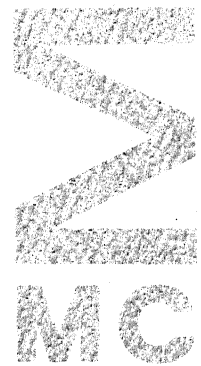


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AFDELING ZUIVERE WISKUNDE
(DEPARTMENT OF PURE MATHEMATICS)

ZN 76/77

AUGUSTUS

A.E. BROUWER

THE t -DESIGNS WITH $v < 18$

amsterdam

1977

**stichting
mathematisch
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2e boerhaavestraat 49 amsterdam

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The t -designs with $v < 18$

by

A.E. Brouwer

Below we list the known and unknown t -designs without repeated blocks for $2 \leq t < k \leq \frac{1}{2}v$, $\lambda \leq \frac{1}{2}\lambda_+ = \frac{1}{2}\binom{v-t}{k-t}$ and $v < 18$.

This table is a preliminary one - we hope to be able to extend it much further, at least up to $v = 24$ and perhaps even to $v = 28$. Comments, additions and corrections and also references for t -designs with $18 \leq v \leq 28$ are very much appreciated.

KEY WORDS & PHRASES: *t*-design

Table of t-designs without repeated blocks,
 $2 \leq t < k \leq v/2, \lambda \leq \lambda/2,$
 A.E. Brouwer, 776529

v	t	k	λ^-	λ^+	group	order	remarks
6	2	3	2	4	PSL5	60	ext of 1-(5,2,2) - unique
7	2	3	1	5	PGL3.2	168	$\lambda=1$: PG(2,2) (Fano) - unique $\lambda=2$: 2 disjoint copies of Fano - unique
8	2	3	6	6			
		4	3	15			see 3-(8,4, $\lambda/3$).
8	3	4	1	5			$\lambda=3$: 4 nonisomorphic systems [NANDI] ext of 2-(7,3, λ) - unique $\lambda=1$: Hadamard 3-design
9	2	3	1	7			$\lambda=1$: AG(2,3) unique - has (2 different) packings of 7 disjoint copies [KIRKMAN]
		4	3	21	C9	9	$\lambda=3$: 3 disjoint copies $\lambda=3$: 11 nonisomorphic systems [STANTON e.a.]
9	3	4	6	6			
10	2	3	2	8	C9c	9	$\lambda=2$
					C10	10	$\lambda=4$
		4	2	28			$\lambda=2$: 3 nonisomorphic systems [NANDI] $\lambda>2$: der of 3-(11,5, λ)
		5	4	56	C9c	9	$\lambda=4$: 6 disjoint copies $\lambda=28$: ext of 1-(9,4,28) $\lambda=4$: 21 nonisomorphic systems [van LINT e.a.]
10	3	4	1	7			der of 4-(11,5, λ)
		5	3	21			$\lambda=1$: AG(2,3) - unique ext of 2-(9,4, λ) $\lambda=3$: 7 nonisomorphic systems [GIBBONS]
10	4	5	6	6			
11	2	3	3	9			der of 3-(12,4, λ)
		4	6	36			der of 3-(12,5, λ)
		5	2	84	C11	11	$\lambda=2$: 2 disjoint copies - unique [RUSAIN] $\lambda=4$: 6 disjoint copies ($5*(\lambda=4)+2*(\lambda=2)$)
					D11	22	all λ except $\lambda=2,6,8$
11	3	4	4	8			der of 4-(12,5, λ)
		5	2	28			$\lambda=2$: does not exist [DEBON, OEFERSCHLUP]
					C11	11	$\lambda>2$
					D11	22	$\lambda>6$
11	4	5	1	7	SA11	55	$\lambda=1$: 2 disjoint copies [WITT] - unique
					S4mS4	24	$\lambda=3$ [BROUWER]
12	2	3	2	10			der of 3-(13,4, λ)
		4	3	45	C11c	11	$\lambda=3$: at least 7 disjoint copies
		5	20	120			der of 3-(13,6, λ)
		6	5	210			see 3-(12,6, $\lambda*2/5$)
12	3	4	3	9			der of 4-(13,5, λ)
		5	6	36	SA11	55	$\lambda=6,18$
					S3*C4	24	$\lambda=12$
		6	2	84			ext of 2-(11,5, λ)
							$\lambda=2$: Hadamard 3-design
12	4	5	4	8	PGL5*E2	120	$\lambda=4$ [DENNISTON]
		6	2	28			$\lambda=2$ does not exist (see 3-(11,5,2)) $\lambda=4,8,12$: see 5-(12,6, $\lambda/4$) $\lambda=14$: ext of 3-(11,5,14) $\lambda=6,10$: unknown
12	5	6	1	7			ext of 4-(11,5, λ) $\lambda=1$: unique [WITT]

