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AFDELING ZUIVERE WISKUNDE
(DEPARTMENT OF PURE MATHEMATICS)

ZN 80/78 JANUARI

A.E. BROUWER

EMBEDDING THE AFFINE PLANE OF ORDER 4 IN A LINEAR SPACE WITH LINES OF SIZE 4 AND 85 POINTS

amsterdam 1978

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2e boerhaavestraat 49 amsterdam

Printed at the Mathematical Centre, 49, 2e Boerhaavestraat, Amsterdam.

The Mathematical Centre, founded the 11-th of February 1946, is a non-profit institution aiming at the promotion of pure mathematics and its applications. It is sponsored by the Netherlands Government through the Netherlands Organization for the Advancement of Pure Research (Z.W.O).

AMS(MOS) subject classification scheme (1978): 05B40

Embedding the affine plane of order 4 in a linear space with lines of size 4 and 85 points

by

A.E. Brouwer

ABSTRACT

We answer a question of professor H. Lenz.

KEY WORDS & PHRASES: *embedding, linear space, block design.*

In [1] professor Lenz proves that for $v \equiv 1$ or $4 \pmod{12}$, $v \geq 49$ there exists a linear space on v points with lines of size 4 and a subspace of 16 points, with the possible exeptions of $v = 49$ and $v = 85$. Here we show that also in these cases such a space exists. (The notation the the usual Hanani-like one.)

PROPOSITION 1. *There exists for* $t \in \mathbb{N}$ *a* $B(\{4,(3t+1)^*\},1;9t+4)$ *design. In particular if* $t \equiv 0$ *or* $1 \pmod 4$ *then there exists a* $B(4,1;9t+4)$ *with a subspace of size* $3t+1$. *In particular there exists a* $B(4,1;49)$ *with a subspace of size* 16.

PROOF. Ray Chaudhuri & Wilson proved the existence of a Kirkman triple system on $6t+3$ points. Completing the $3t+1$ parallel classes of this design with points at infinity yields the first statement; observing that Hanani proved that $u \in B(4,1)$ iff $u \equiv 1$ or $4 \pmod{12}$ yields the second one. Now take $t = 5$. □

PROPOSITION 2. *There exists a* $B(\{4,5\},1;28)$ *with a subspace of size* 5. *Consequently there exists a* $B(4,1;85)$ *with a subspace of size* 16.

PROOF. The implication is well known (and probably due to Hanani) so we only have to prove the first statement. Let $X = I_4 \times \mathbb{Z}_7$ and take the 21 quintuples

$$\{(0,0),(0,1),(1,3),(2,5),(3,4)\} \quad \text{mod } (-,7)$$
$$\{(0,0),(0,2),(1,6),(2,3),(3,1)\} \quad \text{mod } (-,7)$$
$$\{(0,0),(0,4),(1,5),(2,6),(3,2)\} \quad \text{mod } (-,7)$$

and the 28 quadruples

$$\{(0,0),(1,0),(2,0),(3,0)\} \quad \text{mod } (-,7)$$
$$\{(1,1),(1,2),(1,4),(2,0)\} \quad \text{mod } (-,7)$$
$$\{(2,3),(2,5),(2,6),(3,0)\} \quad \text{mod } (-,7)$$
$$\{(3,3),(3,5),(3,6),(1,0)\} \quad \text{mod } (-,7).$$

□

REFERENCE

[1] LENZ, H. *Embedding block designs into larger ones*, Preprint nr. 45, "Kombinatorische Mathematik", Freie Universität, Berlin, Aug. 1977.