562:	4=	60
16966:	5=	90	17502:	20=	56430	17990:	8=	54	18568:	5=	54
16976:	5=	54	17510:	6=	30	17994:	2=	18018	18574:	7=	30
16992:	7=	30	17514:	19=	56430	17998:	10=	90	18576:	3=	114
16998:	30=	56430	17516:	5=	1890	18006:	1=	18018	18582:	18=	56430
17010:	29=	56430	17520:	8=	30	18014:	9=	42	18584:	4=	30
17042:	6=	42	17528:	6=	30	18018:	=	18018	18586:	4=	54
17050:	10=	90	17542:	7=	114	18030:	8=	56430	18594:	21=	2178
17052:	9=	54	17548:	5=	30	18032:	3=	114	18618:	14=	2178
17054:	9=	90	17550:	22=	56430	18052:	8=	1890	18624:	4=	114
17056:	3=	54	17560:	5=	30	18056:	4=	42	18642:	23=	56430
17064:	3=	60	17562:	21=	56430	18060:	1=	24180	18710:	9=	114
17072:	6=	30	17568:	7=	42	18080:	6=	30	18712:	3=	54
17080:	5=	42	17572:	6=	42	18092:	6=	30	18736:	6=	42
17134:	5=	90	17574:	20=	56430	18094:	4=	90	18740:	6=	30
17142:	13=	2178	17578:	8=	114	18108:	8=	30	18758:	6=	90
17146:	3=	54	17584:	2=	126	18112:	4=	42	18768:	9=	30
17154:	12=	2178	17586:	6=	2178	18114:	29=	56430	18786:	23=	56430
17166:	9=	2178	17602:	7=	30	18116:	6=	30	18792:	4=	42

TABLE 7 (cont.)

18794:	6=	30	19408:	7=	1890	19948:	4=	90	20610:	17=	56430
18798:	22=	56430	19416:	9=	30	19968:	10=	30	20622:	34=	56430
18814:	7=	90	19418:	7=	42	20006:	6=	114	20634:	201=	56430
18828:	5=	42	19420:	4=	90	20010:	38=	2178	20646:	19=	2178
18830:	8=	30	19422:	24=	2178	20014:	9=	30	20660:	6=	54
18858:	31=	56430	19426:	9=	42	20020:	7=	114	20662:	13=	30
18876:	5=	126	19436:	4=	126	20028:	5=	42	20702:	6=	90
18884:	8=	54	19440:	8=	30	20042:	6=	90	20704:	4=	90
18912:	9=	30	19444:	7=	2418	20050:	7=	30	20714:	6=	54
18932:	8=	54	19446:	81=	56430	20060:	6=	30	20720:	6=	42
18944:	4=	54	19452:	6=	30	20082:	11=	56430	20732:	4=	90
18946:	6=	54	19464:	9=	30	20090:	9=	30	20766:	18=	56430
18948:	5=	42	19482:	26=	2178	20094:	10=	56430	20768:	5=	90
18962:	7=	126	19488:	7=	42	20108:	5=	42	20778:	17=	56430
18966:	202=	56430	19494:	23=	2178	20140:	7=	90	20790:	16=	56430
19008:	6=	30	19510:	7=	90	20160:	5=	114	20822:	8=	30
19016:	4=	126	19522:	5=	90	20176:	4=	42	20832:	6=	126
19020:	3=	24180	19536:	6=	42	20186:	3=	90	20840:	4=	60
19042:	5=	126	19542:	31=	2178	20202:	31=	56430	20854:	9=	90
19060:	6=	114	19544:	6=	30	20218:	10=	90	20864:	3=	90
19068:	9=	42	19546:	7=	30	20224:	3=	30	20868:	10=	54
19074:	9=	56430	19548:	4=	60	20234:	8=	42	20870:	8=	90
19076:	6=	54	19554:	30=	2178	20256:	5=	42	20874:	39=	56430
19090:	7=	90	19560:	5=	114	20262:	13=	2178	20878:	8=	1890
19092:	7=	114	19562:	4=	42	20282:	6=	30	20892:	6=	30
19096:	6=	54	19566:	29=	2178	20288:	4=	30	20910:	42=	56430
19102:	7=	30	19570:	7=	114	20300:	6=	114	20912:	4=	30
19104:	6=	90	19578:	30=	2178	20302:	7=	30	20922:	32=	2178
19106:	6=	30	19592:	4=	30	20304:	6=	30	20928:	5=	90
19130:	8=	30	19614:	8=	56430	20328:	7=	30	20936:	5=	90
19140:	2=	24180	19632:	5=	114	20330:	8=	90	20944:	7=	90
19146:	19=	56430	19636:	27=	2178	20342:	9=	42	20946:	31=	56430
19158:	18=	56430	19654:	7=	30	20344:	3=	60	20954:	6=	30
19162:	7=	54	19656:	8=	42	20364:	8=	54	20958:	30=	56430
19188:	5=	90	19662:	18=	56430	20370:	5=	56430	20970:	39=	56430
19194:	10=	2178	19674:	32=	56430	20374:	7=	30	20978:	6=	30
19198:	7=	90	19684:	7=	126	20382:	22=	56430	20980:	7=	90
19202:	3=	2418	19696:	5=	30	20388:	7=	42	20982:	188=	56430
19206:	8=	2178	19702:	11=	30	20400:	8=	30	20994:	7=	2178
19208:	10=	30	19710:	7=	2178	20406:	202=	56430	21002:	4=	54
19218:	23=	56430	19716:	7=	54	20416:	5=	30	21006:	6=	2178
19230:	22=	56430	19722:	18=	2178	20418:	13=	56430	21008:	4=	42
19234:	7=	30	19734:	23=	56430	20430:	18=	56430	21018:	30=	2178
19242:	26=	56430	19764:	6=	30	20466:	23=	56430	21020:	7=	114
19246:	4=	90	19790:	8=	90	20468:	8=	30	21026:	5=	90
19256:	4=	30	19800:	5=	1140	20482:	6=	90	21046:	5=	90
19290:	199=	56430	19816:	6=	114	20484:	5=	60	21056:	2=	126
19298:	5=	42	19832:	4=	30	20492:	4=	90	21064:	5=	30
19304:	6=	90	19834:	9=	54	20494:	5=	90	21066:	8=	2178
19320:	11=	30	19852:	2=	2142	20526:	44=	56430	21078:	7=	2178
19324:	5=	126	19864:	5=	54	20548:	5=	42	21094:	7=	30
19348:	5=	42	19878:	12=	56430	20550:	32=	56430	21096:	6=	42
19356:	6=	126	19882:	4=	114	20574:	23=	2178	21116:	5=	1890
19362:	51=	1291050	19886:	4=	42	20578:	5=	42	21118:	6=	1890
19374:	27=	2178	19890:	11=	56430	20582:	8=	90	21144:	9=	30
19378:	5=	42	19900:	5=	42	20586:	18=	2178	21150:	38=	56430
19380:	4=	24180	19906:	7=	30	20590:	9=	42	21152:	6=	30
19384:	3=	42	19928:	5=	114	20592:	4=	114	21178:	4=	54
19386:	26=	2178	19934:	6=	90	20598:	18=	56430	21182:	6=	42
19398:	18=	56430	19944:	5=	126	20600:	6=	90	21192:	6=	90

TABLE 7 (cont.)

