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REKENAFDELING

The Expansion of the Earth's Topography in a Series of Spherical Harmonics

by

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Summary. The Earth's topography is expanded in a series of spherical harmonics up to the 31st degree, thus extending the work of Prey, who gave an expansion up to the 16th degree. The expansion coefficients have been calculated for both the lithosphere and the hydrosphere. The calculations were based on 40824 points on the sphere, where the elevations were read from the most up-to-date maps available. These elevations all entered into the calculations, no smoothing procedure was applied. *)

1. Introduction. In geophysics the problem arises how to find some analytic expression for a function which is experimentally determined over the surface of the Earth. Assuming that the shape of the Earth may be approximated by a sphere, this can be achieved by expanding the function in a series of spherical harmonics. Such an expression for the topography of the Earth has been given by Prey [1].

A reconsideration of Prey's work seemed necessary for the following reasons:

i) Since 1921 much more information has been gathered concerning the depth of the oceans.

ii) In Prey's expansion the highest degree of the spherical harmonics is 16 which is too low for some applications.

iii) If is questionable whether Prey's evaluation of the higher expansion coefficients is meaningful.

The question of improving on Prey's work was raised by Professor F.A. Vening Meinesz in connection with his theory of the origin of the continents, based on convection currents in the Earth's interior [2], [3]. For this he needed certain invariants which can be found from the expansion coefficients. In fact the investigations which led to the present paper were carried out on his request.

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*) Research carried out under the direction of Prof.Dr. Ir. A. van Wijngaarden.

2. Formulas. If the function $F(\vartheta,\varphi)$ on a sphere, where ϑ and φ are spherical coordinates defined by $x=r\sin\vartheta\cos\varphi$, $y=r\sin\vartheta\sin\varphi$, $z=r\cos\vartheta$, satisfies certain conditions, it can be expanded in a series of spherical harmonics

$$(2.1)\qquad F(\vartheta,\varphi)=\sum_{n=0}^{\infty}F_n(\vartheta,\varphi),$$

$$F_n(\vartheta,\varphi)=\sum_{m=0}^{n}(A_n^m\cos m\varphi+B_n^m\sin m\varphi)\overline{P}_n^m(\cos\vartheta).$$

The functions $\overline{P}_n^m(\cos\vartheta)$ are defined by

$$(2.2)\qquad \overline{P}_n^m(\cos\vartheta)=\left[\frac{2n+1}{2}\,\frac{(n-m)!}{(n+m)!}\right]^{\frac{1}{2}}P_n^m(\cos\vartheta)$$

where the $P_n^m(\cos\vartheta)$ are associated Legendre functions according to the usual definition

$$(2.3)\qquad P_n^m(x)=\frac{(-1)^m}{2^n n!}(1-x^2)^{m/2}\left(\frac{d}{dx}\right)^{n+m}(x^2-1)^n,\quad x=\cos\vartheta.$$

The multiplicative factor in (2.2) is chosen such that we have the normalization

$$(2.4)\qquad \int_0^{\pi}\left[\overline{P}_n^m(\cos\vartheta)\right]^2\sin\vartheta\,d\vartheta=1.$$

The coefficients $A_n^m.B_n^m$ follow from

$$(2.5)\qquad A_n^m=\int_0^{\pi}c_m(\vartheta)\overline{P}_n^m(\cos\vartheta)\sin\vartheta\,d\vartheta,\quad B_n^m\int_0^{\pi}s_m(\vartheta)\overline{P}_n^m(\cos\vartheta)\sin\vartheta\,d\vartheta,$$

where $c_m(\vartheta)$, $s_m(\vartheta)$ are Fourier coefficients on a parallel circle

$$(2.6)\qquad c_m(\vartheta)=\frac{1}{\pi}\int_0^{2\pi}F(\vartheta,\varphi)\cos m\varphi\,d\varphi,\quad s_m(\vartheta)=\frac{1}{\pi}\int_0^{2\pi}F(\vartheta,\varphi)\sin m\varphi\,d\varphi,\quad (m\neq 0),$$

$$c_0(\vartheta)=\frac{1}{2\pi}\int_0^{2\pi}F(\vartheta,\varphi)d\varphi\,.$$

If we represent the function $\overline{P}_n^m(\cos\vartheta)$ by the trigonometrical series

$$(2.7)\quad \overline{P}_n^{\,m}(\cos\vartheta) = \sum_k a^m_{n,k}\cos k\vartheta \quad (m \text{ even}), \qquad \overline{P}_n^{\,m}(\cos\vartheta) = \sum_k b^m_{n,k}\sin k\vartheta \quad (m \text{ odd})$$

and if we define the integrals

$$(2.8)\quad (cc)_{km} = \int_0^\pi c_m(\vartheta)\cos k\vartheta\,\sin\vartheta\,d\vartheta, \quad (cs)_{km} = \int_0^\pi s_m(\vartheta)\cos k\vartheta\,\sin\vartheta\,d\vartheta,$$

$$(sc)_{km} = \int_0^\pi c_m(\vartheta)\sin k\vartheta\,\sin\vartheta\,d\vartheta, \quad (ss)_{km} = \int_0^\pi s_m(\vartheta)\sin k\vartheta\,\sin\vartheta\,d\vartheta,$$

we find from (2.5)

$$(2.9)\quad A_n^{\,m} = \sum_k a^m_{n,k}(cc)_{km}, \quad B_n^{\,m} = \sum_k a^m_{n,k}(cs)_{km} \quad (m \text{ even}),$$

$$A_n^{\,m} = \sum_k b^m_{n,k}(sc)_{km}, \quad B_n^{\,m} = \sum_k b^m_{n,k}(ss)_{km} \quad (m \text{ odd}).$$

The summation index k in (2.7) and (2.9) runs through the values $k=0(2)n$ for n even, $k=1(2)n$ for n odd.
If two functions $F(\vartheta,\varphi)$ and $F^*(\vartheta,\varphi)$ are expanded in the same system of spherical harmonics according to (2.1), the following important quantity can be defined

$$(2.10)\qquad Q_n(F,F^*) = \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^\pi d\vartheta\, F_n(\vartheta,\varphi)F_n^*(\vartheta,\varphi).$$

