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AFDELING NUMERIEKE WISKUNDE (DEPARTMENT OF NUMERICAL MATHEMATICS) NW 127/82 APRIL

H.J.J. TE RIELE

RULES FOR CONSTRUCTING HYPERPERFECT NUMBERS

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Rules for constructing hyperperfect numbers *)

by

H.J.J. te Riele

ABSTRACT

Two rules are given by which hyperperfect numbers with k+2 different prime factors can be constructed from certain related numbers with k+1 and with k different prime factors, respectively. By means of these rules many HP's with three and with four, and one with five different prime factors were constructively computed.

It is proved that *all* HP's of the form $p^{\alpha}q$, $\alpha \in \mathbb{N}$, (below a given bound) can be found with one of these two rules or with an additional rule for the construction of certain HP's of the form p^2q. Furthermore, the results are presented of an exhaustive search for all HP's $\leq 10^8$. It turns out that all HP's found could also have been computed (but using much less computer time) with at least one of the rules given here.

Finally, a generalisation of HP's to so-called hypercycles is described.

KEY WORDS & PHRASES: *Hyperperfect numbers*

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*) This paper will be submitted for publication elsewhere.

1. INTRODUCTION

As usual, let $\sigma(n)$ denote the sum of all the divisors of n (with $\sigma(1) = 1$) and let $\omega(n)$ denote the number of different prime factors of n, with $\omega(1) := 0$. The set of prime numbers will be denoted by P. The set of hyperperfect numbers (HP's) is the set $M := \bigcup_{n=1}^{\infty} M_n$, where

$$(1) \qquad M_n := \{m \in \mathbb{N} \mid m = 1 + n\,[\sigma(m) - m - 1]\}.$$

We also define the sets

$$(2) \qquad {}_kM_n := \{m \in M_n \mid \omega(m) = k\}, \quad k, n \in \mathbb{N},$$

and ${}_kM := \bigcup_{n=1}^{\infty} {}_kM_n$; clearly, we have $M_n = \bigcup_{k=1}^{\infty} {}_kM_n$. We will also use the related set $M^* := \bigcup_{n=1}^{\infty} M_n^*$, where

$$(3) \qquad M_n^* := \{m \in \mathbb{N} \mid m = 1 + n\,[\sigma(m) - m]\},$$

and the sets

$$(4) \qquad {}_kM_n^* := \{m \in M_n^* \mid \omega(m) = k\}, \quad k \in \mathbb{N} \cup \{0\}, \quad n \in \mathbb{N},$$

and ${}_kM^* := \bigcup_{n=1}^{\infty} {}_kM_n^*$, so that also $M_n^* = \bigcup_{k=0}^{\infty} {}_kM_n^*$.
It is not difficult to verify that ${}_1M_n = \emptyset$, $\forall n \in \mathbb{N}$, and that

$$(5) \qquad {}_0M_n^* = \{1\}, \quad \forall n \in \mathbb{N} \text{ and}$$

$$ {}_1M_n^* = \begin{cases} \{(n+1)^{\alpha}, \quad \alpha \in \mathbb{N}\}, & \text{if } n+1 \in P, \\ \emptyset, & \text{if } n+1 \notin P. \end{cases}$$

M_1 is the set of perfect numbers (for which $\sigma(m)=2m$). The n-hyperperfect numbers M_n, introduced by MINOLI and BEAR [1], are a meaningful generalization of the even perfect numbers because of the following

RULE 0 ([2]). If $p \in P$, $\alpha \in \mathbb{N}$ and if $q := p^{\alpha+1}-p+1 \in P$ then $p^{\alpha}q \in M_{p-1}$. There are 71 hyperperfect numbers below 10^7 ([2],[3] and [4]). Only one of them belongs to ${}_3M$, all others are in ${}_2M$. In [5] and [6] the present author has constructively computed several elements of ${}_3M$ and two of ${}_4M$.

In section 2 of this paper we shall give rules by which one may find (with enough computer time) an element of ${}_{(k+2)}M_n$ and of ${}_{(k+1)}M_n$ from an element of ${}_kM^*_n$ $(k\geq 0)$, and an element of ${}_kM^*_n$ from an element of ${}_{(k-2)}M^*_n$ $(k\geq 2)$. Because of (5) this suggests the possibility to construct HP's with k different prime factors for any positive integer $k \geq 2$. By actually applying the rules we have found many elements of ${}_3M$, seven elements of ${}_4M$ and one element of ${}_5M$.

