STICHTING
MATHEMATISCH CENTRUM
2e BOERHAAVESTRAAT 49
AMSTERDAM

STATISTISCHE AFDELING

LEIDING: PROF. DR D. VAN DANTZIG
ADVISEUR VOOR STATISTISCHE CONSULTATIE: PROF. DR J. HEMELRIJK

Report S 231

Statistical analysis of part of the data concerning Performance trial nr.VIII on flame radiation

by

A. R. Bloemena

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1. Introduction

Some general aspects concerning the design of PT VIII will be dealt with in a seperate letter. At present we only note, that owing to the experimental set-up a statistical analysis using all the available data is virtually impossible. In consultation with the International Flame Research Foundation it has therefore been decided to deal only with a few smaller questions using a small part of the actual data.

The results stated in this report must be interpreted with the following statement in mind: In case an effect will be found to be significantly different from zero, it may be possible that due to difficulties in the design of the experiment the interpretation of these results is uncertain. This interpretation has to be based on assumptions concerning correct control of variables, stability of furnace conditions, randomization of uncontrollable factors, etc. It is however, far from certain that these conditions were satisfied in this experiment.

For the general set-up of PT VIII we refer to the document nr. F 31/a/20[2].

In this report only R_1-max values of the radiation will be considered.

2. Air versus steam

Flames with flame conditions I (steam as atomizing agent) and XIII (air as atomizing agent) have been observed on four occasions, viz. twice in each of the subexperiments PT VIII A and PT VIII B. In each of these two replicates of the flames have been observed at two different times each. In case observations have been made more than twice, we have used two sets of these measurements, taken at random.

The following subscripts will be used:

$$i = \begin{cases} 1 & \text{for flame condition I} \\ 2 & \text{for flame condition XIII} \end{cases}$$

$$j = \begin{cases} 1 & \text{for sub-experiment PT VIII A} \\ 2 & \text{for sub-experiment PT VIII B} \end{cases}$$

k = 1, 2: replicates

l = 1, 2: measurements on a flame.

Denoting by $\underline{x}_{ijkl}$ the l-th measurement of the k-th replicate in the j-th subexperiment on flame condition i, we can state on the mathematical model:

$$\underline{x}_{ijkl} = \mu + \mu_i + \underline{\epsilon}_{ij} + \underline{\epsilon}_{ijk} + \underline{\epsilon}_{ijkl},$$

where μ_i denotes the contribution to the radiation of the atomizing agent used in the i-th flame condition.

By definition $\sum_i \mu_i = 0$. In the model three random components occur. Of these $\underline{\epsilon}_{ijkl}$ represents the variation in radiation between two measurements on the same setting of the flame, but some time apart. This definition implies $E\underline{\epsilon}_{ijkl} = 0$ for all i,j,k,l, while we shall assume all $\underline{\epsilon}_{ijkl}$ to be equally and normally distributed, while $E\underline{\epsilon}^2_{ijkl} = \sigma^2_w$(within flames).

Regarding the distribution of the $\underline{\epsilon}_{ij}$ and $\underline{\epsilon}_{ijk}$ the same assumptions are made as in the case of $\underline{\epsilon}_{ijkl}$. The component $\underline{\epsilon}_{ijk}$ represents the variation in radiation between different settings of a flame; we shall denote $E\underline{\epsilon}^2_{ijk}$ by σ^2_b (variance between flames). The component $\underline{\epsilon}_{ij}$ represents the variation between sub-experiments. $E\underline{\epsilon}^2_{ij}$ will be denoted by σ^2_e (variance between sub-experiments). Finally we assume all $\underline{\epsilon}_{ij}$, $\underline{\epsilon}_{ijk}$, $\underline{\epsilon}_{ijkl}$ to be distributed independently of each other.

The variance σ^2_e has been introduced in the model, because of the fact PT VIII B has been run half a year after PT VIII A, which might has caused another source of variation. It is therefore desirable to test at the outset the nulhypothesis.

$$H_o : \sigma^2_e = 0 .$$

Under the assumptions stated, this can be done by means of the statistic:

$$\underline{F} = \frac{4}{2} \, \frac{4 \sum_{i,j} (\underline{x}_{ij..} - \underline{x}_{i...})^2}{2 \sum_{i,j,k} (\underline{x}_{ijk.} - \underline{x}_{ij..})^2}$$

which under H_o, has a F-distribution with 2 and 4 degrees of freedom. The test results are stated in Table 1.

