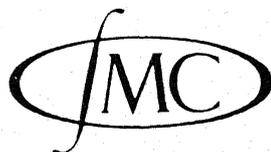


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Fixed fields under automorphism groups of purely transcendental  
field extensions

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Fixed fields under automorphism groups of purely transcendental field extensions <sup>1)</sup>.

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Let  $k$  be a field with characteristic  $p$ . Let  $G$  be a finite transitive permutation group of  $n$  indeterminates  $X_1, \dots, X_n$  over  $k$ . Let  $k_S$  be the field of all symmetric functions in  $X$  over  $k$  and let  $k(G, n, n)$  denote the intermediate field of  $k_S$  and  $k(X_1, \dots, X_n)$  corresponding to  $G$  in the Galois-correspondence. The first argument  $n$  in  $k(G, n, n)$  denotes the number of variables that are permuted transitively under  $G$ , the second  $n$  the transcendence degree of  $k(X_1, \dots, X_n)$ ,  $k_S$  and  $k(G, n, n)$  over  $k$ .

We speak also about fields  $k(G, n, n-1)$ , defined in precisely the same way as  $k(G, n, n)$  except that we require from the  $X_1, \dots, X_n$  that they satisfy one linear relation, viz.  $X_1 + \dots + X_n = 0$ .

The problem is, whether  $k(G, n, n)$  (and also  $k(G, n, n-1)$ ) are purely transcendental over  $k$  or not. If this is the case then we denote this shortly by putting the letters PT before the field under consideration. So PT  $k(G, n, n-1)$  means that  $k(G, n, n-1)$  can be generated by  $n-1$  algebraically independent elements over  $k$ .

Theorem 1.  $PT\ k(G, n, n-1) \implies PT\ k(G, n, n)$

Theorem 2.  $(PT\ k(F, r, r-1) \ \& \ PT\ k(H, s, s-1) \ \& \ p \nmid rs) \implies PT\ k(F \rtimes H, rs, rs-1)$ .

Theorem 3.  $(PT\ k(F, r, r-1) \ \& \ PT\ k(H, s, s-1) \ \& \ p \nmid rs) \implies PT\ k(F \circ H, rs, rs-1)$ , where  $F \circ H$  denotes the wreath-product of the permutation group  $F$  and  $H$ .

Theorem 4. Let  $C$  be the cyclic permutation group with order  $n$ . Let  $k$  contain the  $n$ -th root of unity,  $p \nmid n$ . Then  $PT\ k(C, n, n-1)$ .

Theorem 5. Let  $A$  be an arbitrary abelian group with order  $n$ . Let  $k$  contain the  $m$ -th roots of unity, where  $m$  is the exponent of  $A$ . Then  $PT\ k(A, n, n-1)$  and  $PT\ k(A, n, n)$ , in the case that  $p \nmid n$ .

Theorem 6. If there exists any field  $k$  such that  $PT\ k(G, n, n)$  then all group extensions  $X$  of  $G$  with an arbitrary finite group  $F$ ,  $X/G \cong F$ , can be obtained as a subgroup of  $G \circ F$ .

[1] W. Kuyk, Over het omkeerprobleem van de Galoistheorie, 1960, Amsterdam.

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1) Abstract of the short address held at the I.C.M., Stockholm 1962; some of the theorems are to be found in my dissertation [1].