stichting
mathematisch
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MC

AFDELING ZUIVERE WISKUNDE ZW 1970-010 NOVEMBER

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DEFICIENCY IN INFINITE-DIMENSIONAL MANIFOLDS

2e boerhaavestraat 49 amsterdam

Printed at the Mathematical Centre, 49, 2e Boerhaavestraat, Amsterdam.

The Mathematical Centre, founded the 11-th of February 1946, is a non-profit institution aiming at the promotion of pure mathematics and its applications. It is sponsored by the Netherlands Government through the Netherlands Organization for the Advancement of Pure Research (Z.W.O), by the Municipality of Amsterdam, by the University of Amsterdam, by the Free University at Amsterdam, and by industries.

Contents

1. Introduction. In [17] it was shown that if E is a metric topological vector space (MTVS) which is homeomorphic ($\cong$) to its own countable infinite product E^{ω}, and M is any E-manifold (i.e. M is a paracompact manifold modeled on E), then $M \cong M \times E$. Accordingly we define a subset K of M to have E-deficiency (or to be E-deficient) provided that K is closed and there exists a homeomorphism $h: M \to M \times E$ such that $h(K) \subset M \times \{0\}$.

Such sets have proved to be important to the point-set topology of infinite dimensional manifolds because of results of the following two types.

(1) Negligibility theorem. If ℓ^2 is separable infinite-dimensional Hilbert space, M is an ℓ^2-manifold, and $K \subset M$ is a countable union of ℓ^2-deficient sets, then it was shown in [1] that $M \cong M \setminus K$. More general results have been established in [8].

(2) Homeomorphism extension theorems. If M is as in (1), K_1 and K_2 are ℓ^2-deficient sets in M, and $h: K_1 \to K_2$ is a homeomorphism which is homotopic to id_{K_1} (the identity on K_1), then h can be extended to a manifold homeomorphism [6]. For K_1 and K_2 additionally assumed to be ANR's, a similar result has been established for more general linear spaces than ℓ^2 [11].

In applications it is not easy to recognize that some sets have E-deficiency, thus it becomes desirable to have a coordinate-free topological characterization of E-deficiency in E-manifolds M. Such a characterization was obtained in [3] for $M = \ell^2$ and in [7] it was generalized to M any ℓ^2-manifold. It states (using a notion introduced by Anderson in [3]) that $K \subset M$ (where M is an ℓ^2-manifold) is ℓ^2-definient iff K has Property Z (or is a Z-set), where a set F in a space X has Property Z iff F is closed and for each non-null, homotopically trivial open set U in X, $U \setminus F$ is non-null and homotopically trivial. Among other things this enables us to recognize collared, closed sub-manifolds of M (i.e. bounderies of M) as being ℓ^2-deficient and any closed subset of M which is a countable union of ℓ^2-deficient sets is itself ℓ^2-deficient.

The main result of this paper is the following, which generalizes this characterization of E-deficiency to Fréchet manifolds.

Theorem 1. *Let* $E \cong E^\omega$ *be a Fréchet space,* M *be an E-manifold, and let* $K \subset M$. *Then* K *has E-deficiency iff* K *has Property* Z.

We remark that there are no known examples of infinite-dimensional Fréchet spaces E which do not satisfy the condition $E \cong E^\omega$. Concerning techniques it should be remarked that the proof of the corresponding result for $M = \ell^2$ [3] used the topology of the Hilbert cube I^∞ and the fact that ℓ^2 can be compactified by I^∞ (since ℓ^2 is homeomorphic to the countable product of lines [4]). The proof we give for Theorem 1 also uses the fact that ℓ^2 can be compactified by I^∞.

Using Theorem 1 above and Theorem 1 of [8] we easily obtain the following result.

Corollary. *Let* M *be as in Theorem 1 and let* $K \subset M$ *be a countable union of Z-sets. Then* K *is strongly negligible in* M, i.e. *there exists a homeomorphism* $h: M \to M \setminus K$ *which may be chosen arbitrarily close to* id_M.

(The notion of "arbitrarily close" will be made precise in the next section). We remark that by using different techniques David W. Henderson has recently shown (unpublished) that single Z-sets are strongly negligible in E-manifolds, where $E \cong E^\omega$ is a locally convex (LC) MTVS.

