13

Appl. sci. Res Section A, Vol. **3**

1952

SOME TESTS FOR HYPOTHESES CONCERNING TENSILE STRENGTH OF CLOTH

by J. HEMELRIJK

Mathematical Centre, Amsterdam

Summary

An important quantity for the determination of the quality of a textile product is the tensile strength of the material. This strength may be tested by stretching a strip until it breaks and measuring the maximum tension which is attained during this process. The experiment may be performed in such a way that the time, necessary to attain the breaking point, has a predetermined length. This is called the testing time and it appears that it influences the tensile strength. The present paper gives a number of statistical tests which enable us to investigate certain properties of the relation between the two quantities mentioned. The use of the tests is demonstrated by a numerical example.

§ 1. *Introduction.* In order to obtain information about the distribution of the tensile strength in a lot of about 20 pieces of woollen cloth, a number of samples were taken from these pieces. Each sample was cut into strips of 5 cm or 10 cm width, half of them warpway and the other half weftway. The testing of the strips was done with a dynamometer of the pendulum-type, driven by a motor provided with continuously variable gearing (cf. fig. 1).

The strips are clamped between the jaws of the instrument, and by switching on the motor the pulling jaw is lowered with a constant velocity. The tension produced in the elongated strip is balanced by the upper jaw, fixed to a chain, which in its turn goes partly round and is fastened to an axis that carries and moves the pendulum. The deviation of the pendulum-bar from the vertical is a measure for the load, acting upon the strip. The maximum value of this load attained at the moment of breaking is recorded in kg as the *tensile strength* of the tested strip. Strips, taken from the same sample and cut in the same direction, will stretch to approximately the same length before breaking. The tensile strength, however, is not constant and depends, for instance, upon the velocity of the pulling jaw. By means of testing an additional strip the gear could be adjusted so that the duration of a test had a predetermined length, which we shall call the *testing time*. We shall be concerned mainly with the influence of this factor on the mean of the tensile strength.

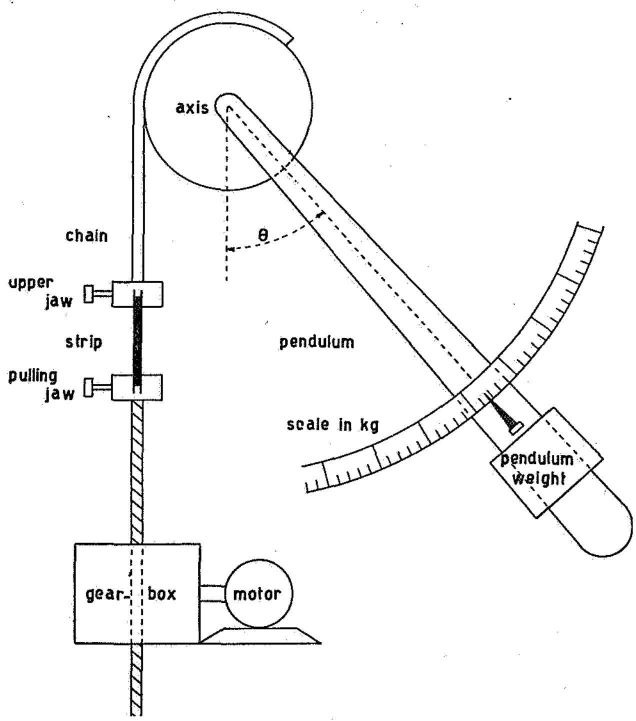

Fig. 1. Dynamometer for textile strength.

According to Mann and Pierce[1]) the influence of the testing-time was expected to be such that, other factors remaining constant, the mean tensile strength should depend linearly on the logarithm of the testing-time. The slope of the regression line of the strength against log time may be independent of the other factors (width of the strip, cutting warpway or weftway, etc.), though these certainly influence the mean value of the tensile strength. For each combination of the other factors, the strength of at least one set of 3 strips, with testing-times 20, 60 or 120 seconds, was determined.

