

MATHEMATISCH CENTRUM
2e BOERHAAVESTRAAT 49
AMSTERDAM
REKENAFDELING

A TABLE OF SOME INTEGRALS

R 245

by

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A TABLE OF SOME INTEGRALS

1. Introduction.

This table contains the values of the integrals

$$1. f_n(x) = \int_0^x v^n \operatorname{tg} v \, dv$$

for $n = 1(1)5$, $x = 0(0.05)1.50$;

$$2. g_n(x) = \int_0^x v^n \operatorname{cotg} v \, dv$$

for $n = 1(1)5$, $x = 0(0.05)2.50$;

$$3. F_n(x) = \int_0^x v^n \operatorname{tgh} v \, dv$$

for $n = 1(1)4$, $x = 0(0.02)1.98$ and

$$4. G_n(x) = \int_0^x v^n \operatorname{cotgh} v \, dv$$

for $n = 1(1)4$, $x = 0(0.02)1.98$.

The error is $1 \cdot 10^{-8}$ in the last three functions, the error in the function $f_n(x)$ is three times as large.

The function $F_n(x)$ and $G_n(x)$ have been computed by the A.R.R.A., the programming is made by miss R.D.M. Mulder. The other functions have been computed already by hand and the checking has been done by making the differences on a National Accounting machine class 31.

2. Some formulae.

Well known are the expansions

$$\begin{aligned} f_n(x) \\ F_n(x) \end{aligned} = \frac{x^{n+2}}{n+2} + \frac{x^{n+4}}{3(n+4)} + \frac{2x^{n+6}}{15(n+6)} + \frac{17 \cdot x^{n+8}}{315(n+8)} + \frac{62x^{n+10}}{2835(n+10)} + \dots$$

for $n > -2$ and $0 \leq x < \pi/2$, and

$$\begin{aligned} g_n(x) &= \frac{x^n}{n} - \frac{x^{n+2}}{3(n+2)} + \frac{x^{n+4}}{45(n+4)} - \frac{2x^{n+6}}{945(n+6)} + \frac{x^{n+8}}{4725(n+8)} + \dots \end{aligned}$$

for $n > 0$ and $0 \leq x < \pi$.

Putting $n = 0$ in the definitions we have

$$\begin{aligned} f_0(x) &= -\log \cos x & 0 \leq x < \pi/2, \\ g_0(\frac{\pi}{2}) - g_0(x) &= -\log \sin x & 0 < x \leq \pi/2, \\ F_0(x) &= \log \cosh x & 0 \leq x < \infty, \\ G_0(x) &= \log \sinh x & 0 < x < \infty. \end{aligned} \tag{2.1}$$

Furthermore we have the relations

$$g_n(\frac{\pi}{2}) - g_n(\frac{\pi}{2} - x) = \sum_{k=0}^n (-1)^k \binom{n}{k} (\frac{\pi}{2})^{n-k} f_k(x) \tag{2.2}$$

for $n \geq 0$ and $0 < x < \pi/2$ and

$$f_n(\frac{\pi}{2} - x) = \sum_{k=0}^n (-1)^k \binom{n}{k} (\frac{\pi}{2})^{n-k} \{g_k(\frac{\pi}{2}) - g_k(x)\} \tag{2.3}$$

for $n \geq 0$ and $0 < x < \pi/2$; in these last formulae one has to use the relation (2.1)

The relation (2.2) can be proved by considering the integral

$$\int_0^x (\frac{\pi}{2} - v)^n \operatorname{tg} v \, dv;$$

using the binomial expansion one finds the right hand side of the equation (2.2) and the substitution $w = \frac{\pi}{2} - v$ gives the left hand side.