In Theorem 2 we establish a homeomorphism extension theorem which generalizes the extension theorem of [6] (which was proved for ℓ^2-manifolds). In Theorem 2' below we give a simplified version of Theorem 2. The more general statement appears in Section 5.

Theorem 2'. *Let* $E \cong E^\omega$ *be a* LCMTVS *and let* M *be an E-manifold. If* K_1 *and* K_2 *are E-deficient subsets of* M *and* $h: K_1 \to K_2$ *is a homeomorphism which is homotopic to* id_{K_1}, *then* h *can be extended to a manifold homeomorphism.*

2. Preliminaries. In this paper all spaces will be assumed to be metric and all homeomorphisms will be assumed to be onto.

Let X and Y be spaces and let $\mathcal{U}$ be an open cover of Y. Then functions $f, g: X \to Y$ are said to be $\mathcal{U}$-close provided that for each $x \in X$ there exists a $U \in \mathcal{U}$ such that $f(x), g(x) \in U$. A function $F: X \times I \to Y$ (where $I = [0,1]$) is said to be limited by $\mathcal{U}$ provided that for each $x \in X$ there exists a $U \in \mathcal{U}$ such that $F(\{x\} \times I) \subset U$.

If X is a space and $F \subset X$ is closed, then by Lemma 3 of [5] there exists an open cover $\mathcal{U}$ of $X \setminus F$ such that if $h: X \setminus F \to X \setminus F$ is any homeomorphism which is $\mathcal{U}$-close to $id_{X \setminus F}$, then h can be extended to a homeomorphism $\tilde{h}: X \to X$ which satisfies $\tilde{h} \mid F = id_F$. Such a cover of $X \setminus F$ will be called normal (with respect to F).

Let X be a space and let $\{f_i\}_{i=1}^{\infty}$ be a collection of homeomorphisms of X onto itself. Then for each $x \in X$ we let $f(x) = \lim_{i \to \infty} f_i \circ \dots \circ f_1(x)$, if this limit exists. If $f(x)$ exists, for all $x \in X$, then we write $f = L\Pi_{i=1}^{\infty} f_i$, and call it the infinite left product of $\{f_i\}_{i=1}^{\infty}$. We now state a convergence criterion for infinite left products. This is essentially a reformulation of West's version [18] of Theorem 4.2 of [4].

Convergence Procedure. Let X be a (topologically) complete space and let $\mathcal{U}$ be an open cover of X. Then to each homeomorphism $f: X \to X$ and each integer $i > 0$ we can assign an open cover $\mathcal{U}_i(f)$ of X such that if $\{f_i\}_{i=1}^{\infty}$ is any collection of homeomorphisms of X onto itself for which f_{i+1} is $\mathcal{U}_i(f_i \circ \dots \circ f_1)$ - close to id_X, for all $i > 0$, then $f = L\Pi_{i=1}^{\infty} f_i$ gives a homeomorphism of X onto itself which is $\mathcal{U}$-close to id_X.

There is one other notion of deficiency which will be useful in the sequel. Let X be a space which is homeomorphic to $X \times \ell^2$ and let $K \subset X$. Then K has ℓ^2-deficiency provided that K is closed and there exists a homeomorphism $h: X \to X \times \ell^2$ which satisfies $h(K) \subset X \times \{0\}$.

We will represent the Hilbert cube I^{∞} as $\Pi_{i=1}^{\infty} I_i$, where each I_i is the closed interval $[-1,1]$. The set $\Pi_{i=1}^{\infty} I_i^{\circ}$, where $I_i^{\circ} = (-1,1)$, will be denoted by s. In [2] it is shown that $s \times I^{\infty} \cong s$ and we have already remarked that $s \cong \ell^2$.

