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Report S 109

Statistical analysis of an epidemiological investigation of  
tuberculosis in Indonesia

by

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Appendix of:

"Tuberculosis, an epidemiological investigation  
in Palembang, South Sumatra, Indonesia",  
by H.P. Z. Driessen, dissertation, Amsterdam 1953.

~~1953~~

1. Introduction.

For 6,019 children in Palembang (partly school children partly clinic children) the following data had been provided:

1. sex
2. age
3. "race"
4. income of the parents
5. the number of living brothers and sisters
6. the number of deceased brothers and sisters
7. ~~residential district (Kampung)~~ ~~place of residence~~
8. the number of rooms ~~in the house~~ *occupied by the family*
9. the result of the VON PIRQUET test.

We distinguished: 1. Indonesian children

2. Chinese children

3. ~~People of~~ *Children* *Originals* ~~Other asiatic countries.~~

The income was divided into six income classes.

The results of the VON PIRQUET test were grouped into five classes: 0 (negative) and 1, 2, 3 and 4 (positive reactions of growing strength).<sup>1)</sup>

*/d* The investigation concerning the school children was carried out per school; the order in which the schools were investigated is known. We do not know the order in which the clinic children were ~~investigated.~~ *examined.*

The object of our analysis was to answer the following questions:

a. Is there a systematic difference between the numbers of positive and negative VON PIRQUET tests for the children from:

1. different kampongs
2. different "races"

1) In the following the results of the VON PIRQUET test will be denoted by:  $R(0)$ ,  $R(1)$ ,  $R(2)$ ,  $R(3)$  and  $R(4)$ .

3. school and clinic children

4. boys and girls.

b. Does the ratio  $\frac{R(0)}{R(0-4)}^2$ , which will be denoted by

*Systematically*  $P_0$ , increase or decrease with:

5. an increasing number of deceased brothers and sisters

6. an increasing number of living brothers and sisters

7. an increasing income.

8. *an increasing number of rooms occupied by the family.*  
c. An estimate of the infection rate.

## 2. Methods of investigation.

### 2.1. Testing of hypotheses.

The testing of a hypothesis  $H_0$  is based upon a number of observations  $x_1, x_2, \dots, x_n$  of one or more random variables <sup>3)</sup> or upon some groups of observations.

The test is executed by means of a statistic  $\underline{u}$ , which is a function of the observations. Supposing the hypothesis  $H_0$  to be true, the probability distribution of  $\underline{u}$  can be calculated.

A set  $Z$  of possible outcomes of  $\underline{u}$  is so chosen that the probability that  $\underline{u}$  assumes a value in  $Z$ , supposing  $H_0$  to be true, is equal to or smaller than a given number  $\alpha$  (the so-called level of significance). This set  $Z$  is called the critical region; the true probability that  $\underline{u}$  assumes a value of  $Z$  if  $H_0$  is true, is called the size of  $Z$ . The size of  $Z$  is not always equal to  $\alpha$  owing to the discrete character of the values which  $\underline{u}$  can take. We choose  $\alpha = 0.05$ .

2) The fraction of non-reactors.

3) A random variable is a variable which possesses a probability distribution. Random variables will be denoted by bold type letters. Values assumed by a random variable are denoted by the same letter in usual type.

The hypothesis  $H_0$  will be rejected on account of the observations  $x_1, \dots, x_n$  if the value of  $\underline{u}$ , calculated from these observations, belongs to  $Z$ . The probability that this will happen if  $H_0$  is true is equal to or smaller than  $\alpha$ .

The result of a test will be expressed in the so-called tail probability  $k$ ; this is the size of the smallest critical region which contains the result. Using a level of significance  $\alpha$ ,  $H_0$  will be rejected if  $k$  is smaller than or at most equal to  $\alpha$ .

For the application of a test some suppositions have always to be made about the probability distributions of the observed variables. In our case the observations should be mutually independent in the statistical sense. This condition is not completely fulfilled, because of one family more than one child may have been involved in the investigation. This may influence the result of tests, where family-factors, like the number of living brothers and sisters, are analysed, but it was impossible to take this into account, owing to a lack of data and this complication had therefore to be ignored.

## 2.2 Wilcoxon's two-sample test.

In this investigation we exclusively used WILCOXON's two-sample test (see the references at the end of this ~~report~~ ~~paper~~). With this method we test the hypothesis  $H_0$  that two samples  $x_1, x_2, \dots, x_n$  and  $y_1, y_2, \dots, y_m$  are observations of two random variables ( $\underline{x}$  and  $\underline{y}$ ) with the same probability distribution.