3. Table of $f_n(x)$.

	n=1	n=2	n=3	n=4	n=5
0	0	0	0	0	0
0.05	0.00004169	0.00000156	0.00000006	0.00000000	0.00000000
0.10	0.00033400	0.00002506	0.00000200	0.00000017	0.00000001
0.15	0.00113010	0.00012720	0.00001527	0.00000191	0.00000025
0.20	0.00268825	0.00040360	0.00006462	0.00001077	0.00000185
0.25	0.00527462	0.00099039	0.00019828	0.00004134	0.00000886
0.30	0.00916629	0.00206663	0.00049671	0.00012431	0.00003199
0.35	0.01465456	0.00385760	0.00108229	0.00031614	0.00009495
0.40	0.02204886	0.00663907	0.00213012	0.00071145	0.00024430
0.45	0.03168122	0.01074287	0.00388051	0.00145890	0.00056383
0.50	0.04391158	0.01656391	0.00665348	0.00278115	0.00119488
0.55	0.05913425	0.02456916	0.01086651	0.00499995	0.00236432
0.60	0.07778587	0.03530910	0.01705464	0.00856765	0.00442251
0.65	0.10035524	0.04943238	0.02589729	0.01410700	0.00789439
0.70	0.12739591	0.06770472	0.03825018	0.02246188	0.01354777
0.75	0.15954227	0.09103355	0.05518675	0.03476257	0.02248501
0.80	0.19753060	0.12050054	0.07805176	0.05251087	0.03626637
0.85	0.24222718	0.15740517	0.10853213	0.07769295	0.05707747
0.90	0.29466633	0.20332412	0.14875248	0.11293146	0.08795955
0.95	0.35610314	0.26019362	0.20140701	0.16169523	0.13313099
1.00	0.42808829	0.33042681	0.26994584	0.22859520	0.19844550
1.05	0.51257717	0.41708479	0.35884688	0.31981637	0.29206146
1.10	0.61209425	0.52413450	0.47402008	0.44375176	0.42544959
1.15	0.7298965	0.65685184	0.62344727	0.61202027	0.61496617
1.20	0.87085918	0.82248105	0.81821732	0.84109264	0.88442252
1.25	1.04126986	1.03137452	1.07431942	1.15511564	1.26951913
1.30	1.25110534	1.29910606	1.41596483	1.59113631	1.82605597
1.35	1.51629947	1.65076461	1.88233398	2.20970755	2.64659805
1.40	1.86513593	2.13084990	2.54312266	3.11931577	3.89885561
1.45	2.35574197	2.83075162	3.54170877	4.54419545	5.93221905
1.50	3.14266348	3.99332525	5.25942209	7.08237291	9.68310851