3. The equivalance of E- and ℓ^2-deficiency. The main result of this section is Theorem 3.1, where we show that in certain spaces E-deficiency and ℓ^2-deficiency are equivalent concepts. A similar proposition was established in [9], where E was additionally assumed to be a Banach space. We remark that the proof we give of Theorem 3.1 below follows in broad outline the proof of the corresponding result of [9], with appropriate modifications being made to overcome the lack of a norm. We will first need a technical result concerning open cones. (By the open cone over a space X (denoted by C(X)) we mean the space $\{v\}\cup(X \times (0,1))$, which is topologized by choosing as a basis the usual topology on $X \times (0,1)$ together with all sets of the form $\{v\}\cup(X \times (0,t))$, for all $t \in (0,1)$. We call v the vertex of the cone). We omit the proof of the lemma, since it is similar to Theorem 5.3 of [17].

Lemma 3.1. Let $E \cong E^\omega$ be a MTVS. Then there exists a homeomorphism $h: E \times [0,1) \times E \to C(E) \times E$ which satisfies the following properties.
(1) $h(E \times \{t\} \times E) = E \times \{t\} \times E$, for all $t \in (0,1)$,
(2) $h(E \times \{0\} \times E) = \{v\} \times E$, where v is the vertex of C(E).

Theorem 3.1. Let $E \cong E^\omega$ be a MTVS, M be an E-manifold, and let $K \subset M$. Then K has E-deficiency iff K has ℓ^2-deficiency.
Proof. Assume K has E-deficiency and let $f: M \to M \times E \times E$ be a homeomorphism such that $f(K) \subset M \times E \times \{0\}$. By the Bartle-Graves-Michael Theorem [15] we have $E \cong (-1,1) \times G$, for some G. Thus

$$E \cong E^\omega \cong (-1,1)^\omega \times G^\omega \cong (-1,1)^\omega \times (-1,1)^\omega \times G^\omega \cong (-1,1)^\omega \times E.$$

Since $(-1,1)^\omega = s \cong \ell^2$ we have $E \cong E \times \ell^2$. Let $g: E \to E \times \ell^2$ be a homeomorphism which satisfies $g(0) = (0,0)$ and let $\tilde{f}: M \to M \times E \times E \times \ell^2$ be defined by $\tilde{f}(x) = (id \times g) \circ f(x)$, where $id \times g: M \times E \times E \to M \times E \times E \times \ell^2$ is defined by $(id \times g)(x,y,z) = (x,y,g(z))$. Then $\tilde{f}$ is a homeomorphism and $\tilde{f}(K) \subset M \times E \times E \times \{0\}$. This implies that K has ℓ^2-deficiency.

On the other hand assume that K has ℓ^2-deficiency. Thus we have a homeomorphism $f: M \to M \times \ell^2$ such that $f(K) \subset M \times \{0\}$. It is easy to modify f to get a homeomorphism $\tilde{f}: M \to M \times E \times [0,1) \times E$ which satisfies

$\tilde{f}(K) \subset M \times E \times \{0\} \times E$. (Use the fact that $\ell^2 \cong \ell^2 \times [0,1)$ [13]). Let h: $E \times [0,1) \times E \to C(E) \times E$ be the homeomorphism described in Lemma 3.1. Then $id_M \times h: M \times E \times [0,1) \times E \to M \times C(E) \times E$ is a homeomorphism and $(id_M \times h)\circ\tilde{f}(K) \subset M \times \{v\} \times E$, where v is the vertex of C(E).

In the proof of Lemma 2 of [12] there is a proof that $E \times \ell^2 \cong C(E \times S_1)$, where $S_1 = \{x\in\ell^2 \mid \|x\| = 1\}$. Since $S_1 \cong \ell^2$ [13] we have $E \cong C(E)$. Thus we can modify $(id_M \times h)\circ\tilde{f}$ to get a homeomorphism g: $M \to M \times E$ which satisfies $g(K) \subset M \times \{0\}$. □

4. Deforming a manifold to an ℓ^2- deficient subset. The main result of this section is Theorem 4.2, which shows how to deform certain manifolds onto subsets which have ℓ^2-deficiency. The following lemma is needed for its proof.