The statistic  $\underline{u}$ , used for this test, is calculated from the observations as follows. First we determine the number of observations in the second sample which are larger than the first observation  $x_1$  in the first sample (if

an observation equals  $x_1$  we count  $\frac{1}{2}$  instead of 1). Let  $u_1$  be this number. Next we determine the number of observations in the second sample which are larger than the second observation  $x_2$  in the first sample (if an observation equals  $x_2$  we count again  $\frac{1}{2}$  instead of 1). Let  $u_2$  be this number. In the same way we determine the numbers  $u_3, u_4, \dots, u_n$  with respect to the observations  $x_3, \dots, x_n$ .

The value  $U$  of the statistic  $\underline{U}$  is now given by:

$$(1) \quad U = u_1 + u_2 + \dots + u_n$$

If the hypothesis tested is true,  $\underline{U}$  has, for large values of  $n$  and  $m$  (e.g. both  $> 10$ ), approximately a normal probability distribution with mean:

$$(2) \quad \mu = \frac{1}{2} mn$$

and variance:

$$(3) \quad \sigma^2 = \frac{mn}{12(m+n)(m+n-1)} \left\{ (m+n)^3 - (t_1^3 + t_2^3 + \dots + t_h^3) \right\},$$

where  $h$  is the number of ties in the pooled samples and  $t_i$  ( $i = 1, 2, \dots, h$ ) the number of observations in the  $i$ -th tie (a tie is a group of equal observations).

If the hypothesis  $H_0$  is not true, the statistic  $\underline{U}$  will assume large or small values according as  $\underline{y}$  is systematically larger or smaller than  $\underline{x}$ .

We reject  $H_0$  if the value of  $\underline{U}$  found in the experiment deviates strongly from  $\mu$ , that is to say if:

$$\frac{|U - \mu|}{\sigma} \geq z_\alpha$$

where  $\alpha$  is the level of significance and  $z_\alpha$  follows from the normal distribution:

$$\frac{1}{\sqrt{2\pi}} \int_{z_\alpha}^{\infty} e^{-\frac{1}{2}x^2} dx = \frac{1}{2}\alpha.$$

The tail-probability is:

$$k = \frac{2}{\sqrt{2\pi}} \int_{\frac{|u-\mu|}{\sigma}}^{\infty} e^{-\frac{1}{2}x^2} dx.$$

Given  $\frac{u-\mu}{\sigma}$ , the value of  $k$  can easily be read from a table of the normal distribution.

If all observations are different,  $t_i = 1$  for all  $i$ 's. The variance of  $\sigma^2$  is then:

$$(4) \quad \sigma^2 = \frac{1}{12} mn(m+n+1).$$

It is easy to prove that the variance of  $\underline{u}$  is reduced by the occurrence of equal observations. If these equal values are not taken into account, the tail probability found will be too large. If this value is already small (especially if it is  $< \alpha$ )  $H_0$  will certainly be rejected and we need not make the more extensive calculations based on (3).

### 2.3. Combination of a number of independent tests

The questions which will be answered by this investigation all concern the association between the sensitivity to the VON PIRQUET test and one of the other data. To investigate this association we must, in order to avoid spurious associations, split the observations into a number of homogeneous groups. For all these groups we investigate the association separately, combining the results afterwards.

We shall illustrate this by means of an example:

Suppose we want to investigate if there is a difference between boys and girls as regards their sensitivity to the VON PIRQUET test. It is known that this sensitivity depends on the age (the fraction  $F_0$  of non-reactors decreases with increasing age). If we now compare boys and girls without dividing the observations into age-homogeneous groups (groups in which the age of the children is constant), spurious association may occur. For example,

"Sensitivity" will be used in this report for the distinction between negative and positive reaction to the Von Pirquet test, not taking into account the intensity of the reaction if it is positive.

if there is no difference in sensitivity between boys and girls, but the boys are older than the girls, the boys will show a greater sensitivity to the VON PIRQUET test than the girls, the only reason for this being the difference in age. On the other hand, working with non-homogeneous groups may also result in not finding really existing associations. We shall therefore ~~work~~, as far as the number of observations allows, work with homogeneous groups and for each of these groups we shall investigate the association between the sensitivity to the VON PIRQUET test and one of the other data. We then obtain for each of these groups:

1. a value for the statistic  $\underline{u}$
2. a value for  $\mu$
3. a value for  $\sigma^2$ .

which will be called  $u_i$ ,  $\mu_i$  and  $\sigma_i^2$  respectively, the index  $i$  indicating the group.