Table 1

Results of the test for $\sigma_e^2 = 0$

slot	F	K
2	9,74	0,03
3	0,30	> 0,50
4	0,01	> 0,50
5	0,13	> 0,50
6	0,73	> 0,50
7	0,98	> 0,40

From table 1 it is quite clear that apart from slot 2 there is no reason at all to reject H_0. At slot 2 there is a slight indication on a non-zero variance between sub-experiments. Because of the general line, prevailing in table 1, the term $\underline{\epsilon}_{ij}$ will be omitted from the mathematical model.

In table 2 the scheme of the analysis of variance to which the model leads is given. The test whether at any one slot the effect of changing the atomizing agent is equal to zero, is again

Table 2

Scheme of the analysis of variance

source of variation	sum of squares	number of degrees of freedom	expectation of the mean sum of squares
atomizing agent	$8\sum_i (x_{i...} - x_{....})^2$	1	$8\sum \mu_i^2 + 2\sigma_b^2 + \sigma_w^2$
between flames	$2\sum_{i,j,k} (x_{ijk.} - x_{i...})^2$	6	$2\sigma_b^2 + \sigma_w^2$
within flames	$\sum_{i,j,k,l} (x_{ijkl} - x_{ijk.})^2$	8	σ_w^2
total	$\sum_{i,j,k,l} (x_{ijkl} - x_{....})^2$	15	

a F-test. The statistic is, as can be seen from table 2

$$\underline{F} = \frac{6}{1} \frac{8\sum_i (\underline{x}_{i...} - \underline{x}_{....})^2}{2\sum_{i,j,k} (\underline{x}_{ijk.} - \underline{x}_{i...})^2}$$

which under H_0 has a F-distribution with 1 and 6 degrees of freedom. The testresults are given in table 3 in the usual way:

I stands for $0{,}01 < k \leq 0{,}05$
II " " $0{,}001 < k \leq 0{,}01$
III " " $k \leq 0{,}001$.

In table 3 have been given as well:
1) the estimate of the general mean
2) " " " the effect of atomizing agent
3) " " " s_b^2 of σ_b^2, together with the results of the test of the hypothesis: $\sigma_b^2=0$
4) " " " s_w^2 of σ_w^2.

The estimate of the effect of the atomizing agent has to be added to (substracted from) the general mean in case this atomizing agent is steam (air). Of course the sign of the estimate has to be taken in consideration. A + sign in the third column of table 3 means that the sum of the observations on steam were higher than on air.

Table 3

Estimates and testresults, regarding the effect of the atomizing agent

slot	general mean	effect atom.agent	s_b^2	s_w^2
2	5,93	-2,20 III	0,462 III	0,071
3	7,93	-1,17 I	0,718 I	0,248
4	6,50	-0,25	1,521 III	0,257
5	4,64	+0,30	1,523 III	0,157
6	3,06	+0,61	0,652 III	0,080
7	2,68	+0,12	0	0,271

From table 3, two conclusions can be drawn:
1. There is an indication that in the first part of the furnace air gives a higher radiation than steam.
2. The variance "between flames" is in general different from zero. We want to stress the importance of designing experiments on radiation in such a way, that the different sources of variation can be measured with sufficient accuracy. In this case this means performing replicates, plaited in in the experiment in a balanced way.

3. An estimate of $2\sigma_b^2 + \sigma_w^2$

In general two observations have been made on one setting of a flame. In testing effects by comparison of flames with different flame conditions one therefore needs an estimate of $2\sigma_b^2 + \sigma_w^2$, with as high precision (= number of degrees of freedom) as possible. The second line of table 2 gives an estimate based on 6 d.f. Another 4 degrees of freedom can be obtained by comparison of:

PT VIII A	flame	XI	and	XI R
	flame	XIV	and	XIV R
PT VIII B	flame	V	and	V R
	flame	IX	and	IX R.