In section 3 necessary and sufficient conditions are given for numbers of the form $p^{\alpha}q$, $\alpha \in \mathbb{N}$, to be hyperperfect. For example, for $\alpha \geq 3$, these conditions imply that there are no other HP's of the form $p^{\alpha}q$ than those characterised by Rule 0. The results of this section enable us to compute very cheaply *all* HP's of the form $p^{\alpha}q$ below a given bound. Unfortunately, we have not been able to extend these results to more complicated HP's like those of the form $p^{\alpha}q^{\beta}$, $\alpha \geq 2$ and $\beta \geq 2$, or $p^{\alpha}q^{\beta}r^{\gamma}$ with $\alpha \geq 1$, $\beta \geq 1$ and $\gamma \geq 1$, etc.

Because of the importance of the set M^* for the construction of hyperperfect numbers, we give in section 4 the results of an exhaustive search for all $m \in M^*$ with $m \leq 10^8$ and $\omega(m) \geq 2$. It turned out that elements of ${}_3M^*$ are very rare compared with ${}_2M^*$, in analogy with the sets ${}_3M$ and ${}_2M$. This search also gave all elements $\leq 10^8$ of M, almost for free, because of the similarity of the equations defining M^* and M.

The paper concludes with a few remarks, in section 5, on a possible generalisation of hyperperfect numbers to so-called hypercycles, special cases of which are the ordinary perfect numbers and the amicable number pairs.

2. RULES FOR CONSTRUCTING HYPERPERFECT NUMBERS

We have found the following rules (we write $\bar{a}$ for $\sigma(a)$):

RULE 1. Let $k \in \mathbb{N}$, $n \in \mathbb{N}$, $a \in {}_kM^*_n$ and $p := n\bar{a} + 1 - n$; if $p \in P$ then $ap \in {}_{(k+1)}M_n$.

RULE 2. Let $k \in \mathbb{N} \cup \{0\}$, $n \in \mathbb{N}$, $a \in {}_kM^*_n$ and $p := n\bar{a} + A$, $q := n\bar{a} + B$, where $AB = 1 - n + n\bar{a} + n^2\bar{a}^2$; if $p \in P$ and $q \in P$ then $apq \in {}_{(k+2)}M_n$.

RULE 3. Let $k \in \mathbb{N} \cup \{0\}$, $n \in \mathbb{N}$, $a \in {}_kM^*_n$ and $p := n\bar{a} + A$, $q := n\bar{a} + B$, where $AB = 1 + n\bar{a} + n^2\bar{a}^2$; if $p \in P$ and $q \in P$ then $apq \in {}_{(k+2)}M^*_n$.

The proofs of these rules don't require much more than the application of the definitions, and are therefore left to the reader. In fact, the proof of Rule 2 was already given in [6], although the rule itself was formulated there less explicitly.

Rule 1 can be applied for $k \geq 1$, but not for $k = 0$ since ${}_0M^*_n = \{1\}$ and $a = 1$ gives $p = 1 \notin P$. For $k = n = 1$ Rule 1 reads: if $p := 2^{\alpha+1}-1 \in P$, then $2^\alpha p \in {}_2M_1$, which is Euclid's rule for finding even perfect numbers. For $k = 1$ Rule 1 is equivalent with Rule 0 given in section 1.

Rules 2 and 3 can both be applied for $k \geq 0$. For instance, for $k = 0$ Rule 2 reads: let $n \in \mathbb{N}$ be given; if $p := n + A \in P$ and $q := n + B \in P$, where $AB = 1 + n^2$, then $pq \in {}_2M_n$. For $n = 1$, 2 and 6 this yields the hyperperfect numbers 2×3, 3×7 and 7×43, respectively. Rule 3 reads for $k = 0$: let $n \in \mathbb{N}$ be given; if $p := n + A \in P$ and $q := n + B \in P$, where $AB = 1 + n + n^2$, then $pq \in {}_2M^*_n$. For $n = 4$ and $n = 10$ we find that $7 \times 11 \in {}_2M^*_4$ and $13 \times 47 \in {}_2M^*_{10}$, respectively.

