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CAN EVERY TYCHONOFF G-SPACE EQUIVARIANTLY BE EMBEDDED IN A COMPACT HAUSDORFF G-SPACE?

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CAN EVERY TYCHONOFF G-SPACE EQUIVARIANTLY BE EMBEDDED IN A COMPACT HAUSDORFF G-SPACE?

by

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ABSTRACT

In this note we present a brief account on some aspects of the so-called compactification-problem, i.e. the problem which is described by the title. The main results imply the fact that a counterexample is probably complicated. Indeed, for an important class of G-spaces, including all locally compact Hausdorff and all homogeneous G-spaces, the answer is in the affirmative.

KEYWORDS & PHRASES: *G-space, equivariant embedding, compactification.*

1. INTRODUCTION

Recall that *a G-space* is a triple $\langle G,X,\pi\rangle$, where G is a topological group, X is a topological space, and π is *an action of* G *on* X. This means that π: $G \times X \to X$ is a continuous mapping satisfying the conditions

(i) $\forall x \in X : \pi(e,x) = x$ (here e denotes the unit of G);

(ii) $\forall (s,t,x) \in G \times G \times X : \pi(t,\pi(s,x)) = \pi(ts,x)$.

Obviously, if π is an action of G on X, then we can define continuous mappings π_x: $G \to X$ and π^t: $X \to X$ by $\pi_x(t) := \pi(t,x) =: \pi^t(x)$ for $(t,x) \in G \times X$. In fact, π^t is a homeomorphic mapping of X onto itself, and $t \mapsto \pi^t$ is a morphism of groups from G into the full homeomorphism group of X. For $x \in X$, the set $\pi_x[G]$ will be called the *orbit* of x in X under G.

If $\langle G,X,\pi\rangle$ and $\langle G,Y,\sigma\rangle$ are G-spaces, then a mapping f: $X \to Y$ is said to be *equivariant* whenever $f \circ \pi^t = \sigma^t \circ f$ for every $t \in G$. A *morphism of G-spaces* is a continuous equivariant mapping; notation: $\langle 1_G, f\rangle: \langle G,X,\pi\rangle \to \langle G,Y,\sigma\rangle$ (we use this clumsy notation merely to distinguish between f as a continuous function - i.e. f as a morphism in the category TOP - and f as an equivariant mapping).

In this way we obtain a category TOP^G, having all G-spaces as its objects and all continuous equivariant mappings as its morphisms.

Let (E) be a property which applies to topological spaces so that it is meaningful to speak of an E space (e.g. Hausdorff, Tychonoff, compact, etc.). Then an E G-space is a G-space $\langle G,X,\pi\rangle$ such that X is an E space. The full subcategory of TOP^G defined by all compact Hausdorff G-spaces, is denoted by COMP^G.

It is well-known (and easy to prove) that COMP^G is a reflective subcategory of TOP^G. (For a systematic treatment of TOP^G from a categorical point of view we refer to the author's thesis, which will appear in [4]). Let $\langle G,X,\pi\rangle$ be any G-space, and let $\langle 1_G, f\rangle: \langle G,X,\pi\rangle \to \langle G,Y,\sigma\rangle$ denote its reflection into COMP^G. If f: $X \to Y$ is a topological embedding, then X is clearly a Tychonoff space. Now the question is whether the converse is true, i.e. is f an equivariant embedding of X in Y if X is a Tychonoff space? We can reformulate the problem by means of the following proposition.

<u>PROPOSITION</u>. *Let* $\langle G,X,\pi\rangle$ *be any* G*-space and let* $\langle 1_G,f\rangle\colon \langle G,X,\pi\rangle \to \langle G,Y,\sigma\rangle$ *be its reflection in* COMP^G. *Then the following statements are equivalent:*

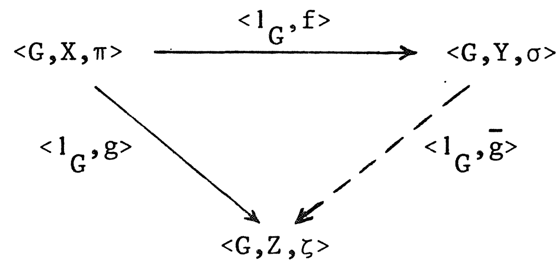
(i) f *is a topological embedding of* X *in* Y.

