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Economic decision problems for flood prevention

by

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1. Introduction.

On February 1st 1953 the South Western part of the Netherlands, and, to a smaller extent, parts of England and Belgium, were struck by a disaster, caused by a flood which exceeded by far in height the highest one which hitherto was known in the history of our country. According to Ir A.G.Maris [3] it caused a loss of over 1800 human lives, over 150.000 hectares of land were flooded, about 9000 buildings were demolished and 38000 damaged; there were 67 breaks of dikes, and hundreds of kilometers of dikes were heavily damaged. The total economic loss is estimated at 1.5 till 2 milliards of guilders.

The government rapidly appointed a committee, consisting of prominent hydraulic engineers, in order to design measures for preventing similar disasters in future. As the domain to be covered by its work should be the delta formed by the rivers Rhine, Meuse and Scheldt, it was called the "Delta-commission". For special questions the committee took several scientific institutions as advisors, like the Central Planning Bureau, The Royal Dutch Meteorologic Institute, the Hydrolic Laboratory of the Technical University at Delft and the Mathematical Centre at Amsterdam, and, of courde, several departments of the Public Works Department itself.

Since then the breaks in the dikes have been closed (already before the winter fell), the land has been reclaimed and drained, and an energetic beginning has been made to repair the other material damage. The Δ-committee has advised the government to close completely four of the six sea-arms. As the entrances to the ports of Rotterdam and Antwerp must remain open, the dikes along these arms have to be heightened.

The mathematical problems raised by the flood belong to three types: 1^o statistical problems, 2^o hydrodynamic problems, and 3^o economic decision problems. At this place I shall leave the problems of the second group, concerning the question which height of sea-level a storm of a given type can cause, completely out of consideration. I shall also not go far into the statistical problems, although something has to be said on them in order to understand the economic problems, which form the subject of this conference.

2. Some remarks on the statistical problem.

Until a few decades ago engineers built dikes to such a height that they were safe against the highest flood hitherto observed at that place. Since then, however, statistical considerations about the frequence of floods of different heights have been introduced. In 1939 the Dutch engineer Wemelsfelder [7] determined the statistical estimate of the (cumulative) distribution of the sealevel-heights, and drew important conclusions from it. In 1940 the government appointed a Stormflood-committee, which also came to the conclusion that no absolute upper limit for the height of a flood exists. (When saying "no" upper limit, I mean, of course, no upper limit which can come into practical consideration. An upper limit of 40 meters, say, would have just the same meaning as an infinite one.)

Hence to every height belongs a positive "exceedance probability". For this reason the expression "flood <u>prevention</u>" in the title of this paper might be considered as somewhat misleading.

Wemelsfelder found that the exceedance frequencies during high-tide at Hook of Holland during the period 1888-1937, when plotted on logarithmic paper, very closely followed a straight line, i.e. that the exceedance probability $1-F(h)$ (where h is the height of high-tide and $F(h)$ the (cumulative) distribution function) is an exponential function $e^{-\alpha h}$ (fig. 1).

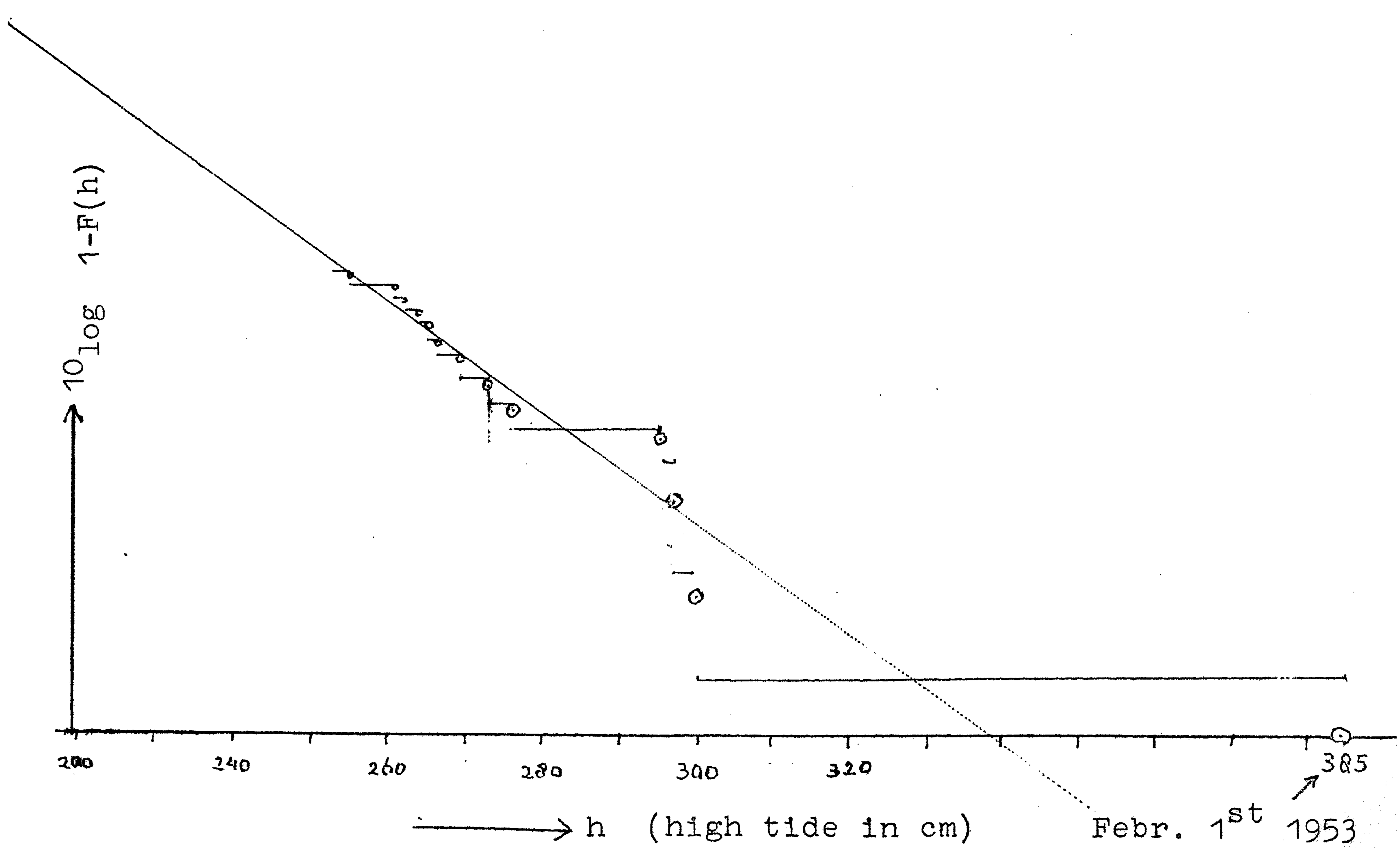

Figure 1.

