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AMSTERDAM

R 487

The Numerical Analysis of the Light-Curve of 12 Lacertae

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1962

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Communication from the Observatory at Utrecht

THE NUMERICAL ANALYSIS OF THE LIGHT-CURVE OF 12 LACERTAE*

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Received June 11, 1962

When this investigation started, two periods were known for 12 Lacertae: $P_1 = 0^d.193\,088\,83$ (STEBBINS (1917); and YOUNG (1925)) and $P_2 = 0^d.197\,358$ (DE JAGER (1953)). A third period, mainly detectable in the radial velocities $P_3 = 0^d.155\,83$ was suggested (DE JAGER (1957)).

The brightness observations obtained during the international observing campaign of 1956 have been subjected to a numerical analysis of the Fourier type, which is described in detail in this paper. This analysis has been carried out by the Mathematical Centre at Amsterdam at the request of the Observatory at Utrecht. A systematic search for periodicities was made between periods of $0^d.08$ and infinity, making use of the X1 computer of the Mathematical Centre. The following periods and relative amplitudes were found. (See table).

The former two periods were already known, but whereas the ratio of their amplitudes was 2/3 according to the 1951–1952 observations (DE JAGER (1953)), this ratio has decreased to about

Periods	*Relative amplitudes*
$P_1 = 0^d.193\,089$	1.000
$P_2 = 0.197\,358$	0.337
$P_3 = 0.182\,127 \pm 0^d.000\,001$	0.315
$P_4 = 25.85 \pm 0.01$	0.319

1/3 in 1956. The initially suggested period of $0^d.155\,83$ could not be found again; it differs considerably from the new third period. Interesting is the long period of nearly 26 days which was hitherto unknown.

1. The method

The brightness observations cover a period of about 4½ months, between July and December 1956. The total number of brightness observations (often being the mean of two or three observations) was 1694. One observed with a non-constant interval of time.

The method for determining the periodicities hidden in the observed brightnesses m_n is based on the following principle: when at any moment of observation t_n the observed brightness m_n is reduced with the sum of the brightness values belonging to k single periodic vibrations, then a maximum reduction (i.e. minimum residual) in the sense of the "least squares", will occur for those k periods (frequencies) of the considered vibrations, which agree (at least approximately) with the periods (frequencies) of the k most important components of the brightness variation. So our aim is to find these k periods.

The brightness belonging to one single periodical vibration with "frequency" f and the amplitude components a and b, is supposed to be "harmonic" and thus can be given at any time t_n by

$$a \cos \pi f t_n + b \sin \pi f t_n \, .$$

When according to the above principle f corresponds to a period P, then we get the following relation between f and P: $f = 2/P$. The corresponding values of a and b which lead to an optimum reduction yield the amplitude c, belonging to the vibration with period P: $c = \sqrt{(a^2 + b^2)}$.

To start with, the given m_n-values are reduced to a mean value zero (reduction with $f_0 = 0$). This means the original m_n is reduced with the mean $a_0 = (1/N) \sum_n m_n$ ($\sum_n$ further means everywhere $\sum_{n=1}^{n=N}$, N is the total number of observations, in this case 1694). Thereupon it was tried to obtain an optimal approximation of the reduced values, which we call m_n', by the sum

$$a \cos \pi f t_n + b \sin \pi f t_n \, , \tag{1}$$

by a suitable choice of f, a and b.

To that end we consider a region of f-values, and for discrete values of f in this region we determine the parameters a and b with the aid of the method of the

* Report R 487 of the Computation Department of the Mathematical Centre, 2de Boerhaavestraat 49, Amsterdam, the Netherlands

"least squares". This means that for a certain value of f, a and b are chosen in such a way that the sum of the squares R^*:

$$R^* = \sum_n \{ m_n' - (a \cos \pi f t_n + b \sin \pi f t_n) \}^2$$

becomes as small as possible. Hence the absolute value of the difference $| \Delta R |$ between $R = \sum m_n'^2$ and R^* becomes as great as possible. This difference is a spectral function of f; in the region considered one should look for that value of f which yields an optimum value of $| \Delta R |$ (periodogram analysis). The value of f belonging to the highest peak in this 'spectrum' and the corresponding values of a and b are called f_1, a_1 and b_1. With these values for f, a and b we get the best possible approximation of m_n' by the above sum, eq. (1).