(ii) *There exist a compact Hausdorff* G*-space* $\langle G,Z,\zeta\rangle$ *and an equivariant embedding* $g\colon X \to Z$ (*i.e. a morphism* $\langle 1_G,g\rangle$ *of* G*-spaces with* g *a topological embedding*).

<u>PROOF</u>.

(i) $\Rightarrow$ (ii): Obvious.

(ii) $\Rightarrow$ (i): By the properties of a reflection there exists a unique morphism of G-spaces $\langle 1_G,\bar{g}\rangle\colon \langle G,Y,\sigma\rangle \to \langle G,Z,\zeta\rangle$ such that the diagram



commutes, i.e. $g = g\circ f$. If g is a topological embedding, then f is a topological embedding as well. □

Thus, our problem is equivalent to the following *compactification problem:* can every Tychonoff G-space equivariantly be embedded in a compact Hausdorff G-space?

Related to this compactification problem is the following *extension problem:* given a Tychonoff G-space $\langle G,X,\pi\rangle$ and a compactification $g\colon X \to Z$ of X (i.e. a dense embedding of X in a compact Hausdorff space Z), which conditions on $\langle G,X,\pi\rangle$, g or Z imply the existence of an action ζ of G on Z making g equivariant?

The compactification problem has been considered earlier in [1]. An improvement on a result of that paper is obtained in the theorem in §3 below. The extension problem for the case G = $\mathbb{R}$ is investigated in [2]. There it has been proved that in general an action of the additive group $\mathbb{R}$ on a normal Hausdorff space X cannot be extended to an action of $\mathbb{R}$ on the Stone-Čech compactification βX of X. On the other hand, if X has a compactification Z so that the remainder $Z \sim X$ is 0-dimensional (cf.[4], theorem VI.30), then every action of $\mathbb{R}$ on X can be extended to an action of $\mathbb{R}$ on Z.

2. COUNTEREXAMPLES ARE NECESSARILY COMPLICATED

We begin with the obvious observation that for a *discrete* group G there is no problem at all. Indeed, if $\langle G,X,\pi\rangle$ is a G-space with G discrete, then continuity of π reduces to the requirement that each $\pi^t\colon X \to X$ is continuous. If X is a Tychonoff space, then each π^t can be extended to a continuous mapping $\bar{\pi}^t\colon \beta X \to \beta X$, and it is easily verified that $\bar{\pi}^e = id_{\beta X}$ and that $\bar{\pi}^s \circ \bar{\pi}^t = \bar{\pi}^{st}$ for all $s,t \in G$. In this way one obtains a continuous action $\bar{\pi}$ of the discrete group G which extends the action π of G on X. If G is non-discrete (e.g. G = $\mathbb{R}$) then one can still define $\bar{\pi}\colon G \times \beta X \to \beta X$, but then $\bar{\pi}$ is not continuous in general (cf. the above mentioned result from [2]).

In order to find a counterexample against an affirmative answer to the compactification problem, one would be inclined to consider spaces with very few compactifications. Now there exist spaces which have only one compactification. But such spaces are necessarily locally compact, and then the following shows that such spaces do not provide us with a counterexample.

PROPOSITION. *Let* $\langle G,X,\pi\rangle$ *be a* G- *space with* X *a locally compact Hausdorff space. Then the action* π *of* G *on* X *can (continuously) be extended to an action* σ *of* G *on the one-point compactification* Z *of* X.

PROOF. Set $Z = X \cup \{\infty\}$ and define $\sigma\colon G \times Z \to Z$ by the rules $\sigma|_{G\times X} := \pi$ and $\sigma(t,\infty) := \infty$ $(t\in G)$. The only thing that needs a proof is continuity of σ on $G \times Z$ at the points (t,∞) $(t\in G)$. To do so, it is sufficient to prove the following statement: for every $t \in G$ and for every compact subset K of X there exists a neighbourhood V of t in G such that $cl_X\ \pi[V \times K]$ is compact.