If the hypothesis  $H_0$  - that there is no association between the sensitivity and one of the other data - is true,  $\underline{u} = \sum_i \underline{u}_i$  will be approximately normally distributed with mean  $\mu = \sum_i \mu_i$  and variance  $\sigma^2 = \sum_i \sigma_i^2$ ;  $\frac{\underline{u} - \mu}{\sigma}$  will be approximately normally distributed with mean 0 and variance 1.

The critical region again consists of large values of  $\left| \frac{\underline{u} - \mu}{\sigma} \right|$ .

### 3. Results

#### 3.1. Comparison of kampongs

Two groups of kampongs could be distinguished:

1. a group of kampongs in which very bad hygienic conditions prevail; these kampongs will be denoted as group I
2. the remaining kampongs which will be denoted as

group II

We want to compare the sensitivity to the VON PIRQUET test of the children of both groups by means of the fraction  $F_0$  of <sup>non</sup> ~~man~~-reactors.

To investigate this the observations were separated into groups which are homogeneous according to age and race. Furthermore, the investigation was carried out separately for school and clinic children. For each of these groups WILCOXON's two sample test was applied (cf. section 2.2) and the results were combined as described in section 2.3.

Table I gives the values of  $\frac{u-\mu}{\sigma}$  and the tail-probabilities for Indonesian and Chinese children separately and combined <sup>4</sup>).

TABLE I  
Comparison of the two groups of kampongs:

"Race"	$\frac{u-\mu}{\sigma}$	Tail-probability
Indonesian	- 2.13	0.03
Chinese	- 3.52	0.0005
Combined	- 3.60	0.0003

From the very small tail-probabilities in table I we see that there is a marked difference between children from the kampongs of group I and those from the other kampongs. The children living in the first group of kampongs have a higher sensitivity to the VON PIRQUET test than the other children.

The possibility of this effect being spurious in spite of our precautions will be investigated further in section 3.6.

4) "Race" 3 is left out of consideration on account of the small number of observations.

3.2. Comparison of the races

To compare the Indonesian children with the Chinese children the observational material was separated into age-homogeneous groups.

Furthermore, the investigation was carried<sup>d</sup> out separately with schoolchildren and with clinic children and separately with the kampong groups I and II (ignoring children with unknown residence). Table II gives the results:

TABLE II  
Comparison of Indonesian and Chinese children:

	kampong group	$\frac{\bar{u}-\mu}{\sigma} 5)$	Tail probability
school children	I	- 1.58	0.1
	II	- 1.54*	> 0.1
clinic children	I	- 0.65*	> 0.5
	II	- 1.54	0.1
All children from both groups of kampongs		- 1.93	0.05

Table II shows that there is a relatively weak indication of a difference between Indonesian and Chinese children in that the former show a greater sensitivity to the VON PIRQUET test.

3.3. Comparison of school and clinic children

Here too we separated into age-homogeneous groups and carried out the investigation separately for the two groups of kampongs. The Chinese school and clinic children could not be compared owing to the small number of observations.

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5) The values with an asterisk are approximations. The true values are closer to zero.

TABLE III  
Comparison of Indonesian school and clinic children:

kampong group	$\frac{u-\mu}{\sigma}$ 6)	tail- probability
I	-1.65 *	> 0.1
II	- 0.37 *	> 0.7
I and II combined	- 1.000 *	> 0.3

It is clear that the observations do not indicate a systematic difference between Indonesian school and clinic children as regards their sensitivity to the VON PIRQUET test.

3.4. Association between the number of deceased brothers and sisters of a child and its sensitivity to the VON PIRQUET test

We tested the hypothesis  $H_0$  that the number of deceased brothers and sisters of a child is stochastically independent of its sensitivity to the VON PIRQUET test. This was done by dividing the children into two groups, the non-reactors and the positive reactors and next applying WILCOXON's two-sample test to the numbers of deceased brothers and sisters of these two groups.