In this paper we are not concerned with the influence of the other factors on the mean of the textile strength, but only with the question of checking the validity of the above mentioned supposition and with testing the equality of the slope of the regression lines when the other factors vary. The random deviations of the tensile strength will be supposed to have a normal probability distribution. Several methods for testing a number of hypotheses will be given, under this and some other assumptions which are formulated in the following section.

The experiment was performed by the "Vezelinstituut T.N.O."; the numerical example, given in § 9, has been based on a part of the experimental results of this institute.

§ 2. *Hypotheses and assumptions.* Let us denote the logarithm of the testing time by t, and the logarithms of the three values used by t_1, t_2 and t_3. Let further n denote the number of triples of indentical strips, tested with testing time t_1, t_2 and t_3, and x_i, y_i and z_i $(i=1, \ldots, n)$ the tensile strength of the i^{th} of these triplets for $t=t_1$, $t=t_2$ and $t=t_3$ respectively.

Then, in order to test hypotheses as mentioned above, we assume x_i, y_i and z_i to be independently normally distributed random variables and we consider the experimental results, obtained by the Vezelinstituut as observations of these random variables; we shall assume one observation of each of these variables to be available. Fig. 2 gives the results of four of these triplets of measurements.

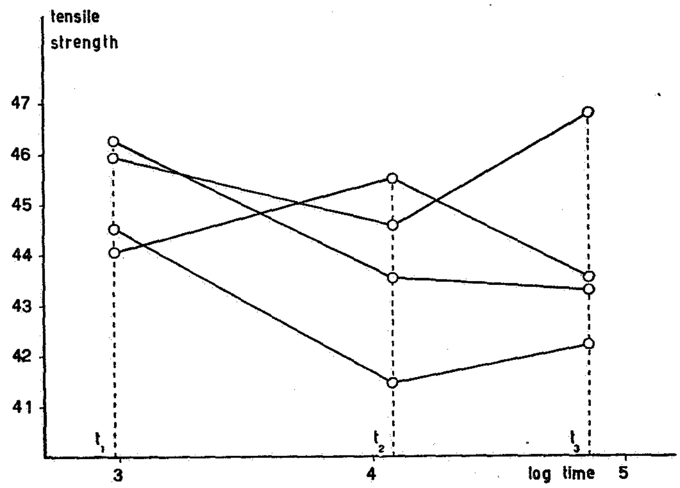

Fig. 2. Four triplets of observations.

Further we assume the standard deviations of x_i, y_i and z_i to be equal to σ_1, σ_2 and σ_3 respectively, where σ_1, σ_2 and σ_3 are unknown, but the same for every i. Under these conditions we shall give methods to test the following hypotheses.

H_1: The regression of tensile strength against t is linear for every i.

H_2: The regression is linear and the slope is the same for every i.

H_3: The regression is linear with the same slope for every i and the ratios σ_1^2/σ_2^2 and σ_3^2/σ_2^2 of the variances of x_i, y_i and z_i are equal to two given numbers k_1^2 and k_3^2 respectively.

§ 3. *General principle of the testing of hypotheses.* The general principle of the testing of hypotheses can roughly be described as an experimental indirect proof: the hypothesis to be tested is first assumed to be true and from this assumption conclusions are drawn about the probability distribution of the random variables or of functions of the random variables which are being observed. If the observed values of these random variables or functions are, in a certain sense, too improbable, the hypothesis is rejected. If this is not the case, the hypothesis can not be rejected on the strength of the given observational material; "non-rejection" is not to be considered as equivalent to "acceptance". A more complete account of the theory may be found in J. Neyman [2]), M. G. Kendall [3]) A. Wald [4]).

Remark: Obviously rejection of H_1 implies rejection of H_2 and H_3, just as rejection of H_2 implies rejection of H_3. Therefore the hypotheses are to be tested in the given order and the process may be stopped as soon as a rejection takes place. Some of the tests, however, may be used for testing hypotheses slightly differing from H_1, H_3 and H_2 (cf. § 8, remark 1).

