Stichting
MATHEMATISCH CENTRUM
2e Boerhaavestraat 49
Amsterdam

R 79

Harmonic Analysis of Earth Tides Measurements

J. Berghuis

A. van Wijngaarden

1952

HARMONIC ANALYSIS OF EARTH TIDES MEASUREMENTS

1. Introduction.

This report contains the results of the harmonic analysis of observations made in order to measure the tidal effects on the gravity, by means of gravimeters placed in a large number of stations all around the earth.

When not mentioned otherwise the measurements started at May 9[th], 1949 at 9^h0^m, G.M.T. and were repeated each quarter of an hour for 14 or 15 consecutive days. The effect to be measured is extremely small, and is, moreover, in many cases obscured by a systematic drift and irregular jumps of the readings.

The large amount of data necessitated a carefull planning of the analysis. The method used is a refinement of a method used by the British Admiralty and worked out by W.L. Scheen. Confer:

A.T.Doodson and H.D.Warburg, "Admiralty Manual of Tides", Hydrographic Department Admiralty, London, 1941.
"The Admiralty Tide Tables, Part III, ibidem, 1941.

W.L. Scheen, "Harmonic Analysis of Tidal Phenomena", Report B.P.M. (Manuscript 1951).

The calculations were performed on behalf of the Geophysical Department of the B.P.M., The Hague, Holland.

2. Harmonic tidal constituents.

A table of harmonic tidal constituents can be found in Doodson and Warburg, Chapter VII. The speed-number of a constituent is defined to be the increment in phase, expressed in degrees per mean solar hour. The so-called relative coefficient is a measure of the amplitude of a given constituent under certain idealised conditions.

The most important constituents are:

Symbol:	Speed-number:	Relative coefficient:
K_1	15.0411	0.531
O_1	13.9430	0.377
P_1	14.9589	0.176
M_2	28.9841	0.908
S_2	30.0000	0.423
N_2	28.4397	0.174
K_2	30.0821	0.115

From this list it is seen that the difference in speed-numbers between K_1 and P_1 and also between K_2 and S_2 is equal to .0821. This means that when K_1 and P_1 or K_r and S_2 have at a certain time the same phase it is only after 182.7 days that they have the same phase again. In order to distinguish reliably between there constituents in observations showing measurements-errors of the same order as the effects to be measured, data of at least a half year should be available. It is, therefore, impossible to distinguish between K_1 and P_1 or between K_2 and S_2 in the underlying data taken over a fortnight only. This difficulty is overcome in this way that K_1 and P_1 are combined into one pseudoconstituent S_1, to which a speed-number 15.0000 is attributed, whereas K_2 is simply identified with S_2 (i.e. the speed-number of K_2 is taken to be 30.0000)

The constituents to be determined are, therefore, S_1, O_1, S_2, M_2 and N_2, the speed-numbers of which are roughly equal to 15, 14, 30, 29 and 28.4 respectively.

3. Harmonic analysis of the constituents.

The harmonic analysis of a time-series T into periodic constituents with given frequencies (or speed-numbers) is best performed by operating on the series T by means of a certain linear operator L_m that is chosen in such a way that the result $L_m T$ is exactly or practically only affected by the amplitude of the k-th periodic constituents C_k. In the ordinary Fourier-analysis this is easy enough because of the orthogonality of all constituents in the interval. In the case that, as here, the frequencies are (practically) relatively irrational an extremely large time-interval ought to be given in order that the same methods might be used. Moreover, there are always a large number of less important constituents left out of consideration that would, anyhow, spoil the result.

Be the given time series T to be analysed

$$T = \sum_{k=1}^{K} S_k \sin f_k t + \sum_{k=1}^{K} C_k \cos f_k t + R, \qquad (3.1)$$

where K is some finite number (here K = 5), and the frequencies f_k are given quantities, and, moreover, R is a remainderterm containing the less important neglected constituents as well as the measurement-errors. Data are available for an equidistant finite set of values of t that by suitable interpretation of f_k may be identified with the integers n = 0,1,2,...,N-1 (here N = 14 x 96 or 15 x 96). Then the operation defined by L_m is

$$L_m T = \sum_{n=0}^{N-1} \lambda_m(n)\, T(n) = \sum_{k=1}^{K} S_k \sum_{n=0}^{N-1} \lambda_m(n) \sin n\, f_k + $$

$$+ \sum_{k=1}^{K} C_k \sum_{n=0}^{N-1} \lambda_m(n) \cos n\, f_k + R_m' \qquad (3.2)$$