In section 3.5 we shall see that the sensitivity to the VON PIRQUET test and the number of living brothers and sisters are stochastically dependent, in such a way that the fraction  $F_0$  of non-reactors gradually increases with an increasing number of living brothers and sisters. In the investigation concerning the association between the sensitivity and the number of deceased brothers and sisters we therefore separated the observations into three groups ac-

6) Cf. footnote 5.

according to the number of living brothers and sisters:

Group I : Number of living brothers and sisters: 0,1 or 2

Group II : Number of living brothers and sisters: 3,4 or 5

Group III: Number of living brothers and sisters:  $\geq 6$ .

The observations were divided into age-homogeneous groups, and the investigation was carried out for the two kampong groups separately. For the Chinese children the number of data concerning the ~~number of~~ living and deceased brothers and sisters was too small ~~in~~ for the test to be applied.

Table IV gives the results.

TABLE IV  
Association between the number of deceased brothers and sisters of a child and its sensitivity to the VON PIRQUET test for Indonesian children:

kampong group	number of living brothers and sisters	$\frac{u-\mu}{\sigma}$ 7) 8)	tail-probability
I	0-2	1.74 *	> 0.08
	3-5	1.58 *	> 0.1
	$\geq 6$	1.42 *	> 0.1
II	0-2	0.40 *	> 0.6
	3-5	2.01	0.04
	$\geq 6$	1.22 *	> 0.2
Results combined for all groups		2.56 **	< 0.01

The last line of table IV shows that the sensitivity to the VON PIRQUET test and the number of a child's deceased brothers and sisters are stochastically dependent,

7) Cf. footnote 5

8) Values with two asterisks are approximations. The true values are farther from zero.

in such a way that the fraction  $F_0$  of non-reactors decreases with an increasing number of deceased brothers and sisters.

A graphical analysis of this phenomenon indicated that this decrease was mainly due to the groups with a small number of deceased brothers and sisters, whereas for the groups with a larger number ( $\geq 2$ ) of deceased brothers and sisters the fraction  $F_0$  of non-reactors seems to be practically independent of this number. This induced us, in the further investigation, to divide the observations concerning the number of deceased brothers and sisters into three groups:

Group I : Number of deceased brothers and sisters: 0

Group II : Number of deceased brothers and sisters: 1

Group III: Number of deceased brothers and sisters:  $\geq 2$ .

### 3.5. Association between the number of living brothers and sisters of a child and its sensitivity to the VON PIQUET test

Here we test the hypothesis  $H_0$  that the number of a child's living brothers and sisters is not associated with its sensitivity to the VON PIQUET test. The test was executed as described in section 3.4.

We again divide the observations into age-homogeneous groups and furthermore into two groups of kampongs and into the three groups mentioned in section 3.4, concerning the number of deceased brothers and sisters. For the Chinese children too few data are available for this test.

The results are shown in table V.

TABLE V

Association between the sensitivity of an Indonesian child to the VON PIRQUET test and the number of its living brothers and sisters:

kampong group	number of deceased brothers and sisters	$\frac{u-\mu}{\sigma}$ 9)	tail-probability
I	0	- 1.66	0.1
	1	0.97 *	> 0.3
	$\geq 2$	1.17 *	> 0.2
II	0	- 1.67	0.1
	1	- 1.25 *	> 0.2
	$\geq 2$	- 2.34 **	< 0.02
Results combined for all groups		- 2.17 **	< 0.03

The last line of this table shows an overall indication of a negative association between the two factors sensitivity and number of living brothers and sisters: the sensitivity increases when ~~this~~ this number decreases. From the rest of the table it is clear, however, that this result is due to kampong group II and not to kampong group I; the results for the latter group are rather nondescript and it would be imprudent to extend the conclusion to this group.

3.6. Further investigation into the difference found in section 3.1 between the children from the two kampong groups

We shall now investigate whether the difference between the kampong groups I and II (see section 3.1) might

9) Cf. footnotes 5 and 8

perhaps originate from a large number of deceased brothers and sisters and (or) a smaller number of living brothers and sisters in group I in comparison with group II. To do this, we compare the numbers of deceased brothers and sisters in the two kampong groups, dividing the observations into age-homogeneous groups and farther into the three aforementioned groups regarding the number of living brothers and sisters. Testing the hypothesis that the distribution of the number of deceased brothers and sisters in the two kampong groups is the same, we find, by means of WILCOXON's two-sample test, the result of table IV.