The proof of this statement is a straightforward compactness argument. Indeed, for every $x \in K$, let U_x be a compact neighbourhood of $\pi(t,x)$ in X. By continuity of π there exist neighbourhoods V_x and W_x of t (in G) and of x (in X) respectively, such that $\pi[V_x \times W_x] \subseteq U_x$. Cover K by finitely many of the W_x's, and let V be the intersection of the corresponding V_x's. Then V is the desired neighbourhood of t. □

REMARK. If G is locally compact, take for V in the preceding proof just a compact neighbourhood of t: then $\pi[V \times K]$ is compact. For locally compact groups this result is well-known, but for non-locally compact groups G I could not trace back the preceding proposition in the literature. (Of course, the compactness argument is standard!).

3. A POSITIVE RESULT

There is a close connection between compactifications of a Tychonoff space X and uniform structures on X. So it is not surprising that the property of a Tychonoff G-space $\langle G,X,\pi\rangle$ of being embeddable in a compact G-space can be expressed in terms of uniformities, as follows:

THEOREM. *Let* $\langle G,X,\pi\rangle$ *be a Tychonoff* G-*space.*
The following statements are equivalent:

(i) $\langle G,X,\pi\rangle$ *can equivariantly be embedded in a compact Hausdorff* G-*space.*

(ii) *There exists a uniformity* $\mathcal{U}$ *on* X, *generating the topology of* X, *for which the family* $\{\pi_x^{\bullet} : x \in X\}$ *is equicontinuous at the point* e *of* G.

PROOF.

(i) ⇒ (ii): Suppose $\langle G,Z,\zeta\rangle$ is a compact Hausdorff G-space and that $X \subseteq Z$ such that $\pi = \zeta|_{G\times X}: G \times X \to X$. Then Z has a unique uniformity $\mathcal{V}$ which is compatible with its topology. A straightforward compactness argument shows that $\{\zeta_z^{\bullet} : z \in Z\}$ is an equicontinuous set of functions from G to Z (i.e. equicontinuous with respect to the uniformity $\mathcal{V}$ on Z). Now it suffices to take for $\mathcal{U}$ the uniformity which is induced on X by $\mathcal{V}$.

(ii) ⇒ (i): First we establish some notation. For any space Y, let $C_c(G,Y)$ denote the space of all continuous functions on G with values in Y, endowed with the compact-open topology. If Y is a uniform space with uniformity $\mathcal{W}$, then this topology on $C_c(G,Y)$ is induced by a uniformity having as a base all sets of the form

$$M(K,\alpha) := \{(f,g) \in C_c(G,Y) \times C_c(G,Y) : (f(t), g(t)) \in \alpha \text{ for all } t \in K\},$$

with $\alpha \in \mathcal{W}$ and with K a compact subset of G. In addition, let $\tilde{\rho}: G \times C_c(G,Y) \to C_c(G,Y)$ be defined by $(\tilde{\rho}^t f)(s) := f(st)$ for $s,t \in G$ and $f \in C_c(G,Y)$. Notice that $\tilde{\rho}^t: C_c(G,Y) \to C_c(G,Y)$ is easily seen to be continuous for every $t \in G$; it is even an autohomeomorphism of $C_c(G,Y)$, due to the fact that $\tilde{\rho}^{st} = \tilde{\rho}^s \circ \tilde{\rho}^t$ for each $s,t \in G$ and $\tilde{\rho}^e$ = identity mapping.

Now suppose condition (ii) is fulfilled. Then the mapping

$$\underline{\pi}: x \mapsto \pi_x: X \to C_c(G,X)$$

sends X into the subset $\underline{\pi}[X]$ of $C_c(G,X)$ which is equicontinuous at e. Then $\underline{\pi}[X]$ is easily seen to be equicontinuous at each point of G, using the fact that $\pi_x(s) = \pi_y(st^{-1})$ with $y = \pi(t,x)$. In addition, observe that $\underline{\pi}$ is a topological embedding of X in $C_c(G,X)$, and that

$$\tilde{\rho}^t \circ \underline{\pi} = \underline{\pi} \circ \pi^t$$

for every $t \in G$ (so if $\tilde{\rho}$ were an action of G on $C_c(G,X)$, $\underline{\pi}$ would be an equivariant embedding of the G-space $\langle G,X,\pi \rangle$ into the G-space $\langle G,C_c(G,X),\tilde{\rho} \rangle$).

Next, recall that the Tychonoff space X can be embedded in some large "cube" $[0,1]^I$, due to the fact that the continuous functions on X with values in the closed interval [0,1] separate points and closed subsets of X. However, we need a little bit more, namely the fact that the functions on X with values in [0,1] and which are *uniformly continuous* with respect to the prescribed uniformity $\mathcal{U}$ in X separate points and closed subsets of X; cf.[4], theorem I.13. It follows that X can be embedded in some cube $[0,1]^J$ by means of a uniformly continuous mapping. This induces a topological embedding F of $C_c(G,X)$ into $C_c(G,[0,1]^J)$ which maps equicontinuous subsets of $C_c(G,X)$ onto equicontinuous subsets of $C_c(G,[0,1]^J)$. Moreover, $\tilde{\rho}^t \circ F = F \circ \tilde{\rho}^t$ for every $t \in G$ (here the left hand $\tilde{\rho}^t$ is an autohomeomorphism of $C_c(G,[0,1]^J)$, and the right hand $\tilde{\rho}^t$ is an autohomeomorphism of $C_c(G,X)$). Resuming, we find that $F \circ \underline{\pi}$ is a topological embedding of X into $C_c(G,[0,1]^J)$, such that $\tilde{\rho}^t \circ (F \circ \underline{\pi}) = (F \circ \underline{\pi}) \circ \pi^t$, and $F \circ \underline{\pi}[X]$ is an equicontinuous subset of $C_c(G,[0,1]^J)$.

Let Z denote the closure of $F \circ \underline{\pi}[X]$ in $C_c(G,[0,1]^J)$. By ASCOLI's theorem, Z is a compact Hausdorff space. In addition, the continuity of each $\tilde{\rho}^t$ and the fact that $F \circ \underline{\pi}[X]$ is mapped into itself by each $\tilde{\rho}^t$ imply that $\tilde{\rho}^t[Z] \subseteq Z$ for each $t \in G$. Since $\tilde{\rho}^e$ is the identity mapping, and $\tilde{\rho}^{st} = \tilde{\rho}^s \circ \tilde{\rho}^t$ $(s,t \in G)$, it follows that $\langle G,Z,\tilde{\rho}|_{G\times Z}\rangle$ is a compact Hausdorff G-space, *provided* $\tilde{\rho}|_{G\times Z}: G \times Z \to Z$ *is continuous*. We shall show now that this is so; then, indeed, $\langle G,X,\pi\rangle$ is equivariantly embedded in the compact Hausdorff G-space $\langle G,Z,\tilde{\rho}|_{G\times Z}\rangle$ by $F \circ \underline{\pi}$.

In order to prove that $\tilde{\rho}|_{G\times Z}: G \times Z \to Z$ is continuous, recall that $F \circ \underline{\pi}[X]$, and consequently, Z, are equicontinuous subsets of $C_c(G,[0,1]^J)$. Consider a typical element $M(K,\alpha)$ of the uniform base of $C_c(G,[0,1]^J)$ with K a compact subset of G and α an element of the uniformity of $[0,1]^J$. Now Z is equi-uniformly continuous on the compact set K, i.e. there exists a neighborhood V of e in G such that $(f(s),f(t)) \in \alpha$ for all $s \in K$ and all $t \in G$ such that $t^{-1}s \in V$, and for all $f \in Z$. This means exactly that $(f,\tilde{\rho}^u f) \in M(K,\alpha)$ for all $f \in Z$ and $u \in V^{-1}$. Since $\tilde{\rho}^s g \in Z$ for every $s \in G$ and $g \in Z$, it follows that $(\tilde{\rho}^s g,\tilde{\rho}^{us} g) \in M(K,\alpha)$ for all $s \in G$, $g \in Z$, and $u \in V^{-1}$. Using this, continuity of $\tilde{\rho}|_{G\times Z}$ on $G \times Z$ is easily proved.

Indeed, if we fix $(s,f) \in G \times Z$, then there is a neighbourhood U of f in Z such that $(\tilde{\rho}^s f,\tilde{\rho}^s g) \in M(K,\alpha)$ for all $g \in U$, because $\tilde{\rho}^s|_Z$ is a homeomorphism of Z. Combining the results, it follows that $(\tilde{\rho}^s f,\tilde{\rho}^{us} g) \in M(K,\alpha^2)$ for all $g \in U$ and $u \in V^{-1}$. This implies continuity of $\tilde{\rho}|_{G\times Z}$ at the point (s,f). □

REMARKS.

(i) The implication (ii) ⇒ (i) occurs in [1], with the additional condition in (ii) that each $\pi^t: X \to X$ be a unimorphism with respect to $\mathcal{U}$. Then for Z the Samuel compactification of X with respect to $\mathcal{U}$ is taken.

(ii) In the above proof the space $[0,1]^J$ can be replaced by the closure of the image of X in $[0,1]^J$. This closure is just the Samuel-compactification of X with respect to $\mathcal{U}$. Notice that it is not clear at all whether Z is the Samuel compactification of X with respect to $\mathcal{U}$.

(iii) If G is locally compact, then $\tilde{\rho}$: $G \times C_c(G,Y) \to C_c(G,Y)$ is continuous for every topological space Y. In this case, the last part of the above proof can be omitted.

We present now an application of the preceding proposition, stating that every homogeneous G-space has a compactification. Here a homogeneous G-space is, by definition, the quotient space G/H of all *left* cosets of a subgroup H of G, with action π of G on G/H defined by

$$\pi^t(sH) := ts\,H \qquad (t,s\in G)$$

(due to the fact that the quotient mapping q: $G \to G/H$ is an open mapping - cf. [3], 5.17 - the mapping π: $G \times (G/H) \to G/H$ turns out to be continuous).

In order to prove our corollary, it is useful to recall that a uniformity on a space can be described by a family of pseudo-metrics. In particular, the right uniformity of a topological group G can be described by a family Σ of right-invariant pseudo-metrics (apply [3],8.2 with "left" replaced by "right"; see also [3], 8.14(a)). Moreover, according to [3], 8.14(a), for any subgroup H of G one can define

$$\sigma^*(q(s),q(t)) := \inf\{\sigma(u,v) \,\vdots\, u \in sH \;\&\; v \in tH\}$$

for every $\sigma \in \Sigma$ and $s,t \in G$. Then $\{\sigma^* \,\vdots\, \sigma \in \Sigma\} =: \Sigma^*$ is a family of continuous pseudo-metrics on G/H just generating its topology. So G/H has a uniformity $\mathcal{U}$ which is compatible with its topology. Notice that the quotient mapping q: $G \to G/H$ is uniformly continuous with respect to this uniformity on G/H and the uniformity on G corresponding to Σ (i.e. the right uniformity on G).

COROLLARY 1. *Let* H *be a closed subgroup of the topological group* G. *Then the homogeneous* G*-space* $\langle G,G/H,\pi\rangle$ *can equivariantly be embedded in a compact Hausdorff* G*-space.*

PROOF. Since H is a closed subgroup of G, the space G/H is a Hausdorff space.

Hence the uniformity $\mathcal{U}$ introduced above is separated (equivalently, G/H is a Tychonoff space). Set x := q(e). Observe that $\pi_x = q$, so according to one of our previous remarks, it follows that $\pi_x: G \to G/H$ is uniformly continuous with respect to the right uniformity on G and the uniformity $\mathcal{U}$ on G/H. Consequently, for every $\alpha \in \mathcal{U}$ there exists a neighbourhood U of e in G such that

$$(*) \qquad (\pi_x(s), \pi_x(ts)) \in \alpha \quad \text{for every } s \in G \text{ and } t \in U.$$

Write y := $\pi(s,x)$ and observe that $\{\pi(s,x) \,\vdots\, s \in G\} = G/H$. So (*) is equivalent to

$$(\pi_y(e), \pi_y(t)) \in \alpha \text{ for every } y \in G/H \text{ and } t \in U.$$

Thus, $\{\pi_y \,\vdots\, y \in G/H\}$ is equicontinuous at e, and we can apply our proposition. □

<u>COROLLARY 2</u>. *Every* G*-space of the form* <G,G×X,μ> *with* G *a topological Hausdorff group,* X *a Tychonoff space and* $\mu^t(s,x) := (ts,x)$ *can equivariantly be embedded in a compact Hausdorff* G*-space.*

<u>PROOF</u>. In view of the preceding corollary there exists a compact Hausdorff G-space <G,Z,ζ> in which the Tychonoff G-space <G,G,λ> can equivariantly be embedded; here $\lambda^t(s) = ts$ for t, s ∈ G. It is clear, that the G-space <G,G×X,μ> can equivariantly be embedded in the G-space <G,Z×βX,σ> where $\sigma^t(z,y) := (\zeta^t z, y)$ for every t ∈ G and (z,y) ∈ Z × βX.[)1] □

<u>REMARK</u>. In the literature, the reflection of the G-space <G,G,λ> in COMP^G – which is by the preceding corollary an equivariant embedding if G is a Hausdorff group – is often referred to as the universal (or greatest) G-ambit. See e.g. [1].