§ 4. *Mathematical formulation of the problem.* Mathematically, the problem takes the following form: *Given* one observation of each of $3n$ random variables x_i, y_i and z_i $(i = 1, \ldots, n)$, which are distributed independently according to the following normal distributions:

x_i with unknown mean μ_{1i} and unknown standard deviation σ_1,
y_i with unknown mean μ_{2i} and unknown standard deviation σ_2, (1)
z_i with unknown mean μ_{3i} and unknown standard deviation σ_3,

to find tests for the hypotheses:

Hypothesis H_1

$$\frac{\mu_{3i}-\mu_{2i}}{\mu_{2i}-\mu_{1i}}=\frac{t_3-t_2}{t_2-t_1}=c \qquad (i=1,\ldots,n) \qquad (2)$$

where c has a *given* value, t_1, t_2 and t_3 being given numbers.

Hypothesis H_2

$$\mu_{3i}-\mu_{2i}=c(\mu_{2i}-\mu_{1i})=cd \qquad (i=1,\ldots,n) \qquad (3)$$

with $c=(t_3-t_2)/(t_3-t_1)$ as in (2) and d an unknown constant.

Hypothesis H_3

$$\mu_{3i}-\mu_{2i}=c(\mu_{2i}-\mu_{1i})=cd \qquad (i=1,\ldots,n) \qquad (3)$$

and

$$\sigma_2^2=\frac{\sigma_1^2}{k_1^2}=\frac{\sigma_3^2}{k_3^2} \qquad (4)$$

where k_1 and k_3 are given constants.

§ 5. *Tests for* H_1. Defining

$$u_i=y_i-x_i \text{ and } v_i=z_i-y_i \qquad (i=1,\ldots,n), \qquad (5)$$

we have, assuming H_1 to be true,

$$\mathcal{E}\,v_i=c.\mathcal{E}\,u_i \qquad (i=1,\ldots,n) \qquad (6)$$

where $\mathcal{E}$ is the mathematical expectation symbol. Now putting (cf. fig. 3)

$$w_i=v_i-cu_i \qquad (i=1,\ldots,n) \qquad (7)$$

we have

$$w_i=cx_i-(1+c)\,y_i+z_i. \qquad (8)$$

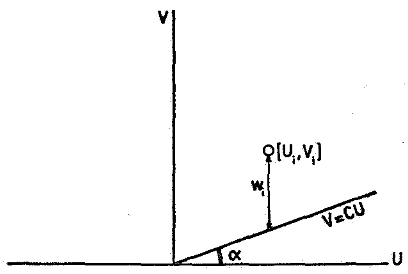

Fig. 3. Geometrical representation of w_i.

Therefore w_i, being a linear combination of independently and normally distributed random variables, is itself normally distributed, with mean

$$\mathcal{E}\, w_i = \mathcal{E}\, v_i - c.\mathcal{E}\, u_i = 0 \tag{9}$$

and variance

$$\sigma^2_{w_i} = c^2\sigma_1^2 + (1+c)^2\sigma_2^2 + \sigma_3^2. \tag{10}$$

These expressions are independent of i, which means that the distribution of w_i is the same for every i; moreover the random variables w_i are distributed independently. From the observed values of x_i, y_i and z_i ($i = 1, \ldots, n$) the corresponding values of w_i may be computed and these n values ought to form a random sample from a normal distribution with zero mean (cf. (9) and variance as given by (10)).

This can be tested in several well known ways:

A. When m denotes the mean of $w_1, \ldots, w_n$ and s^2 the variance:

$$m = m_w = \frac{1}{n} \Sigma\, w_i; \quad s^2 = \frac{1}{n-1} \Sigma\, (w_i - m)^2,$$

then $mn^{\frac{1}{2}}/s$ is distributed according to a Student distribution with $n - 1$ degrees of freedom (cf. M. G. Kendall[5]), p. 239 and M. G. Kendall[3]), p. 98; tables of the Student distribution may be found e.g. in M. G. Kendall[5]), p. 440—441 and E. M. Baldwin[6]).

