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ON PROBLEMS CONCERNING THE QUANTIFICATION
RULES IN MONTAGUE GRAMMAR

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On problems concerning the quantification rules in Montague grammar ^{*)}

by

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ABSTRACT

In Montague grammar variables such as he_1, he_2 ... give rise to two kinds of problems. They can occur in cases where they should not, and they can be lacking in cases where they are wanted. Several proposals for dealing with these problems are discussed and a principle is proposed which guarantees that variables occur just in case they are required. In order to give this principle the form of a restriction on grammars, some new syntactic tools are used, which are borrowed from van Wijngaarden grammars. These tools are also useful for the treatment of verb tenses.

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1. PROBLEMS WITH VARIABLES

MONTAGUE 1973, henceforth PTQ, contains in its vocabulary $he_1, he_2 \ldots$ as expressions of the category of terms. Such words which bear indices, I call syntactic variables (or shortly: variables). On the one hand these variables are not words of English; on the other hand they occur in intermediate stages of PTQ productions of English. Occurrences of these variables are removed by rules like S_3 (relative clause formation) and S_{14} (term substitution); I call them quantification rules. When using these rules two kinds of problems arise.

The first problem is that variables may arise in situations where they are not desired. According to the rules of PTQ, the expression

(1) He_7 runs

is a well-formed expression of the category of sentences. It translates into an expression containing a free variable:

(2) $\underline{run}\ (x_7).$

The word he_7, however, is not an English word; therefore (1) cannot be a correct English sentence. So the PTQ grammar is not adequate in this respect.

The second kind of problems concerns situations in which specific syntactic variables are wanted, but where there are no such occurrences. According to the rules of PTQ, one may apply $F_{14,1}$ to the termphrase *a unicorn* and the sentence *John loves Mary*. Syntactically there are no problems: one has to substitute *a unicorn* for the first occurrence of he_1, and this is quickly done since there are no such occurrences. Thus the sentence

(3)   *John loves Mary*

is produced with translation:

(4) $\lambda P[\exists u[\underline{unicorn}_*(u) \wedge P(^\wedge u)]](^\wedge \lambda x_1\ \underline{love}_*(j,m))$

Notice that this translation contains a vacuous λ-abstraction. Formula (4) reduces to (5):

(5) $\exists \underline{u}[\underline{unicorn}_*(\underline{u}) \wedge \underline{love}_*(\underline{j},\underline{m})]$

So (3) obtains a reading which implies the existence of unicorns. This is, however, not a possible reading of (3). A related problem arises for relative clauses. A relative clause has to contain a pronoun as is demonstrated by the incorrectness of

(6)   *John loves the man such that Mary walks.*

Consider now the following example (GROENENDIJK & STOKHOF 1976). According to PTQ, the sentence

(7)   *John loves the man such that he walks*

can be formed, using $S_{14,1}$, from the termphrase *John* and the sentence

(8)   *He$_1$ loves the man such that he$_1$ walks.*

The subphrase *man such that he$_1$ walks* is formed, using $S_{3,1}$, from *man* and *he$_1$ walks*. With this production, the translation of (7) says that the person who walks, is John. The only possible reading of (7) is, however, that the man who is loved by John is the one who is walking. This demonstrates that a relative clause has to contain a pronoun which refers to the head noun. In other words: if one uses $S_{3,n}$, then the sentence has to contain an occurrence of *he$_n$*.

2. PROPOSALS FROM THE LITERATURE

Although it is not the main theme of his paper, RODMAN 1976 makes some interesting remarks on the signalized problems concerning syntactic variables. His approach is of syntactic nature. Concerning sentences containing unbound syntactic variables he states that they are 'thrown out' because of 'a con-

vention that is implicit in Montague's work, viz, that an expression contain-
ing a free variable is ill-formed' (p. 167). Concerning wanted variables,
his opinion can be concluded from the following. His rule for relative clause
formation is, just as in PTQ, formulated as a rule working for all combina-
tions of nouns and sentences. When he considers a combination of a common
noun with a sentence which does not contain a relativizable pronoun, he writes
'derivation blocks' (p. 170). So there seems to be a general convention as
well for the other kinds of problems. The idea that the derivation should
stop in such a case, can also be found with other authors, but they present it
as explicit conditions on the input sentence (GROENENDIJK & STOKHOF 1976,
KARTTUNEN 1977).

This syntactic approach is attractive, since it has as consequence that,
after a small change of the grammar, the correct structures are produced.
In fact, certain aspects of this approach will figure in the solution pro-
posed in section 3. But in the form presented by Rodman, it is a dangerous
approach since it amounts to a filter; and a filter is a powerful tool which
can also be used for unintended applications. I will demonstrate that
Rodman's throw-out convention is such a dangerous tool. A related argumenta-
tion applies to the solutions to the other problem (blocking-convention as
well as explicit restrictions).

Consider the situation that one wishes to incorporate in the grammar
an obligatory rule which transforms sentences into sentences by performing
some action which we call swopping. Let us assume that we have a formulation
F_{sw} of the swopping action (it is irrelevant for our discussion to know what
swopping really is). The obligatory character of the rule can be obtained
by means of the throw-out filter and some changes in the rules. The rules
which produce a sentence from expressions which are not sentences themselves,
have to be changed. After having formed the sentence, two occurrences of he_7
are written in front of the sentence (below I will explain this number). So
S_4 becomes:

S_4:　　If $\alpha \in P_T$ and $\beta \in P_{IV}$ then $F_4'(\alpha, \beta) \in P_t$,

　　　　where $F_4'(\alpha, \beta) = he_7\ he_7\ F_4(\alpha, \beta)$

　　　　and where F_4 is as in PTQ.