13	2	3	1	11			der of 3-(14,4, λ)
							$\lambda=1$: 2 nonisomorphic systems with autgps of order 39, 6.
		4	1	55	C13	13	disjoint: $4*(\lambda=1)+8*(\lambda=5)$ $\lambda=1$: PG(2,3) - unique
		5	5	165			der of 3-(14,6, λ)
13	3	4	2	10	GA13	156	disjoint: $1*(\lambda=5)+1*(\lambda=10)+3*(\lambda=15)+9*(\lambda=30)$
					GA13	156	$\lambda=4$
					O13	26	$\lambda=2,4$
		5	15	45			der of 4-(14,5, λ)
		6	20	120	GA13	156	
13	4	5	3	9			der of 5-(14,6, λ)
		6	6	36	SA13	76	$\lambda=12$ [KRAMER & MESSEK] $\lambda=6,18$: unknown $\lambda=4$: unknown
13	5	6	4	8			
14	2	3	6	12	SA13n	78	
		4	6	66			der of 3-(15,5, λ)
		5	20	220			der of 3-(15,6, λ)
		6	15	495			der of 3-(15,7, λ)
		7	6	792	SA13n	78	disjoint: $2*(\lambda=6)+2*(\lambda=12)+2*(\lambda=18)+9*(\lambda=36)$
14	3	4	1	11	SA7*E2	21	$\lambda=1$: 2 disjoint solutions [BAYS & DE WECK] 4 nonisomorphic systems with autgps of order 42,14,6,6 [MENDELSON & HUNG]
					SA13n	78	$\lambda=2,3,4,5$
		5	5	55			$\lambda=5$: unknown
					PSL13	1092	$\lambda=10,15,25$
							$\lambda=20$: residual of 4-(15,5,4)
		6	5	165	GA13n	156	
		7	5	330			ext of 2-(13,6, λ)
14	4	5	10	10			
		6	15	45	GA13n	156	!
		7	20	120			$\lambda=20$: unknown
14	5	6	3	9	C13m3n	39	$\lambda=40$: see 5-(14,7,12) $\lambda=60$: ext of 3-(13,6,60)
		7	6	36			!
							$\lambda=12$: ext of 4-(13,6,12)
							$\lambda=6,18$: unknown
14	6	7	4	8			$\lambda=4$: unknown - ext of 5-(13,6,4)
15	2	3	1	13			der of 3-(16,4, λ)
		4	6	78			$\lambda=1$: 80 nonisomorphic systems [HALL & SWIFT]
		5	2	286			der of 3-(16,5, λ)
							$\lambda=2$ does not exist [HALL & CONDOFF]
		6	5	715	PGL5b	120	$\lambda>2$: der of 3-(16,6, λ)
		7	3	1287	PGL5b	120	disj: $(\lambda=5)+(\lambda=10)+(\lambda=20)+(\lambda=40)+5*(\lambda=60)$ for $\lambda>5$ also res of 3-(16,6, $\lambda^2/5$)
							all λ except $\lambda=6,9$
							$\lambda=3$: SEIPD - 5 nonisomorphic systems [PHAT & SHRIKHANDE]
							perm: 29 disjoint copies of $\lambda=3$
15	3	4	12	12			
		5	6	66	A6b	360	$\lambda=12,30$!
					(SA7*C2)n	42	$\lambda=6$!
		6	20	220			$\lambda=18,24$: see 4-(15,5, $\lambda/6$) !
		7	15	495			der of 4-(16,7, λ)
					GA13nn	156	$\lambda=90,105,120,135,165,\dots,225$: see 4-(15,7, $\lambda/3$)
					S6b	720	$\lambda=15,30,45,60,75,90,105,120,135$!
15	4	5	1	11			$\lambda=60,75,90,135,150,165,180,225,240$ [K & M]
							$\lambda=1$: does not exist [MENDELSON & HUNG]
							$\lambda=2,5$: unknown
					PSL5b	60	$\lambda=4$!
					(SA7*C2)n	42	$\lambda=3$!
		6	5	55	PSL13nn	1092	$\lambda=15$!
					SA13nn	78	$\lambda=10$!
							$\lambda=5,20,25$: unknown

