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ECONOMIC DECISION PROBLEMS FOR FLOOD PREVENTION

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1. INTRODUCTION

ON FEBRUARY 1, 1953 the Southwestern part of the Netherlands, and, to a smaller extent, parts of England and Belgium, were struck by a disastrous flood the height of which exceeded by far the highest which hitherto was known in the history of our country. According to the data given by A. G. Maris [5], there was a loss of over 1,800 human lives, over 150,000 hectares of land were flooded, about 9,000 buildings were demolished and 38,000 damaged, there were 67 breaks of dikes, and hundreds of kilometers of dikes were heavily damaged. The total economic loss is estimated at 1.5 to 2 billion guilders.

The government rapidly appointed a committee, consisting of prominent hydraulic engineers under the chairmanship of A. G. Maris, in order to design measures for preventing similar disasters in the future. Because the terrain to be covered by its work was to be the delta formed by the rivers Rhine, Meuse, and Scheldt, it was called the "Delta-Commission." To deal with special problems the committee enlisted several scientific institutions as advisors, including the Central Planning Bureau, The Royal Dutch Meteorologic Institute, the Hydraulic Laboratory of the Technical University at Delft, the Mathematical Centre at Amsterdam, and, of course, several departments of the Public Works Department itself.

Since that time the breaks in the dikes have been closed (even before the winter fell), the land has been reclaimed and drained, and an energetic beginning has been made to repair the other material damage. The Delta-Commission has advised the government to close completely four of the six sea-arms. As the entrances to the ports of Rotterdam and Antwerp must remain open, the dikes along these arms must be heightened.

The mathematical problems raised by the flood fall into three categories: (1) statistical problems, (2) hydrodynamic problems, and (3) economic decision problems. The hydrodynamic problems, which concern the height of sea level that a storm of a given type can cause, are here left completely out of account. I shall also not go far into the statistical problems, although something must be said about them in order for one to understand the economic problems, concerning the heightening of existing dikes, which form the subject matter of this article.

2. SOME REMARKS ON THE STATISTICAL PROBLEM

Until a few decades ago engineers built dikes to such a height that they were safe against the highest flood hitherto observed at that place. Since then, how-

¹ The main contents of this paper were presented before the European meeting of the Econometric Society at Uppsala on August 4, 1954.

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ever, statistical considerations about the frequency of floods of different heights have been introduced. In 1939 the Dutch engineer Wemelsfelder [9] estimated statistically the (cumulative) distribution of sea level heights, and drew important conclusions from it. In 1940 the government appointed a Storm-Flood Committee, which also concluded that no absolute upper limit for the height of a flood exists. (When saying "no" upper limit, one means, of course, no upper limit which can come into practical consideration. An upper limit of 40 meters, say, would have the same meaning as an infinite one.)

Hence to every height there belongs a positive "exceedance probability." For this reason the expression "flood prevention" in the title of this article might be considered somewhat misleading.

Wemelsfelder found that the annual exceedance frequencies during high tide at Hook of Holland during the period 1888-1937 followed very closely a straight line when plotted on logarithmic paper, i.e., that the exceedance probability $p(h) = 1 - F(h)$, where $F(h)$ is the cumulative probability distribution of high tides and $p(h)$ is the probability that the height h of high tide will be exceeded in any given year, is ce^{-ah} (Figure 1). Since 1953 Wemelsfelder's hypothesis has been analysed carefully, but no significant deviation from it has been found, at least for Hook of Holland. (Higher up in the estuaries the situation, which quite recently was analysed also by Wemelsfelder [10], is different.) In particular the available statistical material contains no indication that the curve when extrapolated would tend to a vertical asymptote, which would be an absolute upper limit (although physical considerations concerning the influence of friction effects make it plausible that for *very* high values of h the exceedance

