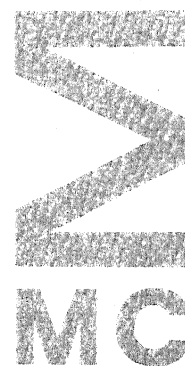


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(DEPARTMENT OF PURE MATHEMATICS)

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DECEMBER

J. VAN DE LUNE

A NOTE ON A PROBLEM OF ERDÖS

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A note on a problem of Erdős

by

J. van de Lune

ABSTRACT

This note contains a method for finding natural numbers  $n$  such that  $2^n$  starts with the same ordered sequence of digits as  $n$  does. Six numbers of this kind are presented.

Recently my attention was drawn to the following observation made by P. Erdős:  $2^6 = 64$  and  $2^{10} = 1024$ . Here we have two examples of the phenomenon that the number  $2^n$  (written, as usual, in the scale of ten) starts with the same ordered sequence of digits as the natural number  $n$  itself.

Let us call a positive integer having this property an Erdős - number.

We use the following notation:  $[x]$  denotes the integral part of the real number  $x$ ;  $\{x\} := x - [x]$  denotes the fractional part of  $x$  and the base ten logarithm of  $x \in \mathbb{R}^+$  will be written as  $\text{LOG } x$ .

From the first principles of our decimal number system it follows that the number of digits  $d(n)$  of a natural number  $n$  is given by

$$d(n) := [\text{LOG } n] + 1$$

and that the first digit  $f(n)$  of  $n \in \mathbb{N}$  is given by

$$f(n) := \left[ \frac{n}{10^{d(n)-1}} \right] = [10^{\text{LOG } n - [\text{LOG } n]}].$$

Some elementary decimal point manipulation shows that  $n \in \mathbb{N}$  is an Erdős - number if and only if

$$n = \left[ \frac{2^n}{10^{d(2^n)-1}} * 10^{d(n)-1} \right] = [10^{\text{LOG } 2^n - [\text{LOG } 2^n] + [\text{LOG } n]}],$$

which subsequently, is equivalent to

$$n \leq 10^{n \text{ LOG } 2 - [n \text{ LOG } 2] + [\text{LOG } n]} < n+1,$$

$$\text{LOG } n \leq n \text{ LOG } 2 - [n \text{ LOG } 2] + [\text{LOG } n] < \text{LOG}(n+1),$$

or, finally

$$0 \leq \{n \text{ LOG } 2\} - \{\text{LOG } n\} < \text{LOG}(n+1) - \text{LOG } n,$$

a formula which is very convenient for finding large Erdős - numbers.

On a small programmable pocket calculator (an HP-65 in our case) we ran the following program:

<u>LBL A</u>	RCL 1
STO 1	1
f LOG	+
STO 2	STO 1
2	f LOG
f LOG	STO 2
STO 3	+
<u>LBL B</u>	$g \times \leq y$
RCL 1	GTO B
RCL 3	RCL 1
*	1
$f^{-1}$ INT	-
RCL 2	R/S
$f^{-1}$ INT	LBL C
-	RCL 1
0	1
$g \times > y$	+
GTO C	STO 1
$g \downarrow$	f LOG
RCL 2	STO 2
CHS	GTO B

Before running this program by pressing key A, one has to key in the number  $n$  one wants to start with. We simply started with  $n = 1$ . At R/S the display showed successively  $n = 6$ ,  $n = 10$  and after quite a while,  $n = 1542$ . After these results the above program was run during an entire weekend without any further result.

After this, the above program was translated into FORTRAN and implemented on a CDC Cyber 73/173. In a few seconds we found the following Erdős - numbers:  $n = 77075$ ,  $n = 113939$  and  $n = 1122772$ .

Finally the same program was run (with double precision arithmetic) for several hours with the result that the previously found Erdős - numbers are the only ones in the range  $n \leq 5 * 10^7$ .

Conclusion: In the range  $n \leq 5 * 10^7$  there are six Erdős - numbers, to wit:

n = 6  
n = 10  
n = 1542  
n = 77075  
n = 113939  
n = 1122772.