Using formula (2.4) and the orthogonality relations of spherical harmonics one can express this integral in terms of $A_n^{\,m}$, $B_n^{\,m}$, $A_n^{\,m*}$ and $B_n^{\,m*}$ by

$$(2.11)\qquad Q_n(F,F^*) = \frac{1}{4}\left[2A_n^{\,o}A_n^{\,o*} + \sum_{m=1}^{n}(A_n^{\,m}A_n^{\,m*} + B_n^{\,m}B_n^{\,m*})\right].$$

It has been shown by one of us [4] that this quantity is independent of the coordinate system on the sphere. By substitution in (2.10) of $F=F^*=\alpha F_1+\beta F_2$ one finds the useful result

$$(2.12)\qquad Q_n(\alpha F_1+\beta F_2,\ \alpha_1 F_1+\beta_2 F_2) =$$
$$= \alpha^2 Q_n(F_1,F_1) + 2\alpha\beta\ Q_n(F_1,F_2) + \beta^2 Q_n(F_2,F_2).$$

3. <u>Computation</u>. We calculated the coefficients A_n^m and B_n^m defined by (2.1), for n=0(1)31, m=0(1)n on the electronic computer ARMAC for both the lithosphere $F_{lith.}(\vartheta,\varphi)$ and the hydrosphere $F_{hydr.}(\vartheta,\varphi)$. The latter was obtained from the former by replacing all positive elevations by zero.

The coordinate system on the Earth was chosen as follows:

$\vartheta = \pi/2$ - Northern Latitude (Northern hemisphere)

$\vartheta = \pi/2$ + Southern Latitude (Southern hemisphere)

φ = Eastern Longitude relative to Greenwich (Eastern hemisphere)

$\varphi = 2\pi$ - Western Longitude (Western hemisphere).

The elevations $F_{i,j} = F(\vartheta_i, \varphi_j^{(i)})$ were given at the points

$$\vartheta_i = \frac{\pi}{750}(3+4i),\ i=0(1)186; \qquad \varphi_j^{(i)} = \pi\left(\frac{2j-1}{N_i} + \frac{1}{72}\right),\ j=1(1)N_i .$$

Thus we had 187 parallels with the approximate zeros of $P_{187}(\cos\vartheta)$ as ϑ_i, each with a number of points N_i varying from 72 near the poles to 360 on the equator. The total number 40,824 of points that entered into the calculations was distributed as in the following list.

N_i	72	144	216	288	360
i	0-25 161-186	26-40 146-160	41-50 136-145	51-65 121-135	66-120

In the neighbourhood of the poles where no precise data were known the elevations were estimated.

In order to facilitate the calculation of the c_m and s_m of (2.6) we introduced a new variable $\bar{\varphi} = \varphi - \alpha_i$, with $\alpha_i = \pi(1/72 - 1/N_i)$. On a parallel $\bar{\varphi}$ takes the discrete values $\bar{\varphi}_j^{(i)} = 2\pi\, j/N_i$, j=1(1)$N_i$ so that for all N_i we have $\bar{\varphi}_{N_i}^{(i)} = 2\pi$.

We can derive from (2.6), using that for a periodic function $f(\varphi)$ holds $\int_0^{2\pi} f(\varphi)d\varphi = \int_{\alpha_i}^{2\pi+\alpha_i} f(\varphi)d\varphi$,

$$(3.1)\qquad c_m(\vartheta) = \bar{c}_m(\vartheta)\cos m\alpha_i - \bar{s}_m(\vartheta)\sin m\alpha_i ,$$

$$s_m(\vartheta) = \bar{s}_m(\vartheta)\cos m\alpha_i + \bar{c}_m(\vartheta)\sin m\alpha_i ,$$

where $\overline{c}_m$ and $\overline{s}_m$ are the shifted Fourier coefficients

$$(3.2)\quad \overline{c}_m(\vartheta) = \frac{1}{\pi}\int_0^{2\pi} F(\vartheta, \overline{\varphi}+\alpha_i)\cos m\overline{\varphi}\, d\overline{\varphi},$$

$$\overline{s}_m(\vartheta) = \frac{1}{\pi}\int_0^{2\pi} F(\vartheta, \overline{\varphi}+\alpha_i)\sin m\overline{\varphi}\, d\overline{\varphi}, \qquad m\neq 0,$$

$$\overline{c}_o(\vartheta) = \frac{1}{2\pi}\int_0^{2\pi} F(\vartheta, \overline{\varphi}+\alpha_i)\, d\overline{\varphi}.$$

These coefficients for m=1(1)31 were computed for each parallel, approximating (3.2) by

$$(3.3)\quad \overline{c}_m(\vartheta_i) = \frac{2}{N_i}\sum_{j=1}^{N_i} F_{i,j}\cos\frac{2\pi mj}{N_i},$$

$$\overline{s}_m(\vartheta_i) = \frac{2}{N_i}\sum_{j=1}^{N_i} F_{i,j}\sin\frac{2\pi mj}{N_i}, \qquad m\neq 0,$$

$$\overline{c}_o(\vartheta_i) = \frac{1}{N_i}\sum_{j=1}^{N_i} F_{i,j}.$$

Application of (3.1) then gave us the $c_m(\vartheta_i)$ and $s_m(\vartheta_i)$.
Next we computed for k=0(1)31 the integrals (2.8); for even m the $(cc)_{km}$ and $(cs)_{km}$, for odd m the $(sc)_{km}$ and $(ss)_{km}$. The Legendre-Gauss quadrature formula for p ordinates reads

$$(3.4)\quad \int_0^{\pi} f(\vartheta)\sin\vartheta\, d\vartheta = \sum_{i=0}^{p-1} W_i\, f(\vartheta_i)\sin\vartheta_i$$

where the abcsissae ϑ_i are the zeros of $P_p(\cos\vartheta_i)$ and the weights are determined by $W_i=2\sin^2\vartheta_i/p^2\left[P_{p-1}(\cos\vartheta_i)\right]^2$.