This can also be written as $2^{-h/a}$, where a is the exceedance probability halving height difference, (i.e. the difference of height, for which the exceedance probability is halved), or shortly, the "halving height"; or also as $10^{-h/a'}$, where a' is the "decimating height". The quantity $\frac{1}{\alpha} = \frac{a}{\ln 2} = \frac{a'}{\ln 10}$ ($\ln 2 \approx 0{,}693$; $\ln 10 = \frac{1}{{}^{10}\log e} \approx 2{,}30$) might be called the "Neperating height", as it is the difference of height, over which the exceedance probability is reduced in the ratio of Neper's number $e \approx 2{,}718$ to 1. Since last year Wemelsfelder's hypothesis has been tested carefully, but no significant deviation from it has been found, at least for Hook of Holland. (Higher up in the estuaries the situation, which quite recently was analysed by Wemelsfelder also, is different.) In particular there is no indication that the curve when extrapolated would tend to a vertical asymptote, which would be an absolute upper limit; on the contrary, the highest floods have a persistant, though not significant tendency to deviate to the right of the straight line. This points to the possibility that the highest floods may be caused by storms of a type different from the ordinary one. In fact, a group of storms having followed paths within a restricted domain, which was selected by Mr C.J.van der Ham in the Meteorological Institute, was analysed in the Mathematical Centre by Prof. J.Hemelrijk with the assistance of Mr H.Kesten en Mr J.Th.Runnenburg, and gave a significantly different straight line. The estimated halving height was raised from 18 to 24 cm, and the .95 confidence limit from 23 to 27 cm.

Nobody doubts, of course, that there is not the slightest certainty that this extrapolation will hold on the long run, but, as no reliable data older than the year 1888 are available, the best thing one can do is to make use of the only result which could so far be reasonably ascertained, whilst avoiding numerous possible pitfalls like dependencies between successive high-tides, occuring during the same storm, etc.

Engineers would prefer to replace the rather arbitrary choice of a .95 or .99 level for the confidence limit by insertion of a "safety factor", traditionally fixed to the value 3, which in this case could best be applied to the exceedance probability p.

3. The decision problem.

I now pass to the economic decision problem. This was put before us in October 1953 by Prof. J.Th.Thijsse at Delft, who also provided us with provisorious estimates of the numerical quantities needed. A provisorious solution was given in December 1953 [1]. The present work was done with the assistance of Mr J.Kriens. As we are no economists, but mathematicians, and as the actual computations of special cases depend upon many local causes, like local variations of depth and currents, wave-rising against the dikes, etc. which can be judged by the engineers only who know the local circumstances, the main task of the Mathematical Centre with respect to this problem is rather of a methodological nature than of giving actual numerical solutions. Work in the latter direction has been done by Ir P.J.Wemelsfelder [8] and quite recently by Prof. J.Tinbergen [4] (on the sea-arm closing project) and Ir F.J.de Vos [6] under Dr Ir J.van Veen on the island of Terschelling. I must add that during the time when most of the work was done we could not benefit from Prof. Tinbergen's experiences, who was abroad.

For these reasons the problem has been treated roughly only, by taking constant average values for quantities which really vary locally or in time.

So we consider a definite part of the country, situated below sea-level and protected against the sea [1)] by surrounding dikes.

Then the economic decision problem can be formulated as follows:

Taking account of the cost of dike-building, of the material losses when a dike-break occurs, and of the frequency distribution of the different sea-levels, to determine the optimal height of the dikes.

For discussing the solution of this problem we assume that the future dikes around the land under consideration will all have the same height H above a given standard level, and we replace the present height (which may vary from place to place) by an average value H_0, so that the amount X by which the dikes have to be heightened, is

$$X = H - H_0$$

The cost $\mathcal{I}$ of heightening the dikes from H_0 to H is a function of X, which can be assumed to be approximately independent of the present height H_0.

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1) We shall speak about 'sea-level', etc. only, not mentioning the fact that part of the surrounding water may be a sea-arm, estuary, river, etc.

The simplest assumption about the possibility of losses is the following, referring to a single polder. Let h at any moment denote the maximum sea-level around the dikes, then no loss is incurred as long as $h \leqq H$; if $h > H$ one can neglect the possibility of partial losses and reckon with a "total loss" only, i.e. assume that all buildings, farms, cattle, industries, etc. contained in the polder are lost. Let V be their total value — we shall call it shortly: the value of the goods (in the polder) and assume that the "consequential loss", e.g. caused by migration costs of the population and cattle, privation of production, etc. etc. are included in it — then the loss S is

$$S = \begin{cases} 0 & \text{if} \quad h \leqq H \\ V & \text{if} \quad h > H \end{cases}$$

The probability distribution of the high-tide sea-level is assumed to be known in the economic problem. By $p(h)$ we denote the probability that any height h will be exceeded at least once during a year.

4. Simple assumptions.

The simplest assumptions to be made are: the value V of the building, etc. and the probability distribution are constant in time; the latter is of the exponential type, as found by Wemelsfelder [7]

$$p(h) = p_0 e^{-\alpha(h - H_0)} \tag{1}$$

where p_0 is the exceedance probability of the present height H_0 of the dikes and $\alpha = \frac{\ln 10}{a'}$, where a' is the "decimating height". Instead of (1) we can also write

$$p(h) = p_0 \cdot 10^{-(h-H_0)/a'} .$$

If a break occurs during one year, it will be assumed that the dikes will have been repaired next year and that the exceedance probability then is the same as before. Neglecting also the probability of repeated breaks (after repair) during one year, the probabilities of losses during different years then are equal and independent. Sea-levels and dike-heights are measured in meters above a given standard level.