4. Table of $g_n(x)$.

	n=1	n=2	n=3	n=4	n=5
0	0	0	0	0	0
0.05	0.04998611	0.00124948	0.00004165	0.00000156	0.00000006
0.10	0.09988884	0.00499166	0.00033267	0.00002495	0.00000199
0.15	0.14962466	0.01120777	0.00111993	0.00012593	0.00001510
0.20	0.19910969	0.01986643	0.00264529	0.00039644	0.00006339
0.25	0.24825953	0.03092357	0.00514303	0.00096296	0.00019240
0.30	0.29698913	0.04432228	0.00883730	0.00198432	0.00047554
0.35	0.34521257	0.05999261	0.01393946	0.00364881	0.00101960
0.40	0.39284288	0.07785132	0.02064540	0.00617060	0.00196932
0.45	0.43979184	0.09780161	0.02913277	0.00978550	0.00351073
0.50	0.48596981	0.11973274	0.03955806	0.01474588	0.00587306
0.55	0.53128549	0.14351969	0.05205366	0.02131494	0.00932907
0.60	0.57564569	0.16902262	0.06672469	0.02976003	0.01419337
0.65	0.61895510	0.19608642	0.08364572	0.04034516	0.02081853
0.70	0.66111602	0.22454005	0.10285734	0.05332258	0.02958876
0.75	0.70202810	0.25419589	0.12436242	0.06892327	0.04091063
0.80	0.74158801	0.28484897	0.14812226	0.08734640	0.05520068
0.85	0.77968914	0.31627610	0.17405237	0.10874755	0.07286929
0.90	0.81622122	0.34823490	0.20201800	0.13322561	0.09430053
0.95	0.85106993	0.38046269	0.23182926	0.16080823	0.11982735
1.00	0.88411646	0.41267529	0.26323586	0.19143571	0.14970157
1.05	0.91523706	0.44456562	0.29592143	0.22494300	0.18405804
1.10	0.94430247	0.47580210	0.32949720	0.26103974	0.22287203
1.15	0.97117736	0.50602693	0.36349513	0.29928809	0.26590927
1.20	0.99571963	0.53485408	0.39736037	0.33907791	0.31266720
1.25	1.01777971	0.56186701	0.43044275	0.37959907	0.36230669
1.30	1.03719969	0.58661614	0.46198747	0.41981047	0.41357242
1.35	1.05381237	0.60861588	0.49112460	0.45840516	0.46470055
1.40	1.06744018	0.62734130	0.51685727	0.49377114	0.51331151
1.45	1.07789399	0.64222435	0.53804835	0.52394694	0.55628581
1.50	1.08497164	0.65264937	0.55340533	0.54657127	0.58961972
1.55	1.08845634	0.65794809	0.56146307	0.55882560	0.60825784
1.60	1.08811481	0.65739373	0.56056397	0.55736847	0.60589800
1.65	1.08369508	0.65019412	0.54883516	0.53825982	0.57476382
1.70	1.07492392	0.63548370	0.52416200	0.49687358	0.50533841
1.75	1.06150389	0.61231412	0.48415713	0.42779595	0.38605169
1.80	1.04310977	0.57964313	0.42612422	0.32470634	0.20291142
1.85	1.01938439	0.53632130	0.34701496	0.18023726	-0.06093395
1.90	0.98993356	0.48107627	0.24337801	-0.01419199	-0.42571662
1.95	0.95432007	0.41249365	0.11129774	-0.26857388	-0.91567445
2.00	0.91205631	0.32899392	-0.05367990	-0.59455187	-1.55980601
2.05	0.86259532	0.22880419	-0.25663763	-1.00571104	-2.39278945
2.10	0.80531968	0.10992329	-0.50339788	-1.51793432	-3.45611039
2.15	0.73952776	-0.02992161	-0.80066078	-2.14984371	-4.79945943
2.20	0.66441634	-0.19332962	-1.15617727	-2.92335255	-6.48248161
2.25	0.57905852	-0.38329558	-1.57896903	-3.86436531	-8.57699318
2.30	0.48237528	-0.60320962	-2.07961135	-5.00367544	-11.16982165
2.35	0.37309833	-0.85742395	-2.67060199	-6.37813423	-14.36650436
2.40	0.24972079	-1.15050787	-3.36684983	-8.03219650	-18.29617190
2.45	0.11043071	-1.48835687	-4.18633395	-10.02000101	-23.11811670
2.50	-0.04698041	-1.87803015	-5.15100806	-12.40822658	-29.03080231