Rule 3 shows a rather curious "side-effect" for $k \geq 1$: if both the numbers p and q in this rule are prime, then not only $apq \in {}_{(k+2)}M^*_n$, but also the number $b := pq$ is an element of ${}_2M^*_{n\bar{a}}$. Indeed, we have

$$\frac{b-1}{\sigma(b)-b} = \frac{pq-1}{p+q+1} = \frac{n^2\bar{a}^2+n\bar{a}(A+B)+AB-1}{2n\bar{a}+A+B+1} =$$

$$= \frac{n^2\bar{a}^2+n\bar{a}(A+B)+n\bar{a}+n^2\bar{a}^2}{2n\bar{a}+A+B+1} = n\bar{a} \in \mathbb{N}.$$

For example, we know that $7 \times 11 \in {}_2M^*_4$. From Rule 3 with $k = 2$, $n = 4$, $a = 7 \times 11$ we find that $7 \times 11 \times 547 \times 1291 \in {}_4M^*_4$; the side-effect is that $547 \times 1291 \in {}_2M^*_{(4\times 8\times 12)} = {}_2M^*_{384}$.

In [5] we gave the following additional

RULE 4. Let $t \in \mathbb{N}$ and $p := 6t-1$, $q := 12t+1$; if $p \in P$ and $q \in P$ then $p^2 q \in {}_2M_{(4t-1)}$.
For example, t=1 and t=3 give $5^2 13 \in {}_2M_3$ and $17^2 37 \in {}_2M_{11}$, respectively.
In section 3 we will prove that with Rules 1, 2 and 4 it is possible to find all HP's of the form $p^\alpha q$, $\alpha \in \mathbb{N}$, below a given bound. We leave it to the reader to find out why there is no rule (at least for $k \geq 1$), analogous to Rule 1, for finding an element of ${}_{(k+1)}M_n^*$ from an element of ${}_kM_n^*$.

From Rules 1 - 3 it follows that elements of ${}_kM_n$ for some given $k \in \mathbb{N}$ may be found from ${}_{(k-1)}M_n^*$ (with Rule 1) and from ${}_{(k-2)}M_n^*$ (with Rule 2) provided that sufficiently many elements of ${}_{(k-1)}M_n^*$ resp. ${}_{(k-2)}M_n^*$ are available; these can be found with Rule 3 and the "starting" sets ${}_0M_n^*$ and ${}_1M_n^*$ given in (5). We have carried out this "program" for the constructive computation of HP's with three, four and five different prime factors.

(i) Construction of elements of ${}_3M_n$. With Rule 1 we found 34 HP's of the form pqr, from numbers $pq \in {}_2M_n^*$ (the smallest one is $61 \times 229 \times 684433 \in {}_3M_{48}$ and the largest one $9739 \times 13541383 \times 1283583456107389 \in {}_3M_{9732}$). The elements of ${}_2M_n^*$ were "generated" with Rule 3 from ${}_0M_n^* = \{1\}$. With Rule 2 we found, from prime powers $p^\alpha \in {}_1M_n^*$, 67 HP's of the form pqr (five of the smallest were given in [5], the largest one is
$8929 \times 79727051 \times 577854714897923 \in {}_3M_{8928}$), 48 HP's of the form p^2qr (the smallest five were given in [5], the largest one is
$7459^2 414994003583 \times 3444400460163740816321\,9 \in {}_3M_{7458}$), 9 of the form p^3qr (the smallest one is given in [5], the largest one is
$811^3 432596915921 \times 89927962885420066391 \in {}_3M_{810}$), 4 of the form p^4qr (the smallest one is $7^4 30893 \times 36857 \in {}_3M_6$, the largest one is
$223^4 553821371657 \times 130059326113901 \in {}_3M_{222}$) and furthermore
$7^6 1340243 \times 2136143 \in {}_3M_6$, $13^7 815787979 \times 11621986347871 \in {}_3M_{12}$ and
$19^8 322687706723 \times 1164084402910006759 \in {}_3M_{18}$.