B. If n is large enough, the χ^2-test may be applied. (cf. e.g. M. G. Kendall[5]), chapter 12).

C. The tests of normality developed by R. C. Geary and E. S. Pearson[7]) (cf. also R. C. Geary[8])) can be applied for small as well as large n.

§ 6. *Tests for* H_2. If the tests, mentioned in § 5, do not lead to rejection of H_1, H_2 is to be tested.

According to H_2 the random variables u_i (as well as v_i) are distributed independently, with unknown mean d (and cd respectively) and unknown standard deviation $\sqrt{\sigma_1^2 + \sigma_2^2}$ (and $\sqrt{\sigma_2^2 + \sigma_3^2}$ respectively). As in the foregoing section, these distributions are the same for every i and therefore the tests B and C of section 5 may be applied to the observed values u_i as well as to the $v_i (i = 1, \ldots, n)$. The test mentioned under A, however, cannot be applied, because d is unknown.

It is reasonable to suspect that in most cases the tests B and C for H_2 will be rather poor. If the slope is not the same for every i, but is itself distributed normally, the power of these tests is only equal to their significance level. The same applies to test C as a test for H_1. In § 8 (test E) an alternative possibility of testing H_2 is given which probably is not very powerful either. The only test for H_2 which certainly may be expected to be powerful is test D (cf. § 8), when the values of k_1 and k_3 in (5) are known. We have not been able to find a powerful test for unknown k_1 and k_3.

Remark: If there is any reason to test a special value of d, i.e. of the common slope of the regression lines mentioned in § 2, this can be done by applying test A of § 5 to the quantity $(m - d)\sqrt{n}/s$.

§ 7. *Tests for* H_3. If H_2 has not already been rejected as a consequence of the tests for H_1 and H_2, additional tests may be developed by considering the following deductions from H_3.
For every i we have (from H_3):

u_i is normally distributed with mean d and variance

$$\sigma_2^2(1 + k_1^2), \tag{11}$$

v_i is normally distributed with mean cd and variance

$$\sigma_2^2(1 + k_3^2). \tag{12}$$

Both u_i and v_i are therefore distributed according to the same (two-dimensional) distribution for every i, but not independently of one another. The simultaneous distribution of u_i and v_i is a two-dimensional normal distribution, u_i and v_i both being linear combinations of normally distributed variables. This probability distribution is obviously the same for every i and every pair (u_i, v_i) is distributed independently of the other pairs (u_j, v_j).

Omitting the index i and using the notation

$$\tilde{u} = u - \mathcal{E}u, \ \tilde{v} = v - \mathcal{E}v, \quad \text{etc.}, \tag{13}$$

we have

$$\mathcal{E}\tilde{u}\tilde{v} = -\mathcal{E}\tilde{v}^2 = -\sigma_2^2,$$
$$\mathcal{E}\tilde{u}^2 = \sigma_2^2(1 + k_1^2), \quad \mathcal{E}\tilde{v}^2 = \sigma_2^2(1 + k_3^2).$$

Therefore the coefficient of correlation ϱ has the value

$$\varrho = \varrho_{u,v} = -\{(1 + k_1^2)(1 + k_3^2)\}^{-\frac{1}{2}}. \tag{14}$$