where R_m' is again some unpredictable remainderterm. Now the functions $\lambda_m(n)$ are to be chosen in such a way that one of the terms $\sum_{n=1}^{N} \lambda_m(n) \sin n f_k$ or $\sum_{n=1}^{N} \lambda_m(n) \cos n\, f_k$ is relatively very large with respect to the others. Then by neglecting R_m' it is seen that if $2K$ operators L_m are defined in such a way that they each favour a different one of the constituents the set of equations (3.2) form a system with (after suitable arrangement) preponderating principal diagonal of the matrix. This matrix can be inverted once for all so that for each given time series (i.e. for the measurements of each station) the vector $L_m T$ has only to be multiplied by a known matrix in order to yield the amplitudes of the constituents. Of course, the functions $\lambda_m(n)$ can be chosen to be the constituents themselves, i.e. $\sin n\, f_k$ or $\cos n\, f_k$. This however, implies that a large number of multiplications has to be performed. This difficulty is overcome by taking instead functions that are either +1 or -1 according to whether $\sin n\, f_k$ or $\cos n\, f_k$ is positive or negative as suggested by Doodson and Warburg. Only summations have then to be performed whereas the loss in accuracy is not very large as long as one of the frequencies is not a small odd multiple of one of the others what is not the case in this particular application. Scheen has pointed out that it is better to use instead functions that take the values +1, 0, -1 only in order to minimise the effect of measurement errors. This does not complicate the computation. On the contrary, the sums extend over fewer terms. The best choice is found by him to be:

$$\lambda_m(n) = -1, \text{ when } -1 \leqslant {\sin \atop \cos}\, n\, f_k \leqslant -\sin \pi/8,$$

$$\lambda_m(n) = 0, \text{ when } -\sin \pi/8 < {\sin \atop \cos}\, n\, f_k < \sin \pi/8, \qquad (3.3)$$

$$\lambda_m(n) = 1, \text{ when } \sin \pi/8 < {\sin \atop \cos}\, n\, f_k < 1.$$

Even with this great simplification one should still have to determine $\lambda_m(n)$ for all the possible cases of m and n. Hence, a second simplification is introduced by performing the operations L_m in two steps, each chosen in a suitable way.

4. Details of the analysis.

The first step consists of applying to the measurements of each day separately, to 96 consecutive values, therefore, a set of four operators L_1, L_2, L_3 and L_4 that are so designed as to pick out precisely the sine- and cosine constituents of speed-numbers 15 and 30. They are defined by the following table. The unity of t is a quarter of an hour.

t	$\lambda_1(t)$	$\lambda_2(t)$	$\lambda_3(t)$	$\lambda_4(t)$
0 - 2	0	1	0	1
3 - 5	0	1	1	1
6 - 9	1	1	1	1
10 - 14	1	1	1	0
15 - 18	1	1	1	-1
19 - 21	1	0	1	-1
22 - 26	1	0	0	-1
27 - 29	1	0	-1	-1
30 - 33	1	-1	-1	-1
34 - 38	1	-1	-1	0
39 - 42	1	-1	-1	1
43 - 45	0	-1	-1	1
46 - 50	0	-1	0	1
51 - 53	0	-1	1	1
54 - 57	-1	-1	1	1
58 - 62	-1	-1	1	0
63 - 66	-1	-1	1	-1
67 - 69	-1	0	1	-1
70 - 74	-1	0	0	-1
75 - 77	-1	0	-1	-1
78 - 81	-1	1	-1	-1
82 - 86	-1	1	-1	0
87 - 90	-1	1	-1	1
91 - 93	0	1	-1	1
94 - 95	0	1	0	1

These sequences are very regular. They have, indeed, a kind of a period of 24. In fact, one has only to distinguish the operators for $t = 0 - 23$. For:

$$\lambda_1(24\text{-}47) = \lambda_2(0\text{-}23),\quad \lambda_2(24\text{-}47) = -\lambda_1(0\text{-}23),$$
$$\lambda_1(48\text{-}71) = -\lambda_1(0\text{-}23),\quad \lambda_2(48\text{-}71) = -\lambda_2(0\text{-}23),$$
$$\lambda_1(72\text{-}95) = -\lambda_2(0\text{-}23),\quad \lambda_2(72\text{-}95) = \lambda_1(0\text{-}23),$$

$$\lambda_3(24\text{-}47) = -\lambda_3(0\text{-}23),\quad \lambda_4(24\text{-}47) = -\lambda_4(0\text{-}23),$$
$$\lambda_3(48\text{-}71) = \lambda_3(0\text{-}23),\quad \lambda_4(48\text{-}71) = \lambda_4(0\text{-}23),$$
$$\lambda_3(72\text{-}95) = -\lambda_3(0\text{-}23),\quad \lambda_4(72\text{-}95) = -\lambda_4(0\text{-}23).$$

The result of these computations is a set of four values for each day, denoted for the n-th day by $L_1'(n)$, $L_2'(n)$, $L_3'(n)$, $L_4'(n)$.

One can compute (cf. Appendix I) the results if the operations are applied to the harmonic constituents S_1, O_1, M_2, S_2, N_2 with unit-amplitude, all starting at the beginning of the day either as a sine or as a cosine-wave. Such a wave has, in general, another phase on the next and following days, due to the fact that the speed-number is not exactly equal to 15 or 30. Hence, tables can be constructed that show the result of applying the operators L_1, L_2, L_3 or L_4 to the basic constituents, on various days, a basic constituent being a constituent of the type S_1, O_1, and so on that start at $t = 0$ either as a sine-wave or a cosine-wave. The results are given in tabel 1 through 8.