The following estimates of $2\sigma_b^2 + \sigma_w^2$, based on 10 degrees of freedom have been found in this way.

slot	2 :	0.633
"	3 :	1,121
"	4 :	2,069
"	5 :	2,037
"	6 :	0.975
"	7 :	0.340.

4. The relation between C/H ratio and the radiation

A number of measurements have been made with the subject of obtaining information on the relation between the C/H ratio of the fuel and the radiation of the flame, generated by this fuel. The flames concerned are the following gasoil-flames:

flames	XIX	XX	XXII	XXIV
C/H of the fuel:	6,47	6,97	7,45	7,66

We shall denote the average radiation of a flame by y_i and its C/H ratio by x_i (i=1 for flame XIX,... , 4 for flame XXIV). If for each x

$$y = \alpha + \beta x + \underline{\epsilon} ,$$

where $\underline{\epsilon}$ is assumed to be a normally distributed variable, with $E\underline{\epsilon}=0$ and for all x $E\underline{\epsilon}^2 = \sigma^2$, regression analysis gives on the basis of the observations (x_i, y_i) (i=1,..,4) estimates a and b for α and β, together with a test for the nulhypothesis $\beta=0$. Results of this (Student's t-) test, and the estimates are given in table 4. The regression lines have been drawn in the figures at the back of this report.

From table 5 the conclusion can be drawn that only for the first two slots an indication exists of a significant relationship between C/H ratio and the radiation. Actually the number of measurements available is definitely insufficient to lead to any valuable conclusion whether or not there is a relationship.

table 4

Results of regression analysis on relation between C/H ratio and radiation

slot	a	b	value of t	k
2	-13.59	2.69	4.50	0.05
3	- 2.79	1.46	5.64	0.03
4	- 0.28	0.65	1.43	0.30
5	- 3.61	1.08	2.05	0.15
6	- 0.00	0.42	1.68	0.25
7	+12.65	-1.34	-1.80	0.25

The question whether the relationship between C/H ratio and radiation changes with the slotnumber has not very much sense now, as no relation between C/H and radiation has been found for the larger part of the furnace.

One could of course plot the found values of b against the slotnumber. A test cannot be applied to examine whether there is any relation between the values of b and the slotnumber, because of the fact that observations on different slots are not independent. The found values of the teststatistic t has been plotted (fig. 1). The diagram actually suggest the existence of a trend, but no proof or disproof can be given on basis of the available measurements.