The swopping rule becomes:

Swop: if $\alpha \in P_t$ then $F_{sw}(\alpha) \in P_{t'}$, where $F_{sw}(\alpha) = \underline{\alpha}$, and $\underline{\alpha}$ is obtained from α
   by first deleting a sequence of two consecuting occurrences of he_7
   and next performing the swopping action.

An expression of the category sentence to which the swopping rule is not
applied, will contain the sequence he_7 he_7 and thus be filtered out according
to the convention. A sentence which has undergone swopping does not contain
such a sequence and will pass the filter. So we used the sequence he_7 he_7
as a mark for the swopping rule. Conjoined sentences will have two such marks
and both sentences will be swopped. One has to be careful that the marks
are not disturbed by other rules. An application of $S_{14,7}$ which substitutes
John for the first he_7 would demolish the mark. Therefore $S_{14,n}$ has to be
changed into

$S_{14,n}$: If $\alpha \in P_T$ and $\phi \in P_{t'}$ then $F'_{10,n}(\alpha, \phi) \in P_{t'}$, where $F'_{10,n}$ is as $F_{10,n}$
   in PTQ with the difference that $F'_{10,n}$ does not change sequences of
   two consecutive he_n's.

Here one sees the use of having added in S_4 two, rather than one occurrence.
A variable used for the usual purposes cannot occur in a sequence of two
consecutive variables, so the ones used for rule ordening can now be recog-
nized. If we would have used just one, this would not have been the case.

  The above example concerns a single obligatory rule. Related methods
can be used to encode any rule ordering. I have objections against this use
of variables. It it not in the style of Montague grammar to have obligatory
rules or any other restriction on the ordering of rules. This is expressed by
Partee in her work concerning the wellformedness constraint (PARTEE 1979).
If one accepts this view, then one should not allow to have arbitrary rule
ordering encoded by the he_n's. And if one does not accept this view and one
wishes to have a prescribed rule-ordering, then one should do so in an ex-
plicit component for the rule ordering (as such is done in transformational
grammars). Variables are introduced for pronouns and scope treatment, not for
rule ordering. That is an abuse of variables which should be made impossible.

  COOPER 1975, explicitly discusses the problems concerning variables and he
presents approaches of a semantic nature. His ideas are especially interest-
ing since they form the fundament for the approach of BACH & COOPER 1978,
in which it is argued that it is possible to have in Montague grammar a

termphrase-sentence analysis of relative clause contructions instead of the
usual common noun-sentence analysis.

We have seen that constructions, in which variables are wanted but are
not there, give rise to vacuous λ-abstraction. Cooper proposes to assign no
truth value to such an expression containing vacuous abstraction. He obtains
this effect by a restriction on the usual definition of interpretation of
$\lambda u\alpha$. '... the function denoted by the abstraction expression $\lambda u\alpha$ is only
defined for entities within its domain if a different assignment to the
variable u will yield a different denotation for α' (p. 246). As he signal-
izes, this has as consequence 'that $\lambda u\alpha$ is undefined not only if α does
not contain a free occurrence of u, but also if α is a tautology. Thus, for
instance, according to this definition, $\lambda u[u=u]$ represents a function which
is undefined for any entity. However, the technique of supervaluation ...
will show these expressions to be defined but not those where α is not a
tautology'. A complicated solution.

One of Coopers proposals concerning free variables is to 'design the
semantics in such a way that any formula containing a free occurrence of a
variable is shown to be truth-valueless' (p. 257). As consequence of this
proposal

(9) $\underline{walk}$ (x)

gets for no point of reference a truth value; it represents the nowhere
defined intension. As a subexpression in the formula:

(10) $\exists x[\underline{man}(x) \wedge \underline{walk}(x)]$

it does have another meaning. So we lost the Fregean principle of composi-
tionality: the meaning of (10) is not built up from the meaning of (9).
A related objection arises if one observes that also

(11) $\underline{talk}(x)$

gets for no point of reference a truth value. So (9) and (11) are intension-
ally equivalent and according to a rule of intensional logic interchangeable

in all contexts. Now (10) constitutes a context in which this is not the
case. So acceptance of this proposal would turn the so basic substitution
law unvalid.

Another approach finds its origin in Universal Grammar (MONTAGUE 1970). It
is practiced by GROENENDIJK & STOKHOF 1976 and it is one of the proposals of
COOPER 1975. In syntax one has a rule which deletes indices, thus delivering
a correct expression. In the semantics, free variables are interpreted by
the current variable assignment. Notice that this leads us away from PTQ
since there a formula is defined to be true for a point of reference if
and only if it is true for every variable assignment (p. 250). So $\underline{run}(x)$
means the same as its universal closure: $\forall x[\underline{run}(x)]$. Therefore this defini-
tion has to be dropped.

There are more problems with this approach. Consider:

(12)   *He runs*   with the translation

(13) $\underline{run}(x)$.

For each variable assignment (13) obtains an interpretation. One of the
possible assignments is that x_7 refers to the utterer of the sentence under
consideration; so (12) is interpreted as *I run*. It may also be the case that
x_7 is the person spoken to, thus meaning *You run,* or it may handle about a
female person, meaning *She runs*. Beside these syntactic inadequacies, there
are also semantic ones; depending on the linguistic and situational context,
the pronoun *he* can refer only to certain individuals (e.g. the person men-
tioned in the last utterance or the person pointed at by the speaker).
None of the authors mentioned above does explicitly signalize these compli-
cations. They can be dealt with by means of a device from Universal Grammar
(MONTAGUE) 1970): considering not all variable assignment functions for eval-
uating a complete sentence, but only a subset thereof. This approach, however,
gives rise to a strange situation. In the light of the above arguments, just a
few assignments for occurring free variables are possible in any particular
situation. The free variables cannot vary and are more like constants. More-
over, the variable assignment function g is by its nature independent of
context or point of reference, whereas the function for interpreting constants

depends on information concerning the time and possible worlds. Why not then
translate such pronouns by constants as is done in BENNETT 1978?