TABLE VI

Comparison of the numbers of deceased brothers and sisters of the Indonesian children in the two kampong groups:

$\frac{u-\mu}{\sigma}$	tail-probability
+ 0.78	0.4

Table VI shows that there is no reason to conclude that the numbers of deceased brothers and sisters of Indonesian children in the two kampong groups are different. Moreover a spurious result in section 3.1 could only have been caused by the numbers of deceased brothers and sisters if the sign of  $\frac{u-\mu}{\sigma}$  had been negative and not positive, as was found in table IV.

If we test in the same way the hypothesis that the distribution of the number of living brothers and sisters is the same in the two kampong groups, we find:

TABLE VII

Comparison of the numbers of living brothers and sisters of Indonesian children in the two kampong groups:

$\frac{u-\mu}{\sigma}$	tail-probability
+ 0.20 <sup>10)</sup>	> 0.8

10) Cf. footnote 5.

There is clearly no reason to fear that the numbers of living brothers and sisters of Indonesian children from the two kampong groups are different and might thus have caused a spurious difference between the kampong groups in section 3.1.

3.7. Association between the income of the parents and the sensitivity to the VON PIRQUET test of the child

Here also we effect a division into age-homogeneous groups, a division into three groups concerning the number of deceased brothers and sisters and into three groups concerning the number of living brothers and sisters. Finally a division into two groups concerning the kampong is used.

We test the hypothesis  $H_0$  that the income distribution for the non-reactors is the same as for the positive reactors.

The results are found in table VIII:

TABLE VIII

Association between the income of the parents and the sensitivity to the VON PIRQUET test of the child:

kampong group	number of deceased brothers and sisters	$\frac{u-\mu}{\sigma}$	tail-probability
I	0	+ 0.03	0.9
	1	+ 0.21	0.8
	$\geq 2$	+ 1.72	0.08
II	0	+ 0.04	0.9
	1	+ 0.07	0.9
	$\geq 2$	- 2.03	0.04
Results combined for all groups		- 0.85	0.4

We find for the children from kampong group I with two or more deceased brothers and sisters a rather small tail-probability in positive direction, that is to say, a decrease of the fraction  $F_0$  with increasing income. For kampong group II we find for the children with two or more deceased brothers and sisters an increase of the fraction  $F_0$  with an increasing income.

This is a contradictory result. Although it is not impossible that the different conditions for the two kampong groups (cf. section 3.1) also result in a difference with respect to the influence of the income of the parents, this conclusion seems rather unacceptable. Although the tail probability is somewhat smaller for group II (third line) than for group I, the difference is too small to attach much importance to it. Thus we do not reach a satisfactory result in this case.

### 3.8. Comparison of boys and girls concerning their sensitivity to the VON PIRQUET test

We use the same division into groups as in section 3.7. The results are listed in table IX according to age, because here the question was whether, and if so for which ages, there is a difference in sensitivity to the VON PIRQUET test between boys and girls.

The investigation could again be carried out exclusively for Indonesian children on account of the small number of data concerning the living and deceased brothers and sisters of Chinese children.

TABLE IX  
Comparison of Indonesian boys and girls concerning their  
sensitivity to the VON PIRQUET test:

age	$\frac{u-\mu}{\sigma}$	tail- probability
6	+ 1.24	0.2
7	- 0.55	0.6
8	- 2.35	0.02
9	- 0.30	0.8
10	+ 1.99	0.05
11	+ 0.16	0.9
12	- 1.48	0.1
13	- 1.63	0.1
14	- 3.24	0.001
15	- 0.54	0.6

Table IX shows in the first place that 14-year-old girls show less sensitivity to the VON PIRQUET test than the boys, which is more or less confirmed by the minus-signs which are found for the 12-, 13- and 15-year groups

For the 8-year group we find an indication in the same direction, while the 10-year group shows a weak indication in the opposite direction. Among ten independent tests it may easily happen by coincidence that one of the tail-probabilities comes close to 0.05, so that these last two indications must be cautiously interpreted

3.9. Association between the number of rooms <sup>occupied</sup> in the  
house and the sensitivity to the VON PIRQUET test

We again divide the observations into age-homogeneous groups, into three groups concerning the number of living brothers and sisters, into three groups concerning the number of deceased brothers and sisters and into two groups regarding the kampongs.

We test the hypothesis  $H_0$  that the distribution of

*occupied by the family*  
the number of rooms ~~in the house~~ of non-reactors is the same as of positive reactors.