We now compute

$$m_n' - (a_1 \cos \pi f_1 t_n + b_1 \sin \pi f_1 t_n) ,$$

and apply to these residuals the same procedure to determine f_2, a_2 and b_2. In principle, one should now try to apply small corrections to f_1 and thus determine a value of f_1 and a corresponding one of f_2 that would yield a minimum residual for a simultaneous reduction of m_n' with components belonging to f_1 and f_2; at the same time the amplitude components a_1, b_1 and a_2, b_2 should still be varied "optimally". The shift of f_1 and f_2 and the consequence of this operation is related to rather intricate perturbation factors. Since the effect of this correction will, in general, be very small, we did not apply them in the present reduction. (As will be shown later, it was not necessary to look for a maximum value of f_1 and f_2, since the first two reductions were performed with components belonging to periods which were already known with great accuracy.) However, in determining the amplitude parameters a and b, we did take into account the mutual influence; thus, f_1 and f_2 having been found we determined those values of a_1, b_1 and a_2, b_2 for which the reduction of m_n' with

$$\sum_{i=1}^{2} (a_i \cos \pi f_i t_n + b_i \sin \pi f_i t_n)$$

yields the smallest possible residual (in the sense of the least squares).

When finally f_1, a_1, b_1 and f_2, a_2, b_2 are determined in the way as indicated above, m_n' is reduced with the corresponding components, and to the reduced values the same procedure has been applied to determine the next components f_3, a_3 and b_3 etc., until finally the values of m_n, after the next reduction, do no longer yield significant periodicities. Analytically this conforms to a more smoothed spectral function ΔR.

The general course of the computation has been such that any j^{th} step of reduction has been performed in such a way that for the values $f_1, f_2 \dots f_j$ found, a simultaneous reduction has been made with parameters a_1, $b_1 \dots a_j$, b_j, so determined that the effect of the reduction is an optimum. Analytically this means that for a simultaneous reduction with $f_1, f_2 \dots f_j$, the corresponding parameters a_i and b_i $(i = 1 \dots j)$ are chosen such that the stationarity conditions $\partial R^*/\partial a_i = \partial R^*/\partial b_i = 0$ for $i = 1 \dots j$ are satisfied, for which

$$R^* = \sum_n \{ m_n' - \sum_{i=1}^{j} (a_i \cos \pi f_i t_n + b_i \sin \pi f_i t_n) \}^2.$$

To satisfy the stationarity conditions the reduction has been made according to the following reduction scheme.

REDUCTION SCHEME I

a. Compute for f_1: a_{11} and b_{11} (according to the method described above, with the aid of the method of least squares).
b. Reduce m_n' with: $a_{11} \cos \pi f_1 t_n + b_{11} \sin \pi f_1 t_n$.
c. Compute (after reduction *b*)) for f_2: a_{21} and b_{21}.
d. Reduce (after reduction *b*)) further with: $a_{21} \cos \pi f_2 t_n + b_{21} \sin \pi f_2 t_n$.
e. Compute (after reduction *d*)) for f_3: a_{31} and b_{31}.
f. Reduce (after reduction *d*)) further with: $a_{31} \cos \pi f_3 t_n + b_{31} \sin \pi f_3 t_n$ etc.

Till finally one has reduced with f_j and the corresponding parameters a_{j1} and b_{j1}.

After that the reduced values of m_n thus found are anew subjected to 'computation' and 'reduction' for f_1, $f_2 \dots f_j$, which yields successively a_{12}, b_{12}; a_{22}, b_{22}; $\dots$; a_{j2}, b_{j2}, etc., until the converging procedure of reduction after the m^{th} cycle does no longer yield any reduction of the m_n-values. When this stage is reached the sums

$$a_i = \sum_{l=1}^{m} a_{il} \text{ and } b_i = \sum_{l=1}^{m} b_{il} \quad (i = 1, \dots, j)$$

are computed. These values a_i and b_i thus satisfy the stationarity conditions for $i = 1, \dots, j$.