The probability density of $\tilde{u}$ and $\tilde{v}$ simultaneously is thus proportional to

$$\exp\left[-\frac{1}{2(1-\varrho^2)}\left\{\frac{\tilde{u}^2}{\sigma_u^2}-\frac{2\varrho\,\tilde{u}\tilde{v}}{\sigma_u\sigma_v}+\frac{\tilde{v}^2}{\sigma_v^2}\right\}\right],$$

which, according to (11), (12), and (14) may be written

$$\exp\left[-C_1\left\{\tilde{u}^2(1+k_3^2)+2\tilde{u}\tilde{v}+\tilde{v}^2(1+k_1^2)\right\}\right] \tag{15}$$

with

$$C_1=\frac{(1+k_1^2)(1+k_3^2)-1}{\sigma_2^2}$$

an unknown constant. We note that, substituting (13), the quadratic terms of u and v in the exponent have the same form as those of $\tilde{u}$ and $\tilde{v}$ in (15). We now apply a transformation of the coordinates of the form

$$\begin{aligned} u &= u'\cos\alpha + v'\cos\varphi,\\ v &= u'\sin\alpha + v'\sin\varphi \end{aligned} \tag{16}$$

with $\tan\alpha = c$ (cf. fig. 4) and φ to be chosen so that the random variables u' and v', resulting from u and v after this transformation, are distributed independently of one another. Fig. 4 shows both coordinate systems.

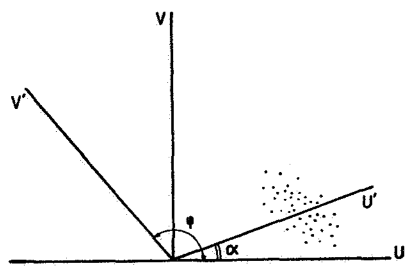

Fig. 4. Transformation of u and v.

Applying the transformation to the quadratic terms of the exponent of the simultaneous probability density of u and v, we have

$$\begin{aligned} u^2(1+k_3^2)+2uv+v^2(1+k_1^2) &=\\ &= u'^2\{(1+k_1^2)\sin^2\alpha+2\sin\alpha\cos\alpha+(1+k_3^2)\cos^2\alpha\}+\\ &+ v'^2\{(1+k_1^2)\sin^2\varphi+2\sin\varphi\cos\varphi+(1+k_3^2)\cos^2\varphi\}+\\ &+ 2u'v'\{(1+k_1^2)\sin\alpha\sin\varphi+(1+k_3^2)\cos\alpha\cos\varphi+\sin(\alpha+\varphi)\}. \end{aligned} \tag{17}$$

It follows from this equation that u' and v' will be distributed independently if φ is chosen such that the coefficient of $u'v'$ is equal to zero. We thus find

$$\tan \varphi = -\frac{\sin a + (1 + k_3^2) \cos a}{\cos a + (1 + k_1^2) \sin a} = -\frac{1 + c + k_3^2}{1 + c + ck_1^2} = g \text{ (say)}. \quad (18)$$

This expression contains no unknown constants; the transformation (16) is thus uniquely determined. We now find from (17) after some reduction, that the probability density of $\tilde{u}'$ is proportional to

$$\exp [- C_1(1 + c^2) \{(1 + k_1^2) c^2 + 2c + (1 + k_3^2)\} \tilde{u}'^2] \quad (19)$$

where C_1 is the unknown constant introduced in (15). The mean of v' being 0 according to (3) and the choice of the u'-axis, the probability density of v' is proportional to

$$\exp [- C_1(1 + g^2) \{(1 + k_1^2) g^2 - 2g + (1 + k_3^2)\} v'^2]. \quad (20)$$

Thus, as a last deduction from H_3, we have

$$\frac{\sigma_{u'}^2}{\sigma_{v'}^2} = \frac{1 + g^2}{1 + c^2} \frac{(1 + k_1^2) g^2 - 2g + (1 + k_3^2)}{(1 + k_1^2) c^2 + 2c + (1 + k_3^2)} = a^2 \text{ (say)}, \quad (21)$$

where again no unknown constants are present any more.

According to (5) and (16) the $3n$ observed values of x_i, y_i and z_i may be transformed into n values of u_i' and n of v_i'. From the derived results it follows, that we may then apply the following

Tests for H_3.