5. Table of $F_n(x)$.

	n=1	n=2	n=3	n=4
0	0	0	0	0
0.02	0.0000027	0.0000000	0.0000000	0.0000000
0.04	0.0000213	0.0000006	0.0000000	0.0000000
0.06	0.0000720	0.0000032	0.0000001	0.0000000
0.08	0.0001705	0.0000102	0.0000007	0.0000000
0.10	0.0003327	0.0000249	0.0000020	0.0000002
0.12	0.0005743	0.0000517	0.0000050	0.0000005
0.14	0.0009111	0.0000956	0.0000107	0.0000012
0.16	0.0013584	0.0001629	0.0000208	0.0000028
0.18	0.0019315	0.0002605	0.0000375	0.0000056
0.20	0.0026456	0.0003965	0.0000634	0.0000106
0.22	0.0035154	0.0005794	0.0001019	0.0000187
0.24	0.0045558	0.0008190	0.0001571	0.0000314
0.26	0.0057609	0.0011256	0.0002339	0.0000506
0.28	0.0072051	0.0015105	0.0003379	0.0000788
0.30	0.0088420	0.0019855	0.0004758	0.0001188
0.32	0.0107053	0.0025635	0.0006552	0.0001745
0.34	0.0128081	0.0032579	0.0008845	0.0002503
0.36	0.0151632	0.0040826	0.0011734	0.0003515
0.38	0.0177833	0.0050525	0.0015325	0.0004845
0.40	0.0206804	0.0061828	0.0019736	0.0006567
0.42	0.0238663	0.0074895	0.0025097	0.0008766
0.44	0.0273525	0.0089891	0.0031549	0.0011542
0.46	0.0311501	0.0106985	0.0039245	0.0015008
0.48	0.0352696	0.0126353	0.0048351	0.0019290
0.50	0.0397213	0.0148172	0.0059047	0.0024534
0.52	0.0445152	0.0172626	0.0071523	0.0030900
0.54	0.0496606	0.0199902	0.0085984	0.0038568
0.56	0.0551757	0.0230192	0.0102649	0.0047738
0.58	0.0610421	0.0263688	0.0121747	0.0058628
0.60	0.0672952	0.0300588	0.0143524	0.0071481
0.62	0.0739339	0.0341090	0.0168236	0.0086560
0.64	0.0809656	0.0385397	0.0196156	0.0104155
0.66	0.0883976	0.0433712	0.0227568	0.0124579
0.68	0.0962366	0.0486240	0.0262769	0.0148170
0.70	0.1044890	0.0543188	0.0302071	0.0175296
0.72	0.1131608	0.0604765	0.0345798	0.0206350
0.74	0.1222578	0.0671180	0.0394289	0.0241757
0.76	0.1317853	0.0742643	0.0447895	0.0281970
0.78	0.1417482	0.0819365	0.0506980	0.0327475
0.80	0.1521512	0.0901556	0.0571920	0.0378788
0.82	0.1629987	0.0989429	0.0643107	0.0436459
0.84	0.1742948	0.1083194	0.0720941	0.0501074
0.86	0.1860431	0.1183062	0.0805840	0.0573249
0.88	0.1982471	0.1289245	0.0898230	0.0653641
0.90	0.2109100	0.1401952	0.0998551	0.0742940
0.92	0.2240348	0.1521395	0.1107255	0.0841876
0.94	0.2376239	0.1647782	0.1224806	0.0951214
0.96	0.2516799	0.1781322	0.1351681	0.1071760
0.98	0.2662048	0.1922221	0.1488366	0.1204362
1.00	0.2812007	0.2070688	0.1635361	0.1349904

	n=1	n=2	n=3	n=4
1.00	0.2812007	0.2070688	0.1635361	0.1349904
1.02	0.2966691	0.2226927	0.1793176	0.1509316
1.04	0.3126117	0.2391144	0.1962332	0.1683566
1.06	0.3290297	0.2563540	0.2143363	0.1873668
1.08	0.3459242	0.2744320	0.2336810	0.2080678
1.10	0.3632961	0.2933682	0.2543230	0.2305698
1.12	0.3811464	0.3131828	0.2763186	0.2549872
1.14	0.3994754	0.3338954	0.2997254	0.2814393
1.16	0.4182838	0.3555258	0.3246019	0.3100497
1.18	0.4375717	0.3780935	0.3510077	0.3409472
1.20	0.4573395	0.4016180	0.3790034	0.3742648
1.22	0.4775871	0.4261184	0.4086506	0.4101406
1.24	0.4983146	0.4516140	0.4400118	0.4487178
1.26	0.5195216	0.4781236	0.4731506	0.4901443
1.28	0.5412081	0.5056662	0.5081314	0.5345731
1.30	0.5633735	0.5342605	0.5450197	0.5821622
1.32	0.5860175	0.5639249	0.5838819	0.6330751
1.34	0.6091395	0.5946780	0.6247853	0.6874800
1.36	0.6327389	0.6265380	0.6677982	0.7455510
1.38	0.6568151	0.6595231	0.7129897	0.8074670
1.40	0.6813672	0.6936513	0.7604298	0.8734126
1.42	0.7063945	0.7289406	0.8101897	0.9435779
1.44	0.7318961	0.7654087	0.8623411	1.0181584
1.46	0.7578712	0.8030733	0.9169568	1.0975554
1.48	0.7843187	0.8419520	0.9741104	1.1813756
1.50	0.8112377	0.8820621	1.0338766	1.2704316
1.52	0.8386273	0.9234211	1.0963308	1.3647418
1.54	0.8664862	0.9660461	1.1615491	1.4645306
1.56	0.8948135	1.0099542	1.2296088	1.5700279
1.58	0.9236081	1.0551624	1.3005879	1.6814701
1.60	0.9528688	1.1016877	1.3745653	1.7990992
1.62	0.9825945	1.1495469	1.4516208	1.9231638
1.64	1.0127840	1.1987566	1.5318350	2.0539182
1.66	1.0434362	1.2493336	1.6152893	2.1916233
1.68	1.0745500	1.3012944	1.7020661	2.3365461
1.70	1.1061241	1.3546554	1.7922486	2.4889603
1.72	1.1381574	1.4094331	1.8859209	2.6491457
1.74	1.1706487	1.4656438	1.9831678	2.8173889
1.76	1.2035968	1.5233037	2.0840751	2.9939828
1.78	1.2370005	1.5824291	2.1887294	3.1792273
1.80	1.2708587	1.6430360	2.2972183	3.3734288
1.82	1.3051702	1.7051405	2.4096299	3.5769005
1.84	1.3399338	1.7687586	2.5260536	3.7899626
1.86	1.3751483	1.8339063	2.6465794	4.0129422
1.88	1.4108127	1.9005994	2.7712981	4.2461733
1.90	1.4469257	1.9688538	2.9003016	4.4899970
1.92	1.4834864	2.0386854	3.0336824	4.7447618
1.94	1.5204934	2.1101097	3.1715341	5.0108232
1.96	1.5579458	2.1831427	3.3139511	5.2885439
1.98	1.5958425	2.2577999	3.4610284	5.5782942