The second step is to apply certain other operators to the $L_1'(n)$, $L_2'(n)$, $L_3'(4)$ and $L_4'(n)$. Therefore again operators of the same kind are used chosen in such a way as to filter out the various components as purely as possible. They are defined by

n	$\mu_1(n)$	$\mu_2(n)$	$\mu_3(n)$	$\mu_4(n)$	$\mu_5(n)$	$\mu_6(n)$	$\mu_7(n)$	$\mu_8(n)$	$\mu_9(n)$	$\mu_{10}(n)$	$\mu_{11}(n)$	$\mu_{12}(n)$
1	1	1	1	1	1	1	1	1	1	0	1	-1
2	1	1	1	1	1	1	1	1	1	-1	1	-1
3	1	1	1	1	1	1	1	1	1	-1	1	1
4	1	1	0	1	1	1	1	1	1	-1	1	1
5	1	1	-1	-1	1	1	1	-1	1	1	1	1
6	1	1	-1	-1	1	1	0	-1	1	1	0	-1
7	1	0	-1	-1	1	1	0	-1	1	1	0	-1
8	1	0	-1	0	1	-1	0	-1	1	1	0	-1
9	1	0	-1	1	1	-1	0	1	1	1	-1	-1
10	1	-1	-1	1	1	-1	-1	1	1	1	-1	1
11	1	-1	0	1	1	-1	-1	1	1	-1	-1	1
12	1	-1	0	-1	1	-1	-1	-1	1	-1	-1	1
13	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	
14	1	-1	1	-1	1	-1	-1	-1				
15	1	-1	1	-1								

In most cases observations during 15 consecutive days were available in some cases only during 14 days whereas in one case only data of 13 days were given. The following table shows the combinations of operators used in these three cases ordened with respect to the basic constituent that this combination tries to extract.

Basic constituent	Number	Combination of operators if data are available during 13 days	14 days	15 days
S_1 cosine	1	M_9L_2	M_5L_2	M_5L_2
S_1 sine	2	M_9L_1	M_5L_1	M_5L_1
O_1 cosine	3	$M_{10}L_2$	M_6L_1	M_6L_1
O_1 sine	4	$M_{10}L_1$	M_6L_2	M_3L_1
S_2 cosine	5	M_9L_4	M_5L_4	M_1L_4
S_2 sine	6	M_9L_3	M_5L_3	M_1L_3
M_2 cosine	7	$M_{11}L_3$	M_7L_3	M_2L_3
M_2 sine	8	$M_{11}L_4$	M_7L_4	M_2L_4
N_2 cosine	9	$M_{12}L_3$	M_8L_4	M_4L_4
N_2 sine	10	$M_{12}L_4$	M_8L_3	M_4L_3

So far, the computations are of a very simple character, only some additions and subtractions of the empirical data and of the first derived quantities being needed. The first part of these computations, i.e. the application of the operators L_1, L_2, L_3 and L_4 on the empirical data can be performed on a National-machine in a very suitable way. The data have to be set only once and the machine forms the required combinations automatically. The second part of the computations handles so few numbers only that the required combinations can easily be made by using an ordinary deskmachine.

The result of the whole computation is a vector, the components of which are closely related to the amplitude of one basic constituent only. The effects of the other compponents are not neglected however. It is first calculated how each basic constituent behaves under the application of the different combination of operators. The basic constituents may be numbered from 1 to 10 as given in the table above. Moreover, the amplitude of the components may be denoted by A_1, A_2, ..., A_{10}. At last the appropriate combinations of operators for a certain number of days be given by Z_1, Z_2, ..., Z_{10}.

Then, one has

$$Z_i T = \sum k \ m_{ik} \ A_k$$

and for the cases 13, 14 and 15 days the matrices (m_{ik}) are given in tables 9, 10 and 11. By inverting, it follows that

$$A_k = (m_{ik})^{-1} \ Z_i T$$

and the inverted matrices $(m_{ik})^{-1}$ in the three cases are given in tables 12, 13 and 14. The principal diagonal is seen to be very large in comparison to the other elements. Hence, it is to be expected that the influence of neglected constituents and measurement-errors is reduced to a minimum.

5. Correction for jumps and drifts.

The measurements show a lot of irregularities that are not caused by the gravity-effect to be measured but by other circumstances such as slow or sudden changes in the characteristics of the measuring apparatus. Calculations have been corrected for these effects. The B.P.M. furnished besides the original data tables of the variations, called drift (slow) and jumps (sudden) at hourly intervals. The correction was performed in this way that these data were subjected to a completely analogous process as the actual measurements with this difference that all the operators were replaced by corresponding ones, relating to hourly data instead of quarter-hourly data. In this way these irregularities are also analysed in the basic constituents and these pseudocomponents serve to correct the originally calculated components. For certain stations these corrections were very essential.