Thus the total reduction for $f_1 \dots f_j$ consists in the reduction of the original m_n with

$$a_0 + \sum_{i=1}^{j} (a_i \cos \pi f_i t_n + b_i \sin \pi f_i t_n) .$$

The reduction vectors (a_1, b_1), (a_2, b_2), ... (a_j, b_j) corresponding to $f_1, f_2, \ldots f_j$ then determine the amplitudes $c_1, c_2, \ldots c_j$ corresponding with the reduction components; here

$$c_i = \sqrt{(a_i^2 + b_i^2)} \quad (i = 1 \ldots j) .$$

The values of f correspond to periods P of the light-variation according to the formula: $f = 2/P$.

After the preceding reduction scheme I it should be possible to construct another reduction scheme (*reduction scheme II*) with the purpose also to correct the constant a_0 (corresponding to $f_0 = 0$) for each reduction, and in such a way that also for a_0 the stationarity condition remains satisfied. In that case the mean of the m_n-values remains 0 after each reduction. To examine the influence of this correction on a further analysis both methods of reduction have been applied in some cases.

The first step is, as was already remarked above, the reduction at $f_0 = 0$. This corresponds to a reduction of the original m_n-values with the average, i.e. with the constant $(1/N) \sum_n m_n$ $(N = 1694)$. The value of ΔR found at this reduction, is equal to $-(\sum_n m_n)^2/N$, as can be easily shown.

COMPUTATION OF a, b AND ΔR FOR A GIVEN f WITH THE AID OF THE METHOD OF THE LEAST SQUARES

$$R = \sum_n m_n^2; \quad F = \sum_n m_n \cos \pi f t_n;$$

$$G = \sum_n m_n \sin \pi f t_n;$$

$$C = \sum_n \cos^2 \pi f t_n; \quad S = \sum_n \sin^2 \pi f t_n;$$

$$D = \sum_n (\cos \pi f t_n \sin \pi f t_n).$$

($\sum_n$ is the summation over $n = 1(1)N$ with $N = 1694$).

Suppose, m_n is approximated by the sum: $a \cos \pi f t_n + b \sin \pi f t_n$.
We minimalize the function

$$R^*(f, a, b) = \sum_n \{ m_n - (a \cos \pi f t_n + b \sin \pi f t_n) \}^2$$

for given f.

A necessary and also sufficient condition to have a minimum is

$\partial R^*/\partial a = \partial R^*/\partial b = 0$, which is equivalent to

$$\begin{cases} \sum_n \{ m_n - (a \cos \pi f t_n + b \sin \pi f t_n) \} \cos \pi f t_n = 0 \\ \sum_n \{ m_n - (a \cos \pi f t_n + b \sin \pi f t_n) \} \sin \pi f t_n = 0. \end{cases}$$

or $\begin{cases} Ca + Db = F \\ Da + Sb = G \end{cases}$, which yields (for $f \neq 0$):

$$a = \frac{FS - GD}{CS - D^2} \quad \text{and} \quad b = \frac{GC - FD}{CS - D^2} .$$

The corresponding reduction is
$\Delta R = R^* - R = a^2C + b^2S - 2aF - 2bG + 2abD =$

$$= \frac{1}{(CS - D^2)^2} \left\{ \begin{array}{l} (FS - GD)^2C + (GC - FD)^2S \\ + 2D(FS - GD)(GC - FD) \\ - 2(F^2S - 2FGD + G^2C)(CS - D^2) \end{array} \right\} =$$

$$= - \frac{1}{CS - D^2} (F^2S + G^2C - 2FGD) =$$

$$= - (aF + bG) .$$

Note: If one introduces $H = F^2 + G^2$ and supposes (since C and S are as a rule $\approx \frac{1}{2}N$) $C = \frac{1}{2}N + \sigma$ and $S = \frac{1}{2}N - \sigma$, then

$$\Delta R = - \frac{F^2S + G^2C - 2FGD}{CS - D^2} =$$

$$= - \frac{\frac{1}{2}NH - \sigma(F^2 - G^2) - 2FGD}{\frac{1}{4}N^2 - \sigma^2 - D^2} .$$