(ii) Construction of elements of ${}_4M_n$. In order to construct elements of ${}_4M_n$ with Rule 1, sufficiently many elements of ${}_3M_n^*$ had to be avaiable. This was realised with Rule 3, starting with elements $p^\alpha \in {}_1M_{(p+1)}$, $p \in P$. The following four HP's with four different prime factors were found:
$3049 \times 9297649 \times 69203101249 \times 5981547458963067824996953 \in {}_4M_{3048}$,

$4201 \times 17692621 \times 7061044981 \times 220478637088071105410940 1 \in {}_4M_{4200}$,
$181^2 5991031 \times 579616291 \times 20591020685907725650381 \in {}_4M_{180}$,
$181^3 1108889497 \times 33425259193 \times 397811517868254406833465492 61 \in {}_4M_{180}$.
By means of Rules 2 and 3 the following three additional elements of ${}_4M_n$ were found:
$1327 \times 6793 \times 10020547039 \times 17769709449589 \in {}_4M_{1110}$ (already in [5]),
$1873 \times 24517 \times 79947392729 \times 80855915754575789 \in {}_4M_{1740}$ (already in [6]),
$5791 \times 10357 \times 222816095543 \times 482764219012881017 \in {}_4M_{3714}$.
(iii) Construction of an element of ${}_5M_n$. We have also constructively computed one element of ${}_5M_n$ with Rule 1. The elements of ${}_4M_n^*$ needed for this purpose were computed from ${}_0M_n^*$ by twice applying Rule 3 (first yielding elements of ${}_2M_n^*$ and next elements of ${}_4M_n^*$). The HP found is the largest one we know of (apart from the ordinary perfect numbers). It is the 87-digit number

$$\begin{aligned} &2095497171870781405883328851321934328974054\\ &07437906414236764925538317339020708786590793 =\\ &4783 \times 83563 \times 1808560287211 \times 297705496733220305347 \times\\ &\times 973762019320700650093520128480575320050761301 \in {}_5M_{4524}. \end{aligned}$$

3. CHARACTERIZATION OF ALL HP's OF THE FORM $p^\alpha q$

The hyperperfect numbers of the form $p^\alpha q$ are characterised by the following

THEOREM. *Let* $m := p^\alpha q (\alpha \in N, p \in P, q \in P)$ *be an hyperperfect number, then*

(i) $\alpha = 1 \Rightarrow (\exists n \in \mathbb{N}$ *with* $m \in {}_2M_n$ *such that* $p = n + A$, $q = n + B$, *with* $AB = 1 + n^2)$;

(ii) $\alpha = 2 \Rightarrow (\exists t \in \mathbb{N}$ *with* $m \in {}_2M_{(4t-1)}$ *and* $p = 6t - 1$ and $q = 12t + 1)$ $\vee$ $(m \in {}_2M_{(p-1)}$ *with* $q = p^3 - p + 1)$;

(iii) $\alpha > 2 \Rightarrow (m \in {}_2M_{(p-1)}$ *with* $q = p^{\alpha+1} - p + 1)$.

PROOF. (i) This case follows immediately from Rule 2 (with k=0).
(ii) If p^2q is hyperperfect, then the number $(p^2q-1)/((p+1)(p+q))$ must be

a positive integer. Consider the function $f(x,y) := (x^2y-1)/((x+1)(x+y))$, $x,y \in \mathbb{N}$. We want to characterise all pairs x,y for which $f(x,y) \in \mathbb{N}$. We can safely take $x \geq 2$ and $y \geq 2$. Let $x \geq 2$ be fixed, then we have for all $y \geq 2$

$$f(x,y) < \frac{x^2y}{(x+1)(x+y)} < \frac{x^2}{x+1} = x-1 + \frac{1}{x+1} .$$

Hence, the largest integral value which could possibly be assumed by f is x-1, and one easily checks that this value is actually assumed for $y = x^3 - x + 1$. So we have found

$$(6) \qquad f(x,x^3-x+1) = x-1, \quad x \in \mathbb{N}, \; x \geq 2.$$

One also easily checks that f is monotonically increasing in y (x fixed) so that

$$(7) \qquad 2 \leq y \leq x^3-x+1 .$$

Now in order to have $f \in \mathbb{N}$, it is necessary that $x + 1$ divides $x^2y - 1$, or, equivalently, that $x + 1$ divides $y - 1$ (since $(x^2y-1)/(x+1) = y(x-1) + (y-1)/(x+1)$). Therefore, we have $y = k(x+1)+1$, with $k \in \mathbb{N}$ and $1 \leq k \leq x(x-1)$ by (7). Substitution of this into f yields

$$f(x,y) = \frac{kx^2+x-1}{(k+1)(x+1)} = x-1 - \frac{x^2-x-k}{(k+1)(x+1)} =: x-1 - g(x,k) .$$