The results are found in table X:

TABLE X  
Association between the sensitivity to the VON FIRQUET test  
and the number of rooms *occupied by the family*

kampong group	number of living brothers and sisters	number of deceased brothers and sisters	$\frac{u-\mu}{\sigma}$	tail-probability
I	0-2	0	- 1.71	0.09
		1	- 0.43	0.7
		$\cong 2$	+ 1.87	0.06
	3-5	0	- 0.04	0.9
		1	+ 0.78	0.4
		$\cong 2$	- 0.77	0.4
	$\cong 6$	0	- 0.67	0.5
		1	+ 1.88	0.06
		$\cong 2$	- 1.10	0.3
II	0-2	0	- 0.13	0.9
		1	- 0.07	0.9
		$\cong 2$	- 0.02	0.9
	3-5	0	+ 1.30	0.2
		1	- 0.47	0.6
		$\cong 2$	- 1.93	0.05
	$\cong 6$	0	0	1.00
		1	+ 0.25	0.8
		$\cong 2$	+ 0.16	0.9
Results combined for all groups			- 1.40	0.2

The combined result does not show an indication of association. Among the 18 independent tests no very small tail probabilities are found and the few small ones can easily originate from fortuitous circumstances. Thus we do

not find an association in this case.

### 3.10. Estimation of the infection rate

In this section the  $\chi^2$ -minimum method is used (cf. e.g. H. CRAMÉR, Mathematical Methods of Statistics, p. 506).

The infection rate (I.R.) is the mean percentage of people of a certain age who will be infected in a certain year. As standard of this infection the VON PIRQUET test is applied.

If one has never been infected the result of the VON PIRQUET test is negative; in the other case for a long time positive. We therefore consider the fraction  $p$  of non-reactors for different ages, i.e.  $p = 1 - \text{I.R.}$

Inasmuch as the children are born uninfected, the fraction of non-reactors of zero age is 1.

Supposing the I.R. to be the same for every age, the fraction of one-year-old non-reactors, apart from random fluctuations, is equal to  $p$ , of two-year-old children equal to  $p^2$ , etc.

The observations do not refer to children of exactly one year old, two years old etc. What we call a group of one-year-old children is in reality a group of children between one and two years of age, i.e. of a mean age of about  $1\frac{1}{2}$  years. So may we get for " $a$ "-year old children (apart from random fluctuations) a mean  $\frac{\text{fraction}}{p^{a+\frac{1}{2}}}$  of non-reactors. From the observations we can calculate for each age " $a$ " the fraction  $f_a$  of non-reactors.

As measure for the deviation of the sample fraction  $f_a$  and the value  $p^{a+\frac{1}{2}}$  (the mathematical expectation on account of the hypothesis that I.R. is constant with regard to age) we use the statistic:

$$n_a \cdot \frac{(f_a - p^{a+\frac{1}{2}})^2}{p^{a+\frac{1}{2}} (1 - p^{a+\frac{1}{2}})},$$

where  $n_a$  is the number of investigated children of age  $a$ .

The sum of their <sup>se</sup> values for all  $h$  age groups is denoted by  $\chi^2$ :

$$\chi^2 = \sum_{a=1}^h n_a \cdot \frac{(f_a - p^{a+\frac{1}{2}})^2}{p^{a+\frac{1}{2}} (1 - p^{a+\frac{1}{2}})}.$$

The true value of  $p$  being unknown, we consider  $\chi^2$  as a function of  $p$  and determine the minimum of this function in the closed interval  $(0,1)$ . The value  $p_m$  of  $p$  which minimises  $\chi^2$  is an estimate for  $1 - I.R.$

At the same time we are able to test the hypothesis: " $p$  is constant", the quantity

$$\chi_m^2 = \sum_{a=1}^h n_a \cdot \frac{(f_a - p_m^{a+\frac{1}{2}})^2}{p_m^{a+\frac{1}{2}} (1 - p_m^{a+\frac{1}{2}})}$$

being asymptotically  $\chi^2$ -distributed with  $h-1$  degrees of freedom.

To apply this method the observations have been divided into two groups:

I children of schoolgoing age, that is to say between 6 and 15 years old

II children not yet going to school, age 0-5 years

The first group again has been divided into two groups as regards the kampongs, into three groups as regards the number of deceased brothers and sisters and into three groups as regards the number of living brothers and sisters. These divisions could not be executed for

group II, the data not being available; this led us to consider group II separately.