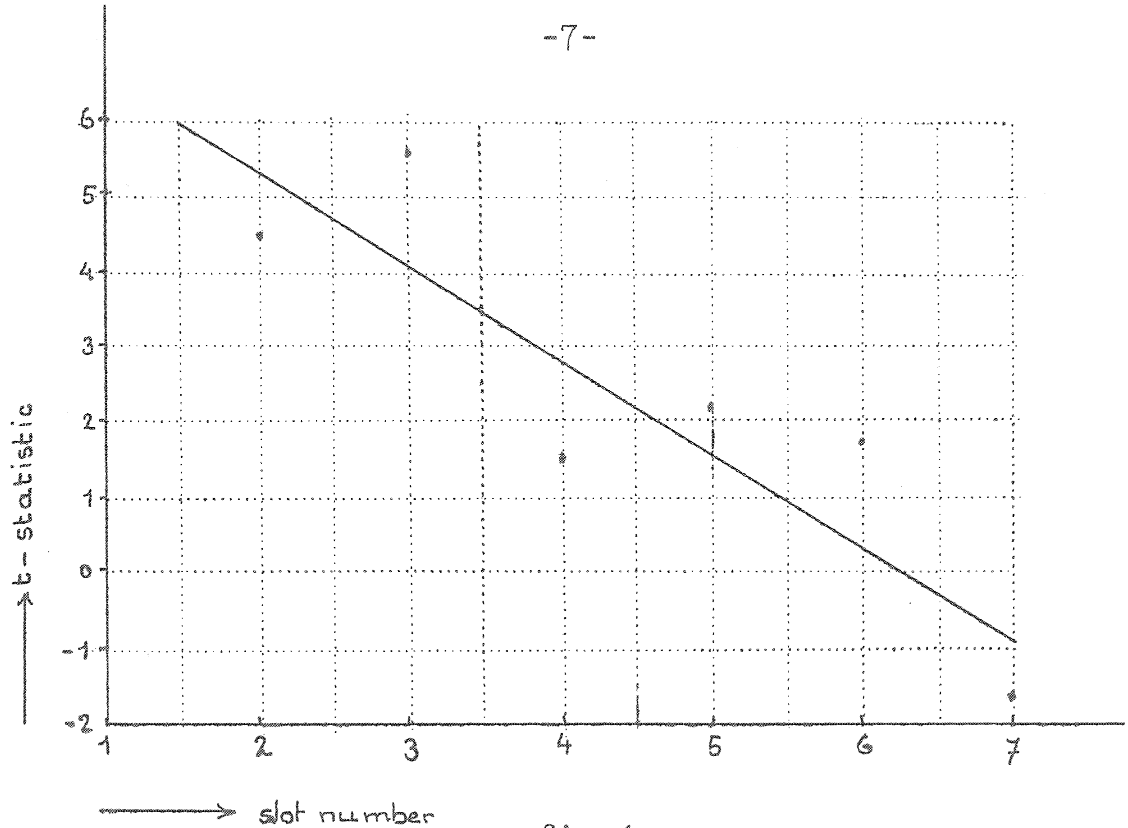


fig.1

Found values of the t-statistic (of table 4), plotted against the slot-number

5. Gasoil - Fuel-oil

The flames, dealt with in section 4 were all generated by means of gasoil. In order to examine whether the type of fuel had any specific influence on the radiation, obtained with a fuel with a certain C/H ratio, a few gasoil flames have been examined. As the C/H ratio of the gasoil flames did not coincide with the C/H ratios of the fuel-oil flames, a direct comparison between flames is not possible. Instead we proceed as follows. Statistical theory (cf [1]) provides a method for determining a confidence belt around a regression line. These confidence limits, as they have been calculated and drawn in figures 2-7 mean:

"If at slotnumber i a measurement will be made on a gasoil flame with a C/H ratio x, then in the long run in about 90% of the cases, the corresponding observed value of R_1 max will fall between the limits indicated for that value of x in the diagram for slot i."

Note that we have calculated 90% confidence limits. The confidence limits may serve as control-limits: if a next measurement on gasoil flame on plotting appeared to fall outside the limits, action might be taken to examine in what respect the flame differed from the other flames (taking account of C/H ratios). In that case there is a probability of 10% that such an action will show that there is no difference.

If on plotting the measurements of the fuel-oil flames less than about 10% of the dots fall outside the limits, one has to draw the conclusion: no difference can be noted between fuel-oil and gasoil flames. In diagrams 2-7 4 out of 18 points are (just) outside the limits. Taking the 18 measurements as indepent observations (which they are not), we can test whether the hypothesis: "Each point has a probability p=0.10 of falling outside the limits" can be rejected. According to the table of a binomial probability distribution we find k=0.10. Therefore we have to conclude that no evidence has been found for a difference between the two types of flames. However the whole "experiment" has been based on seven flames only, which is highly insufficient for a difference (if it is not very large) to show up.

6. PT VIII C

Two flame -conditions, viz. IX and XIII differing only with respect of the density have been studied in a small subexperiment, denoted by PT VIII C. This sub-experiment did not provide an estimate of the source of errors. Therefore in comparing the results of flame IX and those of flame XIII one has to use an external estimate of the error-variance. Table 5, column 2 and 3 state the mean of the observed values.

Each mean in column 2 has a variance $\sigma_b^2 + 1/4\,\sigma_w^2$, as they are all based on five observations. The means in column 3 have a variance $\sigma_b^2 + 1/9\,\sigma_w^2$, being based on nine observations. The difference Δ between both mean values has therefore a variance

$$\sigma_b^2 + \frac{\sigma_w^2}{4} + \sigma_b^2 + \frac{\sigma_w^2}{9} = 2\,\sigma_b^2 + 0{,}36\,\sigma_w^2.$$

As σ_w^2 is smaller than σ_b^2, no great loss is incurred in taking this variance as $2\sigma_b^2 + \sigma_w^2$, of which an estimate has been obtained in section 3.