Economically the simplest way of treatment is to consider the problem as an insurance problem, i.e. to assume that a sum L will be reserved in order to cover all future losses. If the sum L is invested at a rate of interest δ, it must cover the expected values of all future losses, i.e. $p(H)\,V$ each year, and we have

$$L = p(H)\,V \sum_{0}^{\infty} {}^{t} (1 + 0{,}01\,\delta)^{-t} \approx \frac{100\,p(H)\,V}{\delta} \tag{2}$$

$$= p_0\, e^{-\alpha X}\, V\, \frac{100}{\delta} .$$

The total cost of heightening the dikes being $\mathcal{J}(X)$, we now have to determine X so that $\mathcal{J}(X) + L(X)$ is minimal, i.e. that

$$\frac{d\mathcal{J}}{dX} + \frac{dL}{dX} = 0 \tag{3}$$

The left member of (3) being a function of X, this is an equation in X, the solution of which (which is unique in all important cases) gives the optimal heightening X.

If only a relatively small interval of values of X need be considered, $\mathcal{J}$ can be assumed to be a linear function of X:

$$\mathcal{J} = \mathcal{J}_0 + X \tag{4}$$

Here $\mathcal{J}_0$ is the initial cost, to be made as soon as it is decided that the dikes will be heightened over whatever positive height, and K then is the subsequent cost of heightening them over one meter.

Substitution of (2) en (4) gives:

$$\mathcal{J} + L = \mathcal{J}_0 + KX + \frac{100}{\delta}\, p_0\, V\, e^{-\alpha X} \tag{5}$$

so that (3) becomes

$$K - \frac{100\, p_0\, V \alpha\, e^{-\alpha X}}{\delta} = 0$$

or, using $\ln 10 = 2{,}30$,

$$X = \frac{1}{\alpha} \ln \frac{100\, p_0\, V \alpha}{\delta K} \quad ; \quad \frac{X}{a'} = {}^{10}\log \frac{230\, p_0\, V}{\delta K a'} \tag{6}$$

This quantity has a simple interpretation. According to p. 2 K/α is the "Neperating cost", i.e. the cost of heightening the dikes so much that the exceedance probability is "Neperated", i.e. reduced in the ratio $e : 1$. Hence $0{,}01\,\delta K/\alpha$ is the yearly interest of the Neperating cost. Hence (6) states that the optimal dike-heightening, if expressed in the Neperating (or decimating) heights as a unit of length, equals the natural (or decimal, respectively) logarithm of the ratio of the yearly loss expectation to the yearly interest of the Neperating cost.

If a larger range must be admitted for the variation of X, the linear approximation for $\mathcal{Y}(X)$ is no longer valid, because the higher the dikes, the broader a basis is required. If e.g.

$$\mathcal{Y} = \mathcal{Y}_0 + K X^{k}$$

where $k > 1$ (e.g. $k = 2$), (3) becomes a transcendental equation for X :

$$k K X^{k-1} - \frac{100\, P_0 V \alpha\, e^{-\alpha X}}{\delta} = 0 \tag{7}$$

which has the form $y\, e^{y} =$ const. with $y = \frac{\alpha X}{k-1}$ and

$$\text{Const.} = \left(\frac{k(k-1)^{k-1} \delta K}{100\, P_0 V \alpha^{k}} \right)^{\frac{-1}{k-1}} .$$

If e.g.

$$\mathcal{Y} = \mathcal{Y}_0 + K X + \tfrac{1}{2} \kappa K X^2$$

where κ is small (e.g. $\kappa = 0{,}06$), we get instead of (7)

$$K(1 + \kappa X) \frac{100\, P_0 V \alpha\, e^{-\alpha X}}{\delta} = 0$$

which also has the form $y\, e^{y} =$ const. with $y = \alpha X + \frac{\alpha}{\kappa}$ and

$$\text{const.} = \frac{100\, P_0 V \alpha^2}{K \kappa \delta}\, e^{\alpha/\kappa}$$

If κX is small, we can replace $1 + \kappa X$ by $e^{\kappa X}$. This is equivalent with saying that we can write within the domain of practical importance $\mathcal{Y} = \mathcal{Y}_0 + \frac{K}{\kappa}(e^{\kappa X} - 1)$. Then (7) becomes

$$K e^{\kappa X} - \frac{100\, P_0 V \alpha\, e^{-\alpha X}}{\delta} = 0$$

with the solution

$$X = \frac{1}{\alpha + \kappa} \ln \frac{100\, P_0 V \alpha}{\delta \kappa} ,$$

differing from (6) only by the replacement of the denominator α by $\alpha + \kappa$, which, with e.g. $\alpha = 3$, $\kappa = 0{,}06$ is hardly of any importance.

We shall, however, further use the linear approximation (4).

5. Increase of wealth and sinking of the land.

The assumptions made in 4 were to greatly simplified. In the first place the value of the goods is not constant in time. Increasing wealth of the people requires also the number and value of industries, etc. to be increasing. The rate of increase at present can be estimated; estimates due to Prof. Tinbergen vary between 1,5 and 2,5% per annum, which is to be compared with an interest rate of 3,5-4,5% per annum. Nothing is known, of course, about the question whether this rate of increase (which we shall denote by γ) will remain constant in time over a period of a few centuries. On the contrary, past experience points rather to considerable fluctuations in the value of γ. It may even become negative at some times (wearing outs and decay without increase and renewal). Nevertheless, a secular trend throughout many centuries or even millennia seems unmistakable, so, probably the best thing we can do is to assume γ to be constant. The same can be said about the rate of interest δ.

It will be found useful, to introduce the reduced interest factor $\delta' = \delta - \gamma$ and to express time in centuries instead of years. We shall denote by t and τ a time expressed in years and centuries respectively, so that $t = 100\,\tau$. Moreover δ and γ, being expressed in percento per annum, can also be considered as being expressed in "perunum" per century. Taking δ to be the continuous interest rate, the present value of an amount A at time t is $A\,e^{-0{,}01\,\delta t} = A\,e^{-\delta\tau}$, whereas $A\,e^{-\delta'\tau}$ is the present value of an amount which originally was A and has increased to $A\,e^{\gamma\tau}$ by taking part in the increase of wealth.