For large odd values of p one can derive the approximation

$$(3.5)\quad \vartheta_i = \frac{(3+4i)\pi}{2(2p+1)}, \qquad W_i = \frac{2\pi}{2p+1}.$$

We used (3.5) with p=187 so that the integrals (2.8) were computed by means of the formula

$$(3.6)\quad \int_0^{\pi} f(\vartheta)\sin\vartheta\, d\vartheta = \frac{2\pi}{375}\sum_{i=0}^{186} f(\vartheta_i)\sin\vartheta_i.$$

We chose p=187 with a view to obtaining a ϑ-interval of about one degree, exactly representable in not too many figures. In fact we had $\Delta\vartheta = 0\overset{\circ}{.}96$.
The coefficients A_n^m and B_n^m were then calculated according to (2.9). A table of the $a_{n,k}^m$ and $b_{n,k}^m$ occurring in these formulas and a description of the method by which they were calculated is given in a separate report [5].
From the two sets of coefficients, for lithosphere and hydrosphere, we finally evaluated by aid of (2.11) for n=0(1)31 the quantities $Q_n(F_{\text{lith.}}, F_{\text{lith.}})$, $Q_n(F_{\text{hydr.}}, F_{\text{hydr.}})$ and $Q_n(F_{\text{lith.}}, F_{\text{hydr.}})$.

4. Discussion. Comparing Prey's computations with ours we can make the following remarks.

i) It took Prey several years to obtain expansion coefficients of degrees up to 16. Since the amount of work - particularly the number of multiplications - is about proportional to the third power of the highest degree to be calculated, only the use of an electronic computer made it possible to proceed to degrees nearly twice as large.

ii) The normalization (2.2) of the Legendre functions enabled us to work with fixed-point numbers as no factorials occurred in our computation. Multiplication of Prey's coefficients A_n^m and B_n^m by $\left[\frac{2}{n+1}\frac{(n+m)!}{(n-m)!}\right]^{\frac{1}{2}}$ makes them comparable with our values.

iii) Prey calculated on each parallel 32 Fourier coefficients from 32 ordinates. For higher orders, however, these numbers are trigonometrical interpolation coefficients rather than the coefficients of the unlimited Fourier expansion. This is due to the fact that the values numerically obtained are linear composita of the exact coefficients. With N ordinates one has (cf. e.g. Manley [6]) $(c_m)_{\text{numerical}} = c_m + \sum_{k=1}^{\infty}(c_{kN-m} + c_{kN+m})$. Choosing as an example N=32, m=15, one finds $(c_{15})_{\text{numerical}} = c_{15} + c_{17} \ldots$, which is an obviously wrong result for Fourier series as slowly convergent as it turns out to be in the present case. An analogous objection will hold for the ϑ-direction where Prey calculated coefficients up to 16^{th} degree from only 17 ordinates.

In our calculations the parallels near the equator contained 360 points from which we derived only 63 Fourier coefficients. The parallels nearer to the poles contained less points, it is true, but the resulting coefficients are only used after having been multiplied by the factor sin ϑ_i. Actually the number of points on each parallel is roughly equal to 360 sin ϑ_i. Again, from 187 parallels we calculated only coefficients up to the 31st degree.

iv) In order to restrict the number of data entering into the calculations one might apply a smoothing technique. It may be remarked that the expansion coefficients of a function $f(\vartheta,\varphi)$ obtained from the original function $F(\vartheta,\varphi)$ by any smoothing procedure are not identical with the corresponding coefficients of $F(\vartheta,\varphi)$ but in general are functions of all of them. This deviation grows with increasing order and can become considerable. The required coefficients are to be found from a set of equations by a rather laborious calculation. It may be objected to Prey's work that although he applies a smoothing procedure to his data he omits this last step. The capacity of the ARMAC was sufficient to handle all original data and we have preferred the simpler program above reducing the number of data.

For comparison we give here, for degrees up to n=4, a list of Prey's coefficients of the lithosphere reduced to our normalization, together with our values. Further we plotted the topography of the equator from the original data and compared it with reconstructions from Prey's coefficients (fig.1), from our coefficients up to n=16 (as high as Prey, fig.2) and from all our coefficients (fig.3).

5. Acknowledgements. We express our thanks to Professor F.A. Vening Meinesz for suggesting the investigation and for his unabated interest in the progress of the computations, to Professor A. van Wijngaarden for many stimulating discussions on the numerical aspects of the work, to Professor G.J. Bruins and the Geodetical Institute at Delft for supplying us with the data, to Mr H.J. Slettenhaar for technical assistance in operating the ARMAC and finally to all other members of the Computation Department of the Mathematical Centre who took part in the investigation.

n	m	Prey		H.,P.-A. and P.	
		A_n^m	B_n^m	A_n^m	B_n^m
0	0	-3473		-3348	
1	0	+1032		+ 945	
	1	+1305	+ 767	+1188	+ 800
2	0	+ 717		+ 744	
	1	+ 700	+ 702	+ 688	+ 689
	2	- 809	- 198	- 864	- 190
3	0	- 66		- 203	
	1	- 357	+ 139	- 312	+ 187
	2	- 886	+1066	- 893	+ 937
	3	+ 273	+1144	+ 249	+1149
4	0	+ 438		+ 448	
	1	- 354	- 360	- 441	- 463
	2	- 759	+ 162	- 795	+ 142
	3	+ 768	- 287	+ 762	- 278
	4	- 129	+ 982	- 111	+ 965

Comparison of Prey's and our coefficients.