21198:	36=	56430	21734:	8=	30	22252:	4=	90	22768:	7=	30
21200:	6=	30	21740:	6=	114	22256:	4=	30	22770:	28=	2178
21210:	35=	56430	21758:	8=	90	22260:	1=	29580	22782:	202=	56430
21240:	7=	30	21762:	201=	56430	22272:	8=	42	22794:	201=	56430
21252:	10=	42	21768:	11=	30	22290:	199=	56430	22806:	40=	56430
21268:	8=	30	21772:	8=	30	22302:	=	22302	22842:	7=	2178
21272:	6=	30	21774:	89=	56430	22314:	39=	56430	22856:	5=	54
21288:	11=	30	21822:	203=	56430	22316:	3=	2418	22868:	4=	54
21290:	11=	90	21824:	2=	114	22326:	38=	56430	22872:	6=	42
21292:	4=	42	21828:	10=	54	22338:	37=	56430	22888:	8=	90
21306:	30=	2178	21832:	6=	42	22340:	5=	42	22890:	183=	56430
21318:	137=	56430	21834:	202=	56430	22344:	8=	30	22896:	7=	30
21332:	4=	90	21850:	11=	30	22354:	6=	42	22904:	5=	54
21338:	10=	54	21852:	2=	2142	22370:	5=	90	22908:	9=	42
21342:	30=	56430	21858:	21=	56430	22372:	7=	30	22916:	3=	126
21346:	7=	126	21866:	6=	90	22376:	5=	90	22954:	9=	42
21354:	29=	56430	21870:	20=	56430	22386:	24=	2178	22968:	5=	42
21360:	9=	30	21880:	5=	30	22390:	7=	90	22970:	8=	30
21366:	28=	56430	21882:	63=	1291050	22394:	5=	42	22982:	10=	54
21378:	17=	56430	21906:	8=	2178	22412:	4=	90	22988:	7=	2418
21382:	9=	54	21912:	10=	30	22422:	13=	2178	23012:	4=	90
21388:	5=	90	21918:	12=	56430	22434:	14=	2178	23028:	6=	114
21398:	6=	90	21928:	7=	54	22440:	8=	30	23042:	10=	30
21410:	4=	54	21948:	4=	90	22446:	13=	2178	23044:	6=	54
21412:	3=	42	21972:	7=	114	22456:	5=	30	23046:	200=	56430
21432:	8=	30	21978:	24=	2178	22464:	4=	54	23060:	8=	90
21440:	6=	30	21988:	5=	42	22486:	5=	90	23078:	8=	30
21444:	9=	42	21990:	25=	2178	22504:	5=	30	23080:	7=	30
21448:	5=	30	22026:	18=	2178	22506:	31=	56430	23110:	9=	42
21458:	9=	30	22030:	5=	42	22508:	5=	42	23116:	6=	30
21470:	8=	114	22036:	4=	90	22520:	3=	42	23144:	4=	30
21480:	12=	30	22038:	17=	2178	22522:	4=	126	23156:	6=	30
21486:	24=	56430	22040:	6=	54	22530:	36=	56430	23202:	8=	2178
21488:	5=	90	22042:	9=	30	22542:	11=	2178	23208:	7=	42
21498:	23=	56430	22050:	16=	2178	22554:	20=	2178	23224:	5=	90
21500:	4=	90	22052:	3=	90	22556:	4=	126	23226:	10=	2178
21504:	4=	54	22056:	5=	90	22558:	5=	126	23246:	6=	30
21510:	22=	56430	22062:	12=	56430	22566:	15=	2178	23252:	6=	90
21520:	7=	30	22074:	11=	56430	22568:	6=	30	23256:	4=	90
21542:	8=	42	22080:	9=	30	22572:	6=	42	23276:	6=	42
21576:	9=	30	22084:	4=	2178	22578:	14=	2178	23318:	10=	30
21582:	1=	18018	22086:	10=	2178	22590:	29=	2178	23322:	11=	56430
21590:	5=	114	22094:	6=	90	22602:	27=	56430	23334:	30=	2178
21594:	201=	56430	22098:	32=	2178	22604:	6=	30	23336:	8=	54
21600:	6=	30	22104:	5=	30	22614:	26=	56430	23346:	29=	2178
21602:	10=	42	22122:	36=	56430	22626:	25=	56430	23366:	8=	90
21606:	81=	56430	22128:	8=	30	22632:	8=	30	23368:	3=	114
21620:	6=	42	22134:	14=	56430	22648:	5=	30	23380:	7=	30
21626:	12=	54	22138:	5=	30	22650:	10=	2178	23382:	29=	2178
21630:	32=	56430	22152:	10=	30	22664:	5=	90	23386:	11=	30
21648:	5=	42	22154:	8=	30	22674:	9=	56430	23388:	10=	54
21652:	31=	2178	22166:	5=	90	22676:	6=	30	23418:	25=	2178
21672:	3=	114	22170:	20=	2178	22680:	4=	90	23424:	8=	42
21678:	22=	56430	22182:	7=	2178	22686:	8=	56430	23426:	6=	60
21684:	6=	1890	22184:	6=	90	22704:	6=	60	23428:	26=	2178
21690:	21=	56430	22194:	6=	2178	22716:	3=	2958	23444:	6=	126
21700:	7=	42	22222:	7=	30	22718:	6=	30	23456:	4=	54
21702:	200=	56430	22224:	6=	30	22746:	11=	56430	23470:	7=	30
21714:	199=	56430	22242:	86=	56430	22752:	3=	114	23474:	6=	30
21726:	13=	2178	22248:	9=	30	22758:	29=	2178	23476:	25=	2178

TABLE 7 (cont.)

23484:	7=	30	24060:	1=	24180	24554:	6=	30	25078:	5=	90
23490:	17=	56430	24068:	7=	30	24556:	4=	60	25084:	6=	42
23496:	6=	42	24070:	12=	90	24560:	4=	114	25092:	5=	90
23500:	6=	114	24072:	7=	30	24580:	5=	42	25098:	80=	56430
23508:	7=	30	24074:	4=	90	24592:	4=	42	25100:	6=	42
23512:	4=	42	24078:	13=	2178	24616:	5=	42	25104:	4=	30
23536:	3=	60	24080:	5=	54	24620:	6=	30	25110:	37=	56430
23542:	5=	30	24082:	5=	54	24630:	188=	56430	25116:	10=	126
23552:	4=	30	24090:	12=	2178	24642:	20=	56430	25124:	6=	54
23582:	9=	42	24102:	31=	2178	24652:	4=	114	25128:	7=	42
23594:	6=	90	24118:	6=	54	24666:	45=	56430	25158:	32=	56430
23600:	6=	30	24120:	7=	30	24672:	5=	114	25164:	6=	54
23612:	4=	90	24128:	3=	90	24674:	7=	126	25176:	5=	42
23626:	5=	30	24132:	4=	90	24678:	44=	56430	25182:	19=	2178
23634:	9=	2178	24134:	8=	30	24698:	6=	90	25184:	4=	30
23642:	5=	54	24136:	3=	90	24700:	6=	42	25188:	6=	42
23646:	77=	56430	24140:	5=	90	24710:	12=	30	25190:	7=	114
23652:	4=	42	24146:	6=	90	24712:	3=	114	25202:	6=	30
23658:	42=	56430	24160:	5=	30	24716:	3=	2418	25206:	7=	2178
23664:	9=	30	24180	=	24180	24718:	6=	42	25210:	4=	90
23670:	41=	56430	24184:	4=	114	24724:	6=	54	25212:	11=	54
23680:	6=	54	24192:	6=	42	24726:	200=	56430	25218:	6=	2178
23682:	33=	56430	24208:	3=	90	24734:	7=	126	25230:	17=	2178
23688:	8=	30	24212:	10=	54	24740:	3=	126	25240:	4=	54
23694:	32=	56430	24216:	7=	30	24750:	18=	56430	25254:	27=	2178
23696:	6=	54	24234:	22=	56430	24754:	5=	90	25272:	6=	30
23708:	4=	60	24240:	9=	30	24762:	10=	2178	25290:	11=	56430
23712:	6=	30	24246:	7=	2418	24764:	7=	1890	25300:	6=	42
23722:	7=	126	24258:	31=	56430	24770:	10=	54	25302:	8=	56430
23728:	3=	1140	24280:	6=	54	24774:	9=	2178	25304:	3=	90
23742:	24=	2178	24284:	6=	54	24780:	5=	24180	25314:	7=	56430
23744:	3=	42	24292:	35=	56430	24782:	6=	30	25326:	6=	56430
23760:	6=	114	24294:	89=	56430	24786:	8=	2178	25328:	2=	90
23762:	7=	30	24302:	6=	90	24788:	8=	42	25338:	199=	56430
23774:	8=	30	24306:	88=	56430	24792:	5=	114	25352:	7=	90
23818:	7=	54	24312:	7=	42	24824:	3=	42	25376:	4=	42
23820:	3=	24180	24318:	87=	56430	24828:	9=	42	25388:	5=	42
23824:	7=	2418	24326:	8=	30	24838:	5=	60	25390:	9=	90
23828:	5=	42	24328:	7=	54	24870:	31=	2178	25400:	5=	90
23850:	40=	56430	24332:	2=	3522	24882:	78=	56430	25428:	5=	90
23874:	24=	2178	24342:	30=	56430	24916:	7=	2418	25446:	12=	56430
23880:	12=	30	24344:	3=	90	24918:	39=	56430	25454:	8=	30
23890:	9=	30	24352:	3=	42	24926:	7=	114	25458:	11=	56430
23900:	5=	42	24354:	29=	56430	24930:	38=	56430	25468:	7=	42
23928:	7=	30	24372:	6=	42	24936:	10=	30	25470:	10=	56430
23930:	8=	54	24378:	187=	56430	24946:	5=	126	25480:	5=	42
23940:	2=	24180	24400:	5=	54	24960:	6=	42	25484:	5=	126
23942:	8=	126	24402:	43=	56430	24964:	6=	2418	25520:	5=	42
23944:	6=	42	24404:	10=	30	24974:	10=	30	25546:	4=	90
23954:	5=	90	24410:	8=	30	24990:	50=	1291050	25548:	10=	54
23966:	6=	60	24446:	14=	30	25002:	20=	2178	25554:	27=	2178
23972:	7=	30	24458:	6=	90	25024:	3=	30	25560:	6=	1140
23982:	31=	2178	24460:	7=	90	25028:	7=	90	25566:	26=	2178
23994:	37=	56430	24474:	18=	2178	25030:	7=	90	25576:	3=	90
24000:	10=	30	24478:	7=	54	25044:	6=	126	25578:	25=	2178
24008:	10=	30	24480:	7=	30	25046:	5=	90	25592:	6=	30
24014:	5=	90	24486:	17=	2178	25048:	5=	54	25596:	3=	2418
24020:	4=	90	24490:	6=	114	25056:	5=	54	25616:	5=	114
24032:	4=	90	24504:	8=	90	25068:	6=	42	25622:	9=	42
24046:	6=	90	24552:	3=	114	25076:	4=	90	25626:	11=	56430

TABLE 7 (cont.)