It follows that x+1 must divide x^2-x-k or, equivalently, that x+1 must divide k-2. Hence, $k=j(x+1)+2$, with $j \in \mathbb{N} \cup \{0\}$ and $0 \leq j \leq x-2$. Substitution of this into g yields

$$g(x,j(x+1)+2) = \frac{x-2-j}{j(x+1)+3} .$$

This function is decreasing in j, and for $j = 0,1,\ldots,x-2$ it assumes the values:

$$g(x,2) = (x-2)/3,$$

$$g(x,x+3) = \frac{x-3}{x-4} < 1,$$

$$\vdots$$

$$g(x,x(x-1)) = 0.$$

It follows that there is precisely one more possibility (in addition to (6)) for f to be a positive integer, viz., when $j=0$, $k=2$, $y=2x+3$ and $x(\text{mod } 3)=2$. So we have found

$$(8) \qquad f(3t-1,6t+1) = 2t-1, \quad t \in \mathbb{N}.$$

The statement in the theorem now easily follows from (6) and (8).
(iii) As in the proof of (ii) we now have to find out for which values of $x,y \in \mathbb{N}$, $x \geq 2$ and $y \geq 2$, the function $f(x,y) \in \mathbb{N}$, where

$$f(x,y) := \frac{x^{\alpha}y-1}{(x^{\alpha-1}+\ldots+1)(x+y)}, \quad \alpha > 2.$$

For fixed $x \geq 2$ we have

$$f(x,y) < \frac{x^{\alpha}}{x^{\alpha-1}+\ldots+1} = x-1 + \frac{1}{x^{\alpha-1}+\ldots+1}.$$

As in the proof of (ii) we find that $f(x,y) = x-1$ for $y = x^{\alpha+1}-x+1$ and that $2 \leq y \leq x^{\alpha+1}-x+1$. Furthermore, $x^{\alpha-1}+\ldots+1$ must divide $x^{\alpha}y-1$, so that $y = k(x^{\alpha-1}+\ldots+1)+1$, with $1 \leq k \leq x(x-1)$. Substitution of this into f yields a certain function g, in the same way as in the proof of (ii), but in this case g can only assume integral values for $k = x(x-1)$. This implies the statement in the theorem, case (iii). Q.E.D.

It is easy to see that the characterizations given in this theorem are equivalent to Rule 2 (k=0) when $\alpha=1$, to Rule 4 or Rule 1 (k=1) when $\alpha=2$, and to Rule 1 (k=1) when $\alpha > 2$.

This theorem enables us to find very cheaply all HP's of the form $p^{\alpha}q$, $\alpha \in \mathbb{N}$, below a given bound. For example, to find all HP's in M_n of the form pq below 10^8, we only have to check whether $p := n+A \in \mathcal{P}$ and

$q := n+B \in P$ for all possible factorisations of $AB = 1 + n^2$, for $1 \leq n \leq 4999$. This range of n follows from the fact that if $pq \in M_n$ then $pq > 4n^2$. The following additional restrictions can be imposed on n:

(i) n should be 1 or even since, if n is odd and $n \geq 3$ then $n^2 + 1 \equiv$ $\equiv 2 (\mathrm{mod}\ 4)$, so that one of A and B is odd and one of p and q is even and ≥ 4.

(ii) If $n \geq 3$ then $n \equiv 0 \ (\mathrm{mod}\ 3)$ since if $n \equiv 1$ or $2 (\mathrm{mod}\ 3)$ then $n^2 + 1 \equiv 2(\mathrm{mod}\ 3)$, so that one of A and B is $\equiv 1(\mathrm{mod}\ 3)$ and the other is $\equiv 2(\mathrm{mod}\ 3)$; consequently, one of p and q is $\equiv 0(\mathrm{mod}\ 3)$ and > 3.

Hence, the only values of n to be checked are $n = 1$, $n = 2$ and $n = 6t$, $1 \leqslant t \leqslant 833$.