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Opdracht: R 344
Parameters:
Bewerkt: MP
Controle:

Blad: 1
Datum: jan '60
Datum:

		lithosphere		hydrosphere	
n	m	A(n,m)	B(n,m)	A(n,m)	B(n,m)
		DA 0 ZFO DN			
0	0	-3348	+0	-3679	+0
		DA 496 ZFO DN			
31	0	+11	+0	+11	+0
	1	+8	-49	+9	-50
	2	-13	+20	-19	-3
	3	-5	0	-2	+9
	4	+75	-31	+57	-28
	5	-37	+47	-35	+36
	6	-23	+8	-14	+4
	7	-30	+10	-33	-1
	8	+6	+9	-10	+7
	9	+9	0	-6	-18
	10	-23	-20	-40	-3
	11	-15	+43	-12	+40
	12	-36	-12	-15	-7
	13	-11	+14	-15	+5
	14	-32	-15	-32	-21
	15	+6	+15	+4	+23
	16	-24	+38	-20	+26
	17	-56	+47	-73	+45
	18	-36	+13	-37	+6
	19	-1	+24	-3	+4
	20	-4	+4	0	-3
	21	-17	-9	-7	-22
	22	-3	-62	+17	-39
	23	+12	+27	+5	+27
	24	+16	-16	+20	-10
	25	-60	+4	-54	+26
	26	-6	+22	0	+28
	27	+16	+18	0	+4
	28	+29	+38	+29	+36
	29	+10	-40	+30	-42
	30	-3	-24	-4	-10
	31	-8	+5	-8	-3

		lithosphere		hydrosphere	
n	m	A(n,m)	B(n,m)	A(n,m)	B(n,m)
		DA 1 ZFO DN			
1	0	+945	+0	+867	+0
	1	+1188	+800	+1052	+646
		DA 465 ZFO DN			
30	0	+57	+0	+41	+0
	1	-38	+50	-24	+75
	2	-43	+15	-38	+29
	3	-40	-22	-70	-25
	4	-13	+5	+7	+8
	5	-48	-6	-38	-6
	6	0	+19	0	+17
	7	+19	-23	+20	-36
	8	-12	+65	-24	+52
	9	+20	-58	-2	-42
	10	0	-75	-23	-52
	11	-76	-26	-68	-29
	12	+10	-52	+11	-26
	13	+20	+29	+17	+18
	14	+7	+21	+3	+19
	15	-17	-34	-24	-33
	16	-4	+27	-4	+14
	17	-45	+25	-41	+18
	18	-46	+11	-38	+1
	19	-52	+3	-35	0
	20	-3	+50	0	+28
	21	-8	-46	+11	-40
	22	-40	-47	-43	-25
	23	+34	-51	+12	-40
	24	-2	+10	+3	+13
	25	+24	-91	+9	-60
	26	+14	+111	-6	+73
	27	+32	+63	+53	+48
	28	-44	-25	-30	-25
	29	+16	+53	-4	+50
	30	+29	+23	+33	+18

		lithosphere		hydrosphere	
n	m	A(n,m)	B(n,m)	A(n,m)	B(n,m)
		DA 3 ZFO DN			
2	0	+744	+0	+636	+0
	1	+688	+689	+681	+522
	2	-864	-190	-723	-242
		DA 435 ZFO DN			
29	0	-34	+0	-19	+0
	1	-16	-14	-22	-30
	2	+4	+4	+8	-26
	3	-26	-5	-33	-2
	4	+4	-63	+9	-71
	5	-29	+41	-27	+27
	6	+14	+17	+36	+2
	7	+20	-3	+12	+6
	8	-50	-8	-46	-5
	9	-18	-17	-36	-10
	10	+63	-46	+43	-27
	11	-33	+30	-24	+43
	12	-17	-28	-26	-16
	13	-43	-20	-46	-34
	14	-27	-18	-24	-15
	15	-16	-35	-20	-38
	16	+36	-44	+34	-41
	17	+33	-6	+42	-7
	18	+34	-19	+27	-11
	19	0	-28	0	-29
	20	+25	+30	+35	+22
	21	-18	-13	-16	-11
	22	+7	+17	-14	+8
	23	-27	+6	-8	-12
	24	-62	+38	-59	+47
	25	+41	+27	+40	+20
	26	-36	-90	+9	-56
	27	-1	-6	-6	-3
	28	-19	+18	-34	+20
	29	+79	-53	+65	-50

		lithosphere		hydrosphere	
n	m	A(n,m)	B(n,m)	A(n,m)	B(n,m)
		DA 6 ZFO DN			
3	0	-203	+0	-30	+0
	1	-312	+187	-243	+106
	2	-893	+937	-681	+829
	3	+249	+1149	+275	+1075
		DA 406 ZFO DN			
28	0	-24	+0	-42	+0
	1	-42	+4	-42	+3
	2	-17	-26	-1	0
	3	-16	+28	+13	+18
	4	-6	+7	+12	-5
	5	+23	+8	+24	+3
	6	-27	+20	-21	+7
	7	-10	+41	-3	+21
	8	-27	-3	-12	-10
	9	+40	-21	+32	-12
	10	+2	+24	+2	+24
	11	-10	+24	+5	+57
	12	-1	+91	0	+90
	13	+3	+31	+14	+32
	14	-20	+7	-27	+16
	15	-44	-6	-58	-5
	16	+23	-46	+12	-40
	17	+67	-30	+54	-17
	18	-16	-6	-22	-10
	19	-9	+64	-29	+60
	20	0	+6	+5	+1
	21	+25	+64	+30	+53
	22	-22	-40	-10	-44
	23	+32	+7	+40	-19
	24	-26	-101	-11	-82
	25	+24	-113	+19	-91
	26	+77	+12	+53	+2
	27	-19	-1	-13	0
	28	+41	+9	+27	+12

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Controle:

Blad: 2
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Datum:

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 10 ZFO DN			
4	0	+448	+0	+395	+0
	1	−441	−463	−418	−319
	2	−795	+142	−597	+127
	3	+762	−278	+706	−143
	4	−111	+965	−143	+824