6. Performed calculations.

In the way described in the preceding sections the data of all 29 stations were analysed. If measurements covering 15 days were available the calculations were performed for the first 14 days as well as for the first 15 days. The results of the amplitudines and phases of the 5 basic constituents S_1, O_1, S_2, M_2 and N_2 are given in Table A and B. If only one row is given, the measurements covered only 13 or 14 days. If two rows are given, the first is the result of a 14-days analysis, whereas the second one is the result of a 15-days analysis. In one case (10^I, Houston no.10), three rows are given, for 13, 14 and 15 days respectively.

The phases are given with respect to the astronomical tide.

Appendix I:

One can derive the following formulae:

$$L_1'(e^{iat})=\left(\sum_{n=6}^{42} - \sum_{n=54}^{90}\right)e^{iat} =$$

$$=\frac{\{\cos 6a-\cos 54a)+i(\sin 6a-\sin 54a)\}\{(1-\cos a+\cos 36a-\cos 37a)+i(\sin a+\sin 36a-\sin 37a)\}}{4\sin^2\frac{1}{2}a}$$

$$L_2'(e^{iat})=\left(\sum_{n=0}^{18} - \sum_{n=30}^{47} - \sum_{n=48}^{66} + \sum_{n=78}^{95}\right)e^{iat} =$$

$$=\frac{(1-\cos a+i\sin a)(1-\cos 48a-i\sin 48a)\{(1-\cos 19a-\cos 30a+\cos 48a)-i(\sin 19a+\sin 30a-\sin 48a)\}}{4\sin^2\frac{1}{2}a}$$

$$L_3'(e^{iat})=\left(\sum_{n=3}^{21} - \sum_{n=27}^{45} + \sum_{n=51}^{69} - \sum_{n=75}^{93}\right)e^{iat} =$$

$$=\frac{\{(\cos 3a-\cos 27a+\cos 51a-\cos 75a)+i(\sin 3a-\sin 27a+\sin 51a-\sin 75a)\}\cdot\{(1-\cos a+\cos 18a-\cos 19a)+i(\sin a+\sin 18a+\sin 19a)\}}{4\sin^2\frac{1}{2}a}$$

$$L_4'(e^{iat})=\left(\sum_{n=0}^{9} - \sum_{n=15}^{23} - \sum_{n=24}^{33} + \sum_{n=39}^{47} + \sum_{n=48}^{57} - \sum_{n=63}^{71} - \sum_{n=72}^{31} + \sum_{n=87}^{95}\right)e^{iat}$$

$$=\frac{eb+cd}{4\sin^2\frac{1}{2}a}$$

with

$e = (1-\cos 24a+\cos 48a-\cos 72a)+i(-\sin 24a+\sin 48a-\sin 72a)$

$b = (1-\cos a+\cos 9a-\cos 10a)+i(\sin a+\sin 9a-\sin 10a)$

$c = (1-\cos 15a+\cos 39a-\cos 63a+\cos 87a)+i(-\sin 15a+\sin 39a-\sin 63a+\sin 87a)$

$d = (1-\cos a+\cos 8a-\cos 9a)+i(\sin a+\sin 8a-\sin 9a)$

Defining now

$u = -k \cdot 360^\circ + 96 \cdot a^\circ$ for $k = 1,2,$

such that $|u|$ is minimal, one finds for $L_k(n)$ applicated on the constituent, which starts as a:

cos a t: $L_k'(n) = \mathrm{Re}\, L_k'(e^{iat})\cos n u - \mathrm{Im}\, L_k'(e^{iat})\sin n u.$

sin a t: $L_k'(n) = \mathrm{Im}\, L_k'(e^{iat})\cos n u + \mathrm{Re}\, L_k'(e^{iat})\sin n u.$

TABLE I.

$L_1'(n)$ when the constituents start as a sine wave.

n	S_1	O_1	S_2	M_2	N_2
1	57.91350	57.54362	0	- 2.361178	- 3.720010
2	57.91350	46.44738	0	- 1.939999	- 2.186674
3	57.91350	26.39374	0	- 1.172789	0.247948
4	57.91350	1.25005	0	- 0.196390	2.580372
5	57.91350	- 24.13470	0	0.815038	3.849240
6	57.91350	- 44.86508	0	1.681090	3.531558
7	57.91350	- 56.94319	0	2.247290	1.758267
8	57.91350	- 58.03978	0	2.412647	- 0.739732
9	57.91350	- 47.94336	0	2.147666	- 2.932834
10	57.91350	- 28.60105	0	1.499611	- 3.917105
11	57.91350	- 3.74298	0	0.584075	- 3.286855
12	57.91350	21.83690	0	- 0.435642	- 1.301854
13	57.91350	43.20552	0	- 1.377653	1.219733
14	57.91350	56.24195	0	- 2.073937	3.238582
15	57.91350	58.43207	0	- 2.400298	3.922576

TABLE II.

$L_1'(n)$ when the constituents start as a cosine wave.