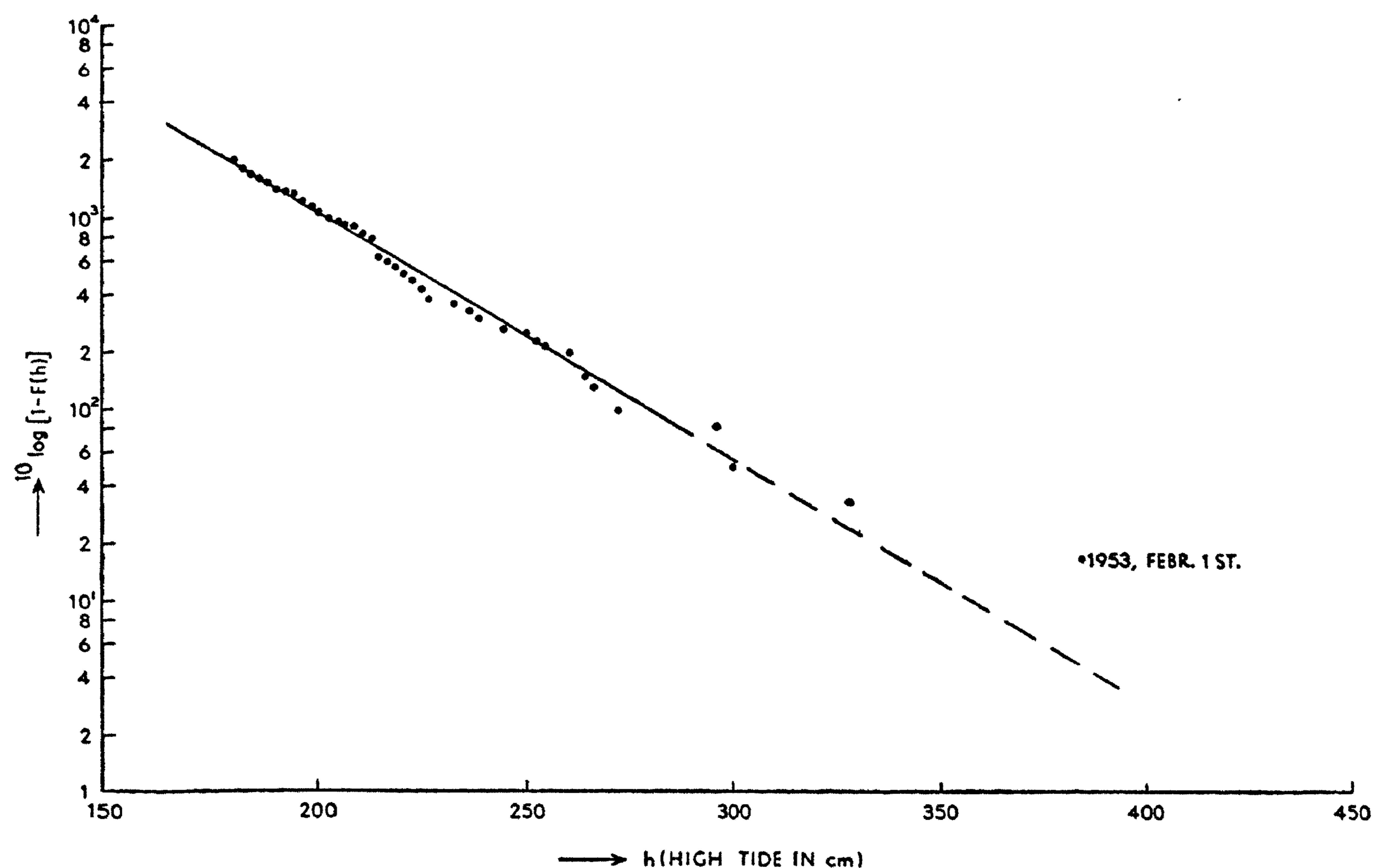


FIGURE 1

probability will decrease more rapidly); on the contrary, the highest floods have a persistent tendency to deviate to the *right* of the straight line. This suggests that the highest floods might be caused by storms of a type different from the ordinary ones. In fact, a group of storms that had followed paths within a restricted band of geographical latitude, which was selected by C. J. van der Ham in the Meteorological Institute, was analysed in the Mathematical Centre by J. Hemelrijk with the assistance of H. Kesten and J. Th. Runnenburg, and gave a clearly different straight line. The estimated halving height was raised from 21 to 25 cm., and the .95 confidence limit from 24 to 26 cm. The estimated Wemelsfelder-line is then (at Hook of Holland) $h = 2.03 - 0.75 \log p$.

Nobody denies, of course, that there are no grounds to assume that this extrapolation will hold in the long run, but, as no reliable data older than the year 1888 are available, the best thing one can do is to make use of the only result which could so far be reasonably ascertained, while avoiding numerous possible pitfalls such as dependencies between successive high tides, occurring during the same storm, etc.

We also tried a logarithmically-normal and a Gumbel distribution [2], the latter for the year maxima.² The first one, however, fits rather badly and the second one, being based on considerably less data, certainly does not give a better approximation than the exponential function.

3. THE DECISION PROBLEM

I now pass to the economic decision problem. This was put before us in October, 1953 by the director of the Hydraulic Laboratory at Delft, J. Th. Thijsse, who also provided us with provisional estimates of the numerical quantities needed. A provisional solution was given in December, 1953 [1]. As this study has been done, not by economists, but by mathematicians, and as the actual computations of special cases depend upon many local causes, e.g., local variations of depth and currents, wave-rising against the dikes, etc., which can be judged only by the engineers who know the local circumstances, the main task of the Mathematical Centre with respect to this problem was to provide a methodological procedure rather than to give actual numerical solutions. Work in the latter direction has been done by P. J. Wemelsfelder [10] and by J. Tinbergen [6] (on the sea-arm closing project), and F. J. de Vos [8] and W. C. Bischoff van Heemskerck [3], both working under J. van Veen, the Secretary of the Delta-Commission, on the island of Terschelling and on Central Holland respectively. I must add that during the time when most of the present work was done we could not benefit from Tinbergen's experiences, as he was abroad.

For these reasons the problem has been treated roughly only, by taking constant average values for quantities which really vary locally or in time.

We therefore consider a definite part of the country, situated below sea level and protected against the sea³ by surrounding dikes. The economic decision

² The year maxima form a subpopulation which, with a few exceptions, is contained in van der Ham's one.

³ We shall ignore the fact that part of the surrounding water may be a sea-arm, estuary, river, etc.

problem can then be formulated as follows: *Taking account of the cost of dike-wilding, of the material losses when a dike-break occurs, and of the frequency distribution of different sea levels, determine the optimal height of the dikes.*

In discussing the solution to this problem we assume that the future dikes round the land under consideration will all have the same height H above a given standard level,⁴ and we replace the present height (which may vary from place to place) by an average value H_0 , so that the amount X by which the dikes must be heightened is

$$X = H - H_0.$$

When speaking about the "height" of a dike, we do not mean the actual height of its crown, but the height of the "critical sea level," i.e., the sea level at which the dike may break (which can be lower than the height of the crown). The cost I of heightening the dikes from H_0 to H is a function of X , which can be assumed to be approximately independent of the present height above the level of the polder.

The simplest assumption about the possibility of losses is the following, referring to a single polder. Let h at any moment denote the sea level along the dikes (assumed to be everywhere the same), then no loss is incurred as long as $h \leq H$; if $h > H$ one can neglect the possibility of partial losses and reckon with a "total loss" only, i.e., assume that all buildings, farms, cattle, industries, etc. contained in the polder are lost. Let V , which we shall call the value of the goods (in the polder), be their total value and assume that the "consequential loss," e.g., the migration costs of the population and cattle, privation of production, etc. are included in it. The loss S is then

$$S = \begin{cases} 0 & \text{if } h \leq H, \\ V & \text{if } h > H. \end{cases}$$

The probability distribution of the high-tide sea level is assumed to be known in the economic problem. By $p(h)$ we denote the probability that any height h will be exceeded at least once during a year.

4. SIMPLE ASSUMPTIONS

The simplest assumptions to be made are: the value V and the probability distribution $p(h)$ are constant in time; the latter is assumed to be of the exponential type found by Wemelsfelder [9],

$$1) \quad p(h) = ce^{-\alpha h} = p_0 e^{-\alpha(h-H_0)}$$

where $p_0 = ce^{-\alpha H_0}$ is the exceedance probability of the present height H_0 of the dikes. We note that $p(h)$ does not depend on p_0 and H_0 separately, but only on the combination $p_0 e^{\alpha H_0} = c$. If a break occurs during one year, it will be assumed that the dikes will have been repaired by the next year and that the exceedance probability then is the same as before. Neglecting also the

⁴ This restriction is not essential; H may be made variable along the dikes so that the exceedance probability is everywhere the same.

probability of repeated breaks (after repair) during one year, the probabilities of losses during different years may then be assumed to be equal and independent.

It is simplest economically to treat the problem as an insurance problem, i.e., to assume that a sum L will be reserved in order to cover all future losses. If the sum L is invested at a rate of interest δ (not expressed as a decimal), it must cover the expected values of all future losses, $p(H)V$, each year, and we have

$$(2) \quad L = p(H)V \sum_{t=0}^{\infty} (1 + 0.01\delta)^{-t} \approx 100p(H)V/\delta = 100p_0 e^{-\alpha X} V/\delta.$$

The total cost of heightening the dikes being $I(X)$, we now have to determine X so that $I(X) + L(X)$ is minimal, i.e., that

$$(3) \quad \frac{dI}{dX} + \frac{dL}{dX} = 0.$$

The left side of (3) being a function of X , this is an equation in X , the solution of which (which is unique in all important cases) gives the optimal heightening X .

If only a relatively small interval of values of X need be considered, I can be assumed to be a *linear* function of X :

$$(4) \quad I = I_0 + kX.$$

Here I_0 is the initial cost, to be made as soon as it is decided that the dikes will be heightened, and k then is the subsequent cost of heightening them per meter.

Adding (2) and (4) gives:

$$(5) \quad I + L = I_0 + kX + 100 p_0 V e^{-\alpha X} / \delta$$

so that (3) becomes

$$k - 100 p_0 V \alpha e^{-\alpha X} / \delta = 0$$

or

$$(6) \quad X = \frac{1}{\alpha} \ln \frac{100 p_0 V \alpha}{\delta k}.$$

If a larger range must be admitted for the variation of X , the linear approximation for $I(X)$ is no longer valid, because the higher the dikes, the broader a base that is required. Under more general assumptions than that of linearity a similar method can be followed, although the mathematics becomes slightly more complicated. But, at least in the case of the dikes protecting Central Holland, the part of the curve actually used in the decision problem is very nearly linear, so that we will use the linear approximation (4) throughout the analysis (albeit with a slightly different interpretation of the constant I_0).

5. INCREASE OF WEALTH AND SINKING OF THE LAND

The assumptions made in Section 4 were too greatly simplified. In the first place the value of the goods, V , is not constant in time. We assume instead that it increases at the same rate as national wealth. This rate of increase at present has been estimated by Tinbergen to be between 1.5 and 2.5 per cent per annum which is to be compared with an interest rate of 3.5–4.5 per cent per annum. Nothing is known, of course, about the question of whether this rate of increase (which we shall denote by γ) will remain constant in time over a period of a few centuries. On the contrary, past experience points rather to considerable fluctuations in its value, and it may for some periods even be negative. Nevertheless, a secular trend throughout many centuries or even millennia seems unmistakable, and we are led in the absence of better knowledge to assume γ to be constant. The same remarks apply to the rate of interest, δ .

It will be found useful to introduce the reduced interest rate $\delta' = \delta - \gamma$ and to express time in centuries instead of years. Time in years we denote by t and time in centuries by τ , so that $t = 100 \tau$. Moreover δ and γ , originally defined in per cent per annum, can also be redefined as in "per unum" per century. Taking δ to be the *continuous* interest rate, the present value of an amount A at time t is $A e^{-0.01\delta t} = A e^{-\delta\tau}$, whereas $A e^{-\delta'\tau}$ is the present value of an amount which has increased from A to $A e^{\gamma\tau}$ by taking part in the increase of wealth.

In the second place the exceedance probability distribution is *not* constant in time. For about 9,000 years the Netherlands have been slowly sinking into the sea. This possibly is an equilibrating readjustment of the earthcrust to the loss of load caused by the melting away of the Fennoscandian icecap about 10,000 years ago. According to a recent investigation by F. A. Vening Meinesz [7] we have sunk since that time about a hundred meters, but the worst seems to be over: we shall sink only about 3.80 meters more, and much more slowly, so that the greatest depth will be reached in about 5,000 years. At present the rate of sinking, which at the beginning must have been about 2 meters a year, is only about 20 cm. a century. It is counteracted partly by a rising of the Alpine Foreland, the rate of which, however, is not known. Moreover, the sea level is constantly rising because of the melting away of the Greenland icecap. This is a much more rapid, but also a rather short-run, phenomenon; it may be over in another 500 years. Finally, apart from this relative sinking of the land with respect to the sea level, account must be taken of the considerable sinking of the crown of a dike with respect to its foot.

All things taken together we must reckon with a slow sinking away of the dikes into the sea at a rate which is not precisely known, but which may be estimated at 0.7 meters per century. The numerical value, however, is admittedly very uncertain. It will be denoted by η , expressed in meters per century. Then the exceedance probability of a dike, now heightened to H meters becomes a function of time, so $p(H)$ changes to $p(H, \tau)$. After τ centuries the height H will have fallen to $H - \eta\tau$, or

$$(7) \quad p(H, \tau) = p_0 e^{-\alpha((H-\eta\tau)-H_0)} = p_0 e^{-\alpha(H-H_0)+\beta\tau},$$

where

$$(8) \quad \beta = \alpha\eta.$$

The expression (7), however, is not self-consistent. In fact, being an exponential function of time, it would increase indefinitely, whereas, being a probability, it must remain ≤ 1 .

After a sufficiently long time, indeed, the dikes—if nothing further were done to them—would sink into the sea again, the probability of a flood would become and remain = 1, and (7) would lose its validity. Hence not only our solution of the decision problem, but also the dikes themselves must be adapted to this sinking away. We shall therefore assume that periodically, with a fixed period of T centuries, the dikes will be regenerated by heightening them by the amount ηT which has been lost during this period. Mathematically one might prefer a *continuous* renewal of the dikes, but this is technically impossible, as one cannot heighten a dike yearly by a few millimeters. A reasonable choice might be $\eta T = 1$ (meter), i.e., $T = 1/\eta$. Let the costs of these successive renewals be R_1, R_2, R_3, \dots . These increase slowly because the work must be done on an increasing height above the polder level.⁵

The total dike building cost now becomes: (1) the cost of the present heightening of X meters, viz., $I_0 + kX$, (2) after times $T, 2T, 3T, \dots$, a heightening of ηT meters at a cost R_1, R_2, R_3, \dots , the present value of which is $R_1e^{-\delta T}, R_2e^{-2\delta T}, R_3e^{-3\delta T}, \dots$. The sum of this series will be called J .

What is important here is that J is approximately a constant, *independent* of X , the small dependence upon X being something we either can neglect or compensate for by a small increase of k . The total building cost is therefore

$$(9) \quad I = I_0 + kX + J.$$

The value of the polder after a time τ becomes, because of the increase of wealth, $Ve^{\gamma\tau}$. The probability of a flood occurring in any year is given by (7), so the expected loss during any given year is $p_0Ve^{-\alpha X}e^{(\beta+\gamma)\tau}$, and its present value is $p_0Ve^{-\alpha X}e^{(\beta+\gamma)\tau}e^{-\delta\tau}$. This must be summed over all years of the first period $0 \leq \tau \leq T$, i.e., integrated over t with $dt = 100 d\tau$. The result is

$$(10) \quad p_0Ve^{-\alpha X} \int_0^T e^{(\beta+\gamma-\delta)\tau} 100d\tau = 100p_0Ve^{-\alpha X} \frac{1 - e^{-(\delta'-\beta)T}}{\delta' - \beta}.$$

This is the present value of the total loss expectation during the first period. During each subsequent period we get the same result, multiplied for the $(n+1)^{\text{st}}$ period by $e^{\gamma nT}$ (because of increasing wealth) and by $e^{-\delta nT}$ (to discount back to the present). Hence the result is the product of (10) and

$$\sum_0^{\infty} e^{-n\delta'T} = \frac{1}{1 - e^{-\delta'T}}.$$

⁵ These terms, of course, are very uncertain, as new technological methods may be found for protecting the polders. On the other hand, their influence is small if a not too small value of T is chosen.

This gives us

$$(11) \quad I + L = I_0 + J + kX + \frac{100p_0 V e^{-\alpha X}}{\delta' - \beta} \frac{1 - e^{-(\delta' - \beta)T}}{1 - e^{-\delta' T}}$$

$$= J_0 + J + kX + \frac{kC}{\alpha} e^{-\alpha X},$$

where

$$(12) \quad C = \frac{100p_0 V \alpha}{(\delta' - \beta)k} \cdot \frac{1 - e^{-(\delta' - \beta)T}}{1 - e^{-\delta' T}}$$

is always positive (with $p_0, V, \alpha, k, \delta', \beta, T$ positive).

Differentiation with respect to X gives the condition for an extremum

$$(13) \quad k - kC e^{-\alpha X} = 0$$

or

$$(14) \quad X = \frac{1}{\alpha} \ln C.$$

It is easily seen that this extremum is actually a minimum. We notice that the left-hand side of (10) is

$$V \int_0^T p(H - \eta\tau) \cdot e^{(\gamma - \delta)\tau} 100 d\tau,$$

so that linearity of $\log p(h)$ is only used in the interval $H - \eta T \leq h \leq H$. This has an important consequence for the case when $p(h)$ is somehow known to deviate from the exponential function (1) for values of h greater than those hitherto observed. As the function $\log p(h)$ proposed for extrapolation does not deviate much from linearity in the relatively short interval $(H - \eta T, H)$ we can, once H is known to a rough approximation, use the same methods as are used here upon replacing the initial straight line representing $\log p(h)$ by the practically straight part of the extrapolated curve and by using, of course, the altered values of α and p_0 .

6. THE DOUBTFUL CONSTANTS

We have already mentioned the fact that several of the constants entering into the problem are rather badly known. For most of them only rough point and interval estimates are available. The interval estimates are not confidence intervals in the strict sense, but have only the meaning that competent workers in the fields think it may be taken for granted that the constants are contained in these intervals. It is, moreover, impossible to improve the estimates within a reasonable time.

So the best thing we can do is to ascertain that our solution will hold under the most unfavourable circumstances which must be considered to be realistic. This means that the problem is considered as a minimax problem: cost is minimized on the supposition that parameter values are such as to maximize it.

Now, from (13) and (14) it is obvious that C and hence X increase if p_0 or V increases or if k decreases (always assuming that other constants remain unaltered).

The dependence upon α , η , and δ' is somewhat more complicated, but after some calculation it is seen that C increases if α and δ' decrease or η increases.

So in order to remain on the safe side we must take the highest reasonable estimates of p_0 , V , and η and the lowest ones of k , α , and δ' . The (lowest) upper bound for H which may be considered safe with respect to all constants separately is obtained from the values $\alpha = 2.6$; $p_0 = 0.0038$ (from $H_0 = 4.25$); $\eta = 1$; $V = 10^{10}$ guilders; $\delta' = 2$; $k = 42.10^6$ guilders; $T = 0.75$ century, leading (with a factor 2 for ideal values, cf. Section 7) to $H = 6.73$ meters. The combination of these extreme values for all constants, however, is rather pessimistic. Several reasonable combinations of values lead to the conclusion that roughly 6.00 meters may be considered as a reasonable estimate of a sufficiently safe height.

We have already discussed the fact that we have only very crude estimates of most of the "constants"⁶ entering into our problem. These constants are of three different types. First, η is a physical constant, which will become known better in the course of time by continued geological research. p_0 and α , the constants of the statistical distribution of the high-tide sea levels, are of a similar nature and may become better known by further meteorological and oceanographical research and by the mathematical and statistical treatment of the data. From improved estimates of these constants β can then also be determined more accurately.

The second group of constants are those which describe the present economic situation. These are: (1) the "commercial" constants giving the costs of dike-building, viz., I_0 and k ; and (2) the "value" V at this moment of the goods on the land, which can be determined from "national economic" data, although considerable difficulty may be involved in estimating the "consequential loss" which we have assumed to be contained in V (for a first rough estimate by multiplying the actual value of the goods by a constant factor of 1.2). From these constants, together with those of the first group, other fundamental constants like the present loss-expectation per annum, p_0V , can be derived.

The situation is somewhat more different with regard to the "secular" economic quantities δ and γ , upon which $\delta' = \delta - \gamma$ depends. As to γ , if we wish to ascertain whether the value V of present goods *really* has increased after a time τ by $e^{\gamma\tau}$, we must have a stable unit in which the values at two different times both can be expressed. It is evident that ordinary monetary units, because of fluctuations and of the secular trend in depreciation of money, do not satisfy this condition. This is an index number problem and it is by no means certain that an index can be defined with sufficient precision so that the quantitative comparison of values that are centuries apart has any meaning. For short periods the difficulty is not so serious as a physical index might be used. But with changes in technology it becomes difficult to extrapolate the trend of a physical index over long periods, e.g., centuries. The situation is fully as complicated with regard to

⁶ Here and further, when we speak about more precise determination of the constants, we mean, of course, also better knowledge about their *deviations* from constancy.

the interest-factor δ used to compute the present value of future amounts. There does not seem to be any adequate market measure of the rate of interest to be used in evaluating investment projects which are as long-term in character as this one and for which there is a comparable amount of uncertainty inhering in the evaluation of both the probabilities and values of long-term gains or losses. It is, moreover, particularly difficult to see why the degree of security the dikes grant our posterity during a long period should depend so heavily upon the value which a fluctuating rate of interest has at a rather haphazardly chosen moment, especially as the cost of dike construction and maintenance must actually be spread out over time. This is, however, forced upon us by the lack of sufficiently precise economic data.

There may, moreover, be serious question as to the suitability of the framework for decision-making which is employed here. We have proposed to minimize the sum of the cost of heightening the dikes plus the mathematical expectation of the (discounted) losses which may still occur after the dikes have been heightened. One may challenge the relevance of our using this mathematical expectation, not only on the grounds that it takes no account of the dispersion or other moments of the probability distribution of future losses (does not introduce an appropriate utility function), but also because probability principles are here used in a case where there may not seem to be an adequately large number of comparable social risks to make the concept of mathematical expectation a suitable basis for social choice. It is mainly in order to keep the analysis as simple as possible that we have, nevertheless, accepted this method.

7. HUMAN LIVES, IDEAL VALUES, AND THE VALUE OF CONTROL

Unlike Sir William Petty [4], who, when discussing the wealth of the Kingdom, calculates the worth of a human life at £69, most modern statisticians are not readily inclined to consider human lives and material goods as commensurable in value. As, however, the possible loss of human lives, quite apart from the loss of material goods, should somehow justify an increase of height of the dikes, some decision about its importance must be made.

Perhaps the best that can be done is not to evaluate human lives themselves, but to see how much the state is willing to spend in order to save a given number of human lives. To obtain a definite figure statistically, one could determine how much the state pays (or induces its citizens to pay) in *other* cases for a similar purpose, e.g., the abolition of unguarded railway crossings, the prevention of factory accidents, the prevention of other traffic accidents, etc. These amounts, if taken per head, vary greatly. They become very large in a few cases which appeal greatly to the public imagination, but in many other cases, where this is not the case (even leaving war out of account), relatively small amounts which could have prevented the loss of many human lives are refused. For this reason it seems undesirable in a case where a conscious decision on this subject is to be made to base it on an actually prevailing average rather than on a figure which can be considered as desirable and which would provide a guiding norm for future cases as well. This, however, does not help us to a definite figure. If, just to try a

figure, we would double the figure for material losses, this would imply in the case of the 1953 flood, when material losses were about 1.5 to 2 billion guilders and when about 1,800 lives were lost, an amount of about 100,000 guilders per head, an amount which certainly goes far beyond any sum which would be acceptable (e.g., based on existing practice of life insurance) as a norm for all cases. On the other hand, any sum which seems acceptable would lead to a barely perceptible increase of height—and therefore one which would be impossible technically and unacceptable emotionally. It does not make sense to increase the dikes by an extra centimeter to account for the value of human lives. The least one could do would be to add to the value of the land and buildings the amount which has been “invested” in human beings, in the form of food and other material goods necessary for their upbringing, together with the labour spent on their education by parents and teachers. But, apart from being far from easy to compute, this still would leave the emotional factor unaccounted for. A similar argument holds for other ideal values such as cultural goods.

There are other factors also hard to compute which, nevertheless, must somehow be accounted for. In the first place, damage might be so serious that it becomes practically impossible to reclaim the land. In that case it would be incorrect to reckon only with the value of land and buildings, as consequential loss would become all important. Moreover, the greater the area of land lost, the smaller the resources available for moving people and cattle, for feeding them, for finding work for them, for trying to reclaim the lost land or part of it, and for rebuilding its industry.

It is, however, hardly possible to estimate (1) the probability of such an occurrence and (2) the loss caused by it. The only thing that can be said is that the curve representing the loss as a function of the area flooded is not linear (not even approximately, even if the distribution of wealth were homogeneous), but must be of the type shown in Figure 2. At some unknown point, where remaining resources for reclaiming part of the land become insufficient, a rather sharp discontinuity must arise, whereas further on the curve must tend to an asymptote, corresponding with the case where the whole national existence becomes impossible. Nothing more, however, can at present be said about this.

In the second place it is incorrect simply to add the expected loss and the in-

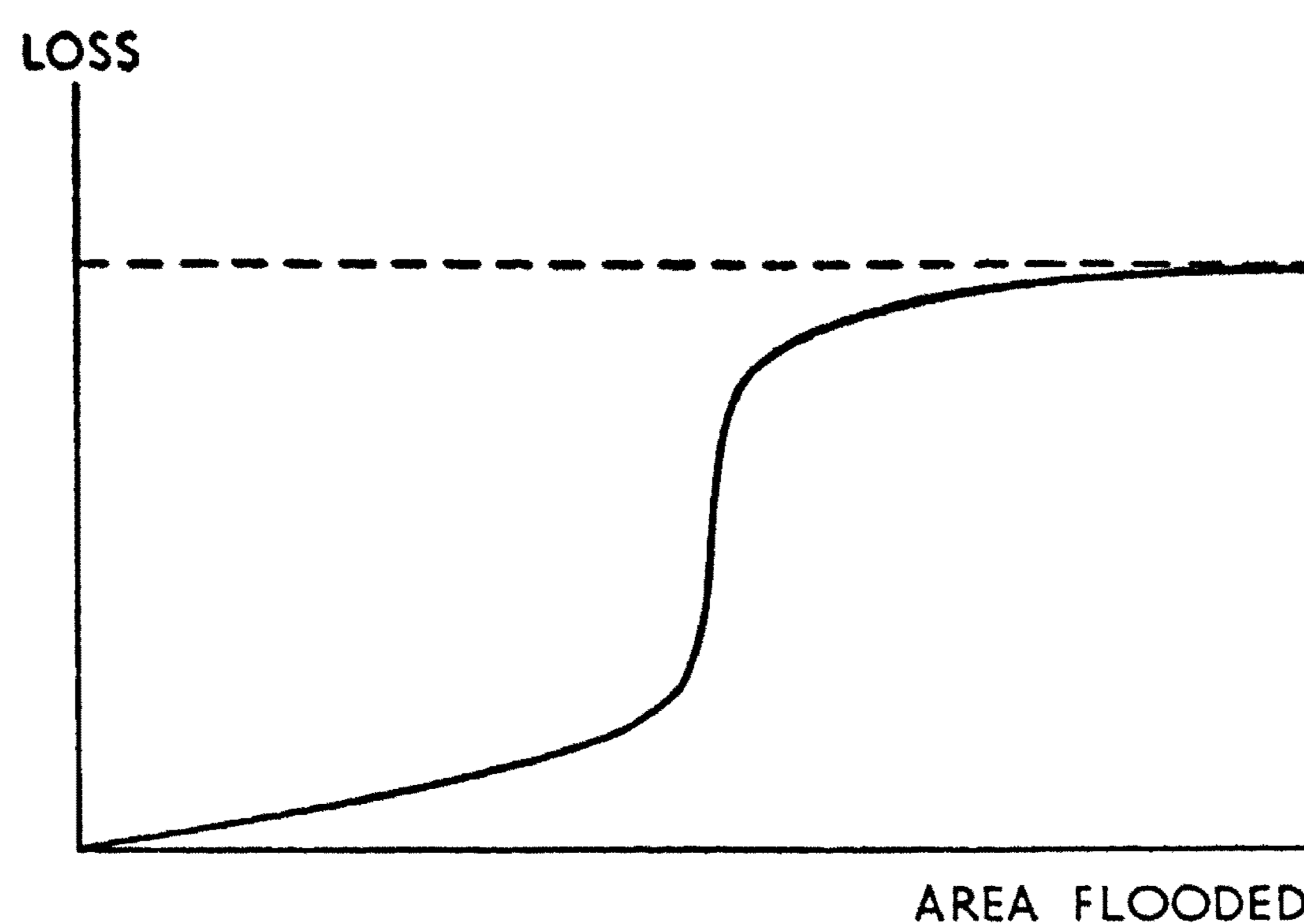


FIGURE 2

vestment in dike-building. In fact, when one has the choice of incurring the cost of one million guilders either in dike-building or in flood damage, one will doubtless choose the former. One will surely be willing to spend a multiple of the amount that would be lost by a flood if the flood can thereby be prevented. It is, however, almost impossible to say how large this factor must be. It serves to avoid a hardly measurable kind of consequential loss, consisting of the disorganization caused by a flood, the increased probability of diseases, the psychological, social, and economic "shock," the possibility that this might occur when one is least prepared for it (e.g., during a war), etc.

It seems that the best thing that can be done is to select, more or less arbitrarily, a factor by which the value V is to be multiplied in order to cover simultaneously the value of human lives and that of "control." This factor cannot be determined on mathematical, statistical, or economic grounds. Its determination requires a decision by the responsible authorities rather than by scientists, at least as long as no better scientific methods and data are available. In fact, even the determination of the values of the badly known constants to be considered as best estimates is rather arbitrary, so that part of this ultimate decision already hides itself behind the selection of these constants.

In conclusion, it can only be hoped that the scientific treatment of the more tractable portion of this problem will be of value to those who must make these decisions in the light of a much wider range of consideration, and that there may be some instructional merit in the present study of benefit to those who must consider other problems of social investment which are comparably vast in scope.

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