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 378 ZFO DN			
27	0	+2	+0	+11	+0
	1	+4	+68	0	+30
	2	+9	+36	+4	+23
	3	−3	+4	+10	−11
	4	−6	−6	−1	−15
	5	0	+42	−24	+16
	6	+3	+39	+7	+24
	7	−79	−17	−65	−22
	8	−5	+14	0	+17
	9	−40	+33	−27	+10
	10	−5	+3	+8	+12
	11	−24	−32	−19	−6
	12	−91	−41	−69	−44
	13	+79	+3	+41	+15
	14	+54	+17	+39	+16
	15	−78	+28	−21	+31
	16	−17	+74	−28	+75
	17	+35	+24	+29	+24
	18	+1	+18	+1	+11
	19	−34	−39	−39	−33
	20	−33	−7	−32	−10
	21	−24	+1	−14	+20
	22	−17	−43	−29	−52
	23	−33	−36	−19	−28
	24	−32	−85	−35	−71
	25	−2	+74	−12	+41
	26	−66	−94	−47	−64
	27	+1	+3	+2	+2

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 15 ZFO DN			
5	0	−762	+0	−576	+0
	1	−94	−205	−101	−107
	2	+1	−285	+32	−238
	3	+267	+85	+193	+205
	4	+1040	−144	+894	−48
	5	−97	+420	−61	+315

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 351 ZFO DN			
26	0	+48	+0	+18	+0
	1	−42	−94	−40	−78
	2	−45	+25	−46	+49
	3	+42	+46	+47	+27
	4	−21	−59	−20	−47
	5	0	+64	+7	+57
	6	−24	−16	−40	−18
	7	−65	−2	−54	−22
	8	−13	+29	−32	+5
	9	−38	0	−28	−13
	10	−80	−35	−74	−13
	11	−10	−38	−3	−53
	12	−64	−106	−36	−107
	13	−65	+2	−54	+7
	14	+10	+10	+19	−6
	15	−110	+43	−89	+46
	16	−27	+24	−25	+36
	17	−58	−14	−45	−28
	18	−50	−3	−46	+10
	19	−14	−40	−15	−28
	20	+52	−41	+27	−38
	21	−35	−36	−40	−15
	22	+112	+32	+90	+31
	23	+8	+77	+11	+43
	24	+67	+111	+82	+92
	25	−14	−56	−10	−29
	26	−36	+34	−28	+33

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 21 ZFO DN			
6	0	+323	+0	+166	+0
	1	+39	−283	+40	−135
	2	+75	−292	−12	−242
	3	+121	+363	+54	+387
	4	+402	−359	+268	−299
	5	−247	−395	−176	−420
	6	+64	+54	+117	+62

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 325 ZFO DN			
25	0	−41	+0	−22	+0
	1	+4	−19	+4	−20
	2	+48	+23	+23	+4
	3	−52	−29	−86	−25
	4	+37	−6	+15	−13
	5	−61	−13	−63	−18
	6	+6	+18	0	0
	7	+45	−33	+28	−29
	8	+68	+11	+51	+3
	9	−101	+77	−89	+63
	10	+34	−49	+11	−47
	11	−17	−33	−10	−57
	12	−34	+26	−12	+28
	13	−12	−89	−11	−90
	14	−60	+7	−38	−10
	15	+4	−27	+13	−14
	16	−16	−82	−9	−71
	17	−24	−30	−11	−27
	18	+32	−49	+22	−48
	19	+26	+19	+7	+10
	20	−83	−39	−90	−42
	21	+21	+3	−5	+15
	22	−124	+51	−92	+50
	23	+56	−13	+76	−3
	24	−6	−3	+6	+17
	25	+46	−8	+46	−7

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 28 ZFO DN			
7	0	−401	+0	−373	+0
	1	+34	+294	−30	+250
	2	+331	+46	+168	+82
	3	−170	+24	−214	−6
	4	−527	+65	−568	+55
	5	+33	−24	+59	−114
	6	−110	−215	−58	−208
	7	−28	−189	+3	−122

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 300 ZFO DN			
24	0	−25	+0	−35	+0
	1	−51	−10	−45	+14
	2	−12	−71	+4	−58
	3	−77	+35	−40	+64
	4	−3	0	+4	−4
	5	−42	−50	+1	−25
	6	+31	−29	+18	−43
	7	−26	+5	−20	−18
	8	+79	−31	+54	−56
	9	+5	−42	+6	−31
	10	+60	+59	+29	+51
	11	+79	−21	+80	−21
	12	+18	−10	+12	−5
	13	−9	+6	−13	−6
	14	−47	−48	−33	−59
	15	+66	−89	+62	−82
	16	+32	−60	+29	−66
	17	−82	−27	−76	−31
	18	+26	+16	+33	+10
	19	−77	−26	−74	−27
	20	−66	+38	−45	+28
	21	+9	+93	+5	+103
	22	+100	−4	+97	+29
	23	+24	0	+38	−1
	24	+46	−8	+43	−11

MATHEMATISCH CENTRUM
REKENAFDELING
2e Boerhaavestraat 49 - Telefoon 56643
AMSTERDAM-O.

Opdracht: R 344
Parameters:
Bewerkt: mB
Controle:

Blad: 3
Datum: Jan '60
Datum:

DA 36 ZFO DN

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
8	0	+5	+0	-79	+0
	1	0	-50	-39	-62
	2	+262	+42	+212	+90
	3	+77	+108	+90	+59
	4	-42	+163	-4	+166
	5	-145	+6	-103	-84
	6	+180	+143	+199	+118
	7	+380	+77	+273	+108
	8	-225	-266	±184	-251

DA 276 ZFO DN

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
23	0	-45	+0	-6	+0
	1	+3	+37	+6	+41
	2	-33	+59	-22	+32
	3	+15	-27	+23	-12
	4	-57	+34	-32	+33
	5	0	-8	+21	+3
	6	-102	+27	-79	+4
	7	-24	+20	-34	+19
	8	+18	+6	-9	-2
	9	+38	-47	+33	-33
	10	0	-38	-21	-42
	11	+47	+72	+37	+73
	12	+44	+27	+7	+32
	13	-2	+11	-5	-15
	14	+60	-17	+75	-26
	15	+61	+6	+46	-7
	16	-1	+23	0	+3
	17	+31	-46	+38	-35
	18	+10	-22	+7	0
	19	-28	+21	-7	+10
	20	+38	-125	+55	-87
	21	+25	+21	-10	+20
	22	-102	-18	-65	-14
	23	+33	+10	+32	-3