In column 6 we have given a 95% confidence interval for the difference Δ . Only for the second slot the value 0 is not included in the confidence interval. This means that only at slot 2 there is a slight indication of an actual difference between the two flames.

Emphasis must be placed on the fact that the width of the confidence intervals (and therefore the uncertainty in the determination of Δ) is due to the set-up of the experiment.

Table 5

Comparison of flames IX and XIII of subexperiment PT VIII C

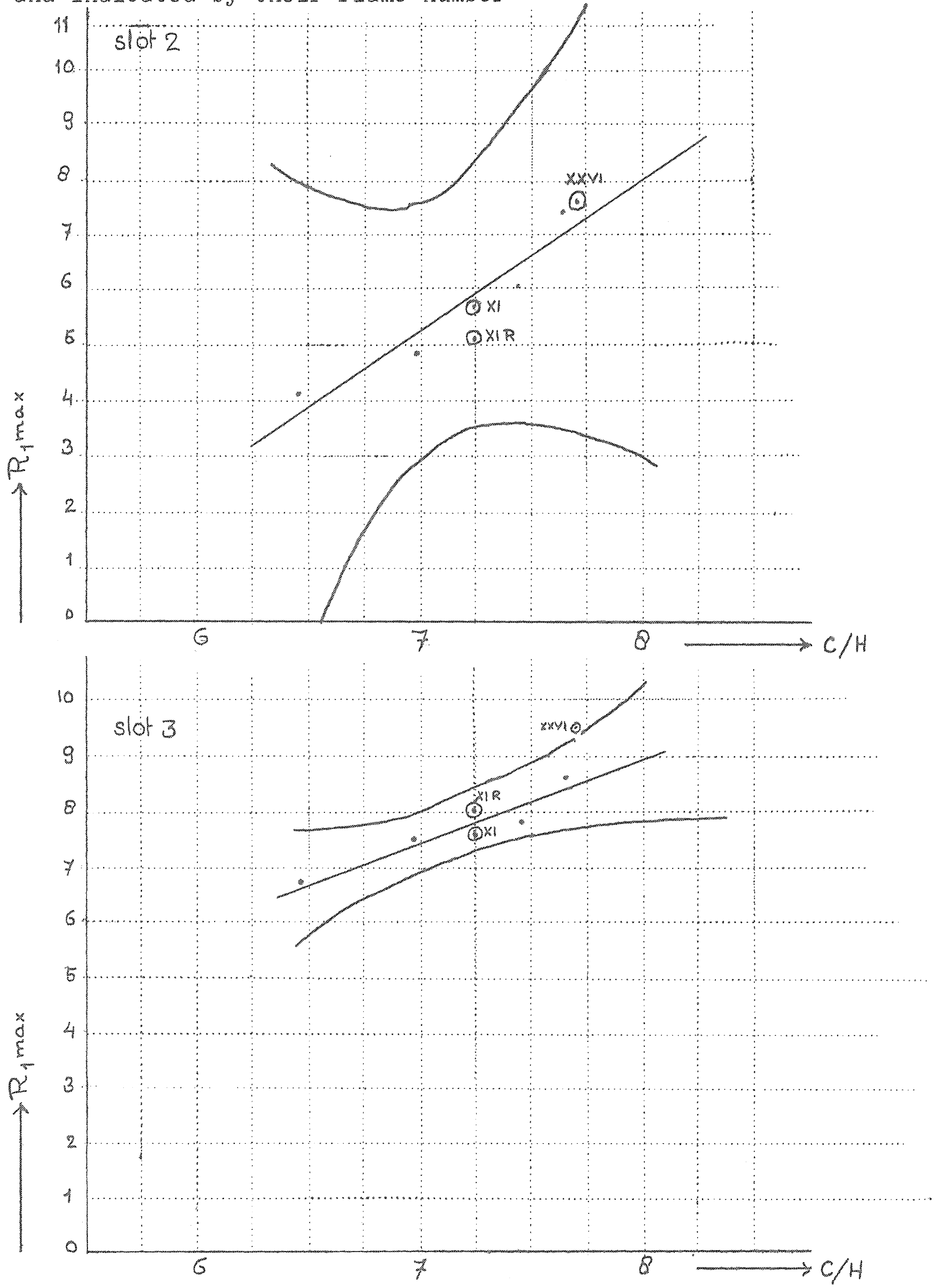
1 slot	2 fl. IX	3 fl.XIII	4 Δ	5 s	6 confidence limits
2	9.18	11.48	-2.30	0.796	-4.07 and -0,53
3	10.39	11.58	-1.19	1.059	-3.55 and +1.17
4	8.41	7.65	+0.76	1.438	-2.44 and +3.96
5	5.18	3.99	+1.19	1.427	-1.99 and +4.37
6	3.79	3.19	+0.60	0.987	-1.60 and +2.80
7	2.68	2.54	+0.14	0.583	-1.16 and +1.43

