THE ARRA
1. **Foundation and aims; generalities.**

The Mathematical Centre at Amsterdam is an institute where modern mathematics and their applications are systematically practised. It was founded in February 1946 in order to coordinate "pure" and "applied" mathematics and to fill the existing gap between the two which was strongly felt in our country. The idea of such a centre was already developed during the war. From the beginning the promoters were convinced of the fact that it would be wrong to cultivate intensively only particular branches of mathematics and to neglect the other branches, and that accordingly the only right way would be to found the new institute on a very broad base. Even the most refined electronic computer can not work if it does not receive well-formulated reasonable orders and for the formulation of these orders a staff of mathematicians, highly educated in many different branches, is necessary. So, only by development of many abstract methods and their practical applications at the same time, the greatest possible effect could be expected. Hence, the first task was the training of a group of enthusiastic workers with a good mathematical education and with much interest in the applications.

Immediately after the war the Government of the Netherlands, the Municipality of Amsterdam and some big industries could be convinced of the necessity of a Mathematical Centre built on these principles. The best arguments could be found in the necessity of a rapid industrialization of the Netherlands and in the facts that no perfectionating of industry and economy is possible without research and that every research needs mathematics. The Board of Trustees of the Mathematical Centre consisted of representatives of the Government, of the Municipality of Amsterdam, of university education and of the industry, with a mathematician, Prof. Dr J. A. Schouten as
Secretary. The Board of Directors consisted at first of Prof. Dr J. G. van der Corput as Director, Prof. Dr J. F. Koksma as Secretary and Prof. Dr D. van Dantzig. Moreover there is a Committee of Advice consisting for the greater part of members of the mathematical departments of the Netherlands universities.

The revenues of the Mathematical Centre come from the Government by the intermediation of the two government organizations for Z.W.O. (pure scientific research) and for T.N.O. (applied scientific research), from the Municipality of Amsterdam, which supplies also the building at a moderate rent, from contributions given by some great industries and from paid orders performed for industries, institutions or private persons. In the beginning housing in a very small building, the Mathematical Centre grew and another building had to be occupied. At the moment (1954) the building 2nd Boerhaavestraat 49 contains 25 rooms and is going to be enlarged soon according to the constantly growing number of workers (85 at the end of 1953).

2. Departments.

In accordance with the principles outlined above, the Mathematical Centre consists of several different departments working under the guidance of the members of the Board of Directors. Of this Board the Director, Prof. Dr J. A. Schouten, who succeeded Prof. Dr J. G. van der Corput after his departure for the U.S.A., and Prof. Dr J. F. Koksma direct the department of pure mathematics including the library. The department of applied mathematics (in the sense of mathematical physics) and that of mathematical statistics work under Prof. Dr D. van Dantzig. Prof. Dr J. Hemelrijk is the chief of the statistical consultation. Prof. Dr A. van Wijngaarden, who
became a member of the Board of Directors in 1951, is the head of the computation department. The work in all these departments can be brought under the heads education, service and research.

3. Education.

It is a first principle that the Mathematical Centre never intrudes on the working field of the universities. All courses and lectures are mainly meant for special training for those who have finished their university studies or as a help for people who wish to learn mathematics but can not study at a university because they have a full-time job. The latter courses are held in the evenings. Courses and lectures are not necessarily held in Amsterdam but can be held wherever they are wanted. The Hague, Eindhoven, Rotterdam and Sittard (in the mining district) had already their courses. They have been held mostly by collaborators of the Mathematical Centre or by members of the Committee of Advice. Topics are chosen according to what is required and they can be taken from the whole field of pure and applied mathematics. The auditories consist of secondary school teachers, assistants of universities, specialists from the industry, etc. There are also some colloquia which always have a well-defined special program and are meant for specialists from different organizations. For instance there is a permanent colloquium on electronic computing machines attended by specialists of the Philips works, of the research laboratories of the Netherlands Postal and Telecommunication Services, and of the Mathematical Centre. From other colloquia we mention e.g. a colloquium on conformal representations, meant for hydrologists and a colloquium on econometrics held in collaboration with other institutes. Nowadays many students have to earn their living by doing
all kinds of paid jobs and studying only in the time that can be spared. Students in mathematics who have passed their first examination (roughly equivalent to graduates) may do paid work as a research assistant at one of the departments of the Mathematical Centre, while taking courses towards their final degree at the university. Regarding the status of the assistants, it must be pointed out that the Mathematical Centre confers no degrees of any kind and is not concerned at all with students at the level of "undergraduate"; on the other hand as a rule it does not employ full-time research assistants. Research assistants work on a part-time basis and receive in this way an additional training. Often they stay on at the Mathematical Centre as collaborators after having finished their studies.