25628:	5=	42	26202:	20=	2178	26792:	5=	1890	27326:	8=	90
25638:	10=	56430	26228:	5=	42	26798:	4=	114	27330:	57=	1291050
25642:	7=	90	26234:	6=	30	26816:	2=	126	27344:	4=	114
25650:	9=	56430	26238:	23=	2178	26824:	4=	90	27350:	8=	30
25656:	4=	90	26242:	4=	42	26832:	7=	90	27360:	7=	30
25666:	7=	30	26250:	22=	2178	26836:	10=	1890	27392:	3=	54
25674:	78=	56430	26256:	6=	30	26866:	7=	90	27396:	7=	30
25680:	6=	42	26264:	3=	42	26876:	5=	1890	27398:	5=	126
25686:	13=	2178	26266:	11=	30	26880:	5=	54	27402:	8=	2178
25698:	7=	56430	26278:	7=	30	26894:	7=	42	27404:	3=	3966
25708:	7=	30	26284:	8=	30	26896:	3=	1890	27414:	7=	2178
25710:	6=	56430	26286:	77=	56430	26920:	6=	54	27436:	7=	30
25722:	9=	2178	26306:	8=	90	26928:	5=	42	27446:	5=	60
25732:	8=	42	26318:	5=	90	26940:	12=	30	27450:	43=	56430
25748:	8=	30	26328:	6=	60	26982:	1=	18018	27460:	6=	114
25762:	7=	90	26334:	21=	56430	26994:	21=	56430	27462:	186=	56430
25770:	136=	56430	26346:	53=	1291050	26996:	4=	42	27464:	4=	114
25772:	4=	90	26358:	52=	1291050	27000:	6=	30	27504:	7=	30
25796:	9=	90	26372:	4=	90	27018:	189=	56430	27514:	6=	114
25798:	10=	90	26392:	4=	90	27030:	11=	2178	27518:	12=	30
25800:	6=	42	26406:	21=	2178	27036:	4=	60	27530:	10=	30
25816:	3=	114	26430:	85=	56430	27042:	29=	56430	27548:	6=	2418
25820:	6=	42	26438:	8=	126	27044:	7=	114	27554:	7=	90
25824:	5=	54	26444:	5=	126	27046:	8=	114	27570:	75=	56430
25828:	5=	42	26454:	25=	2178	27048:	6=	30	27582:	16=	56430
25854:	26=	56430	26466:	24=	2178	27052:	4=	90	27594:	15=	56430
25868:	6=	2418	26472:	10=	30	27054:	28=	56430	27606:	13=	2178
25872:	10=	30	26480:	6=	30	27060:	11=	30	27636:	9=	54
25878:	39=	2178	26482:	6=	126	27076:	6=	30	27638:	11=	90
25890:	76=	56430	26496:	4=	90	27078:	198=	56430	27642:	20=	2178
25912:	5=	54	26510:	8=	90	27082:	9=	90	27654:	13=	2178
25920:	10=	90	26520:	9=	42	27088:	2=	1890	27664:	4=	90
25938:	2=	18018	26524:	6=	54	27090:	197=	56430	27670:	9=	30
25978:	9=	90	26544:	6=	54	27102:	23=	2178	27674:	9=	42
25986:	17=	56430	26560:	5=	30	27112:	9=	30	27680:	4=	1890
25992:	5=	42	26564:	3=	42	27114:	22=	2178	27686:	9=	42
25998:	42=	56430	26584:	3=	90	27126:	21=	2178	27692:	7=	2418
26010:	10=	2178	26586:	90=	56430	27128:	5=	114	27702:	40=	56430
26012:	6=	2178	26588:	7=	1890	27146:	9=	42	27716:	5=	54
26032:	4=	126	26590:	12=	90	27166:	6=	42	27720:	8=	30
26040:	13=	30	26596:	3=	126	27180:	14=	90	27736:	5=	30
26068:	5=	42	26598:	13=	2178	27184:	5=	126	27760:	4=	90
26072:	4=	42	26610:	33=	56430	27192:	10=	30	27762:	41=	56430
26082:	39=	56430	26612:	4=	90	27196:	4=	90	27764:	7=	30
26086:	9=	42	26616:	8=	30	27202:	9=	90	27774:	14=	56430
26088:	12=	30	26624:	8=	30	27208:	5=	30	27786:	24=	2178
26102:	11=	42	26626:	8=	90	27224:	6=	30	27788:	6=	30
26108:	5=	42	26636:	5=	54	27230:	7=	90	27794:	4=	54
26116:	4=	2178	26642:	8=	30	27232:	3=	54	27806:	10=	54
26120:	6=	54	26656:	5=	54	27234:	29=	56430	27828:	8=	30
26126:	7=	126	26664:	12=	30	27236:	6=	30	27838:	5=	60
26128:	4=	90	26668:	5=	42	27242:	9=	42	27852:	7=	114
26140:	5=	90	26678:	8=	30	27256:	5=	30	27854:	6=	30
26148:	6=	90	26704:	4=	1890	27276:	7=	42	27860:	8=	90
26150:	8=	30	26714:	9=	42	27282:	26=	2178	27882:	15=	2178
26158:	4=	2418	26716:	7=	54	27290:	12=	30	27886:	6=	42
26170:	7=	30	26728:	7=	30	27294:	25=	2178	27894:	26=	56430
26178:	10=	2178	26742:	79=	56430	27306:	24=	2178	27896:	3=	90
26184:	8=	30	26754:	78=	56430	27308:	3=	90	27906:	25=	56430
26190:	9=	2178	26766:	13=	1890	27310:	7=	90	27912:	8=	42

TABLE 7 (cont.)

27914:	5=	114	28552:	5=	30	29084:	6=	54	29500:	5=	54
27918:	24=	56430	28572:	10=	42	29102:	9=	42	29512:	5=	90
27970:	6=	42	28590:	18=	2178	29108:	5=	42	29530:	8=	126
27986:	10=	30	28594:	10=	90	29114:	4=	126	29532:	3=	42
27994:	3=	114	28604:	7=	1890	29120:	4=	126	29538:	6=	54
28002:	31=	56430	28624:	2=	2418	29140:	7=	30	29542:	6=	30
28006:	7=	30	28644:	9=	126	29154:	78=	56430	29544:	5=	42
28008:	8=	30	28646:	8=	30	29162:	10=	90	29546:	32=	56430
28026:	36=	56430	28650:	22=	56430	29166:	23=	2178	29722:	9=	30
28032:	5=	42	28660:	7=	30	29172:	8=	30	29730:	23=	2178
28056:	8=	30	28698:	199=	56430	29174:	6=	30	29742:	21=	2178
28072:	5=	42	28710:	198=	56430	29178:	27=	2178	29754:	20=	2178
28094:	5=	90	28722:	13=	2178	29182:	6=	42	29756:	6=	42
28102:	11=	30	28726:	6=	90	29190:	29=	2178	29760:	6=	114
28112:	6=	90	28732:	4=	90	29202:	72=	56430	29762:	8=	126
28124:	5=	126	28734:	12=	2178	29210:	10=	42	29772:	8=	42
28128:	11=	30	28746:	11=	2178	29214:	25=	2178	29776:	1=	1878
28146:	31=	56430	28766:	8=	90	29224:	5=	42	29780:	6=	30
28148:	5=	42	28768:	5=	90	29250:	42=	56430	29808:	3=	114
28150:	5=	60	28780:	8=	114	29256:	12=	30	29812:	4=	54
28152:	8=	42	28782:	18=	56430	29268:	7=	30	29816:	6=	90
28158:	30=	56430	28794:	29=	56430	29286:	31=	2178	29828:	10=	30
28182:	28=	56430	28796:	3=	2958	29292:	5=	60	29832:	10=	30
28184:	8=	90	28804:	6=	54	29294:	8=	30	29840:	4=	90
28212:	6=	42	28806:	28=	56430	29304:	6=	90	29878:	11=	30
28218:	63=	1291050	28808:	4=	90	29318:	8=	30	29880:	6=	1140
28226:	11=	90	28810:	6=	90	29322:	32=	56430	29892:	5=	90
28228:	22=	2178	28814:	11=	42	29328:	10=	30	29906:	7=	42
28230:	62=	1291050	28818:	27=	56430	29334:	79=	56430	29910:	15=	2178
28242:	25=	56430	28840:	6=	114	29342:	10=	126	29932:	3=	2178
28252:	8=	1890	28842:	38=	2178	29346:	78=	56430	29942:	8=	90
28258:	11=	30	28854:	54=	1291050	29366:	12=	54	29948:	2=	1878
28260:	3=	24180	28862:	14=	30	29372:	6=	42	29960:	4=	42
28268:	7=	1890	28866:	31=	56430	29380:	7=	30	29966:	9=	114
28272:	5=	42	28876:	9=	30	29382:	13=	2178	29968:	1=	1890
28282:	11=	30	28886:	5=	126	29394:	14=	2178	29972:	4=	90
28284:	6=	126	28892:	2=	2142	29400:	12=	30	29982:	198=	56430
28290:	5=	56430	28896:	8=	42	29418:	12=	2178	29984:	4=	42
28294:	6=	42	28930:	6=	90	29420:	5=	90	29996:	3=	126
28308:	5=	90	28938:	51=	1291050	29428:	5=	42	30000:	6=	30
28324:	3=	42	28944:	4=	30	29430:	11=	2178	30014:	6=	30
28328:	5=	126	28958:	8=	90	29440:	5=	30	30020:	8=	30
28332:	8=	42	28964:	3=	90	29452:	2=	2142	30038:	11=	90
28336:	7=	30	28966:	7=	54	29454:	34=	56430	30064:	5=	42
28346:	6=	90	28984:	5=	90	29456:	3=	90	30072:	12=	30
28362:	14=	2178	28992:	4=	30	29460:	1=	29580	30084:	4=	2958
28398:	13=	2178	29004:	7=	126	29466:	33=	56430	30088:	9=	54
28400:	4=	30	29010:	80=	56430	29468:	5=	42	30096:	7=	30
28410:	12=	2178	29020:	6=	42	29474:	5=	90	30106:	7=	90
28414:	7=	90	29034:	27=	2178	29476:	8=	2418	30108:	11=	54
28432:	6=	1890	29042:	5=	90	29480:	6=	30	30116:	10=	54
28444:	6=	54	29046:	25=	2178	29492:	5=	114	30118:	4=	2418
28454:	8=	30	29050:	9=	90	29520:	6=	1140	30120:	8=	42
28456:	7=	54	29058:	24=	2178	29530:	6=	54	30126:	186=	56430
28460:	6=	90	29060:	4=	126	29544:	7=	30	30130:	8=	42
28512:	9=	30	29068:	5=	42	29548:	6=	114	30138:	185=	56430
28528:	7=	30	29070:	27=	2178	29568:	11=	30	30146:	2=	3774
28530:	199=	56430	29074:	6=	42	29572:	6=	2418	30150:	184=	56430
28538:	8=	30	29080:	5=	30	29580:	=	29580	30160:	7=	30
28550:	8=	90	29082:	20=	2178	29584:	2=	1878	30162:	41=	56430

TABLE 7 (cont.)