D. If $s_{u'}^2$ and $s_{v'}^2$ are the variances of the n values of u_i' and of v_i' respectively, then according to (21) and the independence of u' and v' the quantity

$$F = s_{u'}^2 / a^2 s_{v'}^2, \quad (22)$$

can be tested by the F-test of Snedecor (which is the same as the z-test of Fisher), cf. e.g. G. W. Snedecor [9]), p. 218—225, R. A. Fisher [10]), p. 242—247, P. G. Hoel [11]), p. 150—154 and p. 250—253.

If F proves to be significantly larger than 1, this merely is an indication, that (3) or (4) or both are not satisfied. If, however, F is found to be significantly smaller than 1, this can never be explained by (3) not being satisfied. This may be seen from fig. 4 where a scatter diagram has been sketched in the supposition that (3) is satisfied. Deviations from (3) will have the effect of scattering the points more widely, and possibly in an irregular way, along the u'-axis. Thus F would increase. Deviations from (4), however, may increase F as well as decrease it. Therefore, if F is too small, this should in the first place be taken as indicating that (4) is not satisfied. It would not, of course, exclude the possibility of (3) not being satisfied either.

E. From the n pairs of values of u_i and v_i, deduced by (5) from the original observations, the correlation coefficient r can be computed in the usual way and the value found can be tested against the theoretical value (14) e.g. with the aid of the tables of F. N. David [12]). Whatever are the values of k_1 and k_3, according to (14) ϱ is always negative. If a large positive value of r is found, this cannot be a consequence of deviations of k_1 and k_3 from their presumed values; such a value may, however, originate from (3) not being satisfied. For, the effect of scattering the points in fig. 4 more widely along the u'-axis is to increase r. A positive value of r, significantly differing from ϱ, is thus in the first place an indication that (3) is not satisfied, which, of course, does not exclude the possibility of (4) not being satisfied either. If r is positive and differs significantly from 0, (3) may be rejected for all values of k_1 and k_3. Conversely, if r is significantly smaller than ϱ, this indicates in the first place that (4) is not satisfied.

§ 8. *Remarks*. 1. If H_1 is rejected because the result of test A is significant, (cf. § 5), it may occur that test B and C are not significant. This means, that the hypothesis H_1'

$$\frac{\mu_{3i} - \mu_{2i}}{\mu_{2i} - \mu_{1i}} = c' \quad (i = 1, \ldots, n), \tag{2'}$$

where c' is an unknown constant, is not rejected, the value $(t_3 - t_2)/$ $/(t_2 - t_1)$ being excluded for c' on account of test A. The regression is then not linear. With the unknown constant c' instead of the known constant c, the test for H_2, mentioned in § 6, may still be applied to test H_2 and test E (cf. § 7) may be used to test H_3. Test D, however, cannot be applied any more since (18) now contains the unknown constant c'. This situation occurs in the example of § 9.

2. When all these tests are applied, the fact should be taken into account that the probability of finding a significant result in one of these cases, if the hypothese are all true, is larger than the signifi-

cance level used would suggest. This probability increases with the number of tests applied, but it is difficult to say how much, because of the interdependence of the tests. It will never, of course, exceed the sum of the significance levels of all tests used; as a matter of fact it may be expected to be considerably smaller than this sum. However, applying the tests in the order in which they are given here, and considering the results critically, it will certainly be possible to get a good idea of which parts of the hypotheses, if any, are to be rejected and which are not.

§ 9. *Example.* A set of 48 triplets of observations has been taken from the experimental results of the "Vezelinstituut T.N.O." (cf. table I).