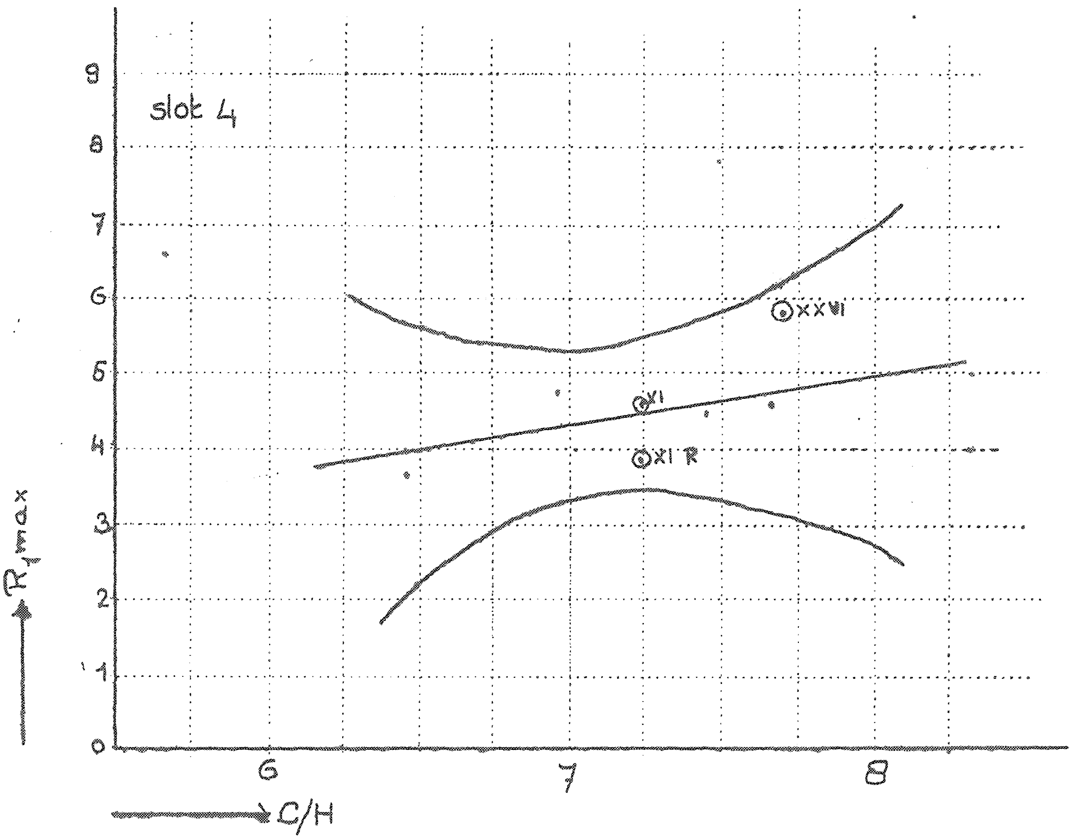
Literature:

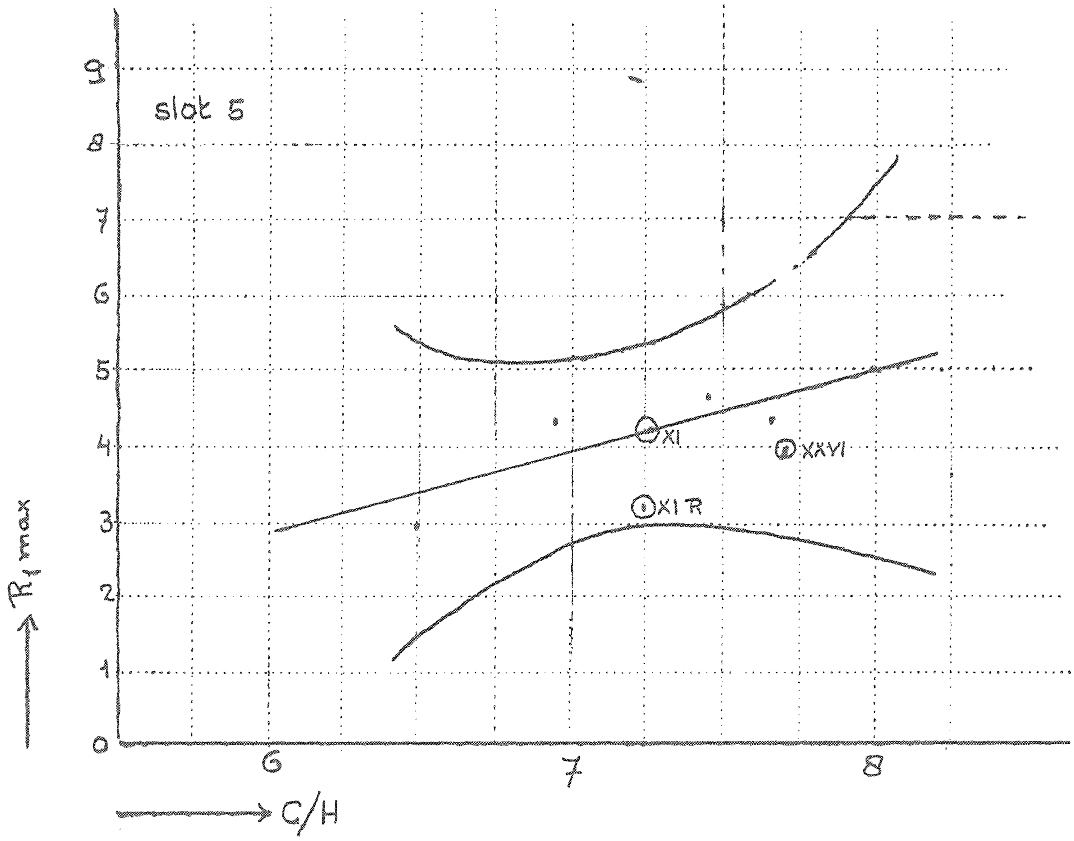
[1] HALD: Statistical Theory with engineering applications, John Wiley, New York, 1952.

