MATHEMATISCH CENTRUM
2e BOERHAAVESTRAAT 49
AMSTERDAM

AFD. TOEGEPASTE WISKUNDE

Report TW 52

The influence of some uniform windfields upon a sea having the form of a semi-infinite strip

by

H.A. Lauwerier

June 1958

The Mathematical Centre at Amsterdam, founded the 11th of February 1946, is a non-profit institution aiming at the promotion of pure mathematics and its applications, and is sponsored by the Netherlands Government through the Netherlands Organization for Pure Research (Z.W.O.) and the Central National Council for Applied Scientific Research in the Netherlands (T.N.O.), by the Municipality of Amsterdam and by several industries.

Table of Contents

§1. Introduction

In this report we shall consider the influence of some uniform windfields upon a rotating shallow sea which has the form of a semi-infinite strip [1)]

$$0 < x < \pi \qquad\qquad 0 < y < \infty .$$

The differential equations are [2)]

$$\begin{cases} (\frac{\partial}{\partial t} + \lambda)u - \Omega v + \frac{\partial \zeta}{\partial x} = U(t) \\ (\frac{\partial}{\partial t} + \lambda)v + \Omega u + \frac{\partial \zeta}{\partial y} = V(t) \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \zeta}{\partial t} = 0. \end{cases} \qquad 1.1$$

The boundary conditions are

$$\begin{cases} x = 0 \text{ and } x = \pi & u = 0 \\ y = 0 & v = 0 \end{cases} .$$

The study of this model may contribute to the understanding of the behaviour of the North Sea under a storm. The North Sea has roughly the form of a rectangle with three coasts and one side bordering to the ocean. In the model considered here the influence of the ocean is left out of account. However, the study of the rectangular model will be the subject of a later report.

Much attention will be paid to the elevation of the sea at the "south" coast y=0, in particular at the midway point $x = \pi/2$. With reference to the geography of the North Sea this corresponds roughly to the Dutch Coast.

The numerical case worked out in this report refers to approximate values for the actual conditions of the North Sea. The following units and numerical constants will be taken

x,y	135 km	u,v	91 km/h
t	1.5 h	ζ	65 m
λ =0.14		Ω =0.71	

----

1) Research carried out under the direction of Prof.Dr D. van Dantzig.

2) Cf TW 31.

If the Laplace transforms of u,v etc. for the full t range $-\infty < t < \infty$ are denoted by the same letters, if necessary writing $\zeta(p)$ etc. the equations 1.1 become

$$\begin{cases} (p+\lambda)u - \Omega v + \dfrac{\partial \zeta}{\partial x} = U(p) \\ (p+\lambda)v + \Omega u + \dfrac{\partial \zeta}{\partial y} = V(p) \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + p\zeta = 0 \end{cases} \qquad 1.2$$

with

$$\begin{cases} x = 0 \text{ and } x = \pi \qquad u=0 \\ y = 0 \qquad v=0 . \end{cases}$$

From 1.2 we obtain by elimination of v and ζ

$$(\Delta - q^2)u = -p(U + \frac{\Omega}{p+\lambda} V) , \qquad 1.3$$

where

$$q^2 = p(p+\lambda) + \frac{\Omega^2 p}{p+\lambda} . \qquad 1.4$$

In a similar way we have

$$(\Delta - q^2)v = -p(V - \frac{\Omega}{p+\lambda} U) , \qquad 1.5$$

and

$$(\Delta - q^2)\zeta = 0 . \qquad 1.6$$

§ 2. Solution of the problem

The system 1.2 has an elementary solution u_o, v_o, ζ_o which does not depend on y and which satisfies the conditions at x=0 and $x=\pi$.

We have from 1.3

$$\frac{\partial^2 u_o}{\partial x^2} - q^2 u_o = -p(U + \frac{\Omega}{p+\lambda} V)$$

with $u_o=0$ for x=0 and for $x=\pi$,

so that

$$u_o = \frac{p}{q^2}\left(U + \frac{\Omega}{p+\lambda} V\right)\left(1 - \frac{\operatorname{ch} q(x-\frac{\pi}{2})}{\operatorname{ch} \frac{q\pi}{2}}\right). \qquad 2.1$$

Next we have

$$v_o = \frac{p}{q^2}\left(V - \frac{\Omega}{p+\lambda}U\right) + \frac{p\,\Omega}{(p+\lambda)q^2}\left(U + \frac{\Omega}{p+\lambda}V\right)\frac{\operatorname{ch} q(x-\frac{\pi}{2})}{\operatorname{ch}\frac{q\pi}{2}}, \qquad 2.2$$

and

$$\varphi_o = \frac{1}{q}\left(U + \frac{\Omega}{p+\lambda}V\right)\frac{\operatorname{sh} q(x-\frac{\pi}{2})}{\operatorname{ch}\frac{q\pi}{2}} \quad . \qquad 2.3$$

The solution of 1.2 satisfying the conditions at $x=0$ and $x=\pi$ and at $y=0$ may be represented by

$$u = u_o + u_1 \qquad v = v_o + v_1 \qquad \varphi = \varphi_o + \varphi_1 \; . \qquad 2.4$$

The functions u_1, v_1, φ_1 satisfy the homogeneous differential equations

$$\begin{aligned} (p+\lambda)u_1 - \Omega\, v_1 + \frac{\partial \varphi_1}{\partial x} &= 0 \\ (p+\lambda)v_1 + \Omega\, u_1 + \frac{\partial \varphi_1}{\partial y} &= 0 \\ \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + p\varphi_1 &= 0 \; . \end{aligned} \qquad 2.5$$

with the boundary conditions

$$u_1 = 0 \quad \text{at} \quad x = 0,\; x = \pi$$

$$v_1 = -v_o(x) \text{ at } y = 0 \; .$$

The elementary solutions of 2.5 satisfying the conditions at $x=0$, $x=\pi$ are

the Kelvin wave

$$\begin{cases} u = 0 \\ v = \exp\left\{ s(x-\frac{\pi}{2}) - y\sqrt{p^2+\lambda p} \right\} \\ \varphi = \sqrt{\frac{p+\lambda}{p}} \exp\left\{ s(x-\frac{\pi}{2}) - y\sqrt{p^2+\lambda p} \right\} \end{cases} , \qquad 2.6$$

the Poincaré waves, $k=1,2\ldots$

$$\begin{cases} u = \frac{k^2+s^2}{k\,\nu_k} \sin kx \; e^{-\nu_k y} \\ v = \left(\cos kx + \frac{p\Omega}{k\nu_k}\sin kx\right) e^{-\nu_k y} \\ \varphi = \frac{p+\lambda}{\nu_k}\left(\cos kx + \frac{\Omega\,\nu_k}{(p+\lambda)k}\sin kx\right) e^{-\nu_k y} \end{cases} , \qquad 2.7$$

where

$$s = \Omega\sqrt{\frac{p}{p+\lambda}} \qquad \nu_k = \sqrt{k^2+q^2} .$$

Thus the solution u, v, φ of 1.2 may be represented by

$$\begin{cases} u(x,y) = u_o(x) + \sum_1^\infty \frac{k^2+s^2}{k\nu_k} C_k \sin kx \, e^{-\nu_k y} \\ v(x,y) = v_o(x) + C_o \exp\left\{ s(x-\frac{\pi}{2}) - y\sqrt{p^2+\lambda p} \right\} + \\ \qquad + \sum_1^\infty C_k(\cos kx + \frac{p\Omega}{k\nu_k}\sin kx)e^{-\nu_k y} \\ \varphi(x,y) = \varphi_o(x) + \sqrt{\frac{p+\lambda}{p}} C_o \exp\left\{ s(x-\frac{\pi}{2}) - y\sqrt{p^2+\lambda p} \right\} + \\ \qquad + \sum_1^\infty \frac{p+\lambda}{\nu_k} C_k \left(\cos kx + \frac{\Omega\,\nu_k}{(p+\lambda)k}\sin kx\right)e^{-\nu_k y} . \end{cases} \quad 2.8$$

The coefficients C_k $k=0,1,2...$ may be determined from the condition at $y=0$. This gives

$$C_o \, e^{s(x-\frac{\pi}{2})} + \sum_1^\infty C_k(\cos kx + \alpha_k \sin kx) = -v_o(x), \quad 2.9$$

where

$$\alpha_k = \frac{p\Omega}{k\nu_k} .$$

The condition 2.9 may be converted into an infinite set of linear equations. This may be done by expanding both sides of 2.9 into a cosine series and equating corresponding terms.

We shall introduce the following coefficients

$$a_l = \frac{2}{\pi}\int_0^\pi e^{s(x-\frac{\pi}{2})} \cos lx \, dx ,$$

$$\Gamma_{lk} = \frac{2}{\pi}\int_0^\pi \sin kx \cos lx \, dx ,$$

$$f_l = -\frac{2}{\pi}\int_0^\pi v_o(x) \cos lx \, dx ,$$

where l runs through the integers 0,1,2... and k through 1,2... .

Then from 2.9 we obtain at once

$$\begin{cases} a_o C_o = f_o - \sum_1^\infty \Gamma_{ok}\alpha_k C_k \\ C_l + a_l C_o = f_l - \sum_1 \Gamma_{lk}\alpha_k C_k , \quad l=1,2... \end{cases} \quad 2.10$$

The coefficients a_l, Γ_{lk} and f_l are

$$\begin{cases} a_l = \dfrac{1}{l^2+s^2}\,\dfrac{4s}{\pi}\,\mathrm{sh}\,\dfrac{s\pi}{2}, & l \text{ even}, \\ a_l = \dfrac{-1}{l^2+s^2}\,\dfrac{4s}{\pi}\,\mathrm{ch}\,\dfrac{s\pi}{2}, & l \text{ odd}. \end{cases}$$

$$\begin{cases} \Gamma_{lk} = 0, & k+l \text{ even}, \\ \Gamma_{lk} = \dfrac{4}{\pi}\,\dfrac{k}{k^2-l^2}, & k+l \text{ odd}. \end{cases}$$

$$\begin{cases} f_o = -\dfrac{2p}{q^2}\left(V-\dfrac{\Omega}{p+\lambda}U\right) - \dfrac{4p\Omega}{\pi(p+\lambda)q^3}\,\mathrm{th}\,\dfrac{q\pi}{2}\left(U+\dfrac{\Omega}{p+\lambda}V\right), \\ f_l = \dfrac{-1}{l^2+q^2}\,\dfrac{4p\Omega}{\pi(p+\lambda)q}\,\mathrm{th}\,\dfrac{q\pi}{2}\left(U+\dfrac{\Omega}{p+\lambda}V\right), & l \text{ even}, \\ f_l = 0 & l \text{ odd}. \end{cases}$$

The system 2.10 may be solved by means of an iterative process. If $p\Omega$ is small we have the following approximate solution

$$C_o\left\{a_o - \sum \Gamma_{ok}\,\alpha_k a_k\right\} = f_o,$$

$$l \text{ even} \qquad C_l = f_l - a_l C_o, \qquad\qquad 2.11$$

$$l \text{ odd} \qquad C_l = -a_l C_o.$$

§ 3. Influence of an exponential windfield

In this section we shall consider the exponential windfield [1]

$$\begin{cases} U = 0 \\ V = \sum P_k e^{p_k t}. \end{cases} \qquad 3.1$$

Calculations will be carried through for the particular numerical case [2]

$p_1 = 0.12 \qquad p_2 = 0.18 \qquad P_1 = -0.13 \qquad P_2 = 0.0284$

----

1) U and V are components of the surface stress which is proportional to the square velocity of the wind at sea level.

2) Cf. TW 42.

This windfield reaches its peak at t=18.5, the maximum velocity is 24 m/sec.

In this case the elevation $\zeta(x,y,t)$ may be represented by

$$\zeta(x,y,t) = - \sum P_k Z(x,y,p_k) e^{p_k t} \qquad 3.2$$

where according to 2.3 and 2.8

$$Z(x,y,p) = \frac{\Omega}{(p+\lambda)q} \frac{\operatorname{sh} q(\frac{\pi}{2}-x)}{\operatorname{ch} \frac{q\pi}{2}} + \frac{\Omega}{s} C_o \exp\left\{s(x-\frac{\pi}{2}) - y\sqrt{p^2+\lambda p}\right\} +$$

$$+ \sum_1^\infty \frac{\Omega}{k} C_k \left(\sin kx + \frac{(p+\lambda)k}{\Omega \nu_k} \cos kx\right) e^{-\nu_k y} . \qquad 3.3$$

The coefficients C_k k=0,1,2... are determined by the equations 2.10 with

$$f_o = \frac{2p}{q^2}\left\{1 + \frac{\Omega^2}{(p+\lambda)^2} \frac{\operatorname{th} \frac{q\pi}{2}}{\frac{q\pi}{2}}\right\} ,$$

$$f_l = \frac{1}{l^2+q^2} \frac{2p\Omega^2}{(p+\lambda)^2} \frac{\operatorname{th} \frac{q\pi}{2}}{\frac{q\pi}{2}} , \qquad l = 2,4...$$

$$f_l = 0 , \qquad l = 1,3...$$

In both cases p=0.12, p=0.18 the equations 2.10 may be solved with only a few iterations. We have found

	p = 0.12	0.18		0.12	0.18
C_o	2.881	2.200	C_6	0.006	0.007
C_1	1.853	1.580	C_7	0.046	0.041
C_2	0.056	0.067	C_8	0.003	0.004
C_3	0.247	0.219	C_9	0.028	0.025
C_4	0.013	0.016	C_{10}	0.002	0.002
C_5	0.091	0.080			

For Z(x,y,p) at the coast y=0 we have the following values

	x=0	π/6	π/3	π/2	2π/3	5π/6	π
Z(x,y,0.12)	6.00	5.90	5.72	5.51	5.31	5.14	5.05
Z(x,y,0.18)	4.50	4.40	4.21	4.01	3.81	3.64	3.56

From these data we obtain according to 3.2 for the following table for $\zeta(x,0,t)$ in meters

	x=0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
t=0	0.65	0.64	0.62	0.60	0.58	0.57	0.56
5	1.11	1.09	1.07	1.02	0.99	0.97	0.95
10	1.82	1.79	1.75	1.70	1.64	1.60	1.57
15	2.82	2.78	2.72	2.64	2.57	2.51	2.47
20	3.54	3.90	3.83	3.74	3.66	3.59	3.55
22	4.25	4.21	4.16	4.08	4.01	3.96	3.92
24	4.32	4.30	4.27	4.23	4.20	4.16	4.15
26	3.94	3.95	3.98	3.99	4.01	4.02	4.01

We observe that the maximum values of $\zeta(x,0,t)$ occur practically simultaneously at about t=24. There is a little decrease from x=0 to $x=\pi$ which is obviously due to the rotation of the earth. However, the deviation from the average, which is at $x=\pi/2$ approximately, is of the order of 2% in any direction.

If $\Omega=0$ the elevation ζ does not depend on x. We have in this case

$$Z(x,y,p) = \frac{e^{-y\sqrt{p^2+\lambda p}}}{\sqrt{p^2+\lambda p}} .$$

At y=0 we have accordingly

$$Z(x,0,0.12) = 5.65 \qquad\qquad Z(x,0,0.18) = 4.15$$

and next

t=0	5	10	15	20	22	24	26
ζ=0.56	1.05	1.73	2.69	3.79	4.13	4.26	3.98

There is a surprisingly small difference between these values and those of $\zeta(\pi/2,0,t)$ for $\Omega\neq 0$. The small influence of Ω appears also in the following table, where the maximum values of $\zeta(x,0,t)$ for $\Omega\neq 0$ are compared with the maximum value of ζ for $\Omega=0$.

	x=0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
$\zeta(\Omega=0)$			4.26	=100%			
$\zeta(\Omega\neq 0)$	4.32	4.30	4.27	4.23	4.20	4.16	4.15
deviation	1.5%	1%	-	1%	1.5%	2%	2.5%

In diagram 1 we have drawn $\zeta(\pi/2,0,t)$ for $\Omega \neq 0$, together with the windfield $-V(t)$. It appears that the maximum of the elevation occurs some 8 hours (unit of time is 1.5h) after that of the wind.

Next a number of diagrams is presented in which the situation at a given time is represented by a few lines of equal elevation.

We observe that the influence of the rotation of the earth which is almost negligeable at the "south" coast y=0 becomes more pronounced for increasing y.

§ 4. Approximate analytic expressions

In this section an approximate expression will be derived for the elevation $\zeta(\pi/2,0,p)$ due to an arbitrary uniform windfield.

From 2.8 we obtain

$$\zeta(\pi/2,0,p) = \frac{\Omega}{s} C_o + \sum_{odd} \frac{\Omega}{k} C_k \sin\frac{k\pi}{2} . \qquad 4.1$$

According to 2.11 we have the rather good approximation

$$k \text{ odd} \qquad C_k = -a_k C_o , \qquad 4.2$$

so that in view of

$$-\sum_{odd} \frac{a_k}{k} \sin\frac{k\pi}{2} = \frac{\operatorname{ch}\frac{s\pi}{2} - 1}{s} , \qquad 4.3$$

we obtain

$$\zeta(\pi/2,0,p) \approx \frac{\Omega}{s} \operatorname{ch}\frac{s\pi}{2} C_o . \qquad 4.4$$

From 2.11 we have also in good approximation

$$2 \operatorname{ch}\frac{s\pi}{2} \left\{ \frac{\operatorname{th}\frac{s\pi}{2}}{\frac{s\pi}{2}} + \frac{8ps\Omega}{\pi^2} \sum_{odd} \frac{1}{k^2(k^2+s^2)\nu_k} \right\} C_o = f_o . \qquad 4.5$$

Since

$$\frac{2}{s\pi} \operatorname{th}\frac{s\pi}{2} = 1 - \frac{8s^2}{\pi^2} \sum_{odd} \frac{1}{k^2(k^2+s^2)} \approx 1 - \frac{8s^2}{\pi^2(1+s^2)}$$

f_o may be approximated by

$$\tfrac{1}{2} f_o = - \frac{V}{p+\lambda} \left\{ 1 - \frac{8s^2}{\pi^2(1+q^2)} \right\} + \frac{U}{\Omega} \frac{8s^2}{\pi^2(1+q^2)} \quad , \qquad 4.6$$

and the expression between brackets on the left hand side of 4.5 by

$$1 - \frac{8s^2}{\pi^2} \frac{\nu_1 - \sqrt{p^2+\lambda p}}{(1+s^2)\nu_1} \quad . \qquad 4.7$$

Thus we obtain

$$\mathrm{ch}\, \frac{s\pi}{2} c_o \approx - \frac{V}{p+\lambda} \left\{ 1 - \frac{8ps\Omega}{\pi^2} \frac{1}{(1+q^2)(\nu_1 + \sqrt{p^2+\lambda p})} \right\} + \frac{U}{\Omega} \frac{8s^2}{\pi^2(1+q^2)} \quad . \qquad 4.8$$

Finally for $\varphi(\pi/2,0,p)$ the following approximation is obtained

$$\varphi(\pi/2,0,p) \approx - \frac{V(p)}{\sqrt{p^2+\lambda p}} \left\{ 1 - \frac{8s^2}{\pi^2} \frac{\sqrt{p^2+\lambda p}}{(1+q^2)(\nu_1+\sqrt{p^2+\lambda p})} \right\} + U(p) \frac{8s}{\pi^2(1+q^2)} \quad . \qquad 4.9$$

We notice that for $\Omega \to 0$ the correct expression is obtained.

For p=0.12 and 0.18 we obtain

$$\varphi(\pi/2,0,0.12) = -5.53\ V + 0.31\ U$$
$$\varphi(\pi/2,0,0.18) = -4.04\ V + 0.32\ U \ .$$

The true coefficients of V are 5.51 and -4.01 respectively, so that the approximation is very close. The influence of the component U is rather small. If V=0 and $U = P_1 e^{p_1 t} - P_2 e^{p_2 t}$ with the values of the preceding section we obtain for φ

t	5	10	15	20	22	24
$\varphi(\pi/2,0,t)$	0.05	0.08	0.11	0.11	0.09	0.04 .

§5. Two special cases

From 4.9 we obtain for an arbitrary windfield

$$U = 0 \qquad V = -f(t) \tag{5.1}$$

in a somewhat less close approximation

$$\bar{\zeta}(\pi/2,0,p) \approx \frac{\bar{F}(p)}{\sqrt{p^2+\lambda p}} - \frac{8s^2}{\pi^2}\,\bar{F}(p)\,. \tag{5.2}$$

Inverse Laplace transformation gives

$$\zeta(\pi/2,0,t) \approx e^{-\frac{\lambda}{2}t}\, I_o(\tfrac{\lambda t}{2}) * f(t) - \frac{8\Omega^2}{\pi^2}\, e^{-\lambda t} * f'(t). \tag{5.3}$$

Case a. A step-function windfield

$$f(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0\,. \end{cases}$$

Formula 5.3 gives

$$\zeta(\pi/2,0,t) \approx \int_0^t e^{-\frac{\lambda\tau}{2}}\, I_o(\tfrac{\lambda\tau}{2})\,d\tau - \frac{8\Omega^2}{\pi^2}\, e^{-\lambda t}\,. \tag{5.4}$$

In the numerical case we find for a few t values for ζ in meters

t = 5.7	$\zeta \approx$ 4.74 - 0.18	= 4.56
11.3	8.16 - 0.08	8.08
17.0	10.82 - 0.04	10.78
22.6	13.00 - 0.02	12.98
28.3	14.86 - 0.01	14.85

The influence here of the rotation of the earth is apparently almost if not completely negligible.

Case b. A sinusoidal windfield

$$f(t) = \begin{cases} 0 & t < 0 \\ \sin \omega t & t > 0\,. \end{cases}$$

From 5.3 we obtain

$$\zeta(\pi/2,0,t) \approx \int_0^t e^{-\frac{\lambda\tau}{2}}\, I_o(\tfrac{\lambda\tau}{2})\, \sin\omega\,(t-\tau)\,d\tau \;+$$
$$- \frac{8\omega\Omega^2}{\pi^2} \int_0^t e^{-\lambda\tau} \cos\omega\,(t-\tau)\,d\tau\,. \tag{5.5}$$

For a few values of t we obtain with $\omega = 0.1$ for ζ in meters

t= 0	f(t)= 0	$\zeta \approx$ 0	- 0	= 0
2.8	0.28	0.39	- 0.09	0.30
5.7	0.54	1.41	- 0.15	1.26
8.5	0.75	2.86	- 0.18	2.68
11.3	0.91	4.57	- 0.15	4.42
14.1	0.99	5.91	- 0.13	5.78
17.0	0.99	7.61	- 0.10	7.51
19.8	0.92	8.96	- 0.03	8.93
22.6	0.77	9.90	+ 0.01	9.91
25.5	0.58	10.30	+ 0.08	10.38
28.3	0.31	10.19	+ 0.15	10.34
31.1	0.03	9.51	+ 0.18	9.69

Results

These results refer to a sea which has the form of a semi-infinite strip ($0 < x < \pi$, $y > 0$). In this model the part $0 < y < 2\pi$ represents roughly the basin of the North Sea. The numerical cases studied in this report refer to approximate actual conditions of the North Sea as regards size, average depth, bottom friction, wind-force and Coriolis coefficient.

The elevation at the coast y=0 is almost exclusively due to the longitudinal component V of the wind. In a representative case (cf section 4) the influence of the lateral component U was about 3% of that of V.

The computation of the elevation of the sea which is due to an exponential windfield of the type

$$V = \sum C\ e^{pt}$$

involves the solution of a few rapidly converging systems of an infinite number of linear equations.

In a typical case (cf. 3.1) the rotation of the earth had hardly any influence -about 2%- upon the elevation at the coast y=0 but the Coriolis effect became more and more pronounced as y increased.

For an arbitrary uniform windfield the elevation at $x=\pi/2$, $y=0$ may be approximated by an analytic expression. In two typical cases (cf. section 5) the influence of the rotation of the earth was very small, 2% at most.

The method studied in this report may be extended to non-uniform exponential windfields of the type

$$U,V = \sum C \exp (\alpha x + \beta y + pt) .$$

These will be discussed in a later report.

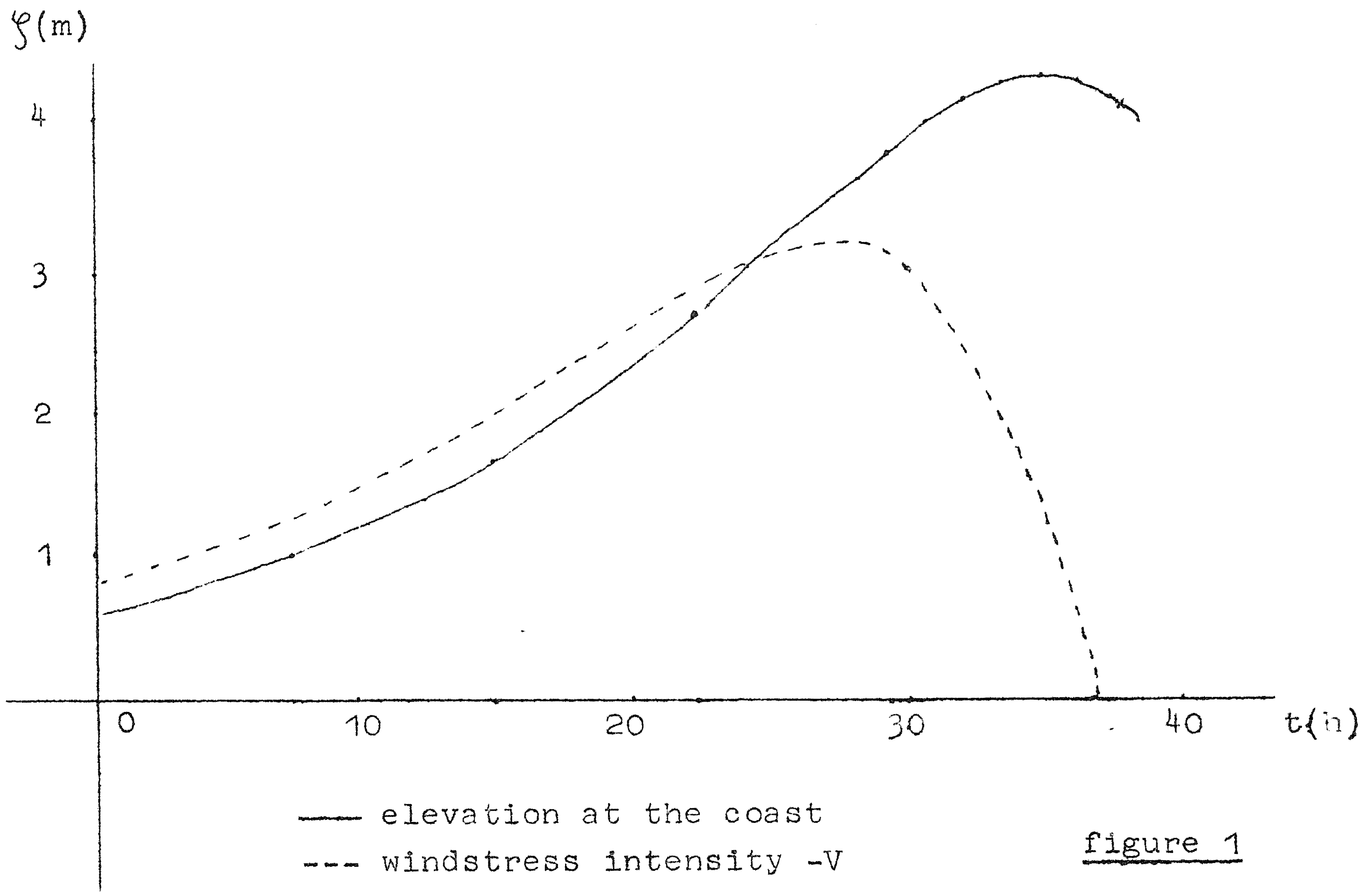

figure 1

Elevation at $x=\pi/2$, $y=0$ due to exponential windfield

$$V = -0.13e^{0.12t} + 0.0284e^{0.18t}$$

$$(-V_{max} = 6.26 \times 10^{-3})$$

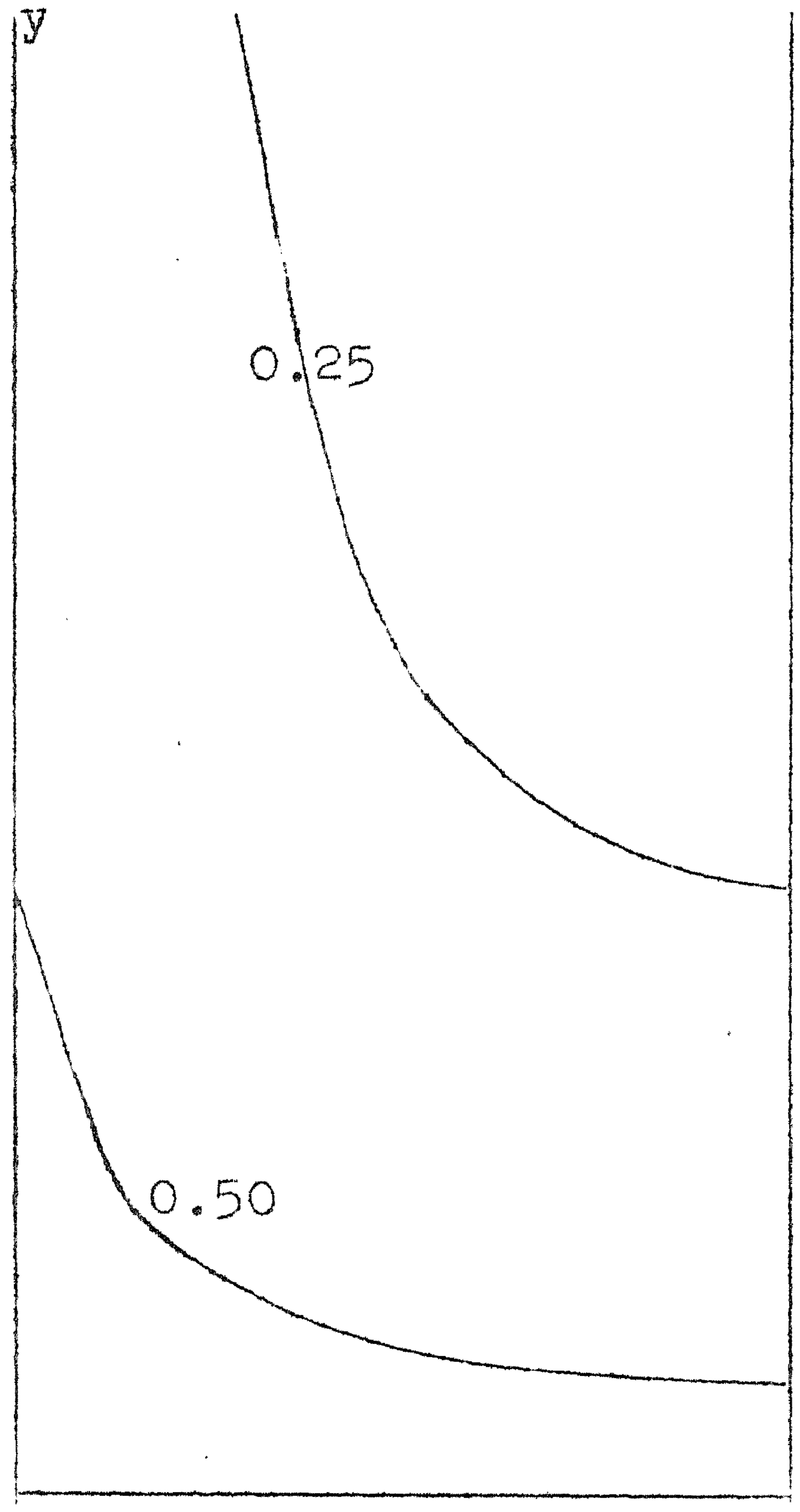

fig.2
t=0h

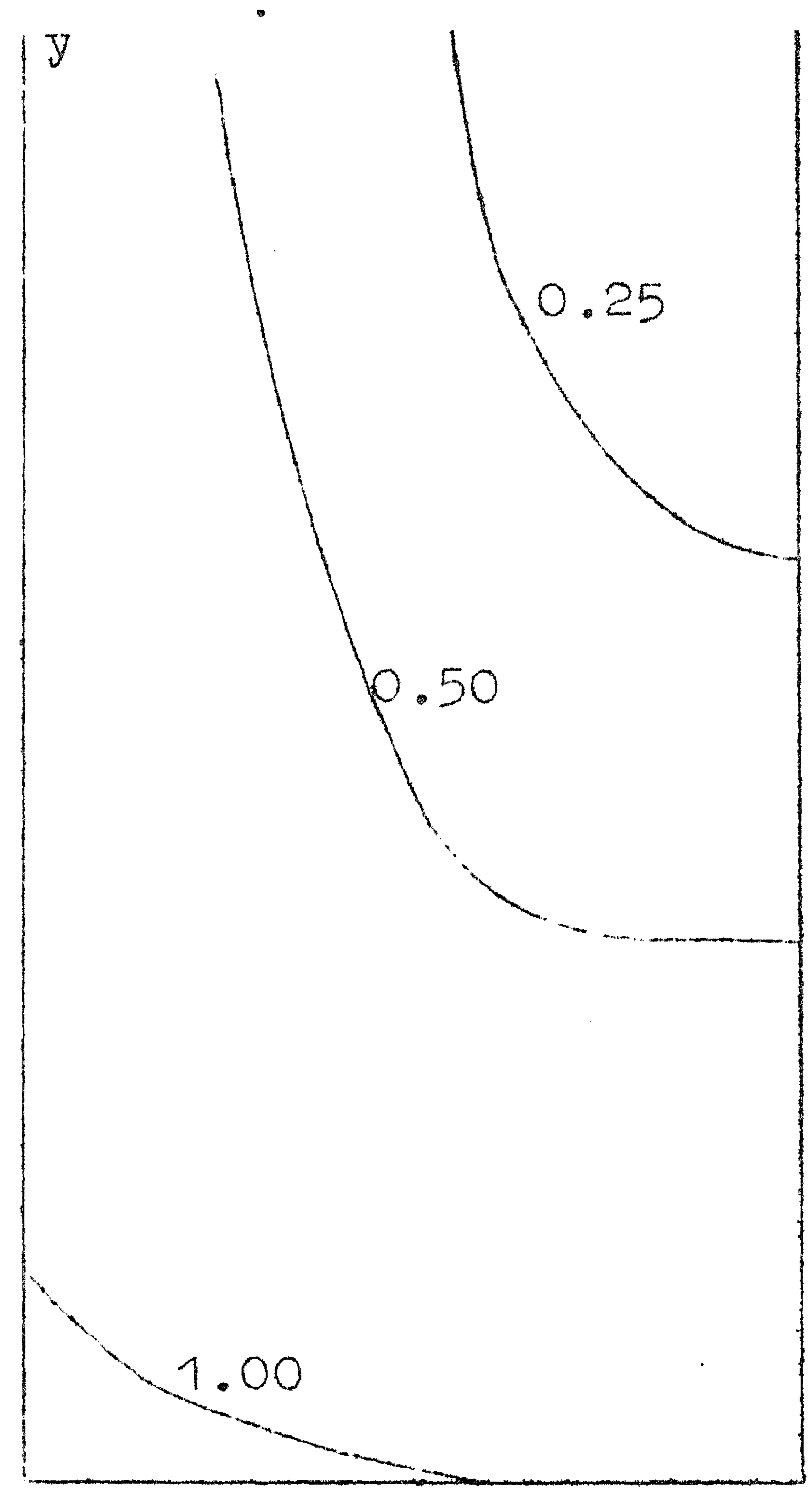

fig.3
t=7,5h

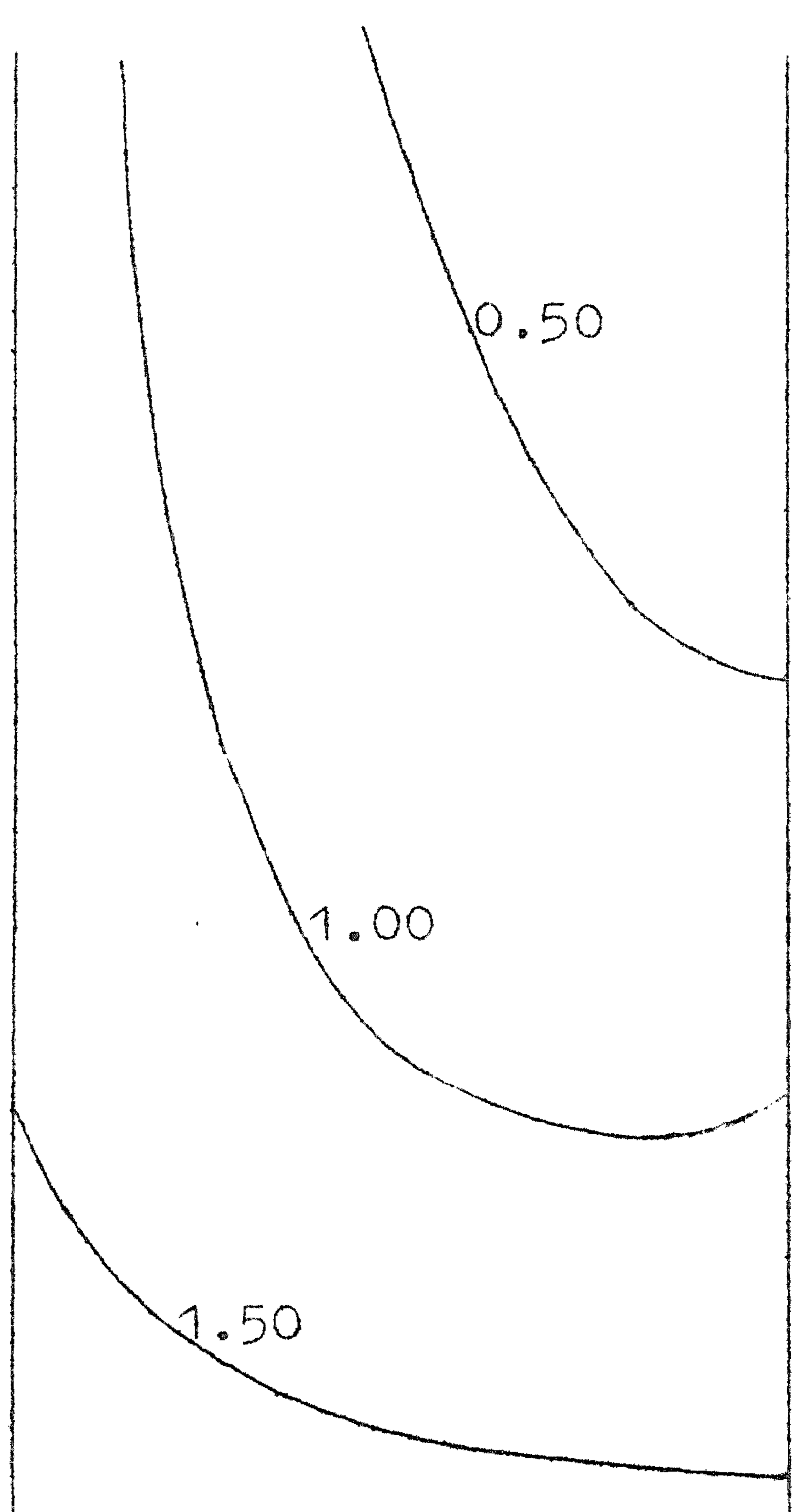

fig.4
t=15h

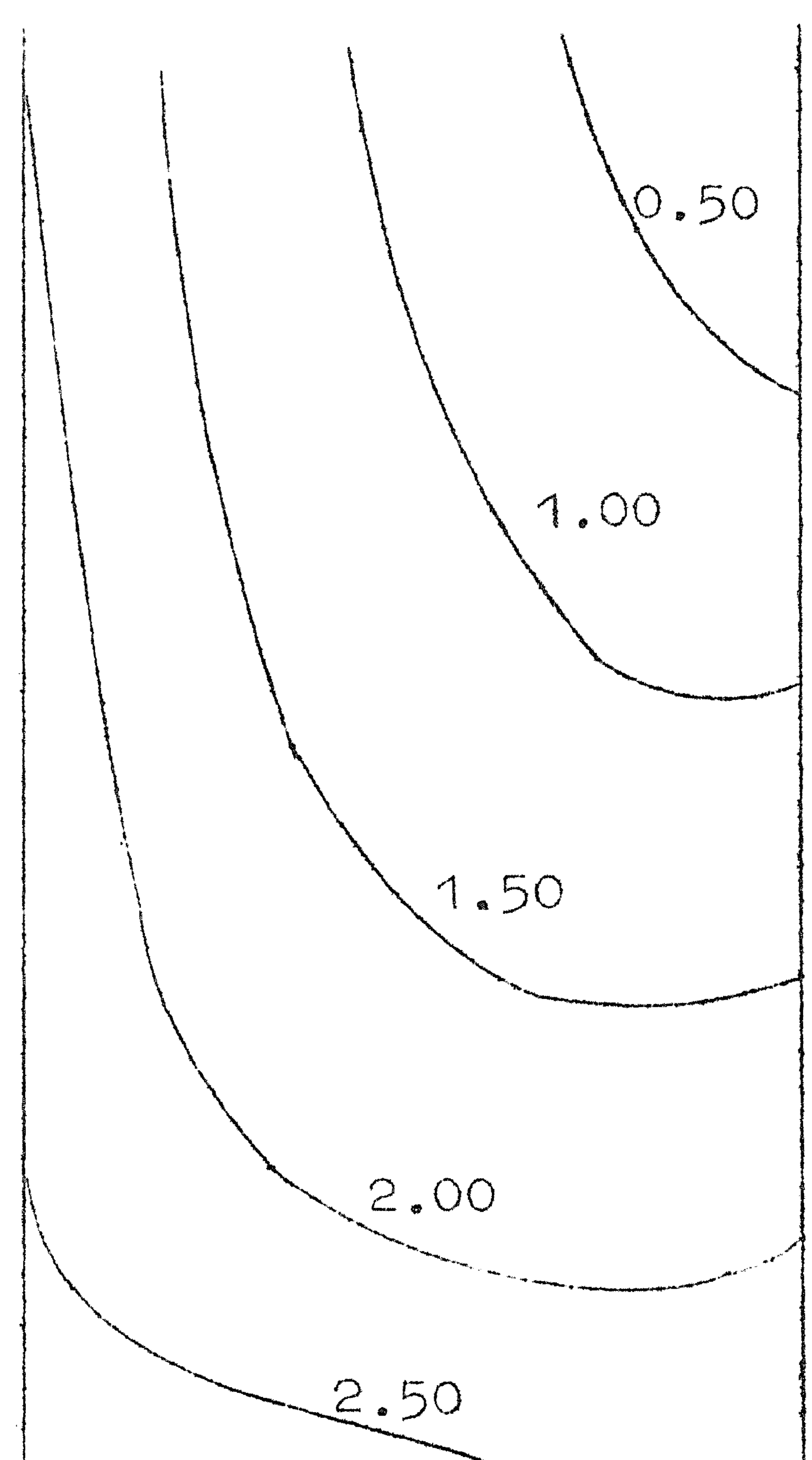

fig.5
t=22,5h

Lines of constant elevation of the sea, due to the exponential wind field V 0.12$e^{0.12t}$ 0.0284$e^{0.18t}$ for various values of t

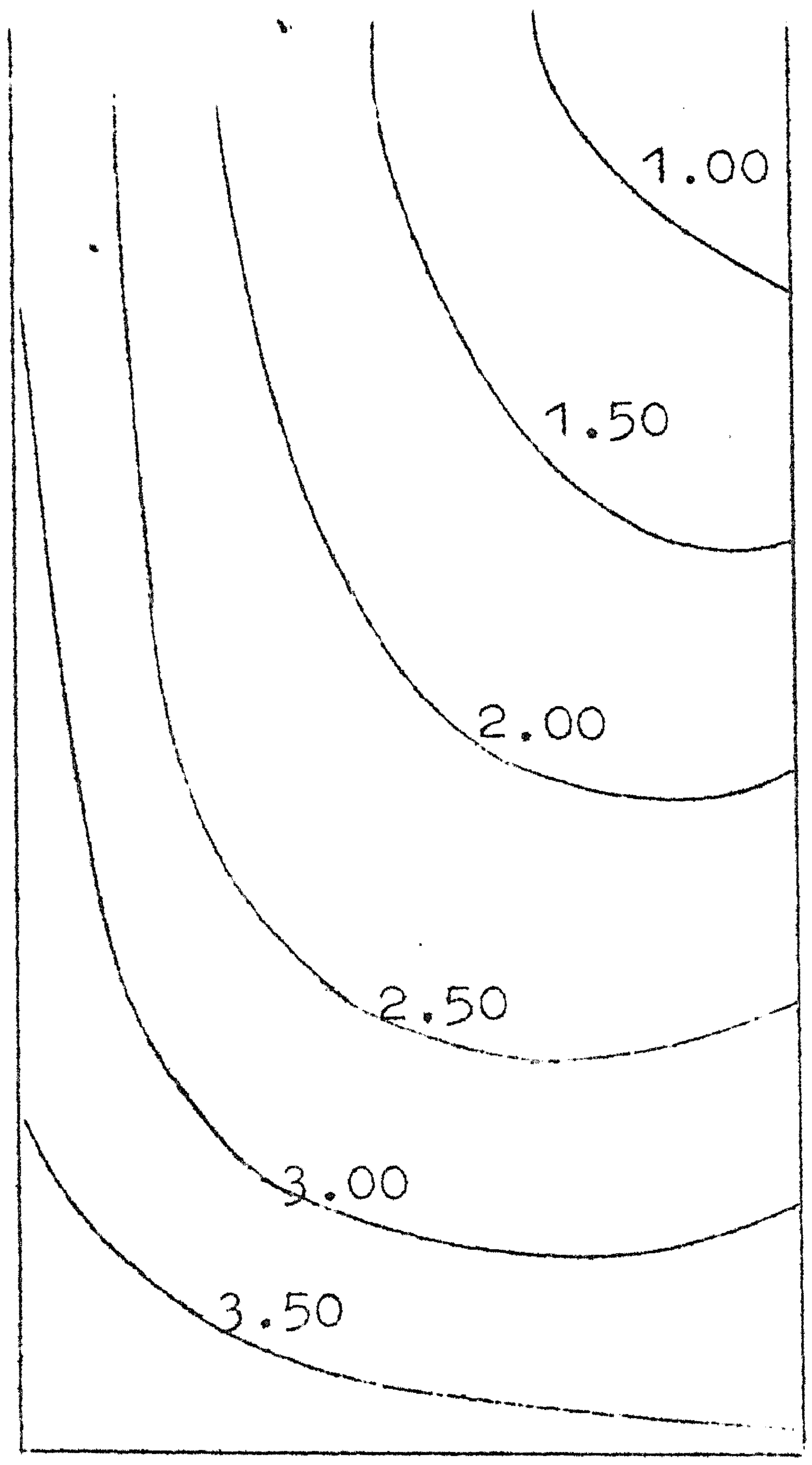

fig.6
t=30h

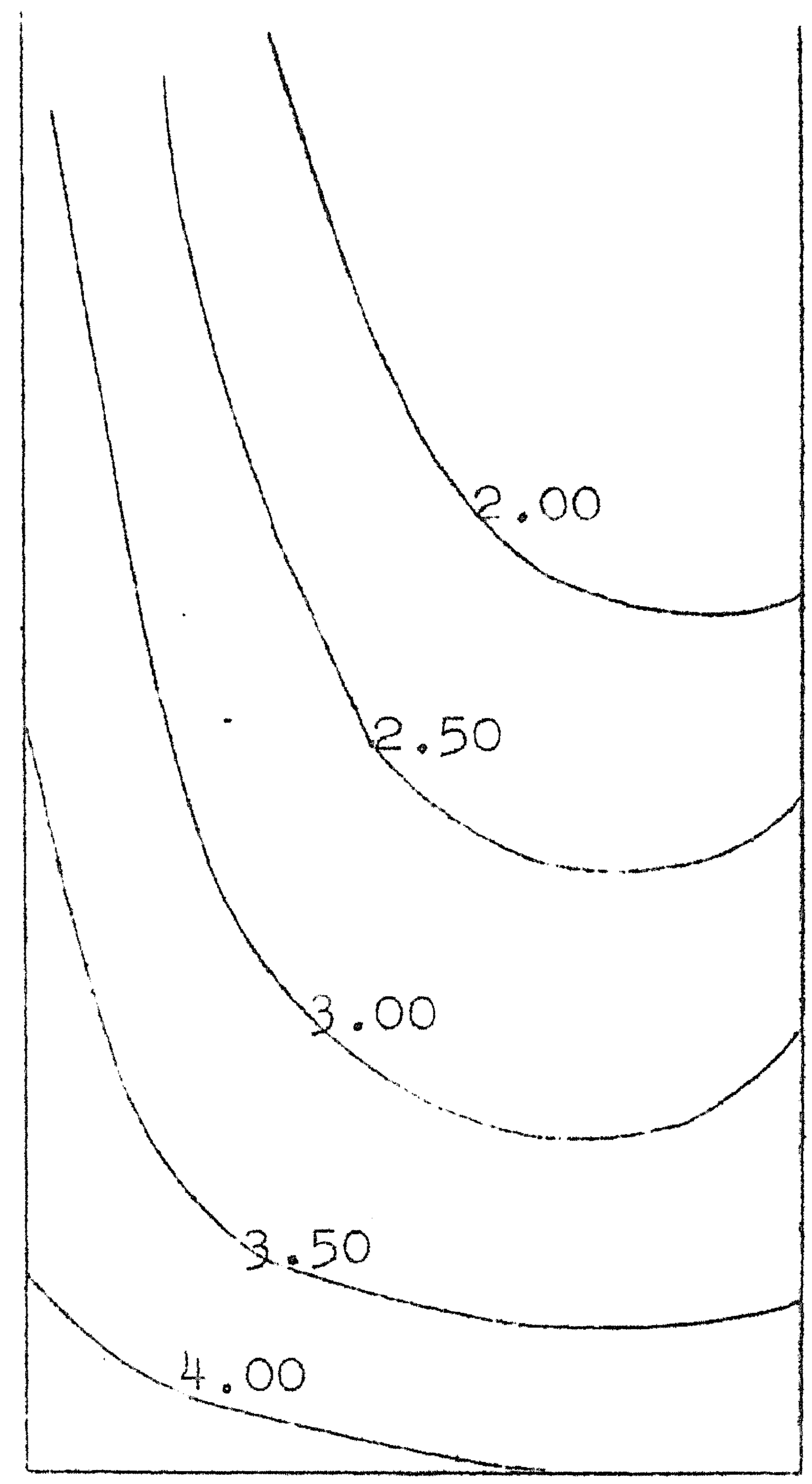

fig.7
t=33h

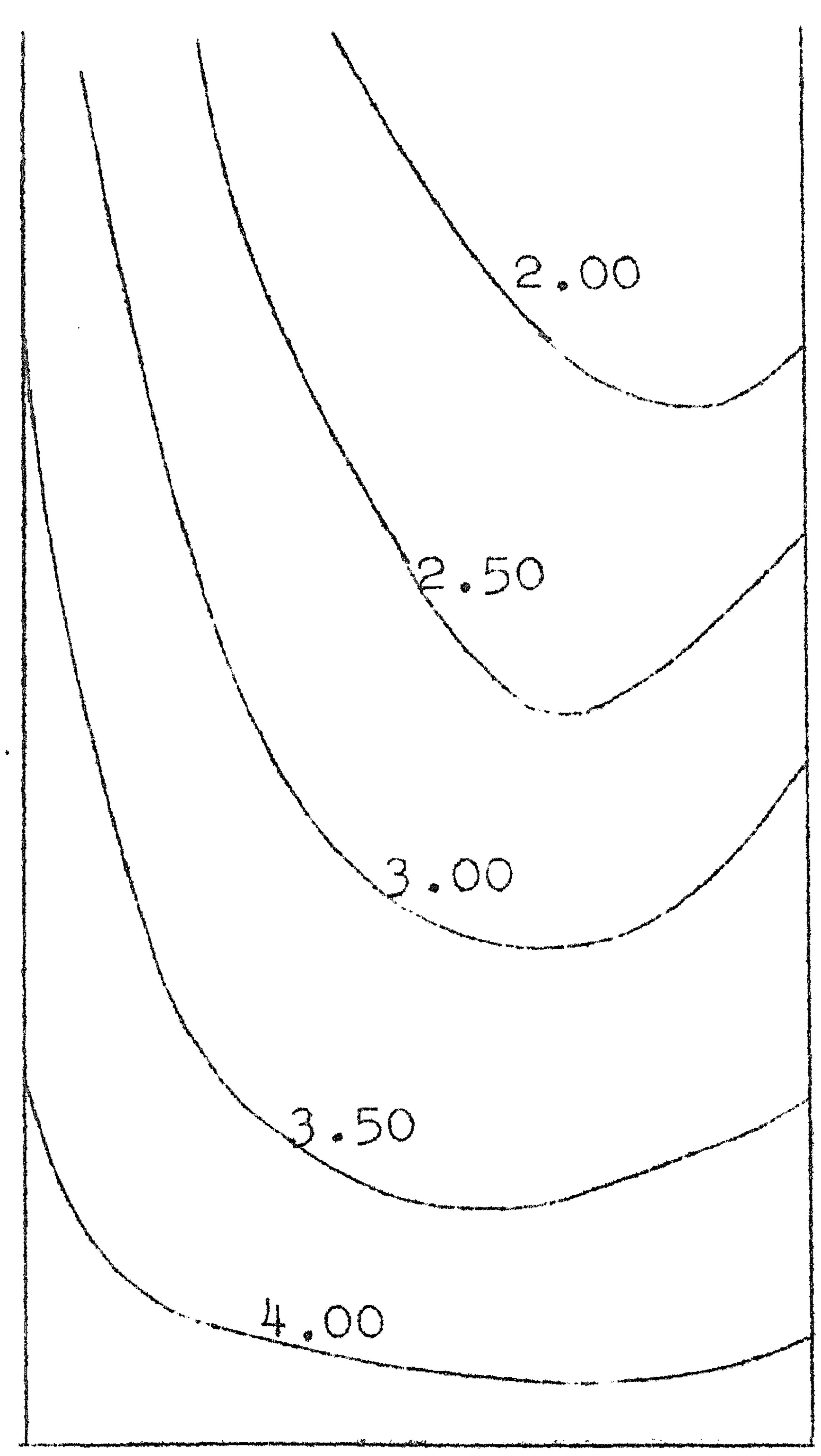

fig.8
t=36h

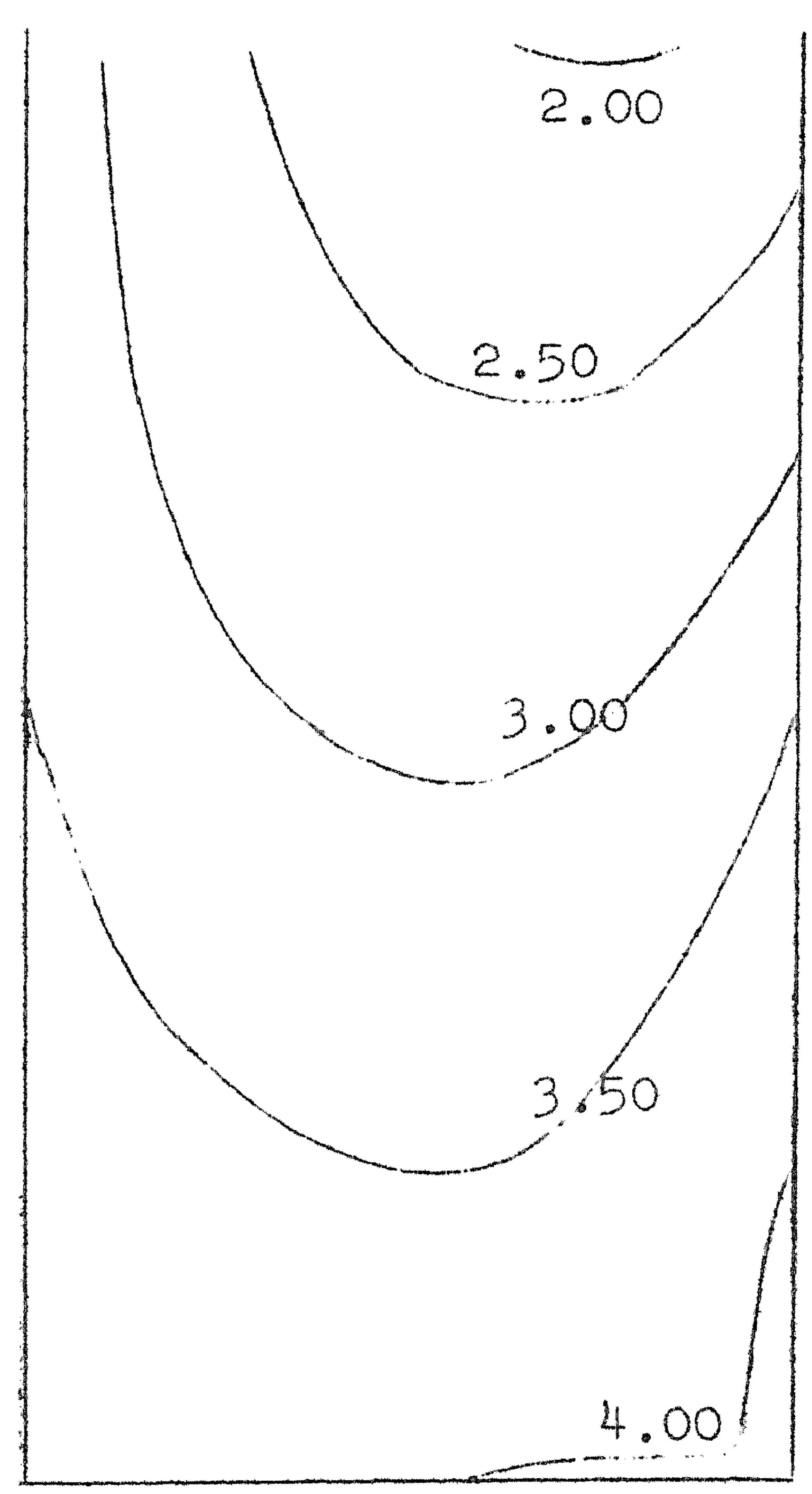

fig.9
t=39h