	S_1	O_1	S_2	M_2	N_2
1	0	12.94860	0	- 0.510159	- 1.260947
2	0	36.35347	0	- 1.439384	- 3.262962
3	0	52.74755	0	- 2.111869	- 3.920075
4	0	58.96924	0	- 2.407666	- 2.961443
5	0	53.81868	0	- 2.274013	- 0.782187
6	0	38.28915	0	- 1.734751	1.719464
7	0	15.37554	0	- 0.886065	3.512401
8	0	- 10.50325	0	0.120665	3.857623
9	0	- 34.35649	0	1.105873	2.612842
10	0	- 51.58406	0	1.893829	0.291121
11	0	- 58.86361	0	2.343988	- 2.150593
12	0	- 54.79128	0	2.376056	- 3.705892
13	0	- 40.15242	0	1.984312	- 3.733727
14	0	- 17.77015	0	1.238632	- 2.222623
15	0	8.03911	0	0.272020	0.204583

TABLE III

$L_2'(n)$ when the constituents start as a sine wave.

n	S_1	O_1	S_2	M_2	N_2
1	0	- 11.75386	0	1.455733	3.270009
2	0	- 33.42765	0	3.694454	7.853791
3	0	- 48.65491	0	5.274205	9.200460
4	0	- 54.49904	0	5.913209	6.754957
5	0	- 49.83299	0	5.497489	1.525250
6	0	- 35.55663	0	4.101196	- 4.333123
7	0	- 14.42315	0	1.973383	- 8.405503
8	0	9.49184	0	- 0.506417	- 9.013370
9	0	31.57633	0	- 2.895888	- 5.906179
10	0	47.57130	0	- 4.768828	- 0.364626
11	0	54.39212	0	- 5.791164	5.327215
12	0	50.72339	0	- 5.780545	8.823328
13	0	37.27262	0	- 4.738865	8.682712
14	0	16.63381	0	- 2.851926	4.963325
15	0	- 7.21285	0	- 0.456295	- 0.801806

TABLE IV

$L_2'(n)$ when the constituents start as a cosine wave.

n	S_1	O_1	S_2	M_2	N_2
1	57.91350	53.2342	0	- 5.737597	- 8.647080
2	57.91350	43.0654	0	- 4.624953	- 4.876774
3	57.91350	24.5913	0	- 2.687368	0.903601
4	57.91350	1.3748	0	- 0.270443	6.311539
5	57.91350	- 22.1067	0	2.194719	9.118036
6	57.91350	- 41.3251	0	4.268415	8.166334
7	57.91350	- 52.5738	0	5.580765	3.848697
8	57.91350	- 53.6837	0	5.897688	- 2.055263
9	57.91350	- 44.4406	0	5.162655	- 7.112103
10	57.91350	- 26.6272	0	3.506773	- 9.237532
11	57.91350	- 3.6786	0	1.225397	- 7.555509
12	57.91350	19.9793	0	- 1.274551	- 2.759317
13	57.91350	39.7842	0	- 3.547159	3.174188
14	57.91350	51.9168	0	- 5.187070	7.799382
15	57.91350	54.0370	0	- 5.901777	9.209889

TABLE V

$L'_3(n)$ when the constituents start as a sine wave.

n	S_1	O_1	S_2	M_2	N_2
1	0	5.163041	57.91350	57.29193	55.52499
2	0	4.167450	57.91350	47.07239	32.63855
3	0	2.368153	57.91350	28.45667	- 3.70091
4	0	0.112162	57.91350	4.76521	- 38.51475
5	0	- 2.165459	57.91350	- 19.77621	- 57.45390
6	0	- 4.025470	57.91350	- 40.79020	- 52.71215
7	0	- 5.109167	57.91350	- 54.52855	- 26.24392
8	0	- 5.207558	57.91350	- 58.54078	11.04130
9	0	- 4.301668	57.91350	- 52.11124	43.77562
10	0	- 2.566200	57.91350	- 36.38674	58.46685
11	0	- 0.335838	57.91350	- 14.17205	49.05969
12	0	1.959291	57.91350	10.57049	19.43148
13	0	3.876568	57.91350	33.42759	- 18.20582
14	0	5.046249	57.91350	50.32229	- 48.33920
15	0	5.242756	57.91350	58.24114	- 58.54852

TABLE VI

$L'_3(n)$ when the constituents start as a cosine wave.

n	S_1	O_1	S_2	M_2	N_2
1	0	1.161797	0	12.37859	18.82098
2	0	3.261775	0	34.92542	48.70310
3	0	4.732717	0	51.24268	58.51117
4	0	5.290953	0	58.41992	44.20257
5	0	4.828825	0	55.17695	11.67491
6	0	3.435456	0	42.09221	- 25.66482
7	0	1.379556	0	21.49959	- 52.42624
8	0	- 0.942391	0	- 2.92786	- 57.57902
9	0	- 3.082598	0	- 26.83308	- 38.99935
10	0	- 4.628324	0	- 45.95214	- 4.34523
11	0	- 5.281475	0	- 56.87484	32.09986
12	0	- 4.916090	0	- 57.65292	55.31429
13	0	- 3.602635	0	- 48.14760	55.72972
14	0	- 1.594410	0	- 30.05431	33.17492
15	0	0.721299	0	- 6.60031	- 3.05366

TABLE VII

$L_4'(n)$ when the constituents start as a sine wave.

n	S_1	O_1	S_2	M_2	N_2
1	0	- 0.3040	0	- 11.7051	- 17.4301
2	0	- 1.2819	0	- 33.4381	- 45.7121
3	0	- 2.0126	0	- 49.2067	- 55.1528
4	0	- 2.3552	0	- 56.1986	- 41.8611
5	0	- 2.2436	0	- 53.1664	- 11.3154
6	0	- 1.6993	0	- 40.6510	23.8942
7	0	- 0.8272	0	- 20.8849	49.2553
8	0	0.2043	0	2.6065	54.3147
9	0	1.1964	0	25.6329	36.9871
10	0	1.9579	0	44.0872	4.4145
11	0	2.3417	0	54.6779	- 29.9777
12	0	2.2739	0	55.5158	- 52.0139
13	0	1.7676	0	46.4514	- 52.6114
14	0	0.9205	0	29.1017	- 31.5240
15	0	- 0.1042	0	6.5611	2.5567

TABLE VIII

L_4' when the constituents start as a cosine wave.

n	S_1	O_1	S_2	M_2	N_2
1	0	2.3510	57.91350	55.1749	52.4219
2	0	1.9941	57.91350	45.4222	31.0204
3	0	1.2526	57.91350	27.5676	- 3.1668
4	0	0.2695	57.91350	4.7959	- 36.0488
5	0	- 0.7655	57.91350	- 18.8312	- 54.0724
6	0	- 1.6529	57.91350	- 39.0995	- 49.8090
7	0	- 2.2216	57.91350	- 52.3937	- 25.0156
8	0	- 2.3618	57.91350	- 56.3426	10.0884
9	0	- 2.0465	57.91350	- 50.2418	41.0343
10	0	- 1.3366	57.91350	- 35.1794	55.0670
11	0	- 0.3689	57.91350	- 13.8423	46.4026
12	0	0.6699	57.91350	9.9639	18.6124
13	0	1.5796	57.91350	31.9929	- 16.8494
14	0	2.1846	57.91350	48.3153	- 45.3663
15	0	2.3683	57.91350	56.0199	- 55.1845

TABLE IX (m_{ik}) 13 days.

1	743.7079		- 62.4065	- 17.1206			9.6943	3.4280	- 10.7212	23.4149
2		743.7079	18.2511	- 67.5929			- 1.5392	3.9038	- 9.7844	- 4.8979
3			-365.8735	- 16.9798			37.7901	4.8296	7.5304	- 73.1400
4			16.7766	-395.9178			- 2.5199	15.3417	30.9460	4.2767
5			- 2.6370	- 0.9820	752.8755		- 91.0131	- 36.2791	69.6850	-137.2088
6			1.6376	- 6.0647		752.8755	37.3469	- 94.7215	146.0419	73.1068
7			40.7872	11.0132			447.6041	176.4819	82.1134	-164.0340
8			6.6041	- 17.7349			171.4361	-430.0801	-154.1126	- 78.2701
9			- 5.1870	8.6855			- 76.7752	75.0638	304.6029	- 36.7357
10			3.6587	2.6738			71.9550	74.1489	- 32.9464	-287.2155

TABLE X (m_{ik}) 14 days

1	800.9190		- 10.4897	- 0.4868			4.5073	0.5760	- 2.9218	28.3782
2		800.9190	0.4810	- 11.3510			- 0.3006	1.8298	- 12.0070	- 1.6594
3			536.5235	22.7346			- 22.4273	- 3.6837	- 1.9045	13.7808
4			23.0099	-495.8096			- 7.0602	55.2433	32.5705	3.3534
5			- 0.4524	- 0.0615	810.7890		- 42.6978	- 7.1770	24.3167	-16[illegible].7330
6			0.0432	- 1.0184		810.7890	7.2925	- 44.[illegible]	173.2169	24.7073
7			39.2990	1.6653			450.8254	74.0484	9.9392	- 71.9192
8			2.3730	- 17.4590			72.8790	-433.5489	- 67.7120	- 9.7591
9			4.6829	- 0.8540			110.0920	- 45.1236	349.1425	-128.7318
10			2.8665	10.2326			47.3205	114.2315	138.7693	369.7320

TABLE XI (m_{ik}) 15 days

1	800.9190		- 10.4897	- 0.4868			4.5073	0.5760	- 2.9218	28.3782
2		800.9190	0.4810	- 11.3510			- 0.3006	1.8298	- 12.0070	- 1.6594
3			536.5235	22.7346			- 22.4273	- 3.6837	- 1.9045	13.7808
4			41.1266	548.7914			1.2080	- 22.1292	- 25.4070	1.1728
5			1.9158	- 0.1657	868.7025		13.3221	- 0.6160	- 30.8658	-166.1760
6			0.7645	4.2243		868.7025	0.6923	13.8419	176.1632	- 33.7809
7			42.0132	- 7.6029			499.5179	- 24.9829	- 12.6720	- 66.0829
8			- 1.6482	- 19.0540			- 22.2404	-480.7610	- 62.3365	11.5784
9			- 0.0472	- 0.5455			- 2.2705	- 49.0782	414.4154	- 76.9738
10			1.2028	- 0.2177			50.9929	- 2.5504	84.2440	439.3219

TABLE XII $(m_{ik})^{-1}$ 13 days

	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$
1	1345									
2		1345								
3	- 234	57	-2709	- 113	28	- 6	242	49	10	37
4	- 47	- 231	120	-2513	- 3	- 19	5	90	52	- 3
5					1328					
6						1328				
7	- 33	14	110	57	357	- 188	2142	567	477	630
8	39	11	145	- 40	- 190	- 383	566	-2231	635	- 506
9	104	60	160	216	- 574	- 708	- 447	-1245	3375	- 816
10	163	- 60	569	- 58	- 721	642	-1382	432	- 879	-3609

TABLE XIII $(m_{ik})^{-1}$ 14 days

	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$
1	1249									
2		1249								
3	26		1853	87	- 8	- 5	- 158	- 23	23	2
4	- 1	- 28	84	-2003	5	- 3	- 9	77	16	26
5					1233					
6						1233				
7	- 8	- 11	94	- 19	102	160	2122	466	- 671	- 164
8	- 26		- 3	- 260	156	- 107	478	-2203	- 180	694
9	40	39	17	119	- 257	- 559	- 122	- 405	2549	- 819
10	- 85	17	- 46	46	498	- 249	389	- 108	751	2405

TABLE XIV $(m_{ik})^{-1}$ 15 days

	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$
1	1249									
2		1249								
3	25	- 3	1863	- 139	1	- 1	- 157	6	2	13
4		26	- 77	1825	- 1	- 6	30	- 73	- 6	- 2
5					1151					
6						1151				
7	- 2		87	- 16	- 73		1960	- 90	- 41	- 220
8	- 1		- 16	- 92		82	- 102	-2040	- 234	45
9	25	35	12	94	- 3	- 478	- 15	- 311	2295	- 440
10	- 77	11	- 42	16	424	3	300	- 14	402	2164

<u>TABEL A</u>

Amplitudines.

No.	Station	S_1	O_1	S_2	M_2	N_2
1	Albuquerque	0.038	0.042	0.037	0.053	0.013
		0.039	0.023	0.038	0.046	0.010
2	Austin	0.049	0.029	0.027	0.068	0.007
3	Beaumont	0.052	0.032	0.032	0.073	0.014
		0.052	0.032	0.032	0.066	0.016
4	Bogota Puert.Cal.	0.013	0.024	0.037	0.070	0.023
5	Columbia Bogota	0.762	0.282	0.453	0.141	0.120
6	Cosoleacaquê	0.046	0.014	0.024	0.074	0.021
	gecorrigeerde waarnemingen	0.014	0.018	0.035	0.072	0.026
7	Edmonton no.29 Alb.	0.054	0.033	0.017	0.029	0.006
		0.054	0.027	0.017	0.026	0.005
8	Edmonton no.49 Alb.	0.050	0.039	0.017	0.030	0.004
		0.050	0.033	0.016	0.029	0.005
9	Den Haag	0.062	0.048	0.022	0.029	0.009
		0.062	0.028	0.021	0.029	0.009
10^{I}	Houston no.10 Texas	0.053	0.027	0.037	0.069	0.024
		0.054	0.029	0.035	0.066	0.024
		0.054	0.031	0.036	0.066	0.026
10^{II}	Houston no.20 Texas	0.051	0.030	0.029	0.066	0.012
		0.051	0.029	0.029	0.067	0.011
11	Maracaibo	0.033	0.014	0.042	0.083	0.028
12	Ottawa Ontario	0.057	0.039	0.023	0.054	0.012
13	Pladjoe	0.046	0.023	0.133	0.110	0.012
14	Salt Lake City	0.085	0.030	0.020	0.059	0.012
		0.085	0.012	0.018	0.063	0.016
15	Sorong no.56	0.009	0.020	0.037	0.092	0.020
16	Sorong no.60	0.016	0.009	0.039	0.096	0.023
17	Toronto	0.051	0.033	0.020	0.046	0.019
		0.051	0.032	0.022	0.048	0.017
18	Trinidad	0.062	0.017	0.050	0.091	0.017
19	Tulsa	0.061	0.019	0.037	0.070	0.017
		0.061	0.025	0.036	0.068	0.012
20	Washington	0.056	0.036	0.025	0.056	0.010
		0.056	0.036	0.022	0.057	0.016

TABEL A (Vervolg)
Amplitudines.