6. Table of $G_n(x)$.

	n=1	n=2	n=3	n=4
0	0	0	0	0
0.02	0.0200009	0.0002000	0.0000027	0.0000000
0.04	0.0400071	0.0008002	0.0000213	0.0000006
0.06	0.0600240	0.0018011	0.0000720	0.0000032
0.08	0.0800569	0.0032034	0.0001709	0.0000102
0.10	0.1001111	0.0050083	0.0003340	0.0000250
0.12	0.1201919	0.0072173	0.0005776	0.0000520
0.14	0.1403047	0.0098320	0.0009182	0.0000964
0.16	0.1604547	0.0128546	0.0013723	0.0001647
0.18	0.1806472	0.0162874	0.0019565	0.0002643
0.20	0.2008875	0.0201331	0.0026879	0.0004035
0.22	0.2211808	0.0243948	0.0035836	0.0005919
0.24	0.2415325	0.0290758	0.0046609	0.0008400
0.26	0.2619476	0.0341797	0.0059376	0.0011595
0.28	0.2824315	0.0397105	0.0074316	0.0015633
0.30	0.3029892	0.0456723	0.0091613	0.0020653
0.32	0.3236260	0.0520699	0.0111452	0.0026807
0.34	0.3443470	0.0589080	0.0134026	0.0034261
0.36	0.3651573	0.0661917	0.0159526	0.0043191
0.38	0.3860620	0.0739266	0.0188153	0.0053789
0.40	0.4070661	0.0821183	0.0220108	0.0066257
0.42	0.4281746	0.0907730	0.0255600	0.0080815
0.44	0.4493925	0.0998969	0.0294841	0.0097694
0.46	0.4707249	0.1094967	0.0338048	0.0117144
0.48	0.4921765	0.1195791	0.0385443	0.0139427
0.50	0.5137523	0.1301515	0.0437256	0.0164823
0.52	0.5354572	0.1412212	0.0493720	0.0193628
0.54	0.5572959	0.1527959	0.0555075	0.0226154
0.56	0.5792732	0.1648837	0.0621566	0.0262733
0.58	0.6013939	0.1774927	0.0693446	0.0303714
0.60	0.6236626	0.1906315	0.0770974	0.0349465
0.62	0.6460840	0.2043088	0.0854415	0.0400374
0.64	0.6686628	0.2185337	0.0944041	0.0456849
0.66	0.6914034	0.2333153	0.1040131	0.0519318
0.68	0.7143103	0.2486633	0.1142972	0.0588233
0.70	0.7373881	0.2645873	0.1252857	0.0664066
0.72	0.7606412	0.2810972	0.1370087	0.0747312
0.74	0.7840739	0.2982034	0.1494972	0.0838491
0.76	0.8076906	0.3159162	0.1627829	0.0938147
0.78	0.8314955	0.3342464	0.1768981	0.1046848
0.80	0.8554929	0.3532046	0.1918762	0.1165190
0.82	0.8796870	0.3728022	0.2077512	0.1293793
0.84	0.9040819	0.3930502	0.2245582	0.1433307
0.86	0.9286816	0.4139603	0.2423329	0.1584408
0.88	0.9534902	0.4355442	0.2611120	0.1747803
0.90	0.9785116	0.4578137	0.2809330	0.1924228
0.92	1.0037499	0.4807808	0.3018343	0.2114448
0.94	1.0292088	0.5044580	0.3238552	0.2319262
0.96	1.0548921	0.5288576	0.3470360	0.2539499
0.98	1.0808037	0.5539922	0.3714179	0.2776023
1.00	1.1069773	0.5798747	0.3970427	0.3029730