TABLE I

x_i, y_i and z_i in kg ($i = 1, \ldots, 48$)								
x_i	y_i	z_i	x_i	y_i	z_i	x_i	y_i	z_i
44.6	41.5	42.3	48.2	41.9	44.8	47.1	44.2	45.9
44.2	41.3	42.7	47.0	42.9	43.4	48.8	44.5	42.9
44.2	45.6	43.6	40.2	39.6	42.2	47.5	43.8	43.9
44.9	42.0	43.7	44.7	41.2	43.7	49.2	43.7	49.2
42.5	41.5	45.2	44.2	40.7	38.7	51.7	43.7	47.2
43.9	43.1	44.5	45.2	40.7	42.0	49.1	45.7	41.7
46.2	43.3	44.2	44.8	40.5	44.7	48.7	44.2	45.7
46.2	43.6	43.7	43.3	41.1	41.1	50.2	43.2	43.5
46.3	43.6	43.4	47.1	44.0	43.3	50.8	44.8	46.4
47.7	43.8	43.5	48.2	43.6	40.2	49.9	47.1	47.7
45.8	43.0	42.9	46.9	44.7	41.2	49.2	44.6	46.0
48.2	43.7	43.3	47.2	46.1	42.4	47.6	43.3	44.0
47.6	41.7	45.0	47.1	45.2	42.3	47.7	44.2	47.2
47.6	40.8	42.7	48.2	44.2	42.8	47.5	43.3	45.2
47.2	43.7	43.6	47.2	45.7	46.3	47.2	43.9	44.7
46.1	38.5	41.1	46.4	44.7	46.9	46.4	42.3	43.5

The three values of the testing time were 20, 60 and 120 seconds, thus

$$t_1 = \log_e 20 = 2.99573,$$

$$t_2 = \log_e 60 = 4.09434,$$

$$t_3 = \log_e 120 = 4.78749$$

and

$$c = \frac{t_3 - t_2}{t_2 - t_1} = 0.6309.$$

The values u_i, v_i and $w_i = v_i - cu_i$, computed from table I and the values of c, are tabulated in table II.

TABLE II

u_i, v_i and $w_i = v_i - cu_i$ ($i = 1, \ldots, 48$)								
u_i	v_i	w_i	u_i	v_i	w_i	u_i	v_i	w_i
—3.1	0.8	2.756	—6.3	2.9	6.875	—2.9	1.7	3.530
—2.9	1.4	3.230	—4.1	0.5	3.087	—4.3	—1.6	1.113
1.4	—2.0	—2.883	—0.6	2.6	2.979	—3.7	0.1	2.434
—2.9	1.7	3.530	—3.5	2.5	4.708	—5.5	5.5	8.970
—1.0	3.7	4.331	—3.5	—2.0	0.208	—8.0	3.5	8.547
—0.8	1.4	1.905	—4.5	1.3	4.139	—3.4	—4.0	—1.855
—2.9	0.9	2.730	—4.3	4.2	6.913	—4.5	1.5	4.339
—2.6	0.1	1.740	—2.2	0	1.388	—7.0	0.3	4.716
—2.7	—0.2	1.503	—3.1	—0.7	1.256	—6.0	1.6	5.385
—3.9	—0.3	2.161	—4.6	—3.4	—0.498	—2.8	0.6	2.367
—2.8	—0.1	1.667	—2.2	—3.5	—2.112	—4.6	1.4	4.302
—4.5	—0.4	2.439	—1.1	—3.7	—3.006	—4.3	0.7	3.413
—5.9	3.3	7.022	—1.9	—2.9	—1.701	—3.5	3.0	5.208
—6.8	1.9	6.190	—4.0	—1.4	1.124	—4.2	1.9	4.550
—3.5	—0.1	2.108	—1.5	0.6	1.546	—3.3	0.8	2.882
—7.6	2.6	7.395	—1.5	2.2	3.146	—4.1	1.2	3.787

Test A (cf. § 5); from table II we find:

$$m_w = \frac{1}{n} \Sigma\, w_i = 2.949; \; s_w^2 = \frac{1}{n} \Sigma\, (w_i - m_w)^2 = 7.417;$$

$$\sqrt{n} \,.\, m_w / s_w = 7.50.$$

Since the number of degrees of freedom is equal to 47, the result is significant even on a level 0.001. H_1, the hypothesis of a linear regression, must therefore be rejected. Indeed this may be guessed at sight from table I.

Test B has not been applied since test C is a more precise one.