In the second place the exceedance probability distribution is not constant in time. Since about 9000 years the Netherlands are slowly sinking into the sea. This is a readjustment of the equilibrium of the earthcrust to the loss of load, caused by the melting away of the Fennoscandian icecap, about 10000 years ago. According to a recent investigation by Prof. Vening Meinesz [5] we have sunk since that time about a hundred meter, but the worst seems to be over: we shall sink only about 3,80 meter more, and much more slowly, so that the greatest depth will be reached in about 5000 years. At present the rate of sinking, which at the beginning must here been about 2 meter a year, is only about 20 cm a century. It is counteracted partly by a rising of the Alpine Foreland, the rate of which, however, is not known. Moreover the sea-level is constantly rising because of the melting away of the Grönland icecap. This is a much more rapid, but also a rather short-run phenomenon; it may be over in another 500 years. Finally, apart from this relative sinking of the land with respect to the sea-level, account must be taken of a relative sinking of the crown of a dike with respect to its foot.

All things taken together we must reckon with a slowly sinking away of the dikes into the sea at a rate which is not precisely known, but which may be estimated at 0,3 m/century. The numerical value, however, is admittedly very uncertain. It will be denoted by η, expressed in meters per century. Then the exceedance probability of a dike, now heightened to H meter becomes a function of time, so $p(H)$ changes over to $p(H,\tau)$. After τ centuries the height H will have fallen to $H-\eta\tau$, or

$$(8) \qquad p(H,\tau) = p_0 \, e^{-\alpha((H-\eta\tau)-H_0)} =$$

$$= p_0 \, e^{-\alpha(H-H_0)+\beta\tau} ,$$

where

$$\beta = \alpha\eta \ . \quad .$$

The expression (8), however, is not self-consistent. In fact, being a probability, it must remain $\leqq 1$, whereas, being an exponential function of time, it would increase indefinit ly.

After a sufficiently long time, indeed, the dikes — if nothing further were done to them — would have sunk into the sea again, and the probability of a flood would become and remain = 1, and (8) will have lost its validity. Hence not only our solution of the decision problem, but also the dikes themselves must be adapted to this sinking away. We shall therefore assume that periodically, with a fixed period of T centuries, the dikes will be regenerated by heightening them by the height ηT they have lost during this period. (Mathematically one might prefer a <u>continuous</u> renewal of the dikes, but this is technically impossible, as one can not heighten a dike yearly by a few millimeters. A reasonable choice might be $\eta T = 1$ (meter), i.e. $T = \frac{1}{\eta}$. Let the cost of these successive renewals be R_1, R_2, R_3, ... They are slowly increasing, because the work has to be done on an increasing height above the ground level.

The total dike building costs become now: 1° the cost of the present heightening over a height X, viz. $I_0 + KX$, 2° at times $T, 2T, 3T, \ldots$ a heightening over ηT at a cost $R_1, R_2, R_3, \ldots$ the present value of which is $R_1 \, e^{-\delta T}, R_2 \, e^{-2\delta T}, R_3 \, e^{-3\delta T}, \ldots$. The sum of this geometric progression will be called J.

The only thing we need is the fact that J is approximately a constant, <u>not</u> depending on X, the small dependence upon X may be neglected, or compensated by a small increase of K. The total building cost is therefore

(9) $$\mathcal{I} = \mathcal{I}_0 + KX + \mathcal{J}.$$

The value of the polder after a time τ has become, because of the increase of wealth: $V e^{\gamma\tau}$. The probability of a flood occurring in one year will have the value (8), so the expected loss during one year is $p_0 V e^{-\alpha X} e^{(\beta+\gamma)\tau}$, and its present value: $p_0 V e^{-\alpha X} e^{(\beta+\gamma)\tau} e^{-\delta\tau}$. This must be summed over all years of the first period $0 \leqq \tau \leqq T$, i.e. integrated over t with $dt =$ $= 100\, d\tau$; the result is

(10) $$p_0 V e^{-\alpha X} \int_0^T e^{(\beta+\gamma-\delta)\tau}\, 100\, d\tau = 100\, p_0 V e^{-\alpha X} \frac{1 - e^{-(\delta'-\beta)T}}{\delta' - \beta},$$

where we have used the form of the last factor, most suitable in the most important case where $\delta' = \delta - \gamma > \beta$. This is the present value of the total loss expectation during the first period. During each subsequent period we get the same result, multiplied for the $(n+1)$st period with $e^{\gamma n T}$ (because of wealth increase) and with $e^{-\delta n T}$ (for taking the present value). Hence the result is the product of (10) with

$$\sum_0^\infty e^{-n\delta' T} = \frac{1}{1 - e^{\delta' T}}$$

and we have

(11) $$\mathcal{I} + L = \mathcal{I}_0 + \mathcal{J} + KX + \frac{100\, p_0 V e^{-\alpha X}}{\delta' - \beta} \frac{1 - e^{-(\delta'-\beta)}}{1 - e}$$

Differentiation with respect to X gives the condition an extremum

(12) $$K - \frac{100\, p_0 V}{\delta' - \beta} \frac{1 - e^{-(\delta'-\beta)T}}{1 - e^{-\delta' T}} \alpha e^{-\alpha X} = 0$$

or $e^{\alpha X} = C$, where

(13) $$C = \frac{100\, p_0 V \alpha}{(\delta' - \beta) K} \frac{1 - e^{-(\delta'-\beta)T}}{1 - e^{-\delta' T}}$$

is always positive (with p_0 V, K, δ', β, T positive). The solution is

(14) $$X = \frac{1}{\alpha} \ln C$$

or also $X = a' \cdot {}^{10}\log C$.

This extremum is easily be seen to be actually a mini… The increasing cost for dike-building at greater height … counted for, like before, by replacing the denominator α … by $\alpha + \kappa$.