30184:	3=	90	30714:	14=	2178	31192:	4=	42	31758:	42=	56430
30200:	4=	30	30718:	5=	60	31200:	7=	30	31818:	38=	2178
30214:	7=	54	30724:	26=	2178	31204:	4=	54	31830:	37=	2178
30224:	6=	90	30726:	13=	2178	31208:	6=	30	31836:	4=	90
30234:	34=	56430	30742:	8=	42	31212:	6=	30	31844:	7=	30
30246:	33=	56430	30760:	7=	30	31242:136=	56430		31852:	4=	114
30250:	4=	42	30780:	14=	90	31254:	21=	56430	31858:	7=	30
30258:	32=	56430	30782:	8=	30	31266:	20=	56430	31860:	13=	90
30266:	5=	114	30788:	7=	30	31274:	10=	54	31872:	6=	42
30276:	5=	54	30794:	6=	30	31278:198=	56430		31876:	6=	30
30278:	6=	90	30798:	9=	2178	31284:	6=	126	31880:	8=	30
30280:	7=	30	30806:	8=	90	31292:	4=	90	31886:	8=	90
30284:	6=	54	30808:	5=	42	31302:	26=	2178	31890:	5=	56430
30288:	6=	114	30818:	8=	90	31308:	7=	30	31908:	5=	42
30294:	8=	2178	30824:	6=	42	31318:	6=	42	31910:	5=	90
30306:	31=	56430	30832:	5=	30	31336:	5=	42	31916:	3=	2958
30318:	30=	56430	30846:	30=	56430	31338:	33=	2178	31920:	8=	114
30364:	6=	54	30856:	3=	114	31344:	8=	30	31928:	6=	30
30376:	5=	42	30858:	24=	2178	31348:	5=	30	31932:	6=	42
30384:	6=	30	30870:	30=	56430	31364:	6=	42	31958:	5=	60
30388:	5=	42	30878:	10=	126	31388:	2=	4158	31960:	5=	30
30398:	11=	90	30880:	5=	42	31394:	10=	30	31974:	31=	56430
30400:	5=	114	30892:	5=	42	31400:	5=	42	31980:	9=	30
30424:	7=	90	30896:	6=	42	31422:	23=	2178	31986:	30=	56430
30426:	22=	2178	30898:	7=	90	31434:	22=	2178	32000:	4=	30
30436:	6=	126	30902:	10=	42	31438:	7=	90	32022:	13=	1890
30452:	6=	30	30908:	6=	2418	31446:	8=	2178	32026:	8=	90
30472:	6=	114	30912:	7=	30	31464:	6=	90	32030:	8=	90
30486:	77=	56430	30918:	31=	2178	31470:	71=	56430	32032:	5=	42
30488:	6=	30	30924:	6=	90	31472:	6=	30	32044:	5=	60
30490:	9=	30	30928:	2=	2418	31480:	5=	30	32046:	20=	56430
30494:	8=	90	30930:	30=	2178	31494:	17=	2178	32048:	5=	114
30498:	76=	56430	30932:	7=	114	31496:	4=	30	32052:	6=	90
30516:	9=	54	30942:	38=	56430	31504:	3=	54	32062:	6=	90
30522:136=	56430		30956:	3=	2178	31518:	21=	56430	32072:	8=	90
30524:	6=	54	30960:	8=	90	31520:	4=	60	32076:	6=	126
30534:135=	56430		30972:	7=	114	31530:	29=	56430	32088:	8=	42
30540:	10=	30	30980:	5=	90	31542:	30=	56430	32100:	2=	24180
30550:	7=	90	30982:	9=	30	31550:	14=	30	32102:	10=	42
30558:198=	56430		31004:	4=	2418	31554:	9=	2178	32108:	7=	2418
30564:	6=	54	31006:	10=	90	31576:	5=	30	32126:	8=	90
30582:	14=	56430	31008:	6=	30	31584:	7=	90	32128:	5=	30
30624:	6=	42	31018:	9=	30	31594:	7=	30	32130	=	32130
30630:	11=	2178	31020:	8=	90	31598:	6=	126	32152:	6=	42
30632:	4=	42	31026:	78=	56430	31614:	35=	56430	32154:	1=	35238
30638:	8=	30	31028:	5=	90	31626:	55=1291050		32162:	10=	54
30642:	10=	2178	31030:	13=	30	31628:	4=	30	32164:	5=	126
30648:	9=	30	31038:	77=	56430	31632:	6=	30	32168:	7=	114
30652:	2=	2142	31050:	29=	56430	31636:	4=	60	32172:	8=	30
30654:	9=	2178	31076:	6=	30	31656:	5=	114	32176:	6=	42
30660:	9=	30	31080:	10=	30	31676:	4=	60	32184:	5=	30
30670:	7=	30	31084:	6=	54	31680:	6=	42	32188:	8=	30
30672:	5=	30	31098:	12=	2178	31688:	4=	90	32200:	5=	42
30674:	10=	126	31100:	6=	90	31702:	7=	90	32202:	24=	56430
30676:	27=	2178	31110:	19=	2178	31704:	5=	42	32208:	6=	42
30678:	13=	2178	31120:	7=	30	31710:	29=	2178	32212:	23=	2178
30680:	6=	30	31122:	5=	56430	31716:	6=	42	32224:	5=	30
30690:	12=	2178	31124:	6=	54	31740:	14=	90	32230:	11=	54
30696:	6=	42	31130:	8=	54	31742:	8=	90	32248:	4=	54
30708:	4=	54	31186:	7=	90	31744:	4=	42	32274:	14=	1890

TABLE 7 (cont.)

32280:	14=	30	32840:	8=	42	33270:135=	56430	33812:	6=	30	
32292:	7=	42	32842:	9=	30	33308:	7=	42	33818:	8=	30
32296:	3=	114	32848:	5=	1890	33312:	5=	42	33822:	15=	2178
32302:	7=	90	32850:	32=	2178	33318:	22=	2178	33828:	7=	42
32306:	5=	90	32852:	6=	30	33322:	5=	42	33834:	22=	2178
32310:	30=	56430	32854:	7=	90	33336:	5=	42	33840:	6=	42
32314:	8=	90	32880:	9=	42	33338:	8=	114	33844:206=	56430	
32340:	1=	39660	32884:	6=	2178	33346:	7=	54	33846:	21=	2178
32344:	3=	90	32892:	5=	90	33352:	5=	42	33858:	20=	2178
32366:	6=	30	32896:	3=	54	33360:	7=	114	33860:	8=	90
32376:	10=	30	32906:	5=	90	33364:	6=	54	33862:	8=	90
32408:	4=	90	32908:	5=	42	33368:	6=	30	33874:	7=	30
32412:	6=	42	32912:	4=	90	33374:	6=	54	33884:	5=	126
32420:	5=	114	32916:	6=	126	33378:197=	56430	33910:	10=	42	
32430:	13=	2178	32922:	45=	56430	33386:	4=	42	33916:	4=	90
32442:	31=	56430	32934:	15=	2178	33390:196=	56430	33920:	5=	126	
32454:	30=	56430	32940:	3=	24180	33396:	7=	54	33926:	6=	90
32466:	5=	56430	32952:	6=	30	33420:	10=	30	33932:	8=	42
32476:	5=	54	32954:	6=	30	33436:	6=	30	33938:	8=	90
32478:	30=	56430	32956:	9=	54	33444:	9=	30	33942:	20=	56430
32490:	29=	56430	32966:	9=	114	33452:	7=	30	33948:	4=	90
32496:	10=	30	32972:	5=	114	33458:	6=	42	33950:	10=	90
32508:	5=	90	32976:	6=	54	33460:	6=	90	33952:	5=	54
32516:	7=	30	32982:	23=	2178	33480:	8=	30	33954:	19=	56430
32534:	8=	42	32986:	5=	30	33482:	4=	114	33956:	5=	54
32544:	5=	126	32998:	7=	90	33494:	8=	90	33966:	18=	56430
32560:	5=	30	33000:	9=	30	33502:	6=	90	33982:	7=	30
32586:203=	56430		33004:	4=	24180	33510:	6=	56430	33988:	3=	30
32592:	7=	42	33006:	22=	56430	33512:	4=	30	33990:	29=	2178
32596:	5=	42	33016:	5=	30	33522:	41=	56430	33992:	4=	42
32598:202=	56430		33018:	21=	56430	33528:	12=	30	34008:	9=	30
32602:	4=	126	33030:	20=	56430	33536:	3=	114	34014:	17=	2178
32614:	9=	42	33044:	6=	90	33540:	9=	30	34018:	7=	90
32618:	6=	90	33050:	5=	54	33582:198=	56430	34026:	16=	2178	
32628:	9=	30	33064:	5=	42	33586:	6=	90	34028:	5=	42
32646:	30=	56430	33066:	8=	2178	33594:	12=	2178	34030:	6=	90
32648:	6=	30	33072:	9=	30	33602:	5=	90	34032:	4=	90
32658:	29=	56430	33082:	6=	126	33604:	6=	54	34036:	4=	90
32664:	10=	30	33084:	3=	3966	33606:	15=	2178	34044:	4=	2958
32670:	28=	56430	33088:	5=	30	33616:	4=	54	34062:	23=	2178
32682:	53=1291050		33092:	6=	42	33638:	8=	30	34094:	11=	90
32714:	10=	90	33096:	8=	30	33656:	6=	30	34106:	4=	54
32728:	7=	90	33102:	5=	18018	33668:	9=	30	34108:	7=	30
32730:	83=	56430	33110:	10=	30	33676:	5=	42	34120:	6=	54
32734:	4=	90	33140:	4=	30	33682:	9=	114	34122:136=	56430	
32742:	26=	2178	33156:	6=	30	33690:	36=	2178	34126:	8=	42
32748:	3=	2142	33158:	8=	30	33692:	4=	90	34136:	4=	42
32750:	4=	54	33162:	13=	56430	33714:	32=	56430	34142:	8=	30
32760:	9=	30	33174:	12=	56430	33718:	9=	30	34144:	3=	90
32778:	8=	2178	33176:	5=	114	33720:	11=	30	34150:	4=	2418
32790:	13=	2178	33196:	4=	90	33726:	62=	56430	34154:	6=	42
32796:	8=	126	33206:	8=	114	33742:	10=	90	34168:	3=	114
32804:	6=	54	33208:	6=	114	33746:	5=	90	34172:	2=	2142
32814:	9=	2178	33210:	19=	56430	33754:	9=	30	34182:	19=	2178
32820:	4=	24180	33214:	7=	90	33756:	7=	126	34184:	4=	30
32824:	5=	30	33216:	4=	60	33776:	3=	54	34188:	9=	42
32826:	12=	2178	33220:	5=	90	33786:	21=	2178	34192:	3=	2178
32834:	5=	114	33246:	7=	2178	33794:	5=	90	34196:	4=	90
32836:	7=	2178	33250:	8=	90	33800:	8=	90	34200:	7=	30
32838:	11=	2178	33264:	5=	114	33804:	7=	30	34206:	9=	2178