Test C. For test C we have to compute

$$a_w = \frac{\Sigma\, | w_i - m_w |}{n s_w} \quad \text{and} \quad (\sqrt{b_1})_w = \left| \frac{\Sigma\, (w_i - m_w)^3}{n s_w^3} \right| .$$

From table II we find

$$a_w = 0.76 \text{ and } (\sqrt{b_1})_w = 0.09.$$

Of these two a_w is not significant on the significance level 0.10, when using a bilateral critical region for a: $(\sqrt{b_1})_w$ is very near its expected value, which is zero. Therefore test C is not significant (cf. R. C. Geary and E. S. Pearson [7]) or R. C. Geary [8])).

According to § 8, remark 1, we may now test hypothesis H'_2 by applying test C to the values u_i and $v_i (i = 1, \ldots, 48)$ seperately. This yields the values

$$a_u = 0.75, \ (\sqrt{b_1})_u = 0.02,$$
$$a_v = 0.76, \ (\sqrt{b_1})_v = 0.48;$$

none of which are significant on the significance level 0.10.

Test E may now be applied according to § 8, remark 1, to test H_3, with the unkown constant c' instead of c. We shall test H_3 with $k_1 = k_3 = 1$. We find for the correlation coefficient r of u_i and v_i $(i = 1, \ldots, 48)$: $r = -0.377$. The mathematical expectation of r is -0.5 if $k_1 = k_3 = 1$; according to the table of F. N. David [12]), p. 49, the value $r = -0.377$ is not significant, even on a significance level 0.20.

We may therefore conclude that the hypothesis H_1, stating the assumedly normally distributed variables x_i, y_i and z_i to have, for every i, standard deviations σ_1, σ_2 and σ_3 respectively and means μ_{1i}, μ_{2i} and μ_{3i}, satisfying the relation

$$\frac{\mu_{3i} - \mu_{2i}}{\mu_{2i} - \mu_{1i}} = \frac{t_3 - t_2}{t_2 - t_1} \qquad (i = 1, \ldots, n), \tag{2}$$

must be rejected.

On the other hand, the hypothesis that $\sigma_1 = \sigma_2 = \sigma_3$ and

$$\frac{\mu_{3i} - \mu_{2i}}{\mu_{2i} - \mu_{1i}} = c' \qquad (i = 1, \ldots, n) \tag{2'}$$

with unspecified c' unequal to c, is not contradicted by the evidence from the observations.

These tests form only a small part of the statistical investigation of the observational material. Since this example was only meant as an illustration of the methods described, we shall not go into any further research of this material.

I want to thank Prof. D. van Dantzig, who suggested the way of approach to the problem and Mr M. de Vries, who carried out the computations of the example. Further thanks are due to the "Vezelinstituut T.N.O." and the department "Afdeling Waarnemingsuitkomsten" of T.N.O. for permission to use the results of the experiment and other cooperation in the preparation of this paper.

Received 19th January, 1952.

REFERENCES

1) Mann, J. C., and F. T. Pierce, J. Textile Inst. **17** (1926) 82.
2) Neyman, J., J. Roy. statist. Soc. **105** (1942) 292.
3) Kendall, M. G., The advanced theory of Statistics II, 2nd ed.; London, 1948.
4) Wald, A., On the principles of statistical inference, 1st ed., Notre Dame; Indiana, 1942.
5) Kendall, M. G., The advanced theory of Statistics I, 3rd ed.; London, 1947.
6) Baldwin, E. M., Biometrika **33** (1943) 362.
7) Geary, R. C., and E. S. Pearson, Tests of normality, 1st ed.; London, 1938.
8) Geary, R. C., Biometrika **28** (1936) 295.
9) Snedecor, G. W., Statistical methods, 4th ed., Ames; Iowa, 1948.
10) Fisher, R. A., Statistical methods for research workers, 10th ed.; London, 1948.
11) Hoel, P. G., Introduction to mathematical statistics, 1st ed.; London, 1948.
12) David, F. N., Tables of the correlation coefficient, 1st ed.; London, 1938.