Since F, G, D and σ are in most cases small as compared to N, we have approximately:

$$\Delta R \approx - \frac{\frac{1}{2}NH}{\frac{1}{4}N^2} = - \frac{2H}{N} .$$

The quantities $|\Delta R|$ and H thus will show approximately the same behaviour, in particular also with respect to the extremes. Since H is a simpler function than ΔR one often considers the extremes of H instead of those of ΔR, but we did not make this assumption.

2. Computations

a) The investigation was performed with the aid of the electronic computer X1. For discrete values of f (with

$$f = \frac{2}{P},$$

P in millidays) we have tabulated for suitable f-regions the following quantities: F, G, D, C, S, a, b and ΔR.

To compute and print these quantities for a given value of f, takes about 23 seconds.

b) We started with the series of 1694 magnitudes m_n at corresponding times t_n (expressed in J. D. from 2 435 675.508 till 2 435 808.430). The magnitudes were given in 3 decimal places and ranged between 0.321 and 0.564.

I. Reduction with $f_0 = 0$. This yields
$a_0 = 0.430\,154$ (= mean of the observed values m_n)
$R = \sum_n m_n^2$, after reduction: 2.460 495.
(With the aid of the relation $\Delta R = -(\sum_n m_n)^2/N$ it is found that the original value of R is of the order 316. The reduction with the mean is more than 99 %).

II. After having reduced m_n with the constant a_0 from I, it was tried to establish a first period from the reduced material. Originally the tabulation was made between $f = 0.0095$ (corresponding to $P \approx 0^d.2105$) and $f = 0.015\,625$ (corresponding to $P = 0^d.128$), increasing first with steps of $\Delta f = 2^{-16}$ (corresponding with a time difference of about 12–30 seconds), and after that in critical regions with smaller Δf finally down to $\Delta f = 2^{-32}$.

From theoretical considerations it could be ascertained that, at least for the region of f-values considered by us, a beginning range of $\Delta f = 2^{-16}$ was sufficiently small to discover "peaks" in the reduction ΔR. The quantity ΔR showed very clear extremes for the following f-values, given for decreasing absolute values of ΔR:

$f \approx 0.010\,358\,0$ ($\rightarrow P \approx 0^d.193\,087\,5$),

the reduction factor

$$\left| \frac{\Delta R}{R} \right|$$

with respect to R is 2.460 495: 53.7 %

$f \approx 0.012\,369$ ($\rightarrow P \approx 0^d.161\,695$);
reduction factor: 16.0 %

$f \approx 0.010\,130\,6$ ($\rightarrow P \approx 0^d.197\,421\,7$);
reduction factor: 8.3 %

$f \approx 0.010\,982$ ($\rightarrow P \approx 0^d.182\,116$);
reduction factor: 8.2 %

$f \approx 0.000\,077\,4$ ($\rightarrow P \approx 25^d.84$);
reduction factor: 3.8 %

Note: The value given in the last line of the above table was discovered in a later stage of the investigation (see part III of this section); in that stage we specially investigated the region of great periods (small f).

It looked reasonable first to reduce for the first f-value, being the most important period in the light-curve. However, it turned out to be rather difficult to determine this f-value with an accuracy sufficient for a suitable reduction, because ΔR and in particular also a and b are very sensitive to very small changes in f, even for $\Delta f = 2^{-32}$, while the stationary region around the searched f-value was furthermore distributed over a relatively large interval of time. For this reason it was not very simple to find a precise optimum f-value in the region around $f = 0.010\,358\,0$, and it was still more difficult to find corresponding values for a and b, which were accurate enough to allow the next step of reduction with a sufficient reliability.