4. EXHAUSTIVE COMPUTER SEARCHES

From the rules given in section 2 it follows that it is of importance to know elements of M^*, when one wants to find elements of M. Therefore, we have carried out an exhaustive computer search for all elements of M^* below the bound 10^8. Because of (5) the search was restricted to elements with at least two different prime factors. A check was done whether $(m-1)/(\sigma(m)-m) \in \mathbb{N}$, for all $m \leq 10^8$ with $\omega(m) \geq 2$. Since the most time consuming part is the computation of $\sigma(m)$, a second check was done (in case $(m-1)/(\sigma(m)-m) \notin \mathbb{N}$) whether $(m-1)/(\sigma(m)-m-1) \in \mathbb{N}$. If so, m was an HP, so that our program also produced, almost for free, all HP's below 10^8. The results are as follows.

Apart from the ordinary perfect numbers, there are 146 HP's below 10^8. Only two of them have the form $p^\alpha qr$ (viz., $13 \times 269 \times 449 \in {}_3M_{12}$ and $7^2 383 \times 3203 \in {}_3M_6$); these were also found in the searches described in section 2. All others have the form characterised in section 3, and could have been found with a search based on that characterisation (using the fact that if $p^\alpha q \in {}_2M_n$, then $p > n$ and $q > n$). A question which naturally arises is the following: are there any HP's which can *not* be constructed with one of the Rules 1, 2 and 4?

There are 312 numbers $m \leq 10^8$ which belong to M^* and which have

$\omega(m) \geq 2$. 306 of them have the form pq and could have been found very cheaply with Rule 3 of section 2. The others are: $7 \times 61 \times 229 \in {}_3M_6^*$, $113 \times 127 \times 2269 \in {}_3M_{58}^*$, $149 \times 463 \times 659 \in {}_3M_{96}^*$, $19 \times 373 \times 10357 \in {}_3M_{18}^*$, $151 \times 373 \times 1487 \in {}_3M_{100}^*$ and $7 \times 11 \times 547 \times 1291 \in {}_4M_4^*$; the second, third and fifth number could not have been found with Rule 3.

5. HYPERCYCLES

A possible generalisation of hyperperfect numbers can be obtained as follows. Let $n \in \mathbb{N}$ be given and define the function $f_n: \mathbb{N}\backslash\{1\} \Rightarrow N$ as follows:

(9) $$f_n(m) := 1 + n[\sigma(m)-m-1], \quad m \in \mathbb{N}\backslash\{1\}.$$

Starting with some $m_0 \in \mathbb{N}\backslash\{1\}$ one might investigate the sequence

(10) $$m_0, f_n(m_0), f_n(f_n(m_0)), \ldots$$

For $n = 1$ this is the well-known aliquot sequence of m_0, which can have cycles of length 1 (perfect numbers), length 2 (amicable pairs) and others. In order to get some impression of the cyclic behaviour for $n > 1$, we have computed, for $2 \leq n \leq 20$, five terms of all sequences (10) with starting term $m_0 \leq 10^6$ and we have registered the cycles with length ≥ 2 and ≤ 5 in the following table.

Table. Hypercycles

i.e., different numbers $m_0, m_1, \ldots, m_{k-1}$ such that $m_k = m_0$ where $m_{i+1} := f_n(m_i)$, f_n defined in (9)

n	k	$m_0, m_1, \ldots, m_{k-1}$
5	2	19461=3.13.499, 42691=11.3881
7	3	925=$5^2$37, 1765=5.353, 2507=23.109
8	2	28145=5.13.433, 66481=19.3499
	3	238705=5.47741, 381969=$3^3$7.43.47, 2350961=79.29759
	4	94225=$5^2$3769, 181153=$7^2$3697, 237057=3.31.2549, 714737=61.11717
	2	3452337=$3^2$7.54799, 17974897=53.229.1481
9	2	469=7.67, 667=23.29
	2	1315=5.263, 2413=19.127
	2	1477=7.211, 1963=13.151
	2	2737=7.17.23, 6463=23.281
10	3	1981=7.283, 2901=3.967, 9701=89.109
12	2	697=17.41, 2041=13.157
	2	3913=7.13.43, 12169=43.283
	2	54265=5.10853, 130297=29.4493
14	2	1261=13.97, 1541=23.67
	3	508453=11.17.2719, 1106925=3.$5^2$14759, 10126397=281.36037
19	2	9197=17.541, 10603=23.461
	4	184491=$3^3$6833, 1688493=3.562831, 10693847=709.15083, 300049=31.9679
	2	5151775=$5^2$251.821, 24124073=89.271057

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