Lemma 4.1. *Let X be a space and let K be a closed subset of $X \times I^\infty$ such that $K \subset X \times s$. Then there exists a homeomorphism $f: X \times I^\infty \to X \times I^\infty$ which satisfies $f(K) \subset X \times \Pi_{i=1}^{\infty} [-\frac{1}{2},\frac{1}{2}]$.*

Proof. For each integer $i > 0$ and each $x\in X$ let

$$f_i(x) = \begin{cases} \text{glb } \{t_i \mid (x,t)\in K\}, & \text{if } K\cap(\{x\} \times I^\infty) \neq \phi \\ 1, & \text{if } K\cap(\{x\} \times I^\infty) = \phi \end{cases}$$

where we adopt the convention that if $t\in I^\infty$, then t_i is the i^{th} coordinate of t. It follows routinely that each $f_i: X \to (-1,1]$ is lower semi-continuous. Thus by Dowker's theorem ([10], page 170) there is a continuous function $g_i: X \to (-1,1)$ which satisfies $-1 < g_i(x) < f_i(x)$, for all $x\in X$. Similarly there is a continuous function $g_i^1: X \to (-1,1)$ which satisfies $-1 < g_i(x) < g_i^1(x) < 1$ and $\tau_i \circ \pi_{I^\infty}(K\cap(\{x\} \times I^\infty)) \subset (g_i(x), g_i^1(x))$, for all $x\in X$, where $\tau_i: I^\infty \to I_i$ and $\pi_{I^\infty}: X \times I^\infty \to I^\infty$ are projections.

For each pair a,b, of real numbers satisfying $-1<a<b<1$, there exists a unique piecewise linear homeomorphism $h_{a,b} : [-1,1] \to [-1,1]$ which satisfies $h_{a,b}(a) = -\frac{1}{2}$, $h_{a,b}(b) = \frac{1}{2}$, and $h_{a,b}$ is linear an each of the intervals $[-1,a]$, $[a,b]$, and $[b,1]$.

Then define $f: X \times I^\infty \to X \times I^\infty$ by $f(x,(t_i)) = (x,(h_{g_i(x),g_i^1(x)}(t_i)))$, for all $(x,(t_i)) \in X \times I^\infty$. Clearly f fulfills our requirements. □

Theorem 4.1. _Let_ $E \cong E^\omega$ _be a_ MTVS, M _be an E-manifold_, $K \subset M$ _be_ ℓ^2-_deficient, and let_ $\mathcal{U}$ _be an open cover of_ M. _Then there exists a homotopy_ $H: M \times I \to M$ _such that_ $H_0 = id$, $H_t|K = id$, _for all_ $t \in I$, $H_1 : M \to M$ _is an embedding such that_ $H_1(M)$ _is_ ℓ^2-_deficient, and_ H _is limited by_ $\mathcal{U}$.

Proof. Since $I^\infty \times s \cong s$ we can use an argument like that used in Lemma 6 of [7] to prove that a closed set $F \subset M$ has ℓ^2-deficiency iff there exists a homeomorphism of M onto $M \times I^\infty$ taking F into $M \times \{0\}$. Thus let $f: M \to M \times I^\infty$ be a homeomorphism such that $f(K) \subset M \times \{0\}$.

Using techniques like those used in Lemma 4.1 we can clearly construct a homotopy $G : M \times I^\infty \times I \to M \times I^\infty$ such that $G_0 = id$, $G_1 : M \times I^\infty \to M \times I^\infty$ is a closed embedding such that $G_1(M \times I^\infty) \subset M \times s$, $G_t|M \times \{0\} = id$, for all t, and G is limited by $f(\mathcal{U})$, the cover of $M \times I^\infty$ induced by f and $\mathcal{U}$. Then define $H : M \times I \to M$ by $H_t(x) = f^{-1} \circ G_t \circ f(x)$. All we have to do is show that $H_1(M)$ has ℓ^2-deficiency.

Note that $f \circ H_1(M)$ is a closed subset of $M \times I^\infty$ which satisfies $f \circ H_1(M) \subset M \times s$. Using Lemma 4.1 there exists a homeomorphism $g: M \times I^\infty \to M \times I^\infty$ such that $g \circ f \circ H_1(M) \subset M \times \Pi_{i=1}^\infty [-\frac{1}{2},\frac{1}{2}]$. From [2] it follows that there exists a homeomorphism $h: I^\infty \to I^\infty \times I^\infty$ such that $h(\Pi_{i=1}^\infty [-\frac{1}{2},\frac{1}{2}]) \subset I^\infty \times \{0\}$. Let $\tilde{h}: M \times I^\infty \to M \times I^\infty \times I^\infty$ be defined by $\tilde{h}(x,y) = (x,h(y))$. Then $\tilde{h} \circ g \circ f \circ H_1(M) \subset M \times I^\infty \times \{0\}$. By our comments above this proves that $H_1(M)$ has ℓ^2-deficiency. □

5. A proof of Theorem 2. The main result of this section is Theorem 2, where we generalize the homeomorphism extension theorem of [6]. The proof we give follows in broad outline the proof given in [6], but there are a few technicalities which have to be overcome in order to make the proof work. We will need the following mapping replacement theorem which resembles Theorem 3.1 of [6].

Lemma 5.1. Let $E \cong E^{\omega}$ be a LCMTVS, M be an E-manifold, X be a space which can be embedded as a closed subset of E, $A \subset X$ be closed, and let $f: X \to M$ be a continuous function such that $f|A$ is a homeomorphism of A onto an E-deficient subset of M. If $\mathfrak{U}$ is any open cover of M, then there exists an embedding $g: X \to M$ such that g(X) is E-deficient, $g|A = f|A$, and g is $\mathfrak{U}$-close to f.

Proof. Using Theorem 4.1 and the fact that E is an AR [16], a proof can be given which is similar to Theorem 3.1 of [6]. □

We will also need the following generalization of Theorem 2 of [7].

Lemma 5.2. Let $E \cong E^{\omega}$ be a MTVS, M be a connected E-manifold, and let $K \subset M$ be an E-deficient set. Then M can be embedded as an open subset of E so that K is taken onto an E-deficient (and therefore closed) subset of E.

Proof. The proof proceeds routinely as in Theorem 2 of [7] provided we note that (1) M can be embedded as an open subset of E, and (2) there exists a homotopy $H: E \times I \to E$ such that $H_0 = id_E$, H_t is a homeomorphism (onto), for $0 \leq t < 1$, and $H_1: E \to E \setminus \{0\}$ is a homeomorphism. The first assertion is just Theorem 4 of [12] and the second assertion follows since a corresponding property is true for ℓ^2 [4] and also since $E \cong E \times \ell^2$ (as was noted in Theorem 3.1). □

Theorem 2. Let $E \cong E^{\omega}$ be a LCMTVS, M be an E-manifold, and let K_1, K_2 be E-deficient subsets of M for which there exists a homotopy $H: K_1 \times I \to M$ such that $H_0 = id_{K_1}$ and $H_1: K_1 \to K_2$ is a homeomorphism. If $\mathfrak{U}$ is an open cover of M such that H is limited by $\mathfrak{U}$, then there exists an ambient

<u>invertible isotopy</u> $G : M \times I \to M$ <u>which satisfies</u> $G_o = id_M$, $G_1|K_1 = H_1$, <u>and</u> G <u>is limited by</u> $St^3(\mathcal{U})$ (<u>the</u> 3^{rd} <u>star of the cover</u> $\mathcal{U}$).
(An isotopy $G : X \times I \to X$ is said to be an <u>ambient invertible isotopy</u> provided that each level is an onto homeomorphism and $G^* : X \times I \to X$, defined by $G_t^*(x) = G_t^{-1}(x)$, is continuous).
Proof. First note that a homeomorphism extension theorem for E (without the limitation by covers) is easy to establish for E-deficient subsets of E. One merely uses the technique of Klee [14], as used in [2]. Thus in the case that $K_1 \cap K_2 = \phi$ we can use Lemma 5.1 and the techniques of [6] to obtain our desired ambient invertible isotopy.