3. THE VARIABLE PRINCIPLE

In the previous section, we have considered several proposals dealing
with the problems concerning variables. The syntactic approach aims at avoid-
ing the problematic cases, but, in doing so, unattractive tools are introduced.
The semantic approaches try to do something useful with the arising problem-
atic cases. These semantic proposals do not give me the impression that the
authors are quite satisfied by such cases, but that they rather try to find
an escape from a situation they would prefer not to be confrontated with. I
will put forward a proposal which combines syntactic and semantic aspects
and which has as consequence that variables arise just in case they are re-
quired.

In my opinion the existing proposals all ignore the special character of
variables. This special character is, in an informal way, expressed by *the
variable principle*:

Syntactic variables correspond closely with semantic variables.

Several arguments for this correspondence can be found in PTQ and related
works. The most immediate is that syntactic variables translate into logical
variables (The term variable he_n is translated as $\lambda PP\{x_n\}$, and the CN-vari-
able one_n, which is discussed in HAUSSER 1979, is translated as P_n). Another
correspondence is that if a syntactic rule removes all occurrences of a syn-
tactic variable, the corresponding translation rule binds the corresponding
logical variable ($S_{3,n}$ and $S_{14,n}$). And rules not binding logical variables
do not remove syntactic variables. The consequence of these correspondences
is that in PTQ an expression contains an indexed syntactic variable if and
only if its translation contains a free logical variable. I consider the
correspondences among syntactic and logical variables not as accidental,
but as fundamental, and propose to follow in Montague grammar the variable
principle. The above formulation of the principle is, of course, rather
informal; a more exact contence is given to it below.

Let us start by recalling that by a syntactic variable is understood a word which bears an index, and that by a syntactic quantification rule is understood a rule that has the effect that all occurrences of a syntactic variable are in some way removed. By the *variable principle* we understand the following requirements:

1a) *The translation of a syntactic variable contains a free occurrence of a logical variable.*

1b) *This is the only way to introduce a free occurrence of a logical variable.*

2a) *If a syntactic rule removes occurrences of a variable, the related translation rule binds the corresponding logical variable.*

2b) *If a translation rule introduces a binder for a free logical variable, the related syntactic rule removes the corresponding syntactic variables.*

3a) *In the complete analysis of a sentence, each syntactic variable has to be removed by a syntactic quantification rule.*

3b) *If a syntactic quantification rule is used, it has actually removed the occurrences of a syntactic variable.*

Clause 3 is formulated for syntactic rules. Since the clauses 1 and 2 force a correspondence of syntactic and semantic rules, clause 3 has an equivalent logical counterpart. This semantic version of clause 3 cannot be formulated as easy as the syntactic one, since it demands for a logical counterpart of the notion 'complete analysis of a sentence'. Therefore I prefer the syntactic formulation.

As the reader has learned from the previous discussion, the principle is intended for the usual syntactic and logical variables. It does not apply to the so-called context variables which are especially introduced for dealing with several aspects of contextual influence (examples are Γ of HAUSSER 1979 and v_c of GROENENDIJK & STOKHOF 1979). Such variables are not interpreted by the variable assignment function g and they might be called context constants as well.

Let us go back to the principle and see what it says about the several proposals we considered. As mentioned before, requirement 1b is obeyed by all existing proposals. This requirement is also proposed by Partee as a constraint on indexed notations (PARTEE 1979). Requirements 1b and 2b are not fulfilled by the proposals of COOPER 1975 and BACH & COOPER 1978.

My abuse-example concerning Rodman's 'throw-out' convention does not obey 1b
nor 2a. Clause 3 makes that the original problems concerning PTQ we discussed
before, do not arise. Restrictions with the same effect as clause 3 are used
in JANSSEN 1976. The conventions of Rodman express the same aims as 3a and 3b,
but now the abuse of variables is prevented by clause 1 and 2. So the vari-
ables arise just in case they are wanted.

The clauses 1 and 2 define restrictions on possible grammars. In order
to make sure that these restrictions are decidable, I should define a format
for rules from which it is possible to check whether these restrictions are
satisfied. Since in all concrete proposals it is easily checked whether these
clauses are satisfied, I will not define such a format. It seems that clause
3 has to be taken as a global constraint on possible derivations. If one has
no objections against such a constraint, then the variable-principle has thus
solved the problems concerning variables. I do have objections against global
constraints. They are even stronger than filters since filters impose con-
straints on the final outcome of a production process, whereas global con-
straints take the whole process into account. As we have seen before, filters
(and thus global constraints) are dangerous since they are so powerful.
Global constraints allow for making the information contained in other parts
of the grammar worthless. In Montague grammar, the syntactic rules provide
for the information which kinds of expressions may be combined to form new
expressions of a certain kind, but global constraints would make it possible
that the rules combine rubbish to rubbish, leaving it to the constraint to
say which combinations make sense. Because of such arguments I would not
like to incorporate global constraints in general. But accepting clause 3 as
the only constraint would be rather ad hoc. Therefore I will describe in the
next section tools which make it possible to consider clause 3 as a restric-
tion on the possible rules in a Montague grammar instead of considering it
as a global one. After having done so, the variable principle can be taken
as a decidable restriction on possible Montague grammars.

4. HYPERRULES AND VARIABLES

In this section I describe a grammar which is in some aspects a gener-
alization of the PTQ syntax. I start with a description of the framework,

some simple examples follow thereafter.