For each of these 18 groups we have calculated the aforementioned estimate of the I.R. and moreover tested the hypothesis that the I.R. is a constant.

We find the results in table XI:

TABLE XI

Estimation of the infection rate and testing the hypothesis that the infection rate is constant for children of 6-15 years:

kampong group	number of living brothers and sisters	number of deceased brothers and sisters	I.R.	tail-probability
I	0-2	0	0.044	0.1
		1	0.055	0.5
		$\geq 2$	0.062	0.6
	3-5	0	0.047	0.4
		1	0.038	0.5
		$\geq 2$	0.064	0.6
	$\geq 6$	0	0.027	0.9
		1	0.041	0.1
		$\geq 2$	0.065	0.8
II	<del>0-2</del>	0	0.053	0.09
	0-2	1	0.055	0.8
		$\geq 2$	0.060	0.2
	3-5	0	0.065	0.7
		1	0.043	0.9
		$\geq 2$	0.054	0.3
	$\geq 6$	0	0.065	0.03
		1	0.035	0.06
		$\geq 2$	0.040	0.4

We find one tail probability  $< 0.05$  among 18 of these tail probabilities, which may be considered as perfectly normal. If we combine the results of these 18 groups by adding the  $\chi^2$ -values we find a tail-probability of 0.6. So there is no reason to conclude that the I.R. is not constant with respect to age.

In the second group of children (0-5 years old) a division according to living and deceased brothers and sisters is not possible on account of the lack of data. A division into two groups concerning the kampongs is possible; but we did not make this division, no differences at all having been found above.

In table XII we find the estimate of the I.R. and the tail probability.

TABLE XII

Estimate of the infection rate and testing of the hypothesis that the infection rate is constant for children of 0-5 years old:

I.R.	tail-probability
0.058	0.99

There is again no reason to conclude that the I.R. is not constant. The median estimate of this I.R. for all groups together is 0.056.

#### 4. Investigation for a trend in the observations.

Up till now we discriminated ~~between~~ between a positive and a negative sensitivity to the VON PIRQUET test.

In the data provided the positive reaction was subdivided into R(1), R(2), R(3) and R(4). Since this subdivision contains subjective elements, we ~~had~~ tried to find out whether this subdivision remained constant during the obtainment of the observations. Particularly we looked for a change in the discrimination between R(1) and R(2-4).

Since we did not know when the clinic children were investigated, we carried out the examination only with data concerning the school children.

For a complete investigation we should have divided the observations according to age, race, kampong, number of living and number of deceased brothers and sisters. But in this way the number of observations per group becomes very small. Therefore we carried out the examination twice: first by not dividing according to living <sup>and deceased</sup> brothers and sisters and secondly by not dividing according to kampong.

For each of the groups the schools are now arranged according to the moment of their investigation and with WILCOXON's two-sample test we tested the hypothesis that the distribution of R(1)-children over the schools arranged in this order is the same as the distribution of R(2-4)-children.

If the observations are not divided according to the number of living and deceased brothers and sisters we find:

TABLE XIII

Investigation of a change in the discrimination between R(1) and R(2-4) in the course of the investigation:  
(no division of the observations according to the number of living and deceased brothers and sisters)

race	kampong group	$\frac{u-\mu}{\sigma}$ 11)	tail-probability
Indonesian	I	3.03 **	< 0.002
	II	4.07 **	< 10 <sup>-4</sup>
Chinese	I	8.09 **	<< 10 <sup>-4</sup>
	II	4.02 **	< 10 <sup>-4</sup>
Results combined for all groups		7.80 **	<< 10 <sup>-4</sup>

11) See footnote 8.

If we do not divide the observations according to kampong we find:

TABLE XIV

Investigation of a change in the discrimination between R(1) and R(2-4) in the course of the investigation:  
(no division of the observations according to kampong)

number of deceased brothers and sisters	number of living brothers and sisters	$\frac{u-\mu}{\sigma}$ 12)	tail- probability
0	0-2	1.19 *	> 0.2
	3-5	0.22 *	> 0.8
	$\geq 6$	0.35	0.7
1	0-2	2.26	0.02
	3-5	0.86 *	> 0.4
	$\geq 6$	- 0.17 *	> 0.8
$\geq 2$	0-2	1.02 *	> 0.3
	3-5	2.13	0.03
	$\geq 6$	0.19 *	> 0.8
Results combined for all groups		2.64 **	< 0.008

It appears from both tables that the fraction of R(1) results ~~decreases~~ markedly decreases with respect to the fraction of R(2-4) results during the time of the obtainment of the observations.