DA 45 ZFO DN

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
9	0	-265	+0	-179	+0
	1	+117	+95	+89	-5
	2	+76	+117	+80	+121
	3	-38	+5	-16	-36
	4	+31	+155	+121	+119
	5	+137	+153	+168	+146
	6	+7	+430	+84	+889
	7	-45	-170	-106	-90
	8	+233	+197	+147	+200
	9	-46	-177	-34	-186

DA 253 ZFO DN

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
22	0	-15	+0	-16	+0
	1	+11	-80	-1	-100
	2	-97	-63	-58	-54
	3	+26	+46	+61	+63
	4	+13	-86	+60	-76
	5	-150	-43	-101	-42
	6	+39	+18	+59	+7
	7	+18	+130	+4	+105
	8	-38	+18	-39	+14
	9	+2	+64	-12	+80
	10	-23	+30	-27	+24
	11	-17	0	-30	+10
	12	+5	-7	-28	-6
	13	+4	+54	+24	+42
	14	+40	+37	+36	+31
	15	+73	-41	+59	-58
	16	-45	-44	-52	-31
	17	+14	-48	+3	-19
	18	+19	-26	+10	-12
	19	+59	-61	+50	-54
	20	-8	+67	+2	+21
	21	-17	+73	-1	+63
	22	-50	+7	-47	0

DA 55 ZFO DN

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
10	0	-82	+0	-68	+0
	1	+156	-45	+127	+4
	2	-345	-203	-233	-183
	3	-198	-174	-177	-102
	4	-185	+15	-91	-32
	5	+60	+58	+66	+76
	6	+141	-136	+142	-135
	7	-3	+97	-33	+162
	8	+161	-172	+153	-174
	9	+298	-13	+270	-43
	10	+199	-133	+155	-119

DA 231 ZFO DN

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
21	0	-59	+0	-50	+0
	1	+3	+51	+10	+28
	2	-22	-47	-49	-49
	3	+72	+2	+7	-46
	4	+38	-7	+55	-10
	5	-29	+50	-36	+38
	6	-87	+72	-59	+46
	7	+16	-37	+6	-64
	8	-75	-22	-52	-12
	9	0	+13	-5	+11
	10	-9	-12	+7	0
	11	+115	-138	+101	-104
	12	+34	+5	+39	+7
	13	-90	+30	-77	+53
	14	+80	+59	+59	+78
	15	+91	+65	+80	+56
	16	+73	-53	+48	-50
	17	-26	+14	-37	+22
	18	+51	-22	+27	-22
	19	-136	-9	-96	±39
	20	-110	+51	-76	+54
	21	-7	-6	-17	0

DA 66 ZFO DN

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
11	0	-103	+0	-39	+0
	1	+288	-148	+240	-84
	2	-43	-89	-35	-82
	3	-9	±64	-19	+5
	4	+104	-156	+143	-148
	5	+143	+62	+139	+76
	6	+7	+6	-39	+5
	7	-50	-149	-52	-100
	8	+149	+128	+130	+120
	9	-49	+148	-9	+62
	10	+128	+101	+130	+81
	11	-141	±40	-157	-9

DA 210 ZFO DN

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
20	0	+19	+0	+7	+0
	1	+65	-37	+79	-100
	2	+41	+24	+12	+29
	3	-95	+46	-84	+37
	4	+2	-149	+7	-122
	5	-123	+4	-83	-9
	6	+78	+29	+97	+18
	7	-50	+52	-49	+8
	8	-15	+32	-20	+40
	9	+88	-39	+83	-30
	10	-74	-40	-45	-17
	11	-12	-41	-26	-26
	12	-31	-24	-12	-20
	13	+119	-27	+97	+5
	14	-64	-102	-66	-89
	15	-46	-66	-34	-70
	16	-7	-6	+1	-23
	17	-49	+50	-46	+31
	18	+24	-2	+44	+8
	19	-6	+57	-14	+44
	20	+35	-57	+28	-39

MATHEMATISCH CENTRUM
REKENAFDELING
2e Boerhaavestraat 49 - Telefoon 56643
AMSTERDAM-O.

Opdracht: R 344
Parameters:
Bewerkt: MP
Controle:

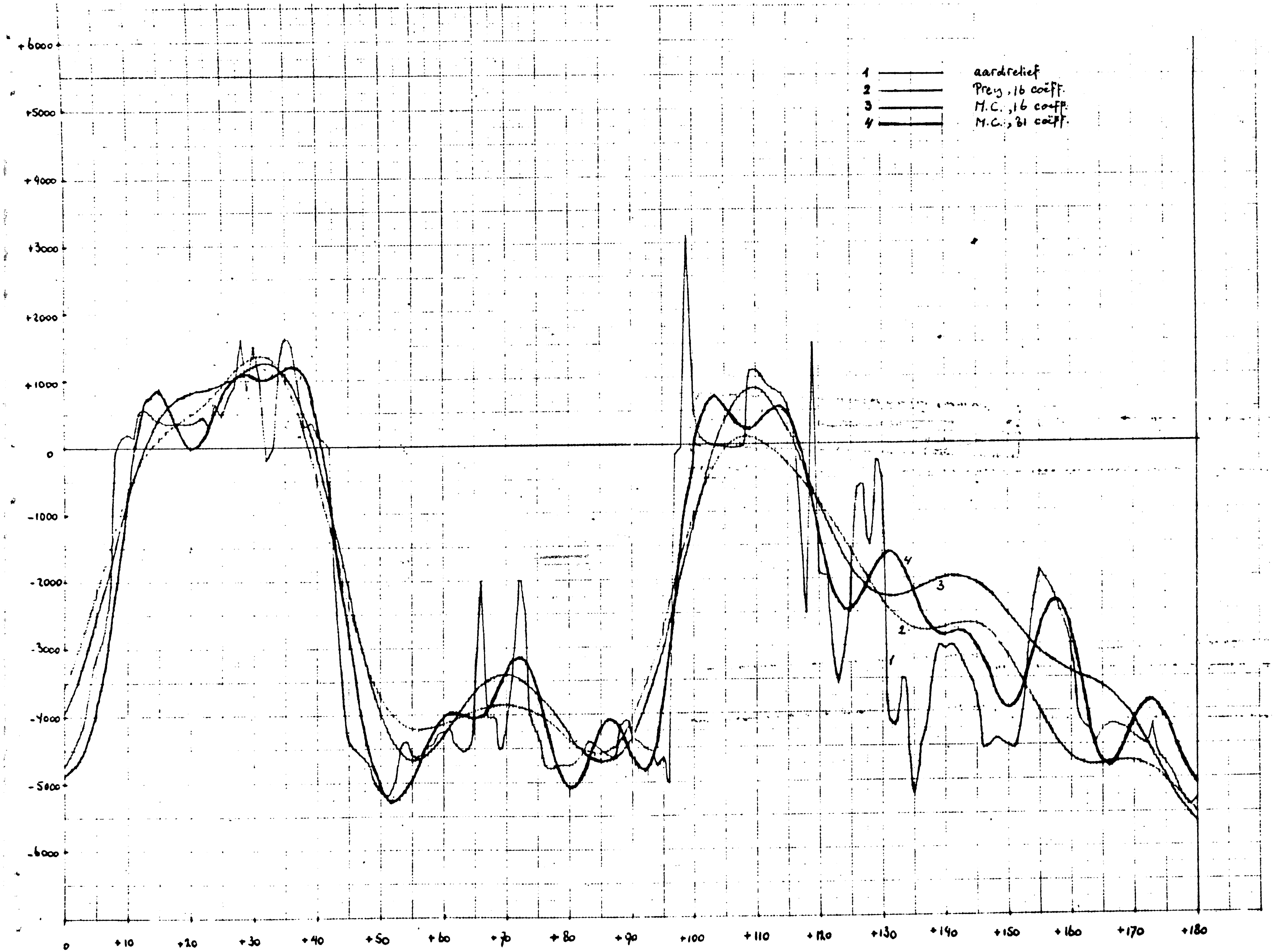
Blad: 4
Datum: Jan 60
Datum:

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 78 ZFC DN			
12	0	−79	+C	−81	+C
	1	+60	−19	+65	+41
	2	−95	+13	−89	+55
	3	−30	−157	+33	−72
	4	−102	+48	−89	+74
	5	−41	−84	−32	−74
	6	+154	−18	+110	−13
	7	−13	−78	+14	−63
	8	−140	+69	−141	+70
	9	+115	+30	+118	−16
	10	−50	−47	−48	−62
	11	+40	+153	+39	+111
	12	−240	−124	−207	−110
		DA 196 ZFO DN			
19	0	−90	−	−103	+C
	1	+20	+6	+34	+31
	2	+16	+57	−30	+37
	3	+16	+11	+10	0
	4	+28	−60	+5	−38
	5	+37	+65	+47	+48
	6	−64	+77	−57	+65
	7	+18	+53	+7	+28
	8	+10	+13	−20	+28
	9	+64	−4	+65	−8
	10	−38	+69	−20	+67
	11	−6	+84	−11	+68
	12	+25	−62	+31	−63
	13	+11	−98	+13	−73
	14	+40	−25	+37	−75
	15	+4	−72	+18	−72
	16	−48	−61	−4	−73
	17	+29	−46	−8	−75
	18	+63	−25	+36	−25
	19	−31	+13	−39	+11

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 91 ZFC DN			
13	0	+107	+C	+66	+C
	1	+79	−11	+75	−17
	2	+2	−169	−85	−133
	3	−163	+101	−124	+93
	4	+2	−25	−10	−6
	5	−5	+68	−16	+49
	6	−63	−54	−62	−39
	7	−20	−143	−3	−180
	8	−42	−1	−5	+17
	9	+5	+209	−21	+179
	10	+33	−192	+26	−156
	11	−141	−146	−161	−95
	12	−90	+96	−74	+43
	13	−161	−9	−133	−26
		DA 171 ZFO DIV			
18	0	−1	+C	+7	+C
	1	+92	−71	+101	−49
	2	+40	+44	+70	+49
	3	−54	−7	+30	+63
	4	−53	−14	−46	−12
	5	−62	+41	−17	+34
	6	−45	−68	−56	−82
	7	−85	+107	−80	+90
	8	+52	+3	+16	−4
	9	−52	+61	−33	+23
	10	−4	+80	+6	+74
	11	−30	−1	−7	−29
	12	+6	−120	+34	−120
	13	+13	−137	+17	−105
	14	+4	−139	+2	−120
	15	−143	−103	−120	−81
	16	+38	+150	+5	+150
	17	−35	−11	−40	−31
	18	+77	−13	+69	−6