TABLE 7 (cont.)

34218:	8=	2178	34636:	3=	2178	35130:	27=	56430	35744:	5=	54
34226:	5=	90	34644:	7=	30	35134:	6=	42	35748:	9=	30
34238:	6=	54	34656:	8=	30	35138:	7=	42	35766:	25=	2178
34250:	5=	90	34658:	8=	54	35162:	3=	126	35778:	12=	2178
34252:	8=	42	34682:	4=	30	35174:	7=	54	35780:	4=	114
34264:	5=	30	34684:	9=	54	35184:	4=	114	35786:	10=	54
34272:	6=	30	34688:	2=	42	35188:	8=	30	35790:	40=	56430
34292:	6=	30	34696:	5=	42	35190:	21=	2178	35796:	4=	3966
34296:	9=	30	34700:	7=	114	35198:	8=	30	35802:	10=	2178
34306:	9=	42	34706:	8=	90	35208:	6=	30	35808:	6=	90
34316:	4=	60	34716:	10=	126	35220:	=	35220	35814:	13=	2178
34332:	10=	42	34722:	28=	2178	35228:	7=	30	35816:	4=	90
34346:	5=	54	34728:	12=	30	35232:	8=	30	35820:	=	35820
34358:	5=	60	34732:	8=	42	35238:	=	35238	35822:	5=	90
34360:	5=	30	34746:	10=	2178	35250:	33=	56430	35836:	4=	60
34364:	4=	2418	34752:	6=	30	35262:	33=	56430	35852:	6=	30
34366:	5=	42	34758:	9=	2178	35264:	6=	90	35854:	11=	126
34368:	5=	114	34760:	6=	42	35276:	4=	42	35862:	11=	2178
34384:	3=	126	34772:	6=	30	35316:	4=	42	35864:	6=	30
34388:	6=	90	34774:	11=	126	35328:	8=	90	35892:	6=	30
34390:	9=	54	34776:	10=	30	35330:	12=	30	35900:	6=	114
34398:	44=	56430	34778:	5=	42	35338:	5=	42	35904:	6=	42
34402:	10=	42	34782:	4=	56430	35340:	=	35340	35906:	6=	30
34404:	11=	126	34792:	7=	54	35342:	8=	90	35910:	25=	2178
34412:	5=	2178	34794:	11=	2178	35344:	2=	2958	35946:	38=	56430
34420:	9=	90	34800:	7=	126	35346:	78=	56430	35954:	8=	30
34428:	5=	90	34816:	4=	126	35382:	17=	2178	35956:	4=	114
34444:	4=	2418	34824:	11=	30	35394:	16=	2178	35958:	15=	2178
34460:	8=	30	34850:	7=	54	35402:	8=	90	35966:	8=	30
34464:	9=	30	34860:	13=	30	35406:	15=	1890	35970:	28=	2178
34466:	10=	30	34868:	9=	30	35410:	7=	90	35974:	9=	54
34492:	7=	42	34870:	9=	30	35414:	9=	30	35992:	3=	126
34510:	6=	126	34878:	31=	2178	35416:	3=	114	36006:	40=	2178
34514:	8=	30	34890:	30=	2178	35418:	27=	2178	36008:	6=	54
34516:	25=	2178	34898:	6=	90	35430:	26=	2178	36014:	10=	42
34528:	3=	114	34902:	39=	2178	35452:	5=	60	36024:	10=	30
34540:	6=	42	34926:	25=	2178	35460:	3=	24180	36030:	13=	2178
34550:	10=	42	34932:	11=	54	35472:	6=	30	36066:	5=	56430
34552:	4=	114	34938:	24=	2178	35490:	65=	1291050	36078:	4=	56430
34554:	137=	56430	34948:	205=	56430	35496:	6=	30	36084:	6=	54
34558:	7=	30	34950:	11=	2178	35508:	6=	60	36090:	197=	56430
34560:	8=	30	34960:	3=	114	35520:	7=	114	36096:	6=	30
34562:	6=	42	34968:	8=	30	35522:	2=	4446	36098:	9=	1890
34564:	24=	2178	34982:	6=	90	35526:	58=	1291050	36102:	21=	2178
34568:	4=	60	34986:	38=	2178	35534:	5=	54	36114:	5=	56430
34570:	10=	42	34996:	5=	114	35536:	5=	1890	36138:	22=	2178
34576:	1=	2178	35002:	8=	114	35598:	77=	56430	36150:	135=	56430
34578:	2=	18018	35008:	3=	42	35602:	9=	30	36152:	6=	30
34584:	9=	30	35010:	22=	2178	35610:	30=	2178	36154:	7=	30
34590:	29=	2178	35044:	26=	2178	35614:	7=	90	36178:	7=	30
34592:	4=	30	35060:	6=	30	35620:	6=	42	36204:	7=	126
34594:	9=	42	35062:	7=	30	35632:	5=	54	36214:	6=	90
34604:	6=	54	35066:	8=	30	35636:	6=	30	36216:	4=	90
34606:	11=	54	35078:	8=	114	35664:	10=	30	36218:	10=	30
34612:	5=	42	35082:	26=	2178	35672:	5=	90	36242:	10=	30
34616:	3=	42	35084:	6=	54	35708:	7=	2418	36244:	7=	54
34622:	6=	90	35088:	10=	30	35712:	4=	90	36250:	10=	30
34626:	29=	56430	35112:	10=	30	35718:	27=	2178	36252:	5=	114
34628:	5=	42	35114:	8=	30	35730:	26=	2178	36258:	197=	56430
34632:	6=	30	35122:	7=	54	35742:	75=	56430	36270:	196=	56430

TABLE 7 (cont.)