Fortunately, however, the period in the region of $f = 0.010\,358\,0$ was already known with very high accuracy from other sources ($P = 0^d.193\,088\,83$ corresponding with $f = 0.010\,357\,927$). We therefore thought it better to perform the reduction with this value as our first f_1. Furthermore, also the second period was known with a fairly great accuracy: $P = 0^d.197\,358$, corresponding with $f_2 = 0.010\,133\,868\,4$. This second period corresponds with the third value of f in our above table. Assuming these values for f_1 and f_2, the reduction has been performed according to the reduction schemes I and II as given in section 1. (For the results, see next page.)

III. With the values of m_n reduced according to the scheme I a further investigation has been made to find other periods. In first instance we examined the region from $f = 0.001$ ($\rightarrow P = 2^d$) increasing with $\Delta f = 2^{-16}$, till $f = 0.02$ ($\rightarrow P = 0^d.1$); in critical regions with a smaller Δf finally down to $\Delta f = 2^{-32}$.

In this region we first found only one real peak; very accurate analysis gave the corresponding f-value: $f_3 =$

RESULTS

a) Reduction according to scheme I (without correction of a_0 for $f_0 = 0$):

f_j	$f_0 = 0$	$f_1 = 0.010\,357\,927$	$f_2 = 0.010\,133\,868$
R:	R before reduction: (see sub I): $R_0 = 2.460\,495$	R after reduction: $R_1 = 1.022\,989$	Reduction factor: $\lvert \Delta R/R_1 \rvert = 58.4\%$
a_j	$a_0 = +\,0.430\,154$	$a_1 = -\,0.038\,299$	$a_2 = +\,0.011\,617$
b_j		$b_1 = -\,0.001\,877$	$b_2 = -\,0.002\,076$
c_j		$c_1 = 0.038\,345$	$c_2 = 0.011\,801$

b) Reduction according to scheme II (including the correction of a_0 for $f_0 = 0$):

f_j	$f_0 = 0$	$f_1 = 0.010\,357\,927$	$f_2 = 0.010\,133\,868$
R	R before reduction: $R_0 = 2.460\,495$	R after reduction: $R_1 = 1.021\,772$	Reduction factor: $\lvert \Delta R/R_1 \rvert = 58.5\%$
a_j	$a_0 = +\,0.429\,105$	$a_1 = -\,0.038\,329$	$a_2 = +\,0.011\,675$
b_j		$b_1 = -\,0.001\,911$	$b_2 = -\,0.002\,119$
c_j		$c_1 = 0.038\,377$	$c_2 = 0.011\,865$

0.010 981 336, the corresponding P-value is $P_3 = 0^{\mathrm{d}}.182\,127\,2$. This peak was situated in the neighbourhood of the fourth f-value, found in II. The reduction factor corresponding to $R = 1.022\,989$ was: 11.4%. The other peak at $f = 0.012\,369$ appeared to be a so-called "spurious periodicity", due to the shadow-effect of other periods. This was clear, since after eliminating the main periodicities, no real extreme was any longer found in the region around this f-value. Also for f-values $f \cong 0.012\,834\,499$ ($\rightarrow P \cong 0^{\mathrm{d}}.155\,83$) and the dubious $f \cong 0.012\,345\,679$ (corresponding to $P \cong 0^{\mathrm{d}}.162$), a period for which some weak indications have been found in a previous analysis of DE JAGER (1957), no clear extreme could be found.

However, it appeared in the course of the reduction, that for fairly small values of f, ΔR showed a much more extreme behaviour, and we therefore examined a region with very small f-values, starting with $f \approx 0$. It turned out that the frequency $f = 0.000\,077\,4$ occurring in the reduction II has some effect on this part of the reduction: the reduction factor corresponding to $R = 1.022\,989$ was about 10%.

Note: Without correction with $f_0 = 0$ the mean of the m_n-values after reduction according to scheme I was of the order $-$ 0.0008. To examine the influence of the correction of a_0 with $f_0 = 0$ in the process of reduction on the further analysis, the analysis was extended with the reduced values, obtained according to the reduction scheme II. This has especially been done in the neighbourhood of $f_3 = 0.010\,981\,336$. It appeared, among other things that f_3 only changed very little after this procedure; the differences occurred only in the eighth decimal place. Also with respect to the corresponding parameters a and b, the difference was very small.