The categories may be compound symbols and it is allowed that
there are an infinite number of them. We can handle an infinite number by
means of a finite number of schemes of rules. These schemes are called hyper-
rules and they constitute the hypergrammar. Such hyperrules are alike PTQ
rules, but they contain variables. By an appropriate substitution for these
variables a hyperrule is transformed into a rule which may actually be used
in a production. The information what is to be substituted for a certain
variable is described by grammatical tools as well. Besides the hyperrules,
we have another set of rules, called metarules. These metarules constitute
a grammar (say context free) in which the variables from the hyperrules
occur as auxiliary symbols. The set of strings of terminal symbols of the
metagrammar which can be produced from a variable, constitutes the set of
possible substitutions for this variable. For each metavariable M we have the
variants $M_1,...M_9$. For these variants we have the same metarules as for M.
If a hyperrule contains occurrences of different variants of a metavariable,
these may be replaced by different terminal productions. The same variants
are of course to be replaced by the same terminal productions. For conve-
nience, we will present the metarules only for one variant (say M) and not
for the others $(M_1,...M_9)$. So our syntactic framework consists of two levels:
the metagrammar and the hypergrammar. This conception of a grammar (and the
related terminology) is due to van Wijngaarden who developped this kind of
grammar for the formal description of the programming language Algol 68
(VAN WIJNGAARDEN 1975). In certain details our framework deviates from the
standard form of a van Wijngaarden grammar (see appendix). A linguistically
oriented example of such a grammar can be found in VAN WIJNGAARDEN 1970.
The main use of the organization of the syntax in two levels is that it
thus provides for a technical handsome, readable, and understandable way
for dealing with a large set of rules.

Compound category symbols and metarules are new in Montague grammar,
but hyperrules are not. Consider the rule for relative clause formation:
$S_{3,N}$: If $\zeta \in P_{CN}$ and $\phi \in P_t$ then $F_{3,N}(\zeta,\phi) \in P_{CN}$ where $F_{3,N}(\zeta,\phi) = \zeta$ *such*
that ϕ'.. and ϕ' comes from ϕ by replacing each occurrence of *he*$_N$... .
This rule cannot be considered as involving the literal expression *he*$_N$ since
the fragment has no such an expression.

The rule rather is a scheme which, for each choice of a number for N, gives a rule. One might add the following eleven metarules to the grammar (a | separates alternatives):

$$N \to 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \mid N0 \mid NN.$$

Note that due to the convention, all occurrences of the metavariable N in $S_{3,N}$ have to be replaced by the same terminal production of N, so by the same representation of a number. The above demonstrates that one does not have to understand numbers in order to handle $S_{3,N}$; one can express everything with pure grammatical tools. Since this is rather obvious, one might omit these metarules. Note that we have the convention to write metavariables in capital (N).

Another example of a PTQ hyperrule is S_1. It reads:

$$B_A \subset P_A \text{ for every category } A.$$

Here A is variable over categories. Its range might be made explicit by means of metarules:

$$A \to \text{Term} \mid \text{Sent} \mid \text{Intr Verb} \mid \dots .$$

A first new example of the framework is the use of compound category symbols for subcategorization. There are arguments for distinguishing among the nouns the mass nouns (e.g. *gold*) and the count nouns (e.g. *ring*). In some respects they behave the same (relative clause formation), and in other respects they behave differently (term formation). Using two separate categories would have as consequence that we have to duplicate the rules for which they behave the same. This is unattractive and can easily be avoided by the use of hyperrules. Let CM be a metavariable which can be replaced by 'Count' as well as by 'Mass'. Then we may have the hyperrules

S_2 if $\alpha \in$ Count Noun then $F_1(\alpha)$, $F_2(\alpha)$, $F_3(\alpha) \in T$ where F_1, F_2 and F_3 are as in PTQ;

$S_{3,N}$ if $\alpha \in$ CM Noun and $\beta \in S$ then $F_{4,N}(\alpha, \beta) \in$ CM Noun.

Note that S_2 is a hyperrule without a metavariable, so it is a rule in its
present form. The rule says that the phrase *a ring* can be produced, but
a gold cannot. Hyperrule S_3 expresses that a count noun with a relative clause
(*ring such that it is in Amsterdam*) remains a count noun and a mass noun with
a relative clause *(gold such that it is in Amsterdam)* remains a mass noun.

The use of a metagrammar is especially clear in case there are more
levels of subcategorization and possibly crosslinks. I present a metagrammar
for the subcategorization which is mentioned in CHOMSKY 1965 (p85). It is
striking that Chomsky uses rewriting rules for the formal representation of
subcategorization as well.

```
COMMON    → COUNT  |  NONCOUNT.
COUNT     → ANIMATE  |  -animate cn.
NONCOUNT  → SGN abstract cn.
ANIMATE   → SGN human cn.
SGN       → +  |-.
```

Again, the auxiliary symbols of the metagrammar are written in capitals.
This metagrammar implicates, according to the substitution convention, that
a hyperrule containing ANIMATE as metavariable, represents 2 actual rules
(for the subcategories +human cn and -human cn) and that a hyperrule contain-
ing COMMON represents 5 actual rules.

Note that up till now, we used the terminology category/subcategory
without defining their difference. On the level of actual rules there is no
distinction between them: they both are indices of sets of expressions.
The above kind of grammar makes it possible to make a formal distinction.
Categories are names from the metagrammar which do not occur at the right
hand side of a metaproduction rule. Subcategories of a certain category
are all names that can be produced from that category using the metarules.
In the sequel I will use in most cases 'category' in the PTQ sense: as an
index of a set of expressions. So it may in many cases be replaced by
'subcategory'.