According to a remark by Ir F.J.de Vos [6], the obtained result (14) van be written in the form

$$X = X_0 + X_1$$

where

(15) $$X_0 = a' \cdot {}^{10}\log C_0 \quad , \qquad C_0 = \frac{100\, p_0 V \alpha}{\delta' K}$$

is the result we should have obtained if there were no sinking of the land (i.e. $\eta = 0$, hence $\beta = 0$) and account is taken of the increase of value (replacement of the interest rate δ by $\delta' = \delta - \gamma$), whereas

(16) $$X_1 = a' \cdot {}^{10}\log f_1 \quad ,$$

with

(17) $$f_1 = \frac{1 - e^{-\delta' T} e^{\beta T}}{1 - e^{-\delta' T}} \cdot \frac{\delta'}{\delta' - \beta}$$

is a correction term, which tends to zero if the renewal time T does. Here $e^{-\delta' T}$ is the value reduction factor, viz. the present value of a unit of money which has participated in the increase of wealth during a period T. Moreover, putting

$$\lambda = \frac{\eta T}{a'}$$

so that λ is the dike-sinking during one period expressed in decimating-heights, we have

$$e^{\beta T} = 10^{\lambda}$$

This factor measures the increase of unsafety (viz. the ratio of the exceeding probabilities) during one period.

X_1 is always less than its limiting value for $T \to \infty$, which if $\beta < \delta'$ is $a' \cdot {}^{10}\log \frac{\delta'}{\delta' - \beta}$. So in this case we may formally take $T = +\infty$ whence

$$C \to C_\infty = \frac{100\, p_0 V \alpha}{(\delta' - \beta) K}$$

$$X \to X_\infty = \frac{1}{\alpha} \ln C_\infty = \frac{1}{\alpha} \ln \frac{p_0 V}{0{,}01 (\delta' - \beta) . K/\alpha}$$

But we must remind that this has a formal meaning only, as we can in no case let T increase above the time when the dikes have sunk unto the present level again, i.e. $T \leq \frac{X}{\eta}$.

If $\beta > \delta'$ we better write instead of (17)

$$X_1 = a' \cdot {}^{10}\log \left(\frac{e^{(\beta - \delta')T} - 1}{1 - e^{-\delta' T}} \cdot \frac{\delta'}{\beta - \delta'} \right)$$

This expression is for large T approximately

$$0,43\, a'(\beta-\delta')T + a'.\,{}^{10}\log \frac{\delta'}{\beta-\delta'}$$

and tends to infinity if T does. For $\beta=\delta'$ (17) simplifies to

$$X_1 = a'.\,{}^{10}\log \frac{\delta' T}{1-e^{-\delta' T}}$$

which tends slowly (logarithmically) to infinity if $T \to \infty$.

6. <u>The doubtful constants</u>.

We have already mentioned the fact that several of the constants entering into the problem are rather badly known. We possess only rough numerical estimates of most of them, and also rough interval estimates. These are no confidence intervals in the strict sense, but have only the meaning that the competent workers in the fields think that it may be taken for granted that the constants are contained in these intervals. It also is not possible to improve the estimates within a reasonable time.

So the best thing we can do is to ascertain that our solution will hold under the most unfavourable circumstances which must be considered to be realistic. This means that the probl is considered as a minimaxproblem: the sum of cost and risk minimized in the supposition that circumstances are such as to maximize it.

Now, from (13), (14) it is obvious that C, hence X increases if p_0 or V increases or if K decreases (always, the other constants remaining unaltered).

The dependence upon α, η and δ' is somewhat more complicated. If $\delta' > \beta$ C increases, in accordance with our expectation if α or δ' decreases or η increases, but in the case $\delta' < \beta$ C can decrease if α or δ' decreases, or η increases. The case $\delta < \beta$ must therefore be considered anomalous, as it is contr to expectation that 1° under increasing α, i.e. decreasing a, i.e. increasing safety of the sea-level distribution the dikes should have to be built higher, 2° under increasing δ' the dikes should be built higher and 3° under increasing η, i.e. increasing unsafeness the dikes should <u>not</u> have to be built higher.

So in order to remain in the normal case on the safe side we have to take the highest reasonable estimates of p_0, V, and η and the lowest ones of K, α and δ'.

As to T, we must not maximize X under its variation, as we can fix it at will.

7. The anomalous case.

This case arises if $\delta' < \beta$, i.e. $\delta - \gamma < \alpha\eta$. The meaning of this inequality is the following. The increase of value of the goods, as measured by γ, added to the increase of unsafeness because of the sinking of the dikes (if left to themselves!), as measured by $\beta = \alpha\eta$, i.e. the dike-sinking per century expressed in the Neperating height as unit, is greater than the increase of a capital by its compound interest. In other words: the risk of the goods increases so rapidly in time, because of increasing value and increasing danger, that it is impossible to insure them (except for a restricted period), as any rese... formed and invested at the rate of interest δ, however large i... might be taken, would on the long run become insufficient to cover the risk. Evidently this case, though anomalous, is by no means impossible. It is not even unrealistic. The value of δ' may be as small as 1,5 (interest rate δ = 3,5; increase of value γ = 2), whereas α almost certainly is near to 3 (halving height a = 0,23, with an 0,95 confidence limit 0,26, yielding $\alpha = \frac{\ln 2}{a} \approx 3{,}0 \geqq 2{,}64$; for the decimating height a' the esti... is 0,77, the confidence limit 0,87; for the Neperating height $\frac{1}{\alpha}$ these numbers are 0,33 and 0,38 respectively). The sinking height η is estimated at 0,2 till 0,5, but might rise to 0,6 or perhaps even higher. Already with α = 3, η = 0,5 and δ' = ... we get $\delta' - \beta = 0$; for this and higher values of β we have the anomalous case.

8. The rentability.

In the simple case dealt with in 4 it is easy to define and compute the rentability of the project.

The total investment in dike heightening is $Y = Y_0 + KX$; the yearly "yield" is the difference between the yearly loss expectation without dike-heightening, i.e. $p_0 V$, and the same with dike-heightening, i.e. $p_0 V e^{-\alpha X}$, hence $p_0 V(1 - e^{-\alpha X})$. Hence the rentability, expressed in percent per annum, is:

$$\rho = \frac{p_0 V (1 - e^{-\alpha X})}{Y_0 + KX}$$

Hence, by (6)

$$\rho = \frac{p_0 V - 0{,}01 \delta K/\alpha}{Y_0 + \frac{K}{\alpha} \ln \frac{100 p_0 V \alpha}{\delta K}}$$

Or, writing $\frac{\alpha p_0 V}{0{,}01\,\delta K} = C'$

(18) $$\frac{\rho}{0{,}01\delta} = \frac{C' - 1}{\frac{\alpha \mathcal{I}_0}{K} + \ln C'}$$

Here C' is the ratio of the yearly risk to the yearly Neperating cost's interest (which we met in (6)). Hence the numerator of (18) is the difference of this ratio and 1, whereas the denominator is the sum of $\ln C'$, i.e. the optimal heightening expressed in Neperating heights, and the ratio of the initial to the Neperating cost.

Under reasonable suppositions (p_0 = 0,02, α = 3, V = 10, K = 1,8, $\mathcal{I}_0$ = 2,6, δ = 3,5, whence C' = 9,5) we find X = 0,75, whence $\frac{\rho}{0{,}01\delta}$ = 1,3, thus ρ = 0,046.

Under the more realistic assumptions where value-increase and dike-sinking are accounted for, the situation is less simple, as the yearly yield now depends on time.

According to p. 10 the yearly yield in decrease of loss-expectation at any time τ during the first period is

$$p_0 V(1 - e^{-\alpha X})\, e^{(\beta+\gamma)\tau}$$

whence the rentability during that year

(19) $$\rho(\tau) = \frac{p_0 V(1 - e^{-\alpha X})}{\mathcal{I}_0 + KX}\, e^{(\beta+\gamma)\tau}$$

This quantity increases in time as the increasing risk (factor $e^{(\beta+\gamma)\tau}$) is reduced proportionally.

In order to express the rentability as a single number, we could take the ratio R of the present value of the total decrease in risk to the total investment.

Under the assumptions, made in 5 the present value of the investments is $\mathcal{I} = \mathcal{I}_0 + \mathcal{J} + KX$, whereas the total present value of the loss expectation when the dikes are heightened with X me... is $L = \frac{KC}{\alpha} e^{-\alpha X}$. Without any heightening the latter amount is $L_0 = \frac{KC}{\alpha}$.

So

$$L_0 - L = \frac{KC}{\alpha}(1 - e^{-\alpha X})$$

and noticing that the weighted average $\bar{\rho}$ of ρ equals $0{,}01\delta R$,

(20) $$\frac{\bar{\rho}}{0{,}01\delta} = R = \frac{\frac{KC}{\alpha}(1 - e^{-\alpha X})}{\mathcal{I}_0 + \mathcal{J} + KX} = \frac{KC - K}{\alpha(\mathcal{I}_0 + \mathcal{J}) + K \ln C} = \frac{C - 1}{B}$$

where

(21) $$B = \frac{\mathcal{I}\alpha}{K}$$

is the ratio of the total investment to the Neperating cost. Under realistic assumptions (e.g. those mentioned above and $\eta = 0,5$, so $\beta = 1,5$; $\gamma = 2$ and $T = \frac{1}{\eta} = 2$) we obtain:

$$\mathcal{C} = 70, \qquad X = 1,42 \qquad \mathcal{B} \approx 9, \text{ whence}$$

$$\frac{\xi}{0{,}01\delta} = R \approx \frac{69}{9} \approx 7{,}7.$$

As an alternative definition of the rentability one could choose the ratio of the difference between the present value of the diminuation in loss-expectation and the present value of the costs, to the present of the costs. We then obtain for the rentability $R - 1$.

9. <u>The interest rate</u>.

We have already discussed the fact that most of the "constants" [2] entering into our problem are only badly known.

They are of three different types.

Firstly η is a physical constant, which will become known better in the course of time by continued geological research. Also p_0 and α, the constants determining the statistical distributions of the high tide sea-levels are of a similar nature and may become better known by further meteorological and oceanographical research and mathematical and statistical treatment of its results. From these constants β can then also be determined more accurately.

The second group of constants, which describes the present economic situation, does not lead to any essential difficulty. These are: i. the "commercial" constants, determining the costs of dike-building, viz. I_0 and K (and, if the correction mentioned in 4 is used, κ), and ii. the "value" V at this moment of the goods on the land, which can be determined from "national economic" data, although some difficulty may be contained in the determination of the "consequential loss", which we have assumed to be contained in V, e.g., for a first rough estimate, by multiplying the actual value with a constant factor, say 1,2. From these constants, together with those of the first group, other fundamental constants like the present loss-expectation per annum $p_0 V$, and the "Neperating cost" K/α can be derived.

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2) Here and further, when we speak about more precise determination of the constants, we mean, of course, rather determination of the <u>functions</u> into which they enter, e.g. better knowledge about their <u>deviations</u> from constancy.

The situation is somewhat different with regard to the "secular" economic quantities δ and γ, upon which $\delta' = \delta - \gamma$ depends. As to γ, if we wish to ascertain whether the value V the goods have at this moment after a time τ really has increased to $e^{\gamma\tau}$, we must have a stable unit in which the values at different times both can be expressed. It is evident that ordinary monetary units, because of fluctuations and of the secular trend in depreciation of money, do not satisfy this condition. One might try to find the solution by means of the concept of buying-power of the money, but it is not certain that this concept can be defined quantitatively with sufficient precision in order that quantitative comparison of values at moments centuries apart becomes meaningful. For short periods the difficulty is not serious, as the increase of value of γ % per year can be determined empirically by the increase of the goods themselves. As, however, existing goods may become useless or useful by changes in technology, the increase of quantity of goods can not easily be extrapolated over long periods, say of a few centuries. The situation is even more complicated with regard to the interest-factor δ, used in computing the present value of future amounts.

Usually computation of the present value just is used as a formal procedure. As soon, however, as we want to interpret this not only as an accountancy procedure, but as describing real economic phenomena, difficulties arise.

In order to interpret this procedure by real economic phenomena, we have, in the beginning of 4, considered our decision problem as an ordinary insurance problem. This entails, that a reserve is formed for covering all future losses, that this reserve is invested at a rate of interest δ, and that the sum of this reserve (formed now) and the investment in dike-building (made now) is minimized.