36296:	6=	30	36768:	8=	90	37362:	22=	2178	37908:	5=	126
36306:	23=	56430	36778:	6=	126	37374:	28=	56430	37914:	12=	2178
36318:	75=	56430	36788:	4=	54	37386:	27=	56430	37916:	4=	90
36320:	4=	42	36790:	6=	42	37406:	10=	54	37918:	8=	126
36324:	6=	90	36796:	7=	54	37420:	7=	30	37922:	10=	30
36330:	74=	56430	36804:	5=	60	37422:	33=	2178	37924:	6=	90
36348:	9=	30	36806:	9=	90	37436:	4=	60	37934:	12=	30
36356:	6=	30	36816:	7=	30	37444:	4=	1878	37942:	10=	30
36370:	5=	126	36818:	5=	114	37446:	22=	2178	37948:	7=	30
36378:	2=	18018	36822:	10=	38682	37450:	9=	90	37962:	40=	2178
36390:	17=	2178	36826:	3=	90	37454:	10=	54	37964:	4=	126
36400:	6=	30	36850:	5=	114	37458:	21=	2178	37974:	25=	2178
36420:	5=	24180	36858:	40=	56430	37464:	7=	30	37976:	8=	90
36422:	8=	42	36870:	39=	56430	37474:	9=	42	37978:	6=	42
36426:	16=	2178	36882:	24=	2178	37506:	34=	56430	37986:	24=	2178
36430:	11=	90	36888:	9=	30	37518:	77=	56430	37994:	10=	54
36452:	6=	114	36892:	6=	90	37520:	2=	1140	38008:	5=	54
36456:	10=	30	36894:	75=	56430	37524:	8=	42	38016:	4=	42
36460:	9=	30	36932:	8=	30	37530:	21=	56430	38026:	5=	126
36466:	6=	1390	36940:	7=	30	37532:	6=	30	38032:	3=	2418
36474:	31=	56430	36948:	5=	114	37536:	8=	42	38036:	8=	30
36482:	6=	90	36954:	46=	56430	37542:	34=	56430	38040:	10=	30
36486:	30=	56430	36984:	12=	30	37554:	33=	56430	38046:	34=	2178
36490:	15=	30	37006:	9=	42	37564:	5=	60	38058:	10=	2178
36494:	8=	30	37014:	30=	56430	37602:	81=	56430	38060:	6=	42
36510:	78=	1291050	37026:	55=	1291050	37610:	8=	90	38068:	8=	30
36514:	9=	30	37036:	6=	90	37616:	3=	114	38070:	9=	2178
36516:	7=	126	37040:	6=	42	37618:	8=	42	38080:	7=	30
36522:	188=	56430	37060:	6=	42	37624:	5=	30	38082:	15=	2178
36524:	5=	2178	37072:	5=	126	37644:	11=	126	38092:	4=	90
36540:	4=	24180	37074:	84=	56430	37652:	11=	2178	38094:	23=	2178
36542:	8=	90	37098:	27=	2178	37674:	10=	2178	38100:	7=	90
36554:	8=	30	37100:	7=	114	37696:	2=	126	38134:	13=	54
36560:	7=	30	37132:	8=	30	37710:	11=	2178	38146:	7=	54
36562:	9=	30	37134:	9=	2178	37722:	16=	2178	38150:	10=	30
36568:	7=	90	37144:	3=	90	37730:	10=	42	38164:	5=	126
36574:	9=	42	37146:	31=	56430	37734:	15=	2178	38172:	8=	42
36580:	8=	114	37148:	9=	30	37740	=	37740	38174:	8=	90
36584:	7=	1890	37166:	5=	54	37758:	26=	56430	38186:	11=	90
36586:	11=	30	37172:	4=	90	37782:	7=	2178	38190:	133=	2178
36598:	7=	30	37176:	5=	114	37784:	6=	114	38196:	8=	30
36600:	9=	30	37180:	5=	90	37790:	5=	42	38202:	57=	1291050
36610:	11=	42	37212:	8=	42	37794:	29=	2178	38204:	4=	2418
36616:	4=	90	37216:	3=	126	37806:	23=	2178	38214:	56=	1291050
36646:	6=	42	37222:	9=	42	37818:	27=	2178	38220:	13=	30
36648:	4=	114	37228:	7=	30	37820:	8=	42	38222:	5=	54
36666:	30=	2178	37236:	8=	90	37824:	8=	30	38224:	2=	2418
36672:	6=	30	37238:	8=	30	37830:	22=	56430	38226:	5=	56430
36678:	31=	2178	37248:	6=	1140	37832:	8=	54	38228:	7=	42
36680:	4=	60	37258:	9=	30	37836:	8=	30	38236:	4=	60
36684:	9=	42	37266:	15=	2178	37840:	4=	90	38274:	31=	56430
36690:	30=	2178	37278:	14=	2178	37848:	9=	30	38280:	14=	30
36694:	7=	30	37292:	6=	30	37858:	9=	114	38284:	5=	126
36704:	4=	30	37316:	7=	30	37866:	53=	1291050	38286:	30=	56430
36714:	61=	1291050	37320:	13=	30	37872:	8=	30	38288:	10=	30
36720:	5=	42	37326:	32=	56430	37876:	3=	114	38292:	11=	54
36736:	3=	114	37332:	5=	60	37878:	52=	1291050	38298:	197=	56430
36748:	7=	30	37338:	31=	56430	37890:	10=	2178	38302:	9=	30
36754:	9=	30	37344:	10=	30	37902:	13=	2178	38304:	5=	90
36762:	76=	56430	37348:	6=	2418	37904:	6=	30	38306:	10=	30

TABLE 7 (concluded)

38318:	7=	126	38828:	5=	42	39268:	29=	56430	39722:	6=	54
38322:	26=	2178	38834:	5=	90	39270:	175=	56430	39724:	4=	60
38332:	6=	30	38836:	9=	54	39296:	2=	114	39732:	6=	42
38334:	56=	1291050	38840:	4=	1878	39298:	5=	54	39740:	6=	42
38346:	55=	1291050	38846:	10=	42	39306:	38=	2178	39748:	5=	42
38352:	9=	30	38856:	10=	30	39312:	3=	114	39754:	9=	90
38354:	10=	30	38860:	6=	42	39316:	28=	56430	39756:	7=	54
38368:	5=	30	38862:	33=	56430	39318:	37=	2178	39772:	8=	42
38408:	6=	126	38866:	5=	126	39320:	5=	90	39774:	15=	2178
38412:	8=	42	38868:	9=	42	39330:	36=	2178	39776:	4=	30
38416:	1=	2418	38870:	11=	42	39334:	7=	54	39780:	=	39780
38428:	5=	42	38878:	5=	54	39354:	134=	56430	39786:	29=	56430
38430:	11=	2178	38880:	5=	42	39362:	8=	126	39788:	2=	2958
38448:	7=	42	38882:	8=	2418	39370:	6=	60	39798:	12=	2178
38460:	9=	30	38884:	7=	2178	39384:	4=	60	39802:	9=	30
38462:	8=	30	38890:	9=	54	39386:	8=	30	39808:	3=	30
38466:	79=	56430	38896:	3=	126	39388:	8=	30	39810:	49=	1291050
38468:	9=	30	38902:	7=	54	39396:	8=	1890	39814:	7=	30
38470:	7=	30	38904:	8=	42	39398:	12=	30	39816:	6=	42
38476:	6=	42	38910:	38=	2178	39402:	203=	56430	39822:	92=	56430
38486:	7=	114	38912:	5=	114	39422:	7=	42	39832:	6=	54
38502:	11=	2178	38924:	5=	2178	39440:	6=	42	39834:	91=	56430
38512:	5=	54	38928:	6=	30	39446:	6=	42	39842:	10=	90
38514:	80=	56430	38932:	6=	2178	39450:	20=	2178	39846:	11=	2178
38520:	6=	1140	38956:	9=	30	39462:	30=	56430	39850:	6=	42
38526:	11=	2178	38976:	9=	42	39470:	8=	30	39864:	7=	42
38528:	6=	30	38984:	6=	30	39474:	29=	56430	39870:	186=	56430
38532:	6=	42	39000:	9=	90	39480:	13=	30	39882:	51=	1291050
38538:	10=	2178	39006:	15=	1890	39500:	6=	114	39888:	5=	42
38550:	15=	2178	39014:	8=	90	39524:	4=	54	39894:	15=	2178
38552:	8=	90	39018:	65=	56430	39530:	11=	42	39908:	4=	114
38554:	7=	30	39034:	13=	54	39534:	83=	56430	39918:	50=	2178
38556:	6=	60	39046:	6=	114	39540:	=	39540	39930:	49=	2178
38568:	12=	30	39048:	6=	114	39548:	7=	2418	39932:	8=	42
38576:	4=	42	39054:	34=	2178	39560:	4=	30	39936:	4=	30
38580:	8=	30	39056:	4=	42	39576:	8=	30	39942:	3=	18018
38602:	7=	90	39058:	7=	90	39582:	12=	56430	39972:	6=	42
38610:	80=	56430	39060:	=	39060	39584:	8=	30	39980:	8=	114
38626:	9=	90	39066:	14=	2178	39592:	5=	42	39998:	11=	90
38632:	4=	54	39070:	11=	54	39594:	61=	1291050			
38642:	6=	126	39076:	6=	2178	39596:	6=	30			
38646:	21=	2178	39082:	7=	30	39598:	9=	30			
38668:	2=	4158	39086:	8=	30	39606:	60=	1291050			
38670:	74=	56430	39096:	4=	90	39626:	7=	114			
38682:	=	38682	39118:	5=	42	39630:	135=	2178			
38696:	6=	90	39128:	6=	90	39634:	7=	30			
38700:	7=	30	39148:	5=	42	39636:	7=	30			
38704:	5=	30	39158:	4=	114	39642:	26=	2178			
38728:	5=	30	39164:	6=	54	39646:	11=	54			
38734:	9=	42	39168:	8=	30	39648:	7=	30			
38744:	5=	54	39184:	3=	42	39652:	7=	30			
38750:	8=	126	39196:	6=	90	39654:	25=	2178			
38760:	10=	90	39220:	6=	42	39656:	4=	114			
38762:	4=	42	39222:	11=	2178	39658:	9=	30			
38772:	8=	42	39240:	6=	114	39660:	=	39660			
38792:	5=	42	39244:	5=	54	39668:	8=	30			
38798:	5=	126	39248:	3=	90	39672:	4=	60			
38802:	5=	56430	39256:	5=	42	39700:	4=	90			
38818:	9=	54	39260:	6=	42	39702:	13=	2178			
38824:	3=	90	39262:	7=	54	39704:	5=	54			

4. Theoretical results

In theorem 4.2 the existence of arbitrary long monotonically increasing UAS-s will be proved. H.W. Lenstra, Jr. proved the same for AS-s, but his proof is unpublished; our proof follows the lines of Lenstra's proof, except for lemma 4.1.

We first mention a congruence property which we shall need in lemma 4.1.; the proof of this property can be found, for instance, in Schuh [18], p. 172.

If a is quadratic non-residue of p^λ (p odd prime, $\lambda \geq 1$) then

$$4.1 \quad a^{\frac{1}{2}\phi(p^\lambda)} \equiv -1 \pmod{p^\lambda},$$

where ϕ is Euler's ϕ -function.

Lemma 4.1.