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 105 ZFO DN			
14	0	−206	+C	−209	+C
	1	+55	+176	+44	+127
	2	−120	−129	−107	−63
	3	0	−98	+16	−90
	4	+39	−114	+63	−75
	5	−106	−37	−119	−82
	6	−49	+105	−44	+122
	7	+2	−51	−4	−105
	8	−16	+29	+18	+28
	9	+102	+10	+79	−12
	10	+165	−37	+131	−15
	11	+35	−197	+45	−168
	12	+28	−162	+45	−124
	13	+124	−50	+115	−58
	14	−138	−30	−143	−44
		DA 153 ZFO DN			
17	0	−18	+C	−17	+C
	1	−110	−162	−79	−92
	2	−87	−72	−77	−87
	3	+95	+81	+106	+98
	4	−71	+22	−65	−8
	5	−127	+40	−112	+46
	6	−103	+51	−69	+18
	7	−10	−53	−12	−41
	8	+56	+12	+39	+9
	9	−1	−118	+25	−120
	10	−99	−11	−107	0
	11	+6	+120	+29	+102
	12	+87	+182	+93	+163
	13	+157	+30	+127	+8
	14	+15	+150	+6	+167
	15	+109	+61	+74	+32
	16	−164	+95	−118	+86
	17	−102	−93	−89	−61

n	m	lithosphere A(n,m)	lithosphere B(n,m)	hydrosphere A(n,m)	hydrosphere B(n,m)
		DA 120 ZFC DN			
15	0	+90	+C	+75	+C
	1	+46	−113	+42	−136
	2	−33	−4	−32	+17
	3	+43	+187	−15	+140
	4	−158	−70	−119	−52
	5	+12	−37	−4	−60
	6	+65	−96	+72	−81
	7	+111	+93	+96	+74
	8	−35	+106	−12	+89
	9	+22	+88	−5	+92
	10	−55	+43	−72	+37
	11	+32	+97	+39	+109
	12	−92	+139	−75	+107
	13	−73	−51	−42	−28
	14	+126	−117	+141	−75
	15	−9	−5	−26	−4
		DA 136 DFO DV			
16	0	−121	+C	−90	+C
	1	+15	+126	+40	+77
	2	−44	−29	+24	−29
	3	−151	−106	−118	−55
	4	−51	+88	−2	+66
	5	−72	+2	−67	+4
	6	−107	+4	−70	−1
	7	+8	−40	+2	−29
	8	−128	+18	−119	−1
	9	−73	−182	−77	−139
	10	+142	−24	+117	−20
	11	+56	+30	+64	+29
	12	+108	+3	+97	−31
	13	+60	+82	+24	+50
	14	−15	−122	−41	−117
	15	+35	+38	+69	+59
	16	−59	−36	−43	−18



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+170
+180
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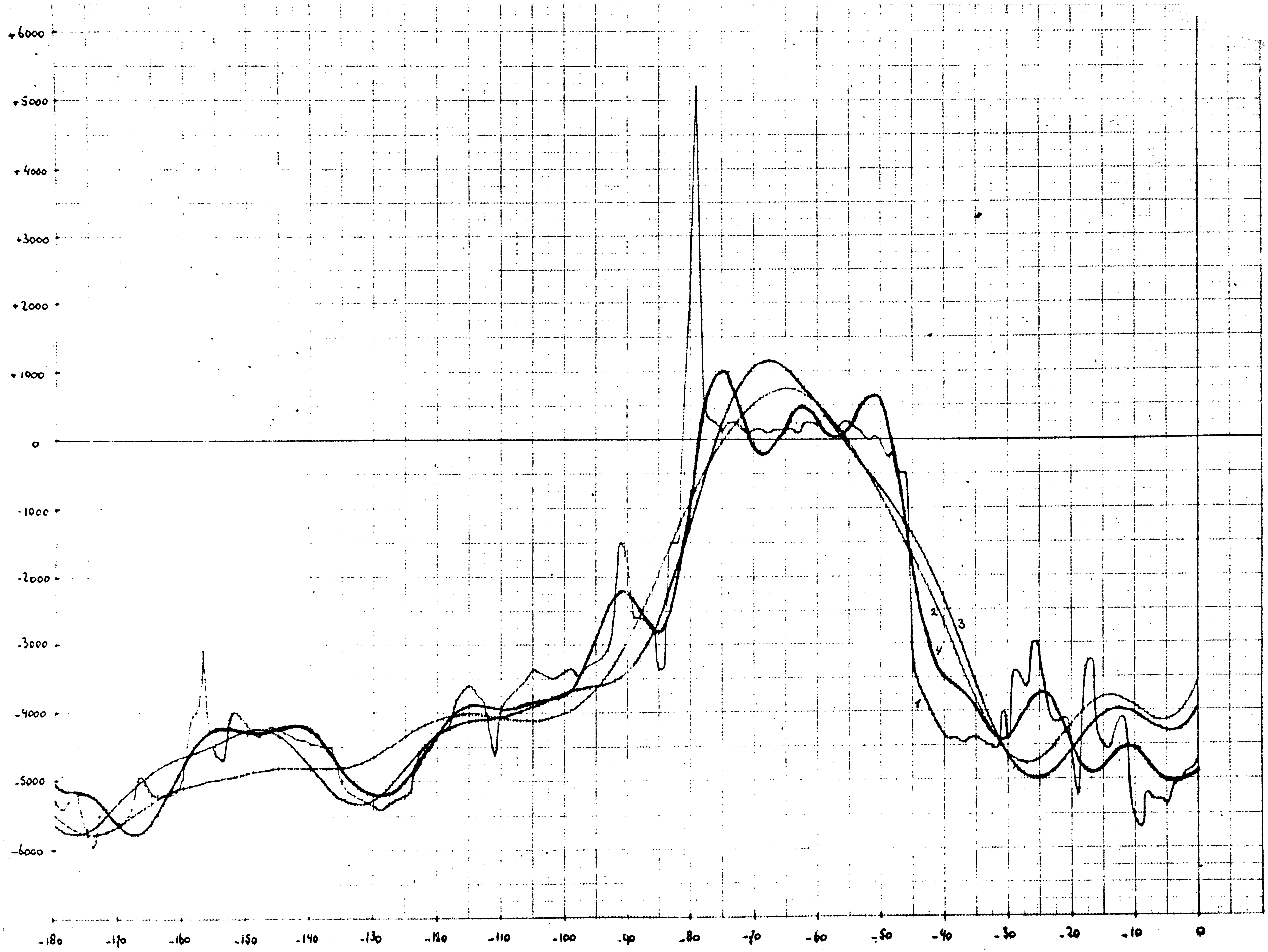
2 Prey, 16 coëff.
3 M.C., 16 coëff.
4 M.C., 31 coëff.



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