Below I will formulate clause 3 of the variable principle within the
syntactic rules. For this purpose I consider a category name as a compound

symbol consisting of a category name as the ones in PTQ, and a bag of integers. A bag (or multiset) is a set in which an element may occur more than once. This bag indicates which indices occur unremoved in the expressions of that category. So he_2 or he_3 is an expression of the category $(T,\{2,3\})$ or equivalently $(T,\{3,2\})$. We write this as he_2 or $he_3 \in (T,\{2,3\})$. Other examples are he_4 or $he_4 \in (T,\{4,4\})$ and John $\in (T,\{\ \})$. The language generated by the grammar is the set of English sentences which contain no indexed terms, so it is the set of expressions of the category $(S,\{\ \})$ or in PTQ terminology $(t,\{\ \})$. The hyperrules we need, contain variables for integers (N) and for bags $(BAG_1,BAG_2,\ldots)$. Let us adopt the following notations:

$BAG_1 \cup BAG_2$ denotes the bag consisting of the bag-union of the bags BAG_1 and BAG_2. So $\{1,2\} \cup \{2,3\} = \{1,2,2,3\}$.

$BAG \underline{w} N$ Is a compound expression indicating that BAG contains element N (w~with).

$BAG - N$ Idem for the bag obtained from BAG by removing all occurrences of N. So $\{1,2\} - 1 = \{2\}$.

The hyperrule corresponding with S_4 reads:

HS_4: If $\alpha \in (T,BAG_1)$ and $\beta \in (IV,BAG_2)$ then $F_4(\alpha,\beta) \in (S,BAG_1 \cup BAG_2)$

This hyperrule states that the syntactic variables in the sentence are the union of syntactic variables in the T-phrase and the IV-phrase. An instance of a rule obtained from HS_4 is:

$$\text{if } \alpha \in (T,\{1,2\}) \text{ and } \beta \in (IV,\{\ \})$$
$$\text{then } F_4(\alpha,\beta) \in (S,\{1,2\}).$$

This rule may be used in the production of He_1 or he_2 runs. Corresponding with S_2, $S_5 \cdots S_{13}$, and S_{17} we have analogous hyperrules. With $S_{14,N}$ corresponds:

$HS_{14,N}$:   if $\alpha \in (T,BAG_1)$ and $\beta \in (S,BAG_2 \underline{w} N)$ then $F_{10,N}(\alpha,\beta) \in$
$\in (S,BAG_1 \cup [BAG_2-N])$.

An instance of $HS_{14,N}$ is:
     if $\alpha \in (T,\{\ \})$ and $\beta \in (S,\{2,3\})$ then $F_{14,2} \in (S,\{3\})$.

This rule may be used in the production of John loves him_3 from John and he_2 loves him_3. The hyperrules corresponding with S_3, S_{14}, and S_{15} are analogous.

One observes that the expression $BAG_2 \underline{w} N$ guarantees that $HS_{14,N}$ is
only applied to sentences containing occurrences of he_N. So the clause 3b
of the variable principle that a quantification rule must remove an actual
occurrence of a variable, can be expressed within the syntactic rules: the rule
only applies to the subcategory consisting of expressions containing such
a variable. So clause 3b can be checked by looking to the rules: does a rule
which removes occurrences of a variable he_N, indeed operate on the related
subcategory of expressions containing he_N, and does it deliver an expression
of a subcategory from which the index in question is indeed removed? So 3b is
now a restriction on the rules of the grammar. Clause 3a that each variable
is removed, is guaranteed by defining the generated languages as being the
set of expressions of category $(S,\{\ \})$ (or in PTQ-like terminology of catego-
ry $(t,\{\ \}))$.

I have implicitly assumed that the reader has some non grammatical know-
ledge since what bags are, and what U, $\underline{w}$ and $-$ mean is not formulated within
the grammar. This can be provided for in the following way. One adds meta-
rules which produce from BAG actual bags and one adds hyperrules which pro-
duce from any instance of BAG-N the bag in question (so from $\{1,2\}-1$ they
produce $\{2\}$). It requires some technics fo formulate such hyperrules; there-
fore they are presented in an appendix.

5. AN APPLICATION TO VERB TENSE

Hyperrules are extremely useful in cases where a lot of closely related
rules are needed. Such a case is constituted by the rules for tense and sign.
In order to demonstrate this, I will present a grammar for the treatment of
the sign and the PTQ tenses of verbs. The grammar deals with the following
phenomena which are not treated in PTQ.
1) Negated iv-phrases are possible:

(14)   *John tries not to run.*

Notice that *try to* cannot be combined with a tensed iv-phrase. This
sentence is also incorporated in the fragment of BENNETT 1976. His solu-
tion is improved in the sense that the iv-phrase

(15)   *not to run slowly*

can, in our fragment, not obtain the reading in which *slowly* has wider
scope than the negation.

2) As in PTQ, tensed variants of conjunctions and disjunctions are possible.
   Syntactic details are changed in order to produce

(16)   *John has walked and talked*

instead of the PTQ version

(17)   *John has walked and talk.*

3) Distinct from PTQ, we allow for conjunction and disjunction of arbitrarily
   tensed sentences and iv-phrases such as

(18)   *A president has walked and will talk.*

The translation of this sentence will express that both actions concern
a present president.

4) The PTQ grammar provides for two readings of the sentence

(19)   *Every president will not walk.*

These are

(20) $\forall x[\underline{president}(x) \to \neg W[\underline{walk}(x)]]$

and

(21) $\neg W[\forall x[\underline{president}(x) \to \underline{walk}(x)]]$

If (21) indeed is a possible meaning of this sentence, then, in my opinion,
also (22) is:

(22) $\neg$ [$\forall x$[_president_(x) $\rightarrow$ W _walk_(x)]].

The grammar presented below produces all these three readings. If one
judges that (21) and (22) cannot be readings of the sentence, a single
hyperrule has to be removed from the grammar.