No.	Station	S_1	O_1	S_2	M_2	N_2
21	Balikpapan	0.014	0.017	0.039	0.087	0.007
22	Hellwan Egypte	0.058	0.030	0.030	0.070	0.018
23	Fort Morgan Colorado	0.050	0.042	0.020	0.047	0.004
		0.058	0.050	0.022	0.043	0.009
	gecorrigeerde waarnemingen	0.052	0.039	0.018	0.051	0.009
		0.052	0.039	0.019	0.049	0.008
24	Honolulu Hawaii	0.042	0.030	0.034	0.089	0.024
		0.042	0.021	0.035	0.093	0.023
25	Muna	0.067	0.024	0.052	0.098	0.034
26	Owerri	0.014	0.012	0.042	0.091	0.021
27	Pasadena	0.102	0.079	0.051	0.114	0.037
		0.102	0.065	0.050	0.112	0.039
28	Panuco 13 dagen	0.047	0.029	0.036	0.081	0.024

TABEL B

Fasehoeken.

No.	Station	S_1	O_1	S_2	M_2	N_2
1	Albuquerque	34°14	61°12	- 24°79	2°00	15°06
		34°36	48°61	- 21°49	2°04	- 14°70
2	Austin	- 17°55	37°35	- 17°82	11°20	101°61
3	Beaumont	- 12°59	14°06	- 33°43	- 3°77	13°09
		- 12°54	11°95	- 31°94	- 3°11	8°60
4	Bogota Puert.Col.	- 21°45	34°12	-188°33	8°88	- 45°38
5	Bogota Columbia	-191°80	170°48	- 62°07	- 87°43	- 61°69
6	Cosoleacaque	66°75	18°70	- 64°95	- 10°29	22°14
	gecorrigeerde waarnemingen	34°41	4°08	- 35°10	16°61	- 49°48
7	Edmonton no.29 Alb.	- 0°70	18°58	- 25°56	- 1°42	5°00
		- 0°50	- 5°30	- 23°12	- 3°65	- 2°13
8	Edmonton no.49 Alb.	- 8°16	21°76	- 22°54	9°34	16°41
		- 7°85	4°16	- 19°32	5°53	9°68
9	Den Haag	- 10°35	25°63	- 29°32	- 10°88	- 15°76
		- 10°10	0°33	- 25°66	- 17°86	- 43°38
10^I	Houston no.10 Texas	1°13	19°28	- 53°02	5°97	19°98
		1°85	17°39	- 49°70	6°40	- 19°67
		1°85	23°38	- 51°10	7°70	- 9°48
10^II	Houston no.20 Texas	- 5°19	20°87	- 20°71	4°51	19°79
		- 5°19	17°52	- 21°25	5°07	22°06
11	Maracaibo	- 40°99	- 42°28	- 15°66	1°36	- 14°93
12	Ottawa Ontario	- 0°07	21°71	- 20°91	13°55	98°81
13	Pladjoe	21°15	99°22	- 43°21	- 10°10	54°54
14	Salt Lake City	3°55	49°53	- 40°29	16°20	29°70
		3°81	8°91	- 45°27	11°29	38°10
15	Sorong No.56	96°41	103°08	1°89	- 8°63	- 41°75
16	Sorong no.60	-230°20	- 37°36	- 10°90	- 3°73	- 7°37
17	Toronto	- 5°18	27°00	- 20°00	6°44	20°04
		- 5°04	1°97	- 19°10	7°95	24°33
18	Trinidad	- 37°68	15°93	- 21°98	2°76	- 18°87
19	Tulsa	-189°62	- 64°52	-201°35	-164°76	-110°34
		-189°66	- 71°39	-198°66	-167°64	-120°39
20	Washington	- 4°09	12°17	- 23°08	10°18	4°74
		- 4°03	12°03	- 17°96	9°27	1°79

TABEL B

Fasehoeken.

No.	Station	S_1	O_1	S_2	M_2	N_2
21	Balikpapan	66°.39	65°.68	0°.54	- 1°.24	12°.52
22	Hellwan Egypte	- 3°.18	4°.18	- 19°.18	- 3°.23	2°.16
23	Fort Morgan Colorado	- 7°.10	13°.30	- 29°.03	10°.07	-119°.59
		- 7°.36	27°.40	- 32°.35	13°.94	-122°.87
	gecorrigeerde waarnemingen	- 6°.58	9°.36	- 21°.15	6°.91	25°.46
		- 6°.56	9°.15	- 20°.54	12°.68	42°.21
24	Honolulu Hawaii	- 14°.29	16°.74	- 14°.89	19°.93	66°.20
		- 14°.12	11°.84	- 12°.21	22°.45	65°.37
25	Muna	10°.83	1°.30	- 37°.34	1°.61	69°.67
26	Owerri	18°.55	- 16°.96	- 32°.20	- 4°.55	- 57°.65
27	Pasadena	-185°.51	225°.74	-182°.69	-168°.52	-147°.92
		-185°.41	221°.22	-181°.44	-168°.02	-150°.77
28	Panuco 13 dagen	20°.96	22°.30	- 15°.03	10°.59	26°.80