The reduction with the three values f_1, f_2 and f_3 was performed according to the two reduction schemes I and II. The result is given in the following tables.

a) Reduction according to scheme I:

f_j	$f_0 = 0$	$f_1 = 0.010\,357\,927$	$f_2 = 0.010\,133\,868$	$f_3 = 0.010\,981\,336$
R	R before reduction: 2.460 495	R after reduction: 0.904 360		Reduction factor: 63.2%
a_j	$a_0 = 0.430\,154$	$a_1 = -\,0.036\,853$	$a_2 = +\,0.012\,557$	$a_3 = +\,0.011\,033$
b_j		$b_1 = -\,0.001\,705$	$b_2 = -\,0.001\,853$	$b_3 = -\,0.004\,767$
c_j		$c_1 = 0.036\,892$	$c_2 = 0.012\,693$	$c_3 = 0.012\,018$

The average of the m_n-values was after reduction: $-$ 0.000 811 7.

b) Reduction according to scheme II:

f_j	$f_0 = 0$	$f_1 = 0.010\,357\,927$	$f_2 = 0.010\,133\,864$	$f_3 = 0.010\,981\,336$
R	R before reduction: 2.460 495	R after reduction: 0.903 253		Reduction factor: 63.3 %
a_j	$a_0 = +\,0.429\,303$	$a_1 = -\,0.036\,879$	$a_2 = +\,0.012\,600$	$a_3 = +\,0.010\,998$
b_j		$b_1 = -\,0.001\,733$	$b_2 = -\,0.001\,888$	$b_3 = -\,0.004\,765$
c_j		$c_1 = 0.036\,920$	$c_2 = 0.012\,741$	$c_3 = 0.011\,986$

IV. With the m_n-values reduced according to scheme I, the analysis was extended. A very great region of f-values has been examined, starting with $f \approx 0$, corresponding to an infinitely long period, down to $f = 0.025$, corresponding to $P = 0^d.08$, i.e. less than two hours. Also in this case we started with $\Delta f = 2^{-16}$; in critical regions the interval has been reduced finally down to $\Delta f = 2^{-32}$. An important extreme behaviour has only been found in the neighbourhood of the same f-value which was already observed in the previous reductions: $f_4 = 0.000\,077\,374\,8$, corresponding to $P = 25^d.8482$. The reduction factor for f_4 corresponding to $R = 0.904\,360$ was more than 10 %.

In the direct neighbourhood of $f = 0.012\,369$, found in reduction II but not in reduction III, we again observed no extreme behaviour, neither was this the case with the other expected f-values mentioned under heading III: the reduction for these f-values was smaller than $1\frac{1}{2}$ %.

Results given by the reduction with f_1, f_2, f_3 and f_4, performed according to the reduction scheme II, are shown by table c.

Note: An interesting behaviour has been noticed in the reduction parameters a_4 and b_4. First a_4 and b_4 were computed for f_4 starting with the values of m_n, found after the reduction III (hence, for $R = 0.904\,360$). There we got $a_4 \approx +\,0.007\,33$ and $b_4 \approx -\,0.007\,47$. However, the above simultaneous reduction with f_1, f_2, f_3 and f_4 yielded especially for the parameter a_4 a non-negligible correction: $a_4 = +\,0.009\,303$. The corresponding value of the amplitudes c_4 thus changed from about 0.010 47 into 0.011 881. A similar behaviour did not show up at f_3 at the simultaneous reduction in III with f_1, f_2 and f_3: there the a- and b-values were first $+\,0.010\,824\,5$ and $-\,0.004\,658\,2$ respectively and later, after correction $+\,0.010\,998$ and $-\,0.004\,765$ resp., with f_0-correction. In the case without f_0-correction we obtained 0.011 033 and $-\,0.004\,767$, resp.

V. Because it did not seem improbable that further peaks would show up after reduction IV, a region of very small f-values has been investigated, from $f = 0$ up to $f = 0.0025$ corresponding to $P = 0^d.8$. In this region we still obtained for some f-values an appreciable reduction, being in the maximal case about 5 % of the last R-value. This might be an indication for the existence of other long periods. However, this point has not been worked out in detail.