There exists a sequence of prime powers $(q_i^{e_i})$ ($i = 1, 2, \dots$) with $e_i \geq 1$, such that

$$q_{i+1}^{e_{i+1}} \equiv -1 \pmod{q_i^{e_i+1}} \quad (i = 1, 2, \dots).$$

Proof

We construct the sequence (q_i) ($i = 1, 2, \dots$) of prime numbers as follows: $q_1 = 2$, $q_2 = 3$, q_{i+1} is a prime $> q_i$, such that q_{i+1} is a quadratic non-residue of q_i ($i = 2, 3, \dots$). This construction is possible, in view of Dirichlet's theorem of the infinitude of primes in the arithmetical progression $kq_i + l$ ($1 \leq l \leq q_i - 1$, l is a quadratic non-residue of q_i). The sequence of exponents (e_i) ($i = 1, 2, \dots$) is constructed by:

$$e_1 = e_2 = 1, \quad e_{i+1} = \frac{1}{2}\phi(q_i^{e_i+1}) \quad (i = 2, 3, \dots).$$

Now q_{i+1} is quadratic non-residue of q_i ($i = 2, 3, \dots$) and, thus, of $q_i^{e_i+1}$; from 4.1, with $a = q_{i+1}$, $p = q_i$, $\lambda = e_i + 1$, it follows that

$$q_{i+1}^{e_{i+1}} = q_{i+1}^{\frac{1}{2}\phi(q_i^{e_i+1})} \equiv -1 \pmod{q_i^{e_i+1}}, \quad (i = 2, 3, \dots).$$

Finally, $3 \equiv -1 \pmod{2^2}$, so that the condition of lemma 4.1 is also satisfied for $i = 1$.

Remark: The way of construction of the sequence $(q_i^{e_i})$ is just an instance; other ways of construction are possible.

Theorem 4.2

For any arbitrary natural number t there exists a UAS with at least t monotonically increasing terms.

Proof

Let $A_t = \{m \mid m \text{ is a natural number, } q_i^{e_i} \mid m, 1 \leq i \leq t\}$, $t \geq 1$, where $(q_i^{e_i})$ is a sequence of prime powers, which satisfy the condition of lemma 4.1 ($q_i^{e_i} \mid m$ means that $q_i^{e_i} \mid m$ and $q_i^{e_i+1} \nmid m$).

If $n_0 \in A_t$ then

$$4.2 \quad s^*(n_0) \in A_{t-1} \text{ and } s^*(n_0) > n_0.$$

Indeed, let the prime factorization of n_0 be given by $n_0 = q_1^{e_1} q_2^{e_2} \dots q_t^{e_t} B$, with $(B, n_0/B) = 1$, then

$$\begin{aligned} s^*(n_0) &= \sigma^*(n_0) - n_0 \\ &= (q_1^{e_1+1} + 1) \dots (q_t^{e_t+1} + 1) \sigma^*(B) - n_0. \end{aligned}$$

From lemma 4.1 it follows that $q_i^{e_i+1} \mid q_{i+1}^{e_{i+1}} + 1$ ($i = 1, 2, \dots, t-1$); since $n_0 \in A_t$, we have $q_i^{e_i} \mid n_0$ ($i = 1, 2, \dots, t-1$), hence $s^*(n_0) \in A_{t-1}$. The second assertion of 4.2 follows from $6 \mid n_0$.

Repeated application of 4.2 results in

$$n_0 < n_1 < \dots < n_{t-1} (\in A_1),$$

with $n_i = s^*(n_{i-1})$ ($i = 1, 2, \dots, t-1$); this proves theorem 4.2.

To conclude this section, we compute the average value of $s^*(n)/n$.

Theorem 4.3

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \frac{s^*(n)}{n} = \frac{\zeta(2)}{\zeta(3)} - 1 \quad (= .3684+).$$

Proof

It is clear that $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \frac{s^*(n)}{n} = \left(\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \frac{\sigma^*(n)}{n} \right) - 1$.

Therefore we compute

$$\sum_{n=1}^N \frac{\sigma^*(n)}{n} = \sum_{n=1}^N \sum_{k|n} \frac{1}{k}.$$

Since

$$\sum_{n=1}^N \sum_{k|n} \frac{1}{k} = \sum_{k=1}^N \frac{1}{k} \sum_{\substack{n=k \\ k|n}}^N 1$$

and

$$\sum_{\substack{n=k \\ k|n}}^N 1 = \left(\left[\frac{N}{k} \right] + O(1) \right) \phi(k) \quad (N \rightarrow \infty),$$

we have

$$\begin{aligned} \sum_{n=1}^N \frac{\sigma^*(n)}{n} &= \sum_{k=1}^N \frac{1}{k} \left[\frac{N}{k} \right] \phi(k) + O(\log N) \quad (N \rightarrow \infty) \\ &= N \sum_{k=1}^N \frac{\phi(k)}{k^3} + O(\log N) \quad (N \rightarrow \infty). \end{aligned}$$

This gives:

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \frac{\sigma^*(n)}{n} = \sum_{k=1}^{\infty} \frac{\phi(k)}{k^3} = \frac{\zeta(2)}{\zeta(3)},$$

which proves the theorem.

Remark: It is well-known [1] that the average value of $s(n)/n$ equals $\zeta(2)-1 = .6449+$.

5. The construction of (unitary) sociable groups from other given (unitary) sociable groups.

A glance at table 3 (p. 5) shows that some unitary sociable groups, like the first two of order 14, seem to be related. This is no accident, as theorem 5.1 will demonstrate. In this theorem (only) we shall speak about a "sociable group of order k with respect to a function \bar{f} ", i.e. a k -tuple of distinct numbers $(n_0, n_1, \dots, n_{k-1})$ with

$$5.1 \quad n_i = \bar{f}(n_{i-1}), \quad (i = 1, 2, \dots, k-1), \quad \bar{f}(n_{k-1}) = n_0.$$

When we apply this theorem, we shall replace \bar{f} by s resp. s^* , and so return to the (ordinary) SG-s (with respect to s) and to the USG-s (with respect to s^*), as defined in section 1.

Theorem 5.1

Let $f(n)$ be a multiplicative number-theoretic function and $\bar{f}(n) = f(n) - n$; let also be given the sociable group of order k with respect to \bar{f} : $(n_0, n_1, \dots, n_{k-1})$. We write (if possible) n_i as:

$$5.2 \quad n_i = am_i, \quad \text{with } (a, m_i) = 1 \quad (i = 0, 1, \dots, k-1) \text{ and } a > 1.$$

If there exists a natural number $b > 1$, $b \neq a$, with

$$5.3 \quad \begin{cases} (b, m_i) = 1 & (i = 0, 1, \dots, k-1) \\ f(b)/b = f(a)/a, \end{cases}$$

then the k -tuple $(bm_0, bm_1, \dots, bm_{k-1})$ is also a sociable group of order k , with respect to \bar{f} .

Proof

Elaboration of 5.1 and 5.2 results in the equations

$$\left\{ \begin{array}{l} (m_0+m_1)/f(m_0) = f(a)/a, (a,m_0) = 1, \\ (m_1+m_2)/f(m_1) = f(a)/a, (a,m_1) = 1, \\ \quad \cdot \\ \quad \cdot \\ (m_{k-2}+m_{k-1})/f(m_{k-2}) = f(a)/a, (a,m_{k-2}) = 1, \\ (m_{k-1}+m_0)/f(m_{k-1}) = f(a)/a, (a,m_{k-1}) = 1. \end{array} \right.$$

5.3 allows us to replace a by b in these equations, but this implies that the k -tuple $(bm_0, bm_1, \dots, bm_{k-1})$ is also a sociable group of order k with respect to the function \bar{f} .

Table 8 presents, for $f(n) = \sigma(n)$ and $f(n) = \sigma^*(n)$, a list of some possible combinations of a and b , i.e. for which $f(a)/a = f(b)/b$. It is clear that, if a, b is a combination, then b, a is also a combination.

In the case $f(n) = \sigma^*(n)$ table 8 contains some combinations a, b which are instances of

$$a = 2^\alpha (\alpha \geq 1), \quad \left\{ \begin{array}{l} b = 2^{\alpha+1} (2^{\alpha+1} + 1), \\ b = 2^{\alpha+2} (2^{\alpha+2} + 1)/3 \end{array} \right.$$

and

$$a = 3^\beta (\beta \geq 1), \quad b = 3^{\beta+1} (3^{\beta+1} + 1)/2;$$

it is easy to see that, for these three combinations, the condition $\sigma^*(a)/a = \sigma^*(b)/b$ is satisfied if

$$2^{\alpha+1} + 1, (2^{\alpha+2} + 1)/3 \text{ and } (3^{\beta+1} + 1)/2$$

are primes or prime powers.

In the case $k = 2$, $f(n) = \sigma(n)$ Hagis, Jr. [10] speaks about "isomeric" pairs of amicable number pairs. Many of those can be found in Escott [7] and García [8]. As an illustration of theorem 5.1 we list two isomeric pairs of amicable number pairs; these correct the erroneous pairs (20) and (38) of García ([8], p. 168), in which the factor 23 has been omitted.

$$\left\{ \begin{array}{ll} 3^6 5 \cdot 23 \cdot 59 \cdot 137 \cdot 547 \cdot 1093 & \begin{array}{l} 13.15802559 \\ 14867.14879 \end{array} & (a=3^6 137 \cdot 547 \cdot 1093) \\ 3^{10} 5 \cdot 23 \cdot 59 \cdot 107 \cdot 3851 & \begin{array}{l} 13.15802559 \\ 14867.14879 \end{array} & (b=3^{10} 107 \cdot 3851) \end{array} \right.$$

$$\left\{ \begin{array}{ll} 3^6 5 \cdot 23 \cdot 59 \cdot 137 \cdot 547 \cdot 1093 & \begin{array}{l} 13.30967919 \\ 6563.138683 \end{array} & (a=3^6 137 \cdot 547 \cdot 1093) \\ 3^{10} 5 \cdot 23 \cdot 59 \cdot 107 \cdot 3851 & \begin{array}{l} 13.30967919 \\ 6563.138683 \end{array} & (b=3^{10} 107 \cdot 3851) \end{array} \right.$$

Another application of theorem 5.1 starts with the amicable number pair 21 of Lee [11]:

$$3^4 7 \cdot 11^2 19^2 127 \begin{array}{l} 359.144779 \\ 911.57149 \end{array}$$

With table 8 we can construct four pairs (we only give the greatest common divisors of the pairs):

$$\begin{array}{ll} 3^3 5^2 19 \cdot 31 & (a=3^4 7 \cdot 11^2 19^2 127, b=3^3 5^2 19 \cdot 31) \\ 3^5 7^2 13 \cdot 19^2 127 & (a=3^4 7 \cdot 11^2, b=3^5 7^2 13) \\ 3^6 5 \cdot 19 \cdot 23 \cdot 137 \cdot 547 \cdot 1093 & (a=3^3 5^2 31, b=3^6 5 \cdot 23 \cdot 137 \cdot 547 \cdot 1093) \\ 3^{10} 5 \cdot 19 \cdot 23 \cdot 107 \cdot 3851 & (a=3^6 137 \cdot 547 \cdot 1093, b=3^{10} 107 \cdot 3851). \end{array}$$

The following theorem presents a method to construct a unitary sociable group from a given ordinary sociable group.