This, however, is a rather unrealistic method. For, contrary to a case of ordinary insurance, the risk is not just one among a large number of similar items. This entails that the reserve will not actually be made, and that, even if it were, it would be somewhat difficult to invest it in such a way that the interest δ were ascertained during centuries, that the security for the investment were not itself endangered by the floods against which the insurance is to be made, and that it could be liquidated at once when a flood disaster occurs and the damages must be settled. The fact that for a real insurance an extra reserve were needed because of the dispersion of the random yearly damage may be left out of consideration here, because of the long period the project refers to.

As a second model for the procedure we could choose, instead of an insurance, a loan for financering the dike-building. If this were done, the successive generations of our posterity would have to pay yearly for 1° interest and redemption of the loan and 2° remaining damage. One could determine the plan of redemption so that the total burden on each generation were constant, and than choose the dike-height so that this constant burden were minimal. This model, which gives, except for negligible quantities, the same result as the first one, has the advantage, that the rate of interest δ could be determined empirically, by the present market for public loans, further that only sums of simultaneous payments have to be formed, and that a real total burden were minimized. The possibility of conversion of the loan, however, can hardly be taken account of. One might perhaps be inclined to omit the redemption, i.e. to make its term infinitely long, but, because of the sinking of the land, this would make an equal distribution of burden impossible, unless it were replaced by 3° renewal of dikes.

If, however, the investment were not paid for by a national loan, but from existing national wealth and taxes, this model also becomes unrealistic. Although one could remark, nations do actually issue loans, if not for one purpose then for another, and that it is irrelevant which part of the state budget is used for which purpose, one might prefer a model depending less specifically upon special methods of financing the project 3). If, however, one tries to define quantitatively how large the detrimant to a definite generation of our descendants is, when we invest an arbitrary amount of national wealth and labour in dike-building instead of other projects, the old difficulties regarding comparison of values for different generations come back again.

The choice of a definite model is not unavoidable, as they do not lead to appreciably different results. The question, nevertheless, seems worth of consideration as it reveals a fundamental difficulty for economic decision problems about long-term projects. For the reasons mentioned the second method still seems to be the most realistic one among those determining a definite interest rate δ with sufficient precision. On the other hand, as the actual rate of interest fluctuates considerably, it

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3) I owe these remarks to Prof. J.Tinbergen.

is difficult to see why the degree of security the dikes grant our posterity during centuries should depend so heavily upon the value this rate has on a rather haphazardly chosen moment.

10. Human lives, ideal values and the value of control.

Unlike Sir William Petty [2], who, when discussing the waelth of the Kingdom calculates the worth of a human life at Ł 69.-.-, most modern statisticians are not readily inclined to consider human lives and material goods as commeasurable in value. As, however, the possible loss of human lives, as apart from loss of material goods, should somehow be accounted for by an extra increase of height of the dikes, some decision about its amount must be taken.

It is, however, not necessary to evaluate human lives themselves, as the booking of an item for this purpose only means that the state is willing to spend a certain amount of money for saving a given number of human lives. In order to get a definite figure one could determine statistically, how much the state pays (or induces its citizens to pay) in other cases for a similar purpose (abolition of unguarded railway-crossings at level; prevention of factory accidents; prevention of other traffic accidents, etc.). These amounts, if taken per head, vary greatly. They become very large in a few cases appealing greatly to public imagination, but in many other cases, where this is not the case, (even when leaving war out of consideration), relatively small amounts which could have prevented loss of many human lives are refused. For this reason it seems undesirable in a case where a conscious decision on this subject is taken, to base it on an actually prevailing average, instead of on a figure which can be considered as desirable and providing a guiding norm for future cases also. This, however, does not help us to a definite figure. If, just to try a figure, we would just double the figure for material losses, this would imply in the case of the 1953 flood, where the latter was about $1,5-2,0 \cdot 10^9$ guilders, whereas about 1800 lives were lost, an amount of about 10^9 guilders per head, an amount, which certainly goes far above any sum which would be acceptable, e.g. based on existing practice of life-insurance, as a norm for all cases. On the other hand, any sum which seems acceptable would lead to a hardly perceptible and therefore technically impossibly and emotionally unacceptibly small increase of height; it does not make sense to increase the dikes with an extra cm for the value of human lives. The least one could do were to add to the value of the land and buildings the amount which has been "invested" in human beings, in the form of food and other material goods necessary for their upbringing, together with the labour spent on their education by their parents and teachers. But, apart from being far from easy to compute, this still would leave the emotional factor unaccounted for. A similar argument holds for ideal values like cultural goods.

There are some more hardly computable factors which nevertheless somehow should be accounted for. In the first place damage might be so serious that it would become practically impossible to reclaim the land. In that case it would become incorrect to reckon with the value of land and buildings only, as consequential loss would become all important. Moreover, the greater the part of the land lost, the smaller the resources for emigrating people and cattle, for feeding them, finding work for them, for trying to reclaim the lost land or part of it, and rebuilding its industry.

It is, however, hardly possible to estimate 1° the probability of such an occurrence, 2° the loss caused by it. The only thing we can say is that the curve representing the loss as a function of the area flooded is not linear (not even approximately, if the distribution of wealth were homogeneous), but must have the type of fig. 2. At some unknown point, where remaining resources for reclaiming part of the land becomes insufficient, a rather sharp discontinuaty must arise, whereas furtheron the curve must tend to an asymptote, corresponding with the case where the whole national existance becomes impossible. Nothing more, however, can at present be said about this.