	n=1	n=2	n=3	n=4
1.00	1.1069473	0.5798747	0.3970427	0.3029730
1.02	1.1333263	0.6065180	0.4239537	0.3301553
1.04	1.1599446	0.6339352	0.4521948	0.3592459
1.06	1.1868056	0.6621397	0.4818108	0.3903450
1.08	1.2139129	0.6911448	0.5128477	0.4235569
1.10	1.2412698	0.7209643	0.5453522	0.4589893
1.12	1.2688797	0.7516117	0.5793723	0.4967541
1.14	1.2967460	0.7831011	0.6149567	0.5369671
1.16	1.3248720	0.8154464	0.6521552	0.5797482
1.18	1.3532608	0.8486617	0.6910186	0.6252212
1.20	1.3819156	0.8827614	0.7315987	0.6735144
1.22	1.4108396	0.9177599	0.7739484	0.7247605
1.24	1.4400358	0.9536717	0.8181214	0.7790964
1.26	1.4695072	0.9905114	0.8641726	0.8366636
1.28	1.4992568	1.0282938	0.9121578	0.8976081
1.30	1.5292874	1.0670337	0.9621340	0.9620807
1.32	1.5596019	1.1067463	1.0141590	1.0302369
1.34	1.5902032	1.1474464	1.0682918	1.1022371
1.36	1.6210938	1.1891493	1.1245924	1.1782466
1.38	1.6522767	1.2318702	1.1831218	1.2584357
1.40	1.6837543	1.2756246	1.2439421	1.3429797
1.42	1.7155292	1.3204277	1.3071164	1.4320594
1.44	1.7476041	1.3662953	1.3727088	1.5258607
1.46	1.7799814	1.4132429	1.4407845	1.6245747
1.48	1.8126635	1.4612861	1.5114099	1.7283983
1.50	1.8456528	1.5104407	1.5846522	1.8375337
1.52	1.8789518	1.5607226	1.6605797	1.9521848
1.54	1.9125625	1.6121476	1.7392619	2.0725772
1.56	1.9464874	1.6647317	1.8207692	2.1989183
1.58	1.9807287	1.7184910	1.9051732	2.3314375
1.60	2.0152884	1.7734414	1.9925464	2.4703659
1.62	2.0501686	1.8295992	2.0829625	2.6159409
1.64	2.0853716	1.8869805	2.1764961	2.7684059
1.66	2.1206992	1.9456016	2.2732229	2.9280106
1.68	2.1567534	2.0054787	2.3732198	3.0950109
1.70	2.1929362	2.0666282	2.4765646	3.2696693
1.72	2.2294495	2.1290665	2.5833362	3.4522544
1.74	2.2662951	2.1928099	2.6936145	3.6430418
1.76	2.3034748	2.2578750	2.8074805	3.8423135
1.78	2.3409904	2.3242781	2.9250164	4.0503581
1.80	2.3788436	2.3920359	3.0463051	4.2674713
1.82	2.4170361	2.4611649	3.1714309	4.4933554
1.84	2.4555695	2.5316817	3.3004788	4.7301196
1.86	2.4944455	2.6036028	3.4335353	4.9762810
1.88	2.5336656	2.6769450	3.5706876	5.2327626
1.90	2.5732314	2.7517249	3.7120239	5.4998953
1.92	2.6131443	2.8279591	3.8576337	5.7780173
1.94	2.6534058	2.9056644	4.0076074	6.0674739
1.96	2.6940173	2.9848575	4.1620364	6.3686180
1.98	2.7349803	3.0655551	4.3210133	6.6818101