If G = ℝ, then the ℝ-spaces of the form <ℝ,ℝ×X,μ> are called *parallel flows*. So by our previous corollary, parallel flows on Tychonoff spaces can equivariantly be embedded in flows on compact Hausdorff spaces.

)[1] Here βX may be replaced by any other compact space in which X can be embedded.

In order to place our next corollary in its proper context, we state two well-known facts. As was observed in the proof of our main theorem, the mapping $\tilde{\rho}\colon G \times C_c(G,Y) \to C_c(G,Y)$ is, in general, not continuous. But if Y is a uniform space and $f \in C_c(G,Y)$, then the restricted mapping

$$\tilde{\rho}\colon G \times \tilde{\rho}_f[G] \to \tilde{\rho}_f[G]$$

is continuous if (and only if) f is right uniformly continuous. In addition, if f[G] is relatively compact in Y, then $\tilde{\rho}_f[G]$ is relatively compact in $C_c(G,Y)$, and $\tilde{\rho}\colon G \times \mathrm{cl}\ \tilde{\rho}_f[G] \to \mathrm{cl}\ \tilde{\rho}_f[G]$ is continuous (see [5], lemma 2.1.8). So in particular, if $f \in RUC^*(G)$, i.e. *if* f *is a right uniformly continuous, bounded, real valued function on* G, *then the* G-*space* $\langle G,\tilde{\rho}_f[G],\tilde{\rho}\rangle$[1] *can equivariantly be embedded in a compact Hausdorff* G-*space*, viz. the closure of $\tilde{\rho}_f[G]$ in $C_c(G)$ (this can also be obtained as a corollary of our main theorem, but then we do not know what that compact G-space looks like; in point of fact, the proof of our theorem is a generalization of the above remark).

Next, we consider a similar G-space $\langle G,\tilde{\rho}_f[G],\tilde{\rho}\rangle$, but now we endow $\tilde{\rho}_f[G]$ with the topology of *uniform convergence*. It is easily seen that this is, indeed, a G-space, provided f is left uniformly continuous.
The elements $f \in C(G)$ such that $\tilde{\rho}_f[G]$ has a compact closure in $C_u(G)$ are called *almost periodic functions*. They have been intensively studied in the past and they still form an interesting field of research, but what is of interest to us in the present context, is the fact that *for each almost periodic function* f *on* G *the* G-*space* $\langle G,\tilde{\rho}_f[G],\tilde{\rho}\rangle$[2] *can equivalently be embedded in a compact Hausdorff* G-*space*, viz. the closure of $\tilde{\rho}_f[G]$ in $C_u(G)$.

In the light of these two facts, the following is a little bit surprising, since on a non-totally bounded group not every uniformly continuous bounded function is almost periodic (see [6], remark 4.3)

)[1] Here $\tilde{\rho}_f[G]$ has the compact-open topology.

)[2] Here $\tilde{\rho}_f[G]$ has the topology of uniform convergence

<u>COROLLARY 3</u>. *If* f $\in$ C(G) *is left uniformly continuous, then the Tychonoff* G-*space* $\langle G, \tilde{\rho}_f[G], \tilde{\rho} \rangle$, *where* $\tilde{\rho}_f[G]$ *has the topology of uniform convergence, can equivariantly be embedded in a compact Hausdorff* G-*space if* f *is also right uniformly continuous.*

<u>PROOF</u>. Set $X = \tilde{\rho}_f[G]$. As in the proof of corollary 2, one shows that $\{\tilde{\rho}_x : x \in X\}$ is equicontinuous at e if for one particular choice of x the mapping $\tilde{\rho}_x : G \to X$ is right uniformly continuous. If X has the uniformity of uniform convergence, then right uniform continuity of $\tilde{\rho}_f : G \to X$ is easily seen to be equivalent to right uniform continuity of $f: G \to \mathbb{R}$. □

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