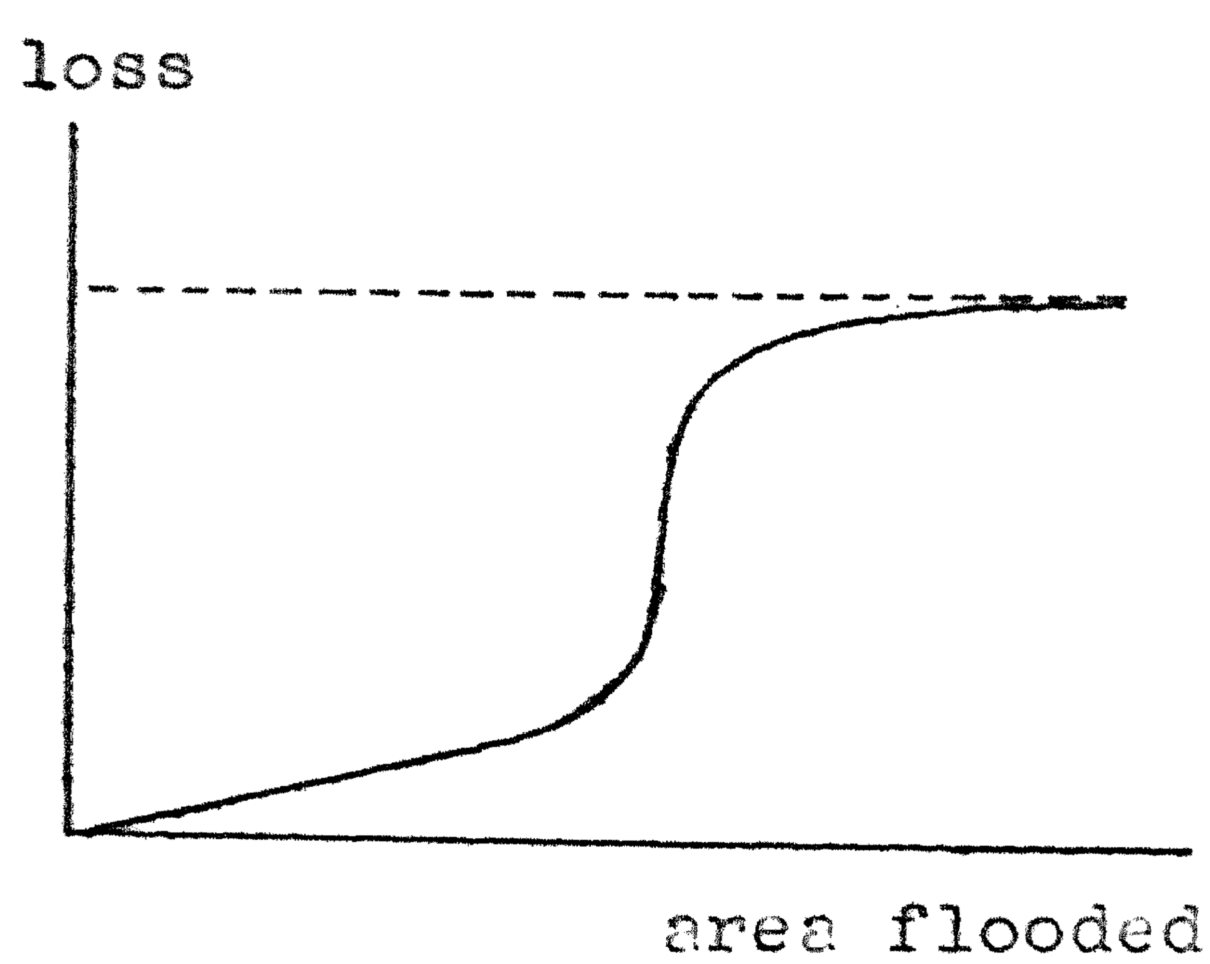

Figure 2.

In the second case it was incorrect that we just have added the expected loss and the investment in dike-building. In fact, when one has the choice between spending one million either in dike-building or in flood-damage, one will doubtless choose the former. One will, of course, even be willing to spend a multiple of the amount if one can thereby prevent the flood. It is, however, almost impossible to say how large this factor must be. It serves to avoid a hardly measurable kind of consequentia loss, consisting of the disorganization caused by a flood, the increased probability of diseases, the psychological, social economic "shock", the possibility that this might occur on a moment that one is least prepared for it (e.g. during a war or an economic depression), etc. etc. Generally speaking one can say that this factor measures the difference in utility of expenditures one has under control and those one has not.

It seems that the best thing we can do is to fix more or less arbitrarily a factor f_3, with which the value V is to be multiplied in order to cover simultaneously the value of human lives and that of control. Like the safety factor f_2, mentioned at the end of 2, which in engineering practice is traditionally fixed on 3, and the period T, or the height ηT over which the dikes may have sunk before they will be renewed, this factor f_3 can not be determined on mathematical, statistical or economical grounds. Its determination is rather a decision which should be made by the responsible authorities than by the scientists, at least as long as no better scientific methods and data are available. In fact, even the determination of the values of the badly known constants which will be considered as best estimates is rather arbitrary, so that we must say that part at least of the ultimate decision hides itself behind the fixation of these constants.

11. Conclusions.

Resuming our results we may say that the solution of the economic decision problem is mathematically simple, but can, at least at present, only partly be made on a purely scientific basis. Some of the constants entering into the result are so badly known, that a fixation of their values is almost a non-scientific decision, some further factors must be inserted either arbitrarily or by tradition. With this proviso we can give the result in the simple form

$$X = \frac{1}{\alpha} \ln C_0 f_1 f_2 f_3$$

where

$$C_0 = \frac{100\, p_0 V \alpha}{\delta' K}$$

depends on the economic quantities V, K, δ' and the unsafety factors p_0 and α only, whereas

$$f_1 = \frac{1 - e^{-(\delta' - \beta)T}}{1 - e^{-\delta' T}} \cdot \frac{\delta'}{\delta' - \beta}$$

depends on the arbitrarily chosen period T, the economic constant δ' and $\beta = \alpha\eta$, η being the geological and soil-mechanical sinking constant. Moreover f_2 is the traditional engineering safety factor, and f_3 is the arbitrary factor for taking account of the values of human lives, cultural goods and control of action.

Notwithstanding this somewhat meagre result, we might venture the remark that the investigation has been sufficiently instructive for looking forward to similar ones referring to other large scale projects usually treated on a non-scientific decision basis. As an example one could think of military defense, which also is not optimal when it is maximal. Its treatment, however, would require an effert to determine relative weights, on a basis as scientifical as possible, of values which currently are accepted as absolute, although they are contradictory, like peace, freedom and welfare.

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