		7	5	165	GA13bd	156	$\lambda=30, 35, 40, 45, 55, 60, 65, 70, 75$! $\lambda=5, 10, 15, 20, 25, 50, 80$: unknown
15	5	6	10	10			$\lambda=15$: unknown
		7	15	45			
15	6	7	3	9			$\lambda=3$: unknown
16	2	3	2	14			der of 3-(17,4, λ)
		4	1	91	S4[C2]	384	der of 3-(17,5, λ) except perhaps for $\lambda=3$ $\lambda=3$
		5	4	364			$\lambda=1$: AG(2,4) - unique
		6	1	1001	A4[C2]	192	der of 3-(17,6, λ) $\lambda>19$ (disj: $\lambda=2, 6, 8, 2*12, 15, 2*24, 32, 36, 6*48, 96$) $\lambda=1$: does not exist [FISCHER] $\lambda=2$: SEIBD - 3 nonisomorphic systems [HUSAIN] $\lambda=3$: nonresidual example [BHATTACHARYA] or residual of 2-(25,9,3) perm: using $\lambda=2, 3$ one finds all $\lambda<65$
		7	14	2002			der of 3-(17,8, λ)
		8	7	3003			$\lambda>28$: see 3-(16,7, $\lambda*5/14$) see 3-(16,8, $\lambda*3/7$)
16	3	4	1	13	GF16am3	80	$\lambda=4, 5$: der of 4-(17,5, λ)
					C3p[C2]	48	$\lambda=1$: 2 disjoint solutions - planes in AG(4,2) $\lambda=1, 3, 4, 6$
		5	6	78			der of 4-(17,6, λ)
		6	2	286			$\lambda=2$: does not exist (see 2-(15,5,2)) $\lambda=6$ or $\lambda>10$: der of 4-(17,7, λ)
					A6bd	360	$\lambda=4, 12, 16, 22, 30, 34, 36, 40, 42, 46, 52, 54, 60$ etc
					PSL7*E2	336	$\lambda=8$ and $\lambda=12$ and all $\lambda>16$!
					GA16	240	$\lambda=6, 10, 16, 20, 26, 30, 36, 40$ etc
		7	5	715			$\lambda=15$ or $\lambda>25$: res of 4-(17,7, $\lambda*2/5$)
					GA16	240	$\lambda=15$ and $\lambda=20$ and all $\lambda>25$
					PGL5bn	120	$\lambda=15, 25, 30$ etc $\lambda=5, 10$: unknown
		8	3	1287			ext of 2-(15,7, λ) $\lambda=3$: Hadamard 3-design
16	4	5	12	12			$\lambda=12, 24$: der of 5-(17,7, λ)
		6	6	66			$\lambda=18$: see 5-(16,6,3)
					GA16	240	$\lambda=30$! $\lambda=6$: unknown
		7	20	220	GA16	240	$\lambda=20, 40, 60, 100$! $\lambda=80$: der of 5-(17,8,80)
		8	15	495			see 5-(16,8, $\lambda/3$) but: $\lambda=75, 240$
					qma16	960	$\lambda=15, 30, 45, 60, 150$: unknown
16	5	6	1	11	PSL7*E2	336	$\lambda=3$! $\lambda=1$: does not exist (see 4-(15,5,1)) $\lambda=2, 4, 5$: unknown
		7	5	55	GA16	240	$\lambda=15$! $\lambda=5, 10, 20, 25$: unknown
		8	5	165			ext of 4-(15,7, λ), but: $\lambda=5, 10, 15, 20, 25, 50, 80$: unknown
16	6	7	10	10			$\lambda=15$: unknown
		8	15	45			
16	7	8	3	9			$\lambda=3$: unknown - ext of 6-(15,7,3)
17	2	3	3	15	GA17	272	disjoint: $(\lambda=3)+2*(\lambda=6)$
		4	3	185	GA17	272	disjoint: $(\lambda=3)+3*(\lambda=6)+7*(\lambda=12)$
		5	5	455	GA17	272	disjoint: $(\lambda=5)+3*(\lambda=10)+21*(\lambda=20)$
		6	15	1365	GA17	272	disjoint: $7*(\lambda=15)+42*(\lambda=30)$
		7	21	3003	GA17	272	disjoint: $7*(\lambda=21)+68*(\lambda=42)$
		8	7	5005	GA17	272	disjoint: $(\lambda=7)+(\lambda=14)+5*(\lambda=28)+85*(\lambda=56)$
17	3	4	2	14	GA17	272	disjoint: $(\lambda=2)+3*(\lambda=4)$
		5	1	91	GA17	272	all λ except $\lambda=1, 3$
					SA17	136	$\lambda=1$: 2 disjoint solutions $\lambda=3$: unknown