This increase may be a result of a shift in the discrimination between R(1) and R(2-4); however it is also possible that the schools investigated later were more infected than those investigated earlier.

If the latter supposition were true we should expect the fraction  $F_0$  of non-reactors also to decrease in the course of the obtainment of the observations.

12) See footnotes 5 and 8.

This we investigated in an analogous way as the decrease of  $\frac{R(1)}{R(1-4)}$  and found:

TABLE XV

Investigation of a decrease of the fraction  $F_0$  of non-reactors in the course of the investigation:  
(no division of the observations according to the number of living and deceased brothers and sisters)

race	kampong group	$\frac{u-\mu}{\sigma} 13)$	tail-probability
Indonesian	I	- 1.26 *	> 0.2
	II	- 1.70 *	> 0.09
Chinese	I	1.01 *	> 0.3
	II	0.68 *	> 0.5
Results combined for all groups		- 1.14 *	> 0.2

If the observations are not divided according to kampong we find:

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13) Cf. footnote 5.

TABLE XVI

Investigation of a decrease of the fraction  $F_0$  of non-reactors in the course of the investigation:  
(no division of the observations according to kampong)

number of deceased brothers and sisters	number of living brothers and sisters	$\frac{u-\mu}{\sigma}$ 14)	tail-probability
0	0-2	1.36 *	> 0.1
	3-5	0.28 *	> 0.7
	$\geq 6$	0.33 *	> 0.7
1	0-2	- 0.22 *	> 0.8
	3-5	- 1.30 *	> 0.2
	$\geq 6$	+ 1.61 *	> 0.1
$\geq 2$	0-2	+ 0.89 *	> 0.3
	3-5	- 1.56	0.1
	$\geq 6$	+ 0.89 *	> 0.3
Results combined for all groups		- 0.83 *	> 0.4

Tables XV and XVI show that there is no reason to conclude that the fraction  $F_0$  of non-reactors increases or decreases with time.

Some find a decrease of the ratio  $\frac{R(1)}{R(1-4)}$  but not of the ratio  $F_0 = \frac{R(0)}{R(0-4)}$ .

These two results together indicate the strong possibility of a shift in the discrimination between  $R(1)$  and  $R(2-4)$ . Owing to lack of time no profound investigation of this symptom could be carried out. The discrimination between  $R(1)$ ,  $R(2)$ ,  $R(3)$  and  $R(4)$  was therefore not used in this statistical analysis.

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14) Cf. footnote 5.

5. Summary of conclusions

1. The kampongs were divided into two groups:

I. Kampongs with bad hygienic conditions

II. Kampongs with better hygienic conditions.

The investigation showed that a positive reaction was clearly more frequent in kampong group I than in kampong group II.

2. A rather slight indication was found of a difference between the Chinese and the Indonesian children. The latter group showed *systematically* more frequently a positive reaction than the former.

The remaining part of the analysis (conclusions 3-9) was only carried out for Indonesian children, because sufficient information was only available about these children

3. There was no indication of a difference between the school children and the clinic children.

4. The sensitivity to the VON PIRQUET test increases with the number of deceased brothers and sisters and decreases with the number of living brothers and sisters.

5. The influence of the income of the parents was not very clear and did not permit of a conclusion.

6. Only for the 14-year-old Indonesian children was a clear indication of a difference between boys and girls found. The sensitivity of the boys was higher. For the 8- and 10-year-old Indonesian children there were weak indications of opposite differences.

7. There was no reason to conclude that the sensitivity to the VON PIRQUET test increases or decreases with the number of rooms *occupied by the family.*

8. It was found that the dependence of the fraction  $F_0$  of non-reactors on age, within groups which are homogeneous in other respects, could be described with sufficient accuracy by means of the introduction of a constant infection

rate. Estimates of the value of I.R. for the different groups are found in tables XI and XII, the median estimate being 0.056.

9. The division of positive reactors into four classes of different strength ( $R(1)$ ,  $R(2)$ ,  $R(3)$  and  $R(4)$ ) seems to have been subject to a shift in the discrimination in the course of the investigation and was therefore not used in the statistical analysis.

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