The main purpose of presenting a grammar for the above phenomena is to
demonstrate the use in syntax of hyperrules. I do not intend to say something
new about semantical concepts concerning verb tenses: the same model and the
same semantical operators for tense are used as in PTQ. This means that we
only deal with the reportive use of tenses (see BENNETT 1977). The semantical
problems raised by the new constructions mentioned above, all have to do with
the scope of the operators W, H and $\neg$. Also the treatment of morphology is
as simple as in PTQ: I do not use features. In the formulation of the hyper-
rules I assume that it is always possible to identify the mainverbs of
iv-phrases. The auxiliary verbs _will, have_ and _do_ may function as mainverbs
as well. The possibility of identification of mainverbs can be obtained by
using recursive definitions as in FRIEDMAN 1979 (or the #-mark of BENNETT
1976 or the subfunctions of PARTEE 1979). Moreover we assume that by related
means the iv-phrase of a sentence can be identified. The syntactic formation
rules do not treat (as is done in PTQ), the action of 'replacing a verb by its
negative future form' as a single syntactic action, but rather as the compo-
sition of 'replacing by its future form' and 'negating' which are performed
by two independent syntactic rules.

The PTQ category t (in our terminology S) will have several subcatego-
ries. For instance, there is a subcategory for each tense. This relationship
subcategory - tense is indicated by the following pairs: sent - present tense,
past sent - simple past, fut sent - future tense. Each negated tense consti-
tutes a subcategory and there is also a subcategory of unspecified tense
(for conjunctions and disjunctions). All subcategories translate into expres-
sions of the same logical type: the type of sentences. For IV-phrases there
is an analogous subcategory system. The intransitive verbs themselves are in
the subcategory iv. The subcategory system is formally described by means
of the following metarules (the symbol λ denotes the empty string):

$$S \twoheadrightarrow \text{TENSED sent.}$$

$$IV \twoheadrightarrow \text{TENSED iv.}$$

$$\text{TENSED} \twoheadrightarrow \text{SGN TNS} \mid \text{tns.}$$

$$\text{SGN} \twoheadrightarrow \text{neg} \mid \lambda.$$

$$\text{TNS} \twoheadrightarrow \text{past} \mid \text{fut} \mid \lambda.$$

The hyperrules presented below replace the PTQ-rules S_4 (IV+T),
S_8 (IV/IV+IV), S_{10} (IV//IV+IV) and S_{17} (tensed variants of S_4). The description
of an hyperrule has the following parts:

H: Information about the categories involved in the hyperrule. We write
        e.g. "$\phi \in$ neg sent" for "ϕ is a phrase of the subcategory neg sent".

T: The translation rule is defined by presenting its output. In this,
        by ϕ', ψ', α' and β' are understood the translations of ϕ, ψ, α and β
        mentioned in the H-part.

F: A specification of the string manipulation function used in the
        H-part. For the ease of reference, the function F will bear the same
        index as the hyperrule in which it is used, even where the same oper-
        ation is used in more than one rule. (This convention and the one for
        T is also practised in DOWTY 1978).

E: -Optionally- an example.

R: -Optionally- some remarks.

H_{101}:   If $\alpha \in$ T and $\beta \in$ TENSED iv then $F_{101}(\alpha,\beta) \in$ TENSED sent.

T_{101}: $\alpha'(^\wedge\beta')$

F_{101}: $F_{101}(\alpha,\beta) = \alpha\underline{\beta}$, where $\underline{\beta}$ is obtained from β by replacing
        β by their third person singular present form, provided that they are
        not yet in some third person singular form.

E_{101}: $F_{101}(John, \; walk) = John \; walks$
        $F_{101}(John, \; will \; walk) = John \; will \; walk.$

R_{101}:   Since there are, according to the metarules, six possible substitu-
        tions for TENSED, one can make six actual rules out of H_{101}.

H_{102}:   If $\alpha \in$ iv then $F_{102}(\alpha) \in$ past iv.

T_{102}: $\lambda x \; H[\alpha'(x)]$.

F_{102}:   Remove all occurrences of the mainverb *do* in α and next replace all

mainverbs in α by their past participle and write *have* in front of the thus changed α.

E_{102}: F_{102}(*walk and talk*) = *have walked and talked*

       F_{102}(*walk and do not talk*) = *have walked and not talked*.

R_{102}:   The above iv yields the reading in which the subject is outside the scope of the tense operator.

H_{103}:   If $\phi \in$ sent then $F_{103}(\phi) \in$ past sent.

T_{103}: $H(\phi')$.

F_{103}:   Apply F_{102} to the iv-phrase of ϕ, next replace mainverb *have* by *has*.

R_{103}:   The subject is inside the scope of the tense operator.

H_{104}:   If $\alpha \in$ iv then $F_{104}(\alpha) \in$ fut iv.

T_{104}: $\lambda x \, W[\alpha'(x)]$.

F_{104}:   Remove all occurrences of the mainverb *do* in α and write *will* in front of the thus changed α.

E_{104}: F_{104}(*walk and do not talk*) = *will walk and not talk*.

R_{104}:   See R_{102}.

H_{105}:   If $\phi \in$ sent then $F_{105}(\phi) \in$ fut sent.

T_{105}: $W(\phi')$.

F_{105}:   Apply F_{104} to the iv-phrase of ϕ.

R_{105}:   See R_{103}.

H_{106}:   If $\alpha \in$ TNS iv then $F_{106}(\alpha) \in$ neg TNS iv.

T_{106}: $\lambda x \, \neg[\alpha'(x)]$.

F_{106}:   If α has just one mainverb and this is *will, have, do,* or *be,* then insert *not* behind it; in all other cases write *do not* in front of α.

R_{106}:   Conjuncted and disjuncted tensed iv-phrases are of the subcategory tns iv. Therefore no instance of H_{106} applies to them.

H_{107}:   If $\phi \in$ TNS sent then $F_{107}(\phi) \in$ neg TNS sent.

T_{107}: $\neg[\phi']$.

F_{107}:   Apply F_{106} to the iv-phrase of ϕ, next replace mainverb *do* by *does*.