3. Summary of the results

Four significant f-values have been found:
$f_1 = 0.010\,357\,927$,
corresponding to $P = 0^d.193\,088\,83$ (known)
$f_2 = 0.010\,133\,868$,
corresponding to $P = 0^d.197\,358$ (known)
$f_3 = 0.010\,981\,336$,
corresponding to $P = 0^d.182\,127 \pm 0^d.000\,001$
$f_4 = 0.000\,077\,374\,8$,
corresponding to $P = 25^d.85 \pm 0^d.01$.

c) Reduction with f_1, f_2, f_3 and f_4, performed according to the reduction scheme II:

f_j	$f_0 = 0$	$f_1 = 0.010\,357\,927$	$f_2 = 0.010\,133\,868\,4$	$f_3 = 0.010\,981\,336$	$f_4 = 0.000\,077\,374\,8$
R	R before reduction: 2.460 495		R after reduction: 0.795 391		Reduction factor: 67.7 %
a_j	$a_0 = 0.426\,738$	$a_1 = -\,0.037\,153$	$a_2 = +\,0.012\,361$	$a_3 = +\,0.010\,833$	$a_4 = +\,0.009\,303$
b_j		$b_1 = -\,0.001\,764$	$b_2 = -\,0.002\,141$	$b_3 = -\,0.004\,469$	$b_4 = -\,0.007\,390$
c_j		$c_1 = 0.037\,195$	$c_2 = 0.012\,545$	$c_3 = 0.011\,719$	$c_4 = 0.011\,881$

Note: It should be noticed that the uncertainties given for f_3 and f_4 are only based on the uncertainties arising from the method of analysis followed and do not incorporate the (presumably small) influences of other non-eliminated perturbation effects. The amplitudes c_1 to c_4 corresponding to the above periods are:

$$\begin{aligned} &\text{for } f_1\text{: } c_1 = 0.037\,195 \\ &\text{for } f_2\text{: } c_2 = 0.012\,545 \\ &\text{for } f_3\text{: } c_3 = 0.011\,719 \\ &\text{for } f_4\text{: } c_4 = 0.011\,881. \end{aligned}$$

The constant a_0, corresponding to $f_0 = 0$, is 0.426 738.

The ratio of the four amplitudes thus found is about 3.00 : 1.01 : 0.95 : 0.96.

Interesting is the ratio between the first two amplitudes, which is about 3 : 1, whereas a previous analysis of observations made in 1951 and 1952 (DE JAGER (1953)) yielded a ratio 3 : 2. It should be examined whether this difference is due to the different methods of analysis or to a real critical change in the star.

The reduction factor $|\Delta R/R|$, with reference to R after reduction I is equal to 67.7%; it is the result of a reduction from $R = 2.460\,495$ to $R = 0.795\,391$.

Earlier suggested periods: $0^{d}.155\,83$ (DE JAGER (1957)) and $0^{d}.162$ (DE JAGER, *private communication*) were not found.

SOME FINAL REMARKS

1. The last analysis, V, necessary for determining other large periods in the light-variation would have yielded the best result if it had been made with Δf-values considerably smaller than those with which the analysis has been made. Though the results of analysis V do not indicate that really important long periods do still occur in the material, a finer analysis would have perhaps been necessary to be able to guarantee this statement with full certainty.

2. It should also be remarked that periods smaller than $0^{d}.08$ have not been examined. However, it seems doubtful, that in this region of the spectrum peaks would still occur.

3. The residual of R corresponds to a mean scatter of the magnitude corrected for all four periods of $0^{m}.02$. This error looks certainly greater than the expected error of the observations. This too might be an indication for the existence of some other periods or it might indicate that the star shows an irregular behaviour superimposed on the regular pattern consisting of the four periods found here.

Acknowledgment

The author is indebted to Professor A. van Wijngaarden and to Professor C. de Jager for helpful criticisms and suggestions.

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