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Fixed fields under automorphism groups of purely transcendental field extensions

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Fixed fields under automorphism groups of purely transcendental field extensions [1)].

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Let k be a field with characteristic p. Let G be a finite transitive permutation group of n indeterminates $X_1, \ldots, X_n$ over k. Let k_S be the field of all symmetric functions in X over k and let k(G,n,n) denote the intermediate field of k_S and $k(X_1, \ldots, X_n)$ corresponding to G in the Galois-correspondence. The first argument n in k(G,n,n) denotes the number of variables that are permuted transitively under G, the second n the transcendence degree of $k(X_1, \ldots, X_n)$, k_S and k(G,n,n) over k.
We speak also about fields k(G,n,n-1), defined in precisely the same way as k(G,n,n) except that we require from the $X_1, \ldots, X_n$ that they satisfy one linear relation, viz. $X_1 + \ldots + X_n = 0$.

The problem is, whether k(G,n,n) (and also k(G,n,n-1)) are purely transcendental over k or not. If this is the case then we denote this shortly by putting the letters PT before the field under consideration. So PT k(G,n,n-1) means that k(G,n,n-1) can be generated by n-1 algebraically independent elements over k.

Theorem 1. PT k(G,n,n-1) $\Longrightarrow$ PT k(G,n,n)
Theorem 2. (PT k(F,r,r-1) & PT k(H,s,s-1) & $p \nmid rs$) $\Longrightarrow$ PT $k(F \times H, rs, rs-1)$.
Theorem 3. (PT k(F,r,r-1) & PT k(H,s,s-1) & $p \nmid rs$) $\Longrightarrow$ PT $k(F \circ H, rs, rs-1)$, where $F \circ H$ denotes the wreath-product of the permutation group F and H.
Theorem 4. Let C be the cyclic permutation group with order n. Let k contain the n-th root of unity, $p \nmid n$. Then PT k(C,n,n-1).
Theorem 5. Let A be an arbitrary abelian group with order n. Let k contain the m-th roots of unity, where m is the exponent of A. Then PT k(A,n,n-1) and PT k(A,n,n), in the case that $p \nmid n$.
Theorem 6. If there exists any field k such that PT k(G,n,n) then all group extensions X of G with an arbitrary finite group F, $X/G \cong F$, can be obtained as a subgroup of $G \circ F$.

[1] W. Kuyk, Over het omkeerprobleem van de Galoistheorie, 1960, Amsterdam.

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1) Abstract of the short address held at the I.C.M., Stockholm 1962; some of the theorems are to be found in my dissertation [1] .