MATHEMATISCH CENTRUM
2e BOERHAAVESTRAAT 49
AMSTERDAM

REKENAFDELING

Leiding: Prof. Dr Ir A. van Wijngaarden

Computation of the Ginzel-Ludwig correction factor K for a three-bladed propeller in a homogenous flow.

by

The Staff of the Computation Department

Report R 195 B

1 9 5 6

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1. On behalf of the Netherlands Ship Model Basin the Ginzel-Ludwig correction factor K [1] was computed for the values

$\sigma = 0.2, 0.5, 0.7, 0.9$

$\lambda = 0.1, 0.2, 0.25, 0.4, 1$

with a profile defined by $1/D = 1.66628\sqrt{1-x} - 1.63267(1-x)$
In the computations the values of the Goldsteinfactor $\varkappa^{(3)}(x)$, given in report R 195 A [2] were used.

2. Following Ginzel-Ludwig [1] the following formulae were used:

$$\bar{x} = (x^2 + \lambda^2)^{\frac{1}{2}}$$

$$\varphi(x) = 2^{-\frac{1}{2}}\bar{x}^{-1}\, 1/D$$

$$\bar{\varphi} = \varphi(2^{-\frac{1}{2}})$$

$$\Gamma_0(x) = 2^{-1}\pi(x/\bar{x})^2\varkappa^{(3)}(x)$$

$$\bar{\Gamma}_0 = 2\int_0^1 x\,\Gamma_0(x)dx$$

$$\bar{\sigma}(x) = 1 + (\sigma - x)\bar{x}^{-1}$$

$$k^2(x) = 4\,\bar{\sigma}(x)\left\{1 + \bar{\sigma}(x)\right\}^{-2}$$

$$f(x) = x\,\varphi(x)$$

$$g(x) = \lambda\varphi(x)$$

$$H(x) = \Gamma_0(x)\,\varphi^{-1}(x)$$

$$\alpha = \frac{\pi}{3} + \bar{\varphi}$$

$$\beta = \frac{\pi}{3} - \bar{\varphi}$$

$$I_1(\sigma,\lambda) = \frac{1}{4\pi}\int_0^1 H(x)\,\frac{\{\sigma f(x) + \lambda g(x)\} - (x-\sigma)\{\sigma f'(x) + \lambda g'(x)\}}{(\lambda^2+\sigma^2)^{\frac{1}{2}}\left\{(x-\sigma)^2 + f^2(x) + g^2(x)\right\}^{\frac{3}{2}}}\,dx$$

$$I_2(\sigma,\lambda) = -\frac{1}{4\pi}\int_0^1 H'(x)\,\frac{x\sigma+\lambda^2}{\pi^2(\lambda^2+\sigma^2)^{\frac{1}{2}}}\left[\frac{1}{1-\bar{\sigma}(x)}\left\{E(k) - E\left(k, \frac{\pi-\varphi(x)}{2}\right) + \right.\right.$$

$$\left.\left. - \frac{k^2\sin\varphi(x)}{2\left(1-k^2\cos^2\frac{\varphi(x)}{2}\right)^{\frac{1}{2}}}\right\} + \frac{1}{1+\bar{\sigma}(x)}\left\{K(k) - F\left(k, \frac{\pi-\varphi(x)}{2}\right)\right\}\right]dx$$

$$I_3(\sigma,\lambda) = \frac{\bar{\Gamma}_0}{4\pi\bar{\varphi}(\sigma^2+\lambda^2)^{1/2}}\sum_{i=1}^{4} C_i$$

$$C_1 = - \frac{\lambda^2 \bar{\varphi} \cos\alpha + \sigma \sin\alpha}{\lambda^2 \bar{\varphi}^2 + \sigma^2 \sin^2\alpha} \left\{ \frac{1 + \sigma \cos\alpha}{(\sigma^2 + 2\sigma\cos\alpha + 1 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}} - \frac{\sigma \cos\alpha}{(\sigma^2 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}} \right\}$$

$$C_2 = \frac{\sigma^2 \sin\beta - \lambda^2 \bar{\varphi} \cos\beta}{\lambda^2 \bar{\varphi}^2 + \sigma^2 \sin^2\beta} \left\{ \frac{1 + \sigma \cos\beta}{(\sigma^2 + 2\sigma\cos\beta + 1 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}} - \frac{\sigma \cos\beta}{(\sigma^2 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}} \right\}$$

$$C_3 = \frac{-\lambda^2 (\sigma + \frac{1}{2}) + \sigma (1 + \frac{1}{2}\sigma)}{\lambda^2 (\sigma^2 + \sigma + 1) + (1 + \frac{1}{2}\sigma)^2} \left\{ \frac{\frac{1}{2} \cdot 3^{\frac{1}{2}} \sigma + \bar{\varphi}(1 + \lambda^2)}{\left[\sigma^2 + \sigma + 1 + \bar{\varphi}^2 (1 + \lambda^2) - 3^{\frac{1}{2}} \sigma \bar{\varphi}\right]^{\frac{1}{2}}} + \right.$$

$$\left. - \frac{\frac{1}{2} \cdot 3^{\frac{1}{2}} \sigma - \bar{\varphi}(1 + \lambda^2)}{\left[\sigma^2 + \sigma + 1 + \bar{\varphi}^2 (1 + \lambda^2) + 3^{\frac{1}{2}} \sigma \bar{\varphi}\right]^{\frac{1}{2}}} \right\}$$

$$C_4 = \frac{2 \lambda^2 \bar{\varphi}}{\sigma (\sigma^2 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}}$$

where primes denote differentiation with respect to x.

3. Results

	$\sigma \downarrow$ / $\lambda \rightarrow$	0.1	0.2	0.25	0.4	1
I_1	0.2	2.6067	2.1293	1.9654	1.5920	.8406
	0.5	2.3414	1.9831	1.8222	1.4236	.6998
	0.7	2.7747	2.2382	2.0287	1.5423	.7220
	0.9	5.4409	3.9466	3.4903	2.5603	1.1496
I_2	0.2	-.1568	-.2062	-.1576	-.0093	+.1034
	0.5	-.3789	-.0532	+.0237	+.1056	+.1054
	0.7	-.4530	+.1605	+.2471	+.2713	+.1391
	0.9	+.8098	+.8207	+.7243	+.5112	+.2036
I_3	0.2	-.3012	+.1581	+.3123	+.5085	+.3798
	0.5	-.1120	-.0577	-.0320	+.0238	+.0596
	0.7	-.0462	-.0277	-.0186	+.0027	+.0238
	0.9	-.0115	-.0066	-.0041	+.0021	+.0106

	$\sigma \downarrow$ \ $\lambda \rightarrow$	0.1	0.2	0.25	0.4	1
I	0.2	2.1487	2.0812	2.1201	2.0912	1.3238
	0.5	1.8504	1.8722	1.8139	1.5530	.8648
	0.7	2.2754	2.3710	2.2572	1.8163	.8849
	0.9	6.2392	4.7607	4.2105	3.0736	1.3638
K	0.2	1.198	1.021	.929	.722	.658
	0.5	1.009	.911	.878	.818	.731
	0.7	.844	.724	.703	.677	.654
	0.9	.460	.463	.467	.472	.478

4. References

[1] I. GINZEL, H. LUDWIG: Zur Theorie der Breitblattschraube. Deutsche Luftfahrtforschung: Untersuchungen und Mitteilungen Nr. 3097, (1944).

[2] Staff of the Computntion Department of the Mathematical Centre, Amsterdam.
R 195 A: Computation of the Goldstein factor $\varkappa$ for a three-bladed propeller in a homogenous flow (1956).