TABLE 8

$f(n) = \sigma(n), \bar{f}(n) = s(n)$		
$f(a)/a$	a	b
16/9	$3^3 5$	$3^2 7 \cdot 13$
416/225	$3^3 5^3$	$3^2 5^2 31$
152/81	$3^4 7 \cdot 11^2$	$3^5 7^2 13$
256/135	$3^3 5^2 31$	$3^6 5 \cdot 23 \cdot 137 \cdot 547 \cdot 1093$
368/243	$3^6 137 \cdot 547 \cdot 1093$	$3^{10} 107 \cdot 3851$
1024/513	$3^3 5^2 19 \cdot 31$	$3^4 7 \cdot 11^2 19^2 127$
448/243	$3^5 7 \cdot 13$	$3^7 5 \cdot 41$
512/297	$3^4 11^3 31 \cdot 61$	$3^6 11 \cdot 23 \cdot 137 \cdot 547 \cdot 1093$
1792/1053	$3^5 13^2 31 \cdot 61$	$3^6 13 \cdot 23 \cdot 137 \cdot 547 \cdot 1093$
$f(n) = \sigma^*(n), \bar{f}(n) = s^*(n)$		
$f(a)/a$	a	b
3/2	2	$2^2 5, 2^3 3$ 1)
5/4	2^2	$2^3 3^2, 2^6 7 \cdot 13$
9/8	2^3	$2^4 17, 2^5 11$
33/32	2^5	$2^7 43$
129/128	2^7	$2^8 257$
513/512	2^9	$2^{11} 683$
2049/2048	2^{11}	$2^{13} 2731$
32769/32768	2^{15}	$2^{16} 65537, 2^{17} 43691$
5/3	$2^2 3$	$2 \cdot 3^2$ 1)
14/9	$2 \cdot 3^3$	$2^2 3^2 5^2 13$
28/15	$2 \cdot 3^3 5$	$2^2 3 \cdot 5^2 13$
7/5	$2^3 3^3 5$	$2^2 5^2 13$
4/3	3	$3^2 5$
80/63	$3^2 7$	$3^3 5 \cdot 7^2$
28/27	3^3	$3^4 41$
28/25	$5^2 13$	$3^2 5^3$
8/7	7	$5^2 7^2 13$
8/5	3.5	$2^2 5^2 7 \cdot 13$

1) The cases $(k=2) a = 2, b = 20$ and $(k=2) a = 12, b = 18$ are theorems 4 and 5 of Hagis, Jr. [10].

Theorem 5.2

Let $(n_0, n_1, \dots, n_{k-1})$ be an (ordinary) sociable group of order k . We write (if possible) n_i as:

$$n_i = am_i \text{ with } (a, m_i) = 1 \quad (i = 0, 1, \dots, k-1), a > 1.$$

Suppose the m_i 's are squarefree.

If there is a number $b > 1$ such that

$$5.4 \quad \left\{ \begin{array}{l} \sigma(a)/a = \sigma^*(b)/b \\ (b, m_i) = 1 \quad (i = 0, 1, \dots, k-1), \end{array} \right.$$

then $(bm_0, bm_1, \dots, bm_{k-1})$ is a unitary sociable group of order k .

Proof

For the given sociable group we have the equations

$$5.5 \quad \left\{ \begin{array}{ll} (m_0 + m_1)/\sigma(m_0) = \sigma(a)/a, & (a, m_0) = 1, \\ \vdots & \\ (m_{k-2} + m_{k-1})/\sigma(m_{k-2}) = \sigma(a)/a, & (a, m_{k-2}) = 1, \\ (m_{k-1} + m_0)/\sigma(m_{k-1}) = \sigma(a)/a, & (a, m_{k-1}) = 1. \end{array} \right.$$

Because of 5.4 and the fact that the m_i 's are squarefree, we can replace in 5.5

$$\sigma(m_i) \text{ by } \sigma^*(m_i) \quad (i = 0, 1, \dots, k-1),$$

$$\sigma(a)/a \text{ by } \sigma^*(b)/b \text{ and}$$

$$(a, m_i) \text{ by } (b, m_i) \quad (i = 0, 1, \dots, k-1).$$

But this implies that the k -tuple $(bm_0, bm_1, \dots, bm_{k-1})$ is a unitary sociable group of order k .

Remark: A special case of theorem 5.2 is: a is squarefree. In that case: $\sigma(a)/a = \sigma^*(a)/a$ and the ordinary sociable group itself is a unitary sociable group. Among the 108 unitary amicable number pairs of Hagis, Jr. [10], there are 76 squarefree ones, extracted from the available lists of amicable number pairs.

Table 9 presents a list of possible combinations of a and b , which satisfy the condition of theorem 5.2 $\sigma(a)/a = \sigma^*(b)/b$.

TABLE 9

$\sigma(a)/a = \sigma^*(b)/b$	a	b
13/9	3^2	$2^2 3^2 5^2$
26/15	$3^2 5$	$2^2 3 \cdot 5^2$
14/9	$3^2 13$	$2 \cdot 3^3$
40/27	3^3	$2^2 3^3 7$
16/9	$3^3 5$	$2 \cdot 3^3 7$
32/25	$5^2 31$	$5^2 7 \cdot 13$
168/125	$5^3 13$	$3 \cdot 5^3$
60/49	$7^2 19$	$5 \cdot 7^2$
40/21	$3^2 7^2 13 \cdot 19$	$2^2 3 \cdot 7$

By use of theorems 5.1 and 5.2 and tables 8 and 9, the author is able to construct more than 600 new unitary amicable number pairs from ordinary amicable number pairs; these will be published in a subsequent report.

An instance of how tables 8 and 9 are to be used to form unitary amicable number pairs, will be given next.

We start with the amicable number pair of Escott [7], nr. 25

$$2 \cdot 5^2 31 \left\{ \begin{array}{l} 17 \cdot 79 \cdot 7109 \\ 79 \cdot 127979 \end{array} \right.$$

We take $a = 5^2 31$ ($m_1 = 2.17.79.7109$, $m_2 = 2.79.127979$), and apply theorem 5.2 for $k = 2$; this yields with table 9: $b = 5^2 7.13$; as $(b, m_1) = (b, m_2) = 1$,

$$(i) \quad 2.5^2 7.13 \quad \left\{ \begin{array}{l} 17.79.7109 \\ 79.127979 \end{array} \right.$$

is a unitary amicable number pair.

Now we apply theorem 5.1 with $f(n) = \sigma^*(n)$, $\bar{f}(n) = s^*(n)$, $k = 2$.

We take $a = 2$ ($m_1 = 5^2 7.13.17.79.7109$, $m_2 = 5^2 7.13.79.127979$) and conclude from table 8: $b = 2^3 3$ ($b = 2^2 5$ is impossible, since $(2^2 5, m_1) \neq 1$).

This yields the second unitary amicable number pair:

$$(ii) \quad 2^3 3.5^2 7.13 \quad \left\{ \begin{array}{l} 17.79.7109 \\ 79.127979 \end{array} \right.$$

This application of theorem 5.1 can be repeated (for this case) five times; we only give the "heads" of the pairs and omit the "tail":

$$\left\{ \begin{array}{l} 17.79.7109 \\ 79.127979 \end{array} \right.$$

which is always the same.

$$\begin{array}{ll} (iii) & 2^5 3.5^2 7.11.13 & (\text{from (ii), } a = 2^3, b = 2^5 11) \\ (iv) & 2^7 3.5^2 7.11.13.43 & (\text{from (iii), } a = 2^5, b = 2^7 43) \\ (v) & 2^8 3.5^2 7.11.13.43.257 & (\text{from (iv), } a = 2^7, b = 2^8 257) \\ (vi) & 2.3^2 5^3 7 & (\text{from (i), } a = 5^2 13, b = 3^2 5^3) \\ (vii) & 2^2 3.5^3 7 & (\text{from (vi), } a = 2.3^2, b = 2^2 3) \end{array}$$

Another illustration of theorems 5.1 and 5.2 will be given for $k = 4$. We start with the sociable group of order 4 (David [5]):

$$2.5 \left\{ \begin{array}{l} 7.19.263.599 \\ 17.59.24593 \\ 19.23.211.251 \\ 17.499.2713, \end{array} \right.$$

from which we construct unitary sociable groups; of course the group itself is a unitary sociable group of order 4, because it is squarefree (theorem 5.2). Repeated application of theorem 5.1 yields the "heads":

$$\begin{array}{l} 2^3 3.5 \\ 2^5 3.5.11 \\ 2^7 3.5.11.43 \\ 2^8 3.5.11.43.257 \end{array}$$

of four new unitary sociable groups of order 4.

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