	6	4	364	GA17	272			
	7	7	1001	GA17	272	all λ except $\lambda=7$		
						$\lambda=7$: unknown		
	8	14	2002	GA17	272			
17	4	5	1	13	GA17	272	disjoint: $2*(\lambda=4)+(\lambda=5)$ [KRAMER]	
							$\lambda=1,2,3,6$: unknown	
							$\lambda=1$: if existing then autgp is e17 [DENNISTON]	
	6	6	78				der of $5-(18,7,\lambda)$ [KRAMER]	
	7	2	286	GA17	272	$\lambda > 10$!		
				PGL16	4080	$\lambda=6,40,46,60,66,100,106,110,116$ [KRAMER]		
						$\lambda=2$: does not exist (see $2-(15,5,2)$)		
						$\lambda=4,8,10$: unknown		
	8	5	715	PGL16	4080	$\lambda=15,80,95,100,115,120,135,160,175,200,215,$ $220,235,240,255,260,275,280,295,320,335,$ $340,355$ [KRAMER]		
				GA17	272	$\lambda=30, \dots, 55, 70, \dots, 95, 120, \dots, 130$!		
						$\lambda=5$: does not exist [DELSARTE]		
						some other values of λ are known:		
						res of $5-(18,8,\lambda*2/5)$: $\lambda=110,150,155,165$ etc		
						der of $5-(18,9,\lambda)$: $\lambda=140$ etc		
						$\lambda=10,15,20,25,55,60,65,85,105,145$ etc: unknown		
17	5	6	12	12				
		7	6	66	GA17	272	$\lambda=12,24$!	
							$\lambda=6,18,30$: unknown	
		8	20	220	PGL16	4080	$\lambda=80$ [KRAMER]	
							all other λ are unknown	
17	6	7	1	11			$\lambda=1$: does not exist (see $4-(15,5,1)$)	
							all other λ are unknown	
		8	5	55			all λ unknown	
17	7	8	10	10				
18	2	3	2	16	Cl70	17	$\lambda=2$: 6 disjoint copies	

Explanation:

abbreviations:

der	derived design
ext	extension
res	residual design
aut	automorphism
gp	group
disj	disjoint
...	including all intermediate admissible values of lambda
BIBD	balanced incomplete block design
SBIBD	symmetric BIBD

group denotations:

E#	identity (as permutation group on # elements) - order 1
C#	cyclic group - order #
D#	dihedral group - order $2 * \#$
S#	symmetric group - order #!
A#	alternating group - order $\#! / 2$
GA#	general affine group - order $\# * (\# - 1)$
GA(d, #)	general affine group on $AG(d, \#)$
SA#	special affine group - order $\# * (\# - 1) / 2$
PGL#	general projective group - order $(\# + 1) * \# * (\# - 1)$
PGL(d, #)	general projective group on $PG(d, \#)$
PSL#	special projective group - order $(\# + 1) * \# * (\# - 1) / 2$
S4mS4	representation of S4 of degree 11 - order 24 (permute points, bipartitions and triples of a 4-set)
Cl3m3	subgroup of GA13 generated by $x \rightarrow x+1$ and $x \rightarrow 3x$ - order 39
Cl15m3	subgroup of GA16 generated by $x \rightarrow x+1$ and $x \rightarrow ax$, where $a^{**5} = 1$ - order 30

Now let G and H be permutation groups acting on sets X and Y , resp. with degrees $m = |X|$ and $n = |Y|$. Then we define:

$G+H$ direct sum acting on $X+Y$ - degree $m+n$ - order $|G| * |H|$
 $G \times H$ direct product acting on $X \times Y$ - degree $m \times n$ - order $|G| * |H|$
 G_0 representation of the abstract group G with degree $m+1$ obtained by adding a point fixed by all group elements (i.e. $G+E_1$) - order $|G|$
 G_b representation of the abstract group G with degree $m \times (m-1)/2$ acting on the unordered pairs of X - order $|G|$
 $G[H]$ group of degree $n \times m$ and order $|G| * (|H| \times m)$ obtained by letting G permute the coordinates of the direct product of m copies of H
 $G \sim H$ group of degree $n \times m$ and order $|G| * (|H| \times m)$ obtained by letting G permute the summands of the direct sum of m copies of H (wreath product)

geometry denotations:

$PG(d,c)$ projective geometry of dimension d over $GF(c)$
 $AG(d,c)$ affine geometry of dimension d over $GF(c)$
 $MG(d,c)$ moebius geometry of dim d over $GF(c)$ ($d=2$: inversive plane)

symbols:

v number of points of the design
 b number of blocks of the design
 k blocksize
 t strength
 λ lambda: each t -set is covered by λ blocks
 λ_- smallest lambda satisfying the divisibility conditions
 λ_+ lambda of the trivial design (each k -set is a block)

remarks:

ext a $t-(2k+1, k, \lambda)$ design is extendable to a $(t+1)-(2k+2, k+1, \lambda)$ design when t is even, or when t is odd and $\lambda = \lambda_+/2$ [AULTOP]
 der derive from a $t-(v, k, \lambda)$ design a $(t-1)-(v-1, k-1, \lambda)$ design
 res the residual of a $t-(v, k, \lambda)$ design is a $(t-1)-(v-1, k, \lambda \times (v-t+1)/(k-t+1) - \lambda)$ design
 group usually the group given is not the full automorphism group of the design (e.g. we give SA11 for the unique $4-(11, 5, 1)$ which has A11 as full automorphism group)
 perm if a $t-(v, k, \lambda)$ exists then it can be chosen disjoint from a given collection of d k -sets when
 $v! > b * d * k! * (v-k)!$
 i.e. when
 $d < \lambda_+/\lambda_-$.
 Hadamard 3-design: a $3-(4r, 2r, r-1)$ exists whenever a Hadamard matrix of order $4r$ exists.
 ! this design (or: the design with smallest λ among these) seems to be new.

List of unknown designs with small v
 A.E. Brouwer, 770828

$v < 11$: all known

$v = 11$: $3-(11,5,2)$ does not exist [DEHON e.a.]
 all others are known

$v = 12$: $4-(12,6,2)$ does not exist
 $4-(12,6,\lambda)$ are unknown for $\lambda = 6,10$
 all others are known

$v = 13$: $4-(13,6,\lambda)$ are unknown for $\lambda = 6,18$
 $5-(13,6,4)$ is unknown (and extendable to $6-(14,7,4)$ [ALLTOP])
 all others are known

$v = 14$: unknown:
 $3-(14,5,5)$, $4-(14,7,20)$, $5-(14,7,\lambda)$ $\lambda=6,18$, $6-(14,7,4)$.

$v = 15$: nonexistent:
 $2-(15,5,2)$ [HALL & CONNOR]
 $4-(15,5,1)$ [MENDELSON & HUNG]
 unknown:
 $4-(15,5,\lambda)$ $\lambda = 2,5$
 $4-(15,6,\lambda)$ $\lambda = 5,20,25$
 $4-(15,7,\lambda)$ $\lambda = 5,10,15,20,25,50,80$
 $5-(15,7,15)$
 $6-(15,7,3)$

$v = 16$: nonexistent:
 $2-(16,6,1)$ [FISCHER]
 $3-(16,6,2)$
 $5-(16,6,1)$
 unknown:
 $3-(16,7,\lambda)$ $\lambda = 5,10$
 $4-(16,6,6)$
 $4-(16,8,\lambda)$ $\lambda = 15,30,45,60,150$
 $5-(16,6,\lambda)$ $\lambda = 2,4,5$
 $5-(16,7,\lambda)$ $\lambda = 5,10,20,25$
 $5-(16,8,\lambda)$ $\lambda = 5,10,15,20,25,50,80$
 $6-(16,8,15)$
 $7-(16,8,3)$

$v = 17$: nonexistent:
 $4-(17,7,2)$
 $4-(17,8,5)$ [DELSARTE]
 $6-(17,7,1)$
 note:
 $4-(17,5,1)$ cannot have automorphisms [DEHNISTOP]
 unknown:
 $3-(17,5,3)$
 $3-(17,7,7)$
 $4-(17,5,\lambda)$ $\lambda = 1,2,3,6$
 $4-(17,7,\lambda)$ $\lambda = 4,8,10$
 $4-(17,8,\lambda)$ $\lambda = 5,10,20,25,60,65,105,145,170,180,185,210, \dots, 330$
 $t-(17,k,\lambda)$ $t > 4$
 but known:
 $5-(17,7,\lambda)$ $\lambda = 12,24$ [BROUWER]
 $5-(17,8,80)$ [KRAMER]

$v = 18$: nonexistent:
 $4-(18,6,1)$ [WITTE]
 $5-(18,8,2)$
 $5-(18,9,5)$

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