Frequently the work of a collaborator of the Mathematical Centre leads to a deeper investigation from which a doctor's thesis may arise under the guidance of any university professor, who of course need not belong to the Mathematical Centre.

As regards the time a full collaborator usually stays at the Mathematical Centre, a very remarkable and unexpected experience presented itself. At first the idea was to let all collaborators stay only a few years in order to get a kind of constant circulation and an uninterrupted supply of well-trained forces for the industry and for other organizations. However, to fulfill the task of the Mathematical Centre with respect to orders from outside it was necessary to have not only semi-skilled men and very soon it became clear that a permanent staff of collaborators was indispensable. In fact there is now such a permanent staff, and besides a regular flow of collaborators, who after staying a few years, get other positions. This is one example of a change of course forced upon the Direction by circumstances. Changes of this kind were and are still frequent because at the moment of the foundation of the Mathematical Centre very few useful examples of mathe-
matical working groups were in existence and many problems of organization could only be solved by experience.

4. Service.

Every industry, laboratory, institution and private person who is in need of mathematical advice, can call upon the service of the Mathematical Centre. The advices are often free of charge, but of course they must be paid for, if the client's aim is a commercial one, or if he gets sufficient financial help from other sources for his investigations. It is an important question how to fix the right amount of what a client has to pay in every special case, but for this a system of scales could be worked out, largely founded on own experience and in general working quite satisfactorily. Problems of this kind are immediately connected with questions as how to find the right equilibrium between paid and non-paid orders, between the number of research assistants and that of collaborators and between the number of long staying collaborators and that of collaborators only staying a short time. Such problems concerning the budget belong to the most difficult part of the task of the Board of Directors.

Clients come to the Mathematical Centre with questions of various kinds. Of course there are always some nonsense questions about the quadrature of the circle, Fermat's theorem, etc., but they can be dealt with very shortly. Most questions are serious and of vital importance for the client. As a rule clients are non-mathematicians, for instance engineers, physicists, physicians, biologists, social workers, etc. In the beginning some kind of systematic propaganda was planned but there was never time nor necessity to organize such a thing, as clients came very soon of their own accord and in so great numbers, that waiting lists are often necessary.
A non-mathematician comes nearly always with questions that are very vague and not yet put in the form of a definite mathematical problem. Conferences with the client are necessary in order to find out what he really wants and how a problem can be formulated. This is the task of the four different departments and here it comes out very clearly that these departments are all necessary.

So, to take for instance the department of mathematical statistics, a difficulty may arise if a client has already made long series of experiments before consulting the Mathematical Centre. Then it may happen that from a statistical point of view so many vital experiments are left out that no definite conclusions can be drawn. Sometimes the whole scheme of experiments was inadequate and a lot of work and valuable time may have been lost. In order to avoid such disappointments, the Mathematical Centre tries to make clear in courses, lectures and conferences that the statistical department should always be consulted before series of experiments are started. In order to enable clients to do their own computing work in cases where this is possible, the statistical department has written a number of leaflets containing short directions for the use of statistical methods. In this way the department frees itself from a great deal of purely mechanical work and at the same time it gives its clients a kind of training in statistical methodology.

The computation department has had similar experiences; often on some numerical problem a great amount of work has been done, that could have been avoided if advice had been asked for at an earlier stage.

In general such advices and remarks of the Mathematical Centre find a ready ear, though they may be at first disappointing, and it can be stated that modern insights are gradually replacing the incorrect ideas on mathematical calculations that reigned not so long before.
The existence of a well organized Mathematical Centre is also very useful in the case of special events. Such an event was the enormous flood disaster that struck the south-western part of the Netherlands in February 1953. In order to find out the best methods for preventing as far as possible a similar calamity in the future, the government appointed the so-called Delta committee (the name refers to the delta of Rhine, Meuse and Scheldt). The action of this committee gave rise to a number of mathematical problems, a great deal of which could immediately be tackled by the Mathematical Centre.

Another event of a more friendly nature was the International Mathematical Congress 1954, which was decided in 1950 to be held in the Netherlands. The preparations for such a big congress are rather complicated and it was greatly advantageous that the Mathematical Centre could put part of its staff and of its organization at the service of the Organization Committee.

5. The building.

A number of rooms is required for management, comptability and administration, including typing and reproduction. Two lecture rooms have been available, but want of space necessitated the transfer of one of them into a working room. The automatic computing machine „ARRA“ occupies one room for itself and two other rooms contain the workshop and the laboratory, necessary for the regular service of the ARRA, for research and for the construction of new electronic machines, two of which are in course of building now. The I.B.M. punchcard machines are placed in another room. For the library there is only one room and the other rooms are occupied by the staff, the collaborators, assistants and working students and 
by the computers. All rooms are very crowded but the planned
enlarging of the building already mentioned above will provide
more room for all departments.

6. The library.

The stock of books (about 2800 for a great part quite
recent ones) is rapidly increasing (with 470 in 1953) and so
is the collection of reprints (about 11,000). The 180 perio­
dicals (increase 25 in 1953) are obtained mostly in exchange
with the Indagationes Mathematicae. The library gives not
only service for the Mathematical Centre itself but also (free
of charge) for all scientists in the Netherlands. This service
extends to giving advice as to the study of literature. It is
greatly appreciated that many authors give copies of their
books or papers as a present to the library.
Several card catalogues, some of them composed by means of the
Mathematical Reviews enable the librarian to give much valuable
advice in a short time. There is a good collaboration with the
other four mathematical libraries in Amsterdam. So the purchase
of unnecessary duplicates can be avoided and catalogues for
books and periodicals can be made uniformly. A central cata­
logue is in the making. Of course there is a reading apparatus
for microfilm and more of them will be installed as soon as
the library gets more room.

7. Contact with foreign countries.

Many contacts were laid by members of the Board of Direc­
tors and by collaborators visiting congresses and other confe­
rences. The Mathematical Centre receives also continuously
foreign scientists (45 in 1953) and many of them stay for some days and give a lecture. Others work during a longer time as guests in the Mathematical Centre. Other very important contacts arise from the exchange of publications with other centres of mathematical activity. In this way much valuable information is exchanged.

8. **Work of the departments.**

a. **Department of pure mathematics.**

In this department research work is done for instance on number theory, analysis, differential geometry and algebra. A great deal of the time is also spent in helping the other departments with questions of a pure mathematical character and to the investigation of literature and manuscripts. It is often asked to develop further existing mathematical methods or to invent new ones.

The administration of the library and the construction of the card catalogues not belonging to the other departments, are part of the task of this department and the same holds for the organization of most courses, lectures and colloquia and the reproduction of lecture notes (also for university lectures). We mention the colloquia on group theory, asymptotic developments, theta-functions, recurring series and theory of partitions. Help is also given to authors of textbooks and of theses and to editors who wish to get advice on manuscripts for publication.

b. **Department of applied mathematics.**

This department covers the ground of what is often called "mathematical physics". Problems present itself as a result of
own initiative or by orders of the other departments or from outside. The field, problems come from, is very vast, for instance help was given to the pure mathematical investigations on asymptotic developments but also to the practical problem of Amsterdam's supply of drinking water. From 1953 on much hydrodynamical work was done for the Delta-committee already mentioned before. We mention here an investigation on tidal movements and another investigation on this line concerning the foundations of the theory of wind working on great masses of water. The numerical results of these investigations, which will be obtained for a great part in collaboration with the computation department may be expected to be of importance for the protection of the Netherlands against future floods.

c. Department of mathematical statistics.

From the beginning the task of this department was the systematic development of modern statistical methods and the stimulation of the application of these methods instead of the older ones used before the war. Therefore a number of young mathematicians had to be instructed and these men found later working fields in and outside the Mathematical Centre. Many publications have had their influence on the development of the science of statistics. We mention here Hemelrijk's ,,symmetry-test'', Terpstra's ,,test against trend'', Theil's investigations on ,,regression-analysis'', van der Vaart's study on ,,Wilcoxon's test'', van Dantzig's and Kemperman's investigations on stochastic processes, and Benard's and van Elteren's ,,generalization of the method of m rankings''.

The department got many orders from outside (about 60 in 1953). Among them the greater ones, requiring a complete statistical analysis, come mostly from other institutes. Among
the smaller ones there are short consults, and others have the form of a continuous contact with investigators, who during some series of experiments, do their own statistical computing work. Many orders give rise to theoretical investigations and lead to publications in reports, which often appear in print. Of the subjects of statistical analyses and consults in 1953, we mention: high-water levels in the North-Sea (for the above mentioned Delta-committee); regression analysis of the power transmitted on a shipspropeller; waiting times of aeroplanes before landing; epidemiological investigation on tuberculosis in Indonesia; insulin-assay; statistical diagnosis of rheumatism; dental care in Netherlands schools; blood-counts; truncation of normally distributed variables in a problem of diagnosis; cancer-mortality in different parts of the Netherlands; technical problems of the blast-furnace industry; etc. Consults are not only asked for by mathematicians and insurance companies, but also by biologists, physicians, economists, physicists, hydrodynamists, engineers of all kinds, etc. The consulting task took such big proportions, that it was necessary to appoint the collaborator Prof. Hemelrijk, extraordinary professor of mathematical statistics in the Technical University of Delft, as Chief of the Statistical Consultation.

There is a close cooperation with the Central Bureau for Statistics, the Economic University of Rotterdam and the Central Planning Bureau. Contacts with the industry could be laid and developed by the founding of an international working group of statistical workers.

d. Computation Department.

The department receives orders of many different kinds (about 75 in 1953). Of recent work we mention here: harmonic analysis of tidal effects in the gravity of the earth connected with
oil prospecting; calculations for the theory of ships' propellers; tabulation of functions for the determination of ship forms (these calculations made it possible to deliver ships about 5 weeks earlier); integrations connected with the radiation pressure in big stars and in star nebulae; calculations of the azimuth from right ascension, latitude and declination; solution of algebraic equations and determination of eigenvalues of matrices for investigations concerning the motion of railway cars and the oscillations of aeroplane wings; filter computing; investigations on basins of rain water under the sea; solution of partial differential equations by relaxation for problems concerning the drinking water supply from the dunes; calculation of the aerodynamic coefficients of oscillating wings in subsonic flow; integrations in connection with investigations on cancer. Many of these calculations were effected by means of desk-machines (15 mechanical and 25 electrical ones in use in 1953) but for others adapted accounting machines (3 in 1953), I.B.M. punch-card machines or the automatic electronic computer ARRA are used. Many of these orders gave rise to independent research work, the results of which appeared in a number of reports and communications. We mention here the following subjects: phase functions; analytical interpolation; summation of divergent series; Fourier coefficients of modular functions; mathematical possibilities of punch-card machines; numbers of Fibonacci; ground-wave propagation; tabellation of Mathieu functions, Fresnel integrals. Problems connected with random numbers were so interesting that a colloquium on recurring series was held, attended by collaborators of all departments. As an example of the collaboration with the department of pure mathematics we mention the problems concerning asymptotic developments. Of course, there is a continuous collaboration with all departments, because the computation department has to do all computing
that is too complicated for the simple equipments of the other departments.

The department designed and built the construction of the automatic electronic computer ARRA (originally built as a relay machine, but now transformed into a full electronic one). It entered into service in the beginning of 1954. It works 24 hours a day, but for the week-ends. It is middle-fast and built up from interchangeable units, so that small disturbances can be eliminated in a very short time by putting in a new unit. One of the bigger orders for this machine concerns the solution of a great number of differential equations for the wave functions arising in the theory of the interaction of atoms. This problem is certainly beyond the reach of a normal sized team of ordinary computers.

The good results obtained with the ARRA brought an order of one of the big Netherlands factories for the construction of a computer on the same lines. This machine will be ready at the beginning of 1955. Meanwhile the laboratory and the workshop are also occupied with the preparations for a new computer, the AERA, much greater and faster than the ARRA and with a far greater storage capacity. This machine is intended for calculations beyond the scope of the ARRA. It will be built also on the principle of interchangeable units.

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In the foregoing pages we have given a very short description of the aims and the working methods of the Mathematical Centre.

It would have been impossible to give more details in a small leaflet like this. But of course investigators seriously interested in the work done by the Mathematical Centre and in the methods used, can always get every information available.