On the other hand assume that $K_1 \cap K_2 \neq \phi$. It follows routinely that K_1 and K_2 are Z-sets, and therefore $K_1 \cup K_2$ is a Z-set. Using an unpublished result of David W. Henderson there exists, for each open cover $\mathcal{U}'$ of M, a homotopy $F : M \times I \to M$ such that $F_o = id_M$, $Cl(F_1(M)) \cap (K_1 \cup K_2) = \phi$ (where Cl denotes closure), and F is limited by $\mathcal{U}'$. Thus by Lemma 5.1 and an appropriate choice of $\mathcal{U}'$, there exists an embedding $F^* : K_2 \times I \to M$ such that $F_o^* = id_{K_2}$, $F^*(K_2 \times I)$ is an E-deficient set in M for which $F_1^*(K_2) \cap (K_1 \cup K_2) = \phi$, and F^* is limited by $\mathcal{U}$.

Using the above remarks there exists an ambient invertible isotopy $G^* : M \times I \to M$ such that $G_o^* = id_M$, $G_1^*|K_2 = F_1^*$, and G^* is limited by $\mathcal{U}$. Note that K_1 and $F_1^*(K_2)$ are disjoint E-deficient subsets of M and $F_1^* \circ H_1 : K_1 \to F_1^*(K_2)$ is a homeomorphism which is homotopic to id_{K_1}, with a homotopy that is limited by $St(\mathcal{U})$. We can once more use the above techniques to find an ambient invertible isotopy $H^* : M \times I \to M$ such that $H_o^* = id_M$, $H_1^*|K_1 = F_1^* \circ H_1$, and H^* is limited by $St^2(\mathcal{U})$. Then the obvious composition $G = (G^*)^{-1} H^*$ fulfills our requirements. □

6. Proof of Theorem 1. The step from E-deficiency to Property Z is straightfoward and resembles Theorem 9.1 of [3]. For the other implication let $K \subset M$ have Property Z and let $h : M \to M \times I^\infty$ be a homeomorphism. Using the representation for I^∞ and s given in Section 2 let $B(I^\infty) = I^\infty \setminus s$. It is shown in [3] that there is a homeomorphism of I^∞ onto itself which sends $B(I^\infty)$ into s. We can obviously write $B(I^\infty) = \bigcup_{n=1}^{\infty} C_n$, where each C_n is compact. Thus using the above comment and the techniques of Theorem 4.1 it follows that each $M \times C_n$ is E-deficient in $M \times I^\infty$.

We will describe a sequence $\{g_i\}_{i=1}^{\infty}$ of homeomorphisms of $M \times I^\infty$ onto itself whose left product $g = L\Pi_{i=1}^{\infty} g_i$ gives a homeomorphism of $M \times I^\infty$ onto itself which satisfies $g \circ h(K) \subset M \times s$. Then we can apply the techniques of Theorem 4.1 to conclude that $g \circ h(K)$ is E-deficient.

Since h(K) is a Z-set in $M \times I^\infty$ we can use the technique of the proof of Theorem 2 to get a homeomorphism $g_1 : M \times I^\infty \to M \times I^\infty$ such that $g_1 \circ h(K) \cap (M \times C_1) = \phi$. Now invoking the Convergence Procedure of Section 2 we need to produce a homeomorphism $g_2 : M \times I^\infty \to M \times I^\infty$ which is $\mathcal{U}$-close to $id_{M \times I^\infty}$, for any prechosen open cover $\mathcal{U}$ of $M \times I^\infty$, and $g_2 \circ g_1 \circ h(K) \cap (M \times (C_1 \cup C_2)) = \phi$. Once more using the fact that $M \times (C_1 \cup C_2)$ is E-deficient and $g_1 \circ h(K)$ is a Z-set, we can use the techniques of Theorem 2 to obtain the desired g_2.

Thus using an inductive procedure we can choose homeomorphisms $g_i : M \times I^\infty \to M \times I^\infty$ so that $g_i \circ \dots \circ g_1\ h(K) \cap (M \times (\bigcup_{n=1}^{i} C_n)) = \phi$ and $g = L\Pi_{i=1}^{\infty} g_i$ gives a homeomorphism of $M \times I^\infty$ onto itself. Since we are able to select each g_i arbitrarily close to $id_{M \times I^\infty}$ we can choose $\{g_i\}_{i=1}^{\infty}$ so that $g \circ h(K) \cap (M \times B(I^\infty)) = \phi$, thus $g \circ h(K) \subset M \times s$ and we are done. □

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