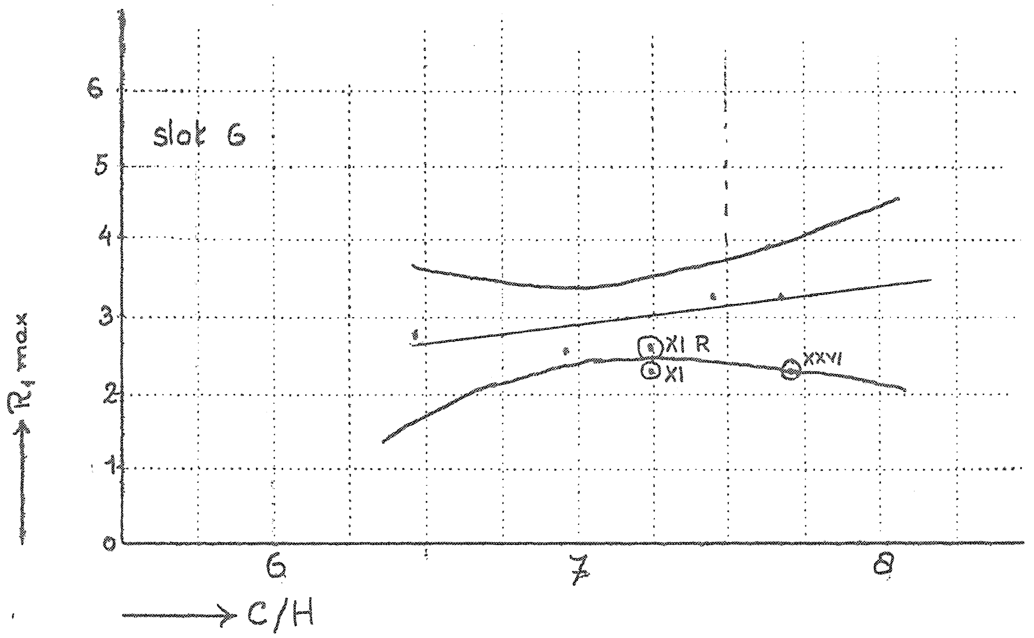
Figures 2-7

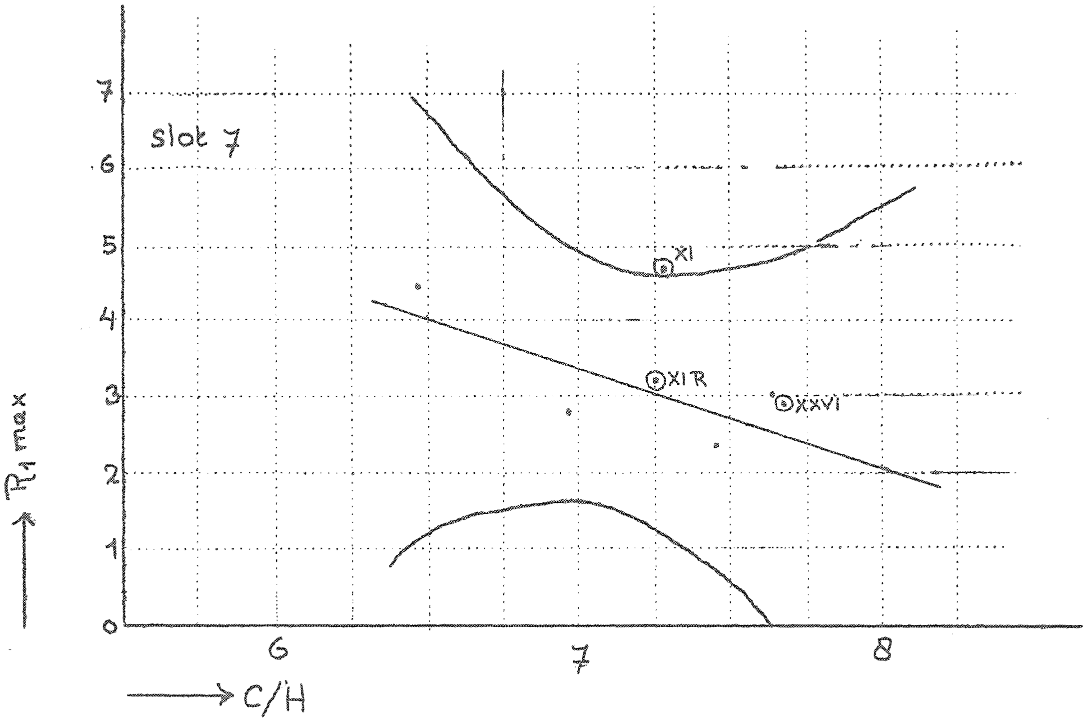
Relationship between C/H ratio and R_1max for gasoil fired flames. The indicated 10%-confidence limits are confidence limits for a single observation. Measurements on fuel-oil flames are plotted and indicated by their flame-number




slot 4
0
1
2
3
4
5
6
7
8
9
6
7
8
C/H
R_1max
XXVI
XI
XI R


slot 5
0
1
2
3
4
5
6
7
8
9
6
7
8
C/H
R_1max
XI
XXVI
XI R


slot 6
0
1
2
3
4
5
6
R1 max
XI R
XI
XXVI
6
7
8
C/H


slot 7
0
1
2
3
4
5
6
7
R1 max
XI
XI R
XXVI
6
7
8
C/H