R_{107}:   Conjuncted and disjuncted sentences are of the subcategory tns sent. Therefore no instance of H_{107} applies to them. We have discussed the ambiguity of sentence (19). If one wishes to have only one reading for such sentences, then H_{107} has to be removed from the grammar.

H_{108}:   If $\alpha \in$ IV//IV and $\beta \in$ SGN iv then $F_{108}(\alpha,\beta) \in$ iv.

T_{108}: $\alpha'(\hat{}\beta')$.

F_{108}: If β starts with *do not* then delete this and insert *not* before the first *to* in the IV//IV-phrase. Otherwise concatenate α and β.

E_{108}: F_{108}(*try to, run*) = *try to run*

F_{108}(*try to, do not run*) = *try not to run*.

H_{109}: If $\alpha \in$ IV/IV and $\beta \in$ iv then $F_{109}(\alpha,\beta) \in$ iv.

T_{109}: $\alpha'(\hat{}\beta')$

F_{109}: $F_{109}(\alpha,\beta) = \beta\alpha$.

E_{109}: F_{109}(*slowly, run*) = *run slowly*.

R_{109}: Since adverbs can only be combined with an untensed and unsigned iv-phrase, a phrase like *do not run slowly* can only have the reading with slowly inside the scope of negation (see remark on (15)).

H_{110}: If $\alpha \in$ iv and $\beta \in$ SGN iv then $F_{110}(\alpha,\beta) \in$ iv.

T_{110}: $\lambda x[\alpha'(x) \land \beta'(x)]$.

F_{110}: $F_{110}(\alpha,\beta) = \alpha$ *and* β.

R_{110}: The asymmetry in this hyperrule makes that the negation in:

(23)   *John will not walk and talk*

applies to *will walk and talk,* and not just to *walk* (cf. FRIEDMAN 1979).

H_{111}: Analogously to H_{110} for disjunction.

H_{112}: If $\alpha \in$ TENSED$_1$iv and $\beta \in$ TENSED$_2$iv then $F_{112}(\alpha,\beta) \in$ tns iv.

T_{112}: see T_{110}.

F_{112}: see F_{110}.

R_{112}: This rule is used in the production of

(24)   *John will not walk and talks*.

(cf. R_{110})

H_{113}: Analogously to H_{112} for disjunction.

H_{114}: If $\phi \in$ TENSED$_1$sent and $\psi \in$ TENSED$_2$sent then $F_{114}(\phi,\psi) \in$ tns sent

T_{114}: $\phi' \land \psi'$.

F_{114}: $F_{114}(\phi,\psi) = \phi$ *and* ψ.

H_{115}: Analogously to H_{114} for disjunction.

These 15 hyperrules describe in a handsome way 164 actual rules, so the use of hyperrules reduced the length of the grammar by a factor 10. If I would also incorporate other tenses such as the past perfect, the benefit would still be more. Note that there is no syntactic problem in replacing H_{112}-H_{115} by two rules using one metavariable for both iv and sent, but then we would not be able to find two corresponding semantic rules.

Below I present three examples. All expressions are preceded by the name of the subcategory they belong to. From this information one easily concludes which actual rule is used.
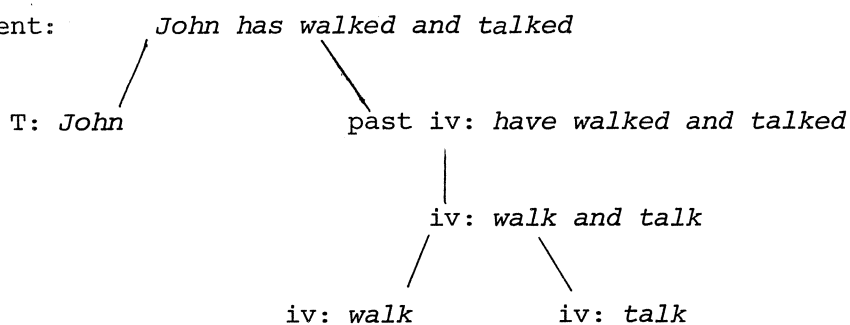
1) sent: *John tries not to run*

        T: *John*   iv: *try not to run*

              IV//IV: *try to*   neg iv: *do not run*

                                        iv: *run*
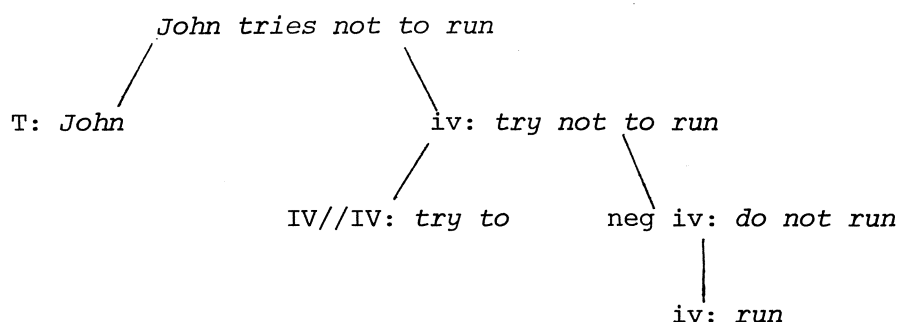
the corresponding translation yields:

$$\underline{try\ to}\ _*\ (j, \lambda u\ ^\neg \underline{run}\ _*\ (u))$$

2) past sent: *John has walked and talked*

        T: *John*   past iv: *have walked and talked*

                             iv: *walk and talk*

              iv: *walk*   iv: *talk*
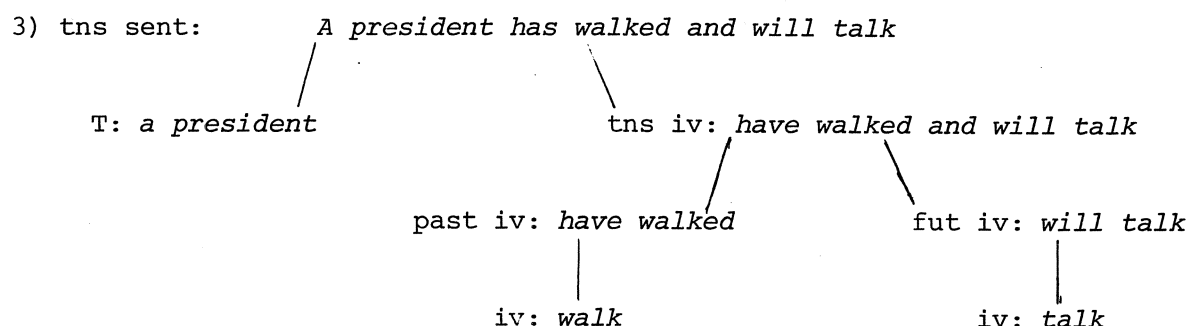
the corresponding translation yields

$$H(\underline{walk}(j) \wedge \underline{talk}(j))$$

An alternative analysis (with the same semantic result) would apply H_{103} to:

sent: *John walks and talks.*

3) tns sent: *A president has walked and will talk*

      /   \

  T: *a president*   tns iv: *have walked and will talk*

               /   \

     past iv: *have walked*   fut iv: *will talk*

         | |

       iv: *walk*   iv: *talk*

translation

$$\exists u[\underline{president}_*(u) \land H\ \underline{walk}_*(u) \land \underline{talk}_*(u)].$$

6. CONCLUSIONS

Two kinds of problems arise in connection with syntactic variables in Montague grammar. The existing proposals for treating these problems are not free of objections, whereas the *variable principle* is able to cope with them. The application of hyperrules, metarules and compound category symbols permits the incorporation of the variable principle as a restriction on possible grammars. These tools are also useful for other purposes (subcategorization, verb tense). The variable principle excludes the complicated NP-S analysis of relative clauses as proposed by Bach & Cooper. Concerning the demonstrative use of pronouns and related phenomena, the principle guides us towards the use of constants as is practised by Bennett.

7. APPENDIX

Rules for bags.

The aim of this appendix is to demonstrate that is possible to describe the bag manipulations used in section 4 by means of pure grammatical tools. Before to do so, some remarks for experts in van Wijngaarden grammars. The hyperrules presented in this paper do not have the format of hyperrules according to the literature, but they can be transformed to such a format. The hyperrule $S_{14,N}$ then obtains the shape

$$X(S,BAG_1 \cup [BAG_2 - N])Y ::=$$

$$X \ F_{10,N}((T,BAG_1),(S,BAG_2\underline{w} \ N))Y.$$

The metanotions X and Y stand for arbitrary contexts (cf. VAN WIJNGAARDEN 1970). The syntactic operation $F_{10,N}$ has to be described by means of other hyperrules. The phrase '$\underline{unless} \ N_1 \ \underline{is} \ N_2$' (used below) is to be considered as a separate hypernotion. The rules below use the rotation technique of VAN WIJNGAARDEN 1974 and the blind alley technique of Sintzoff as practised in VAN WIJNGAARDEN 1975. .

The metarules are (λ denotes the empty string)

$$
\begin{aligned}
BAG \ &\rightarrow \ \{SEQ\} \ | \ \{ \ \} \\
SEQ \ &\rightarrow \ N \ | \ N,SEQ \\
N \ &\rightarrow \ 1 \ | \ 2 \ | \ 3 \ | \ 4 \ | \ 5 \ | \ 6 \ | \ 7 \ | \ 8 \ | \ 9 \ | \ NN \ | \ NO. \\
LSEQ \ &\rightarrow \ SEQ, \ | \ \lambda. \\
RSEQ \ &\rightarrow \ ,SEQ \ | \ \lambda.
\end{aligned}
$$

The hyperrules for union are

$$
\begin{aligned}
\{SEQ_1\} \cup \{SEQ_2\} \ &\rightarrow \ \{SEQ_1,SEQ_2\} \\
\{SEQ_1\} \cup \{ \ \} \ &\rightarrow \ \{SEQ_1\} \\
\{ \ \} \cup \{SEQ_2\} \ &\rightarrow \ \{SEQ_2\} \ .
\end{aligned}
$$

For with

$$\{LSEQ \; N \; RSEQ\} \; \underline{w} \; N \rightarrow \{LSEQ \; N \; RSEQ\}$$

For minus

$$\{SEQ\} - N \rightarrow \{*,SEQ\} - N$$

$$\{ \; \} - N \;\; \rightarrow \{ \; \}$$

$$\{LSEQ \; * \; RSEQ,N_1\} - N_2 \rightarrow \{N_1,LSEQ \; * \; RSEQ\} - N_2 \; \underline{unless} \; N_1 \; \underline{is} \; N_2$$

$$\{LSEQ \; *\} - N_2 \rightarrow \{LSEQ\}.$$

Hyperrules concerning the <u>unless</u> phrase are

$$\underline{unless} \; \underline{false} \rightarrow \lambda$$

$$\textit{0} \; \underline{is} \; \textit{0} \rightarrow \underline{true}; \quad \textit{0} \; \underline{is} \; \textit{1} \rightarrow \underline{false}; \quad \textit{0} \; \underline{is} \; \textit{2} \rightarrow \underline{false} \; ...$$

$$N_1N_2 \; \underline{is} \; N_3N_2 \rightarrow N_1 \; \underline{is} \; N_3; \quad N_1N_2 \; \underline{is} \; N_1N_3 \rightarrow N_2 \; \underline{is} \; N_3.$$

So the phrase '<u>unless</u> N_1 <u>is</u> N_2' reduces to the empty string just in case N_1 is not equal to N_2, otherwise one cannot get rid of the phrase and thus the rule cannot be used in an actual derivation.

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