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ABSTRACT

This note deals with consultation in operations research at the Mathematical Centre in Amsterdam. After a short description of the activities of the MC, in particular of its Department of Operations Research and System Theory, three practical projects are described.

KEY WORDS & PHRASES: *operations research, consultation, simulation, linear programming, combinatorial optimization.*
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The Mathematical Centre (MC) was established in 1946 as a nonprofit foundation for the promotion of mathematics and its applications.

The Institute MC has six scientific departments: pure mathematics, applied mathematics, numerical mathematics, mathematical statistics, operations research and system theory, and computer science. Among the supporting nonscientific departments, the library and the printing office play a role of national importance. The MC is sponsored by the government through the Netherlands Organization for the Advancement of Pure Research (ZWO). Next to pure research, the MC also carries out consulting activities in the private and public sectors. The budget for 1980 amounted to more than thirteen million Dutch guilders, of which about 85% was provided by ZWO. The institute employs around 150 people.

The Foundation MC was recently appointed to coordinate, stimulate and evaluate mathematical research at universities to the extent that it is being financed by ZWO. To this end, a number of research communities in several branches of mathematics were created. This new task is mentioned here to emphasize the central position of the MC within Dutch mathematics, although it is not of primary relevance to the subject matter of this paper: consultation in the area of operations research.

The Department of Operations Research and System Theory is engaged in the investigation of mathematical models and methods that could support optimal actions in decision situations. The motivation originally came from problems in economics and industrial engineering, and today is also found in communication and control and even in the political and social sciences.

These investigations entail the study of a wide range of mathematical subjects, such as complexity theory, combinatorics, probability theory and differential geometry. The unifying element is the potential applicability of the models and methods under investigation. Consequently, the department tries to become involved in projects that lead to original and advanced applications in areas in which it has expert knowledge. Such projects can vary from answering specific questions explicitly to participating in development research, with the purpose of making new theory applicable in practice.

The involvement in practical projects is an essential part of the department's scientific policy. The current research projects and the main application areas are:

- *combinatorial optimization*, i.e., the determination of optimal distribution systems, depot locations, room assignments, timetables, production plans, cutting patterns, and other discrete structures;
- *analysis and control of information flows in networks*, such as computer networks, telecommunication systems and networks of queues;
- *system and control theory*, in particular prediction, filtering, nonlinear control, system identification and time series analysis.

Experience has shown that consultative activities often lead to innovative applications as well as to intriguing mathematical problems and results. This will be illustrated below on three practical projects. They were carried out by A.W.J. Kolen, B.J. Lageweg, L. Stougie, O.J. Vrieze and the authors.

1. Playing for keeps

A consortium of four international contractors had completed a large dredging contract. An inventory of equipment was left over, consisting of 268 items, including crane ships, barges, rock breakers and smaller items such as pipes and spare parts. An independent consultant had established a price for each item. The total value of the inventory was about $24.8 million. Since the inventory could not be sold locally, it was agreed among the partners that each would buy a quarter of the lot. It was also agreed that the allocation of the items to the partners would be determined by an auction, as it was virtually impossible to decide on this by straightforward negotiations.

The auction consisted of 25 rounds. In each of the first 24 rounds each partner would be allotted $0.25 million to buy items or to save money for subsequent rounds. It was not allowed, however, to save an amount exceeding the price of the most expensive unsold item. In the last round the remaining budgets must be spent. The order in which the companies should buy from the inventory in the first round would be determined by drawing lots: $S_1 = (1,2,3,4)$. The order in the subsequent rounds followed from that in the first one: in the second round $S_2 = (2,3,4,1)$, in the third $S_3 = (3,4,1,2)$, and in the fourth $S_4 = (4,1,2,3)$. Then a second cycle of four rounds would follow: S_2, S_3, S_4, S_1. The third cycle would start with S_3, and so on.

Our client, one of the partners, had composed a listing of the items with the price of each item, its attractiveness for the company and guesses of the preferences of the others. The attractiveness was defined as a classification into categories *A* (very attractive) to *E* (scrap). Category *A* contained three expensive crane ships; five to six rounds of saving would be necessary to acquire one. Due to the extended production times for new cranes it was expected that each partner would try to obtain at least one of these.

A program was developed to keep track of purchases and savings of each partner and to provide information, if requested, such as lists of attractive items that could be bought in the present or the next round. This program was run on the company's computer and used on-line by the delegation at the auction.

Our assignment was to develop a strategy to obtain as much attractive equipment as possible. An analysis of the inventory and the rules of the game provided useful information. It was impossible to evade buying from category *E*; minimization of this was selected as the primary objective. The drawing of lots was intended to provide equal opportunities but did not do so; e.g., the company which draws 2 is preceded by company 1 in most rounds and thus is at a disadvantage if both prefer the same items. Another problem was the endgame. It was possible that only expensive items would be left at the end and no partner has sufficient savings to buy. Even if this situation did not occur, at least $0.8 million worth of material would be left at the end, without rules for allocating it.

For this auction, various strategies are conceivable. With the help of a simulation program five strategies were investigated, each with and without going after the cranes. The auction was simulated for over 50 combinations of these basic strategies for the competitors and assumed preferences for the items, and the results were analyzed.

According to the simulations, the outcome for the company would depend to a large extent on the drawing of the lots. At the beginning of the game as little as possible should be saved. It was most advantageous to buy an item requiring several rounds of saving in one of the last few rounds. The round in which to start saving depended upon the outcome of the lottery. While saving, one might not spend more than the competitor with most nearly the same amount of savings. Much attention should be paid to items priced $0.25 million or less, which do not require saving. Finally, the game would probably reach a deadlock, so the partners should define rules for the endgame.

With the help of the bookkeeping program and the selected strategy, our client succeeded in spending only 10% of the budget in category *E* and very high percentages in categories *A* and *B*. This is in spite of an unfavourable starting position and an arbitrary allocation of the leftovers in the last round. The estimates of benefits ranged between $0.25 and $1.50 million. An objective estimate is impossible since the real preferences of the other partners are unknown. One of them purchased different items than was expected. In general, the others tried to optimize in each round, whereas our client pursued an overall optimum.

This consultation was not remarkable by the problem and the results alone, but also because we were given only ten days. Nevertheless, an intensive and effective preparation for the auction was feasible.

2. After the last ride

The public transportation service in one of the major Dutch cities, which operates 16 tramway lines and 270 tramcars, was interested in an optimal allocation of trams to depots. On each line, a number of trams runs between both endpoints. After the last ride at night, each tram goes to one of seven depots. Each depot has a limited capacity. A ride to the depot costs a certain amount that, among other things, depends on the length of the ride. The problem is to allocate trams to depots in such a way that the total costs of the depot rides are minimized.

Three variants of this problem were of interest. The first one represents current practice: all trams of the same line are allocated to the same depot. This stimulates contact among the drivers on one line. The second variant can yield savings: the trams of a line that make their last ride to the same endpoint have to go to the same depot. The lower costs were to be weighed against the inconvenience of dividing personnel. The third variant is the cheapest one: each tram can go to any depot. This was not considered to be a feasible alternative, but it would set an informative lower bound on the minimum total costs.

Out of the seven depots mentioned above, only three do exist in reality. Two of the four fictitious depots are locations at which a depot could be built. Before deciding to do this, one wanted to have a definite estimate of the potential savings as a function of the capacities to be chosen. The other two fictitious depots are in fact two new routes to an existing depot. Building those routes could diminish noise pollution at night, again depending on the capacities to be chosen - i.e., the number of trams that would be allowed to use the routes. One was interested in the relation between operating costs and usage of one or both new routes. All this led to 40 combinations of depot capacities for each variant. Therefore, a total number of 120 problems had to be solved.

The mathematical formulation of these problems was obvious. The third variant is nothing but a linear transportation problem, for which standard techniques yield integral solutions; there is no concern that a tram will be split over several depots. The first and second variants have side constraints to enforce that all trams of the same line (or of the same endpoint) go to the same depot; this required the introduction of a 0-1 variable for each of the $16\times7 = 112$ line-depot combinations (or for each of the $32\times7 = 224$ endpoint-depot combinations).

The resulting integer linear programming problems were solved by the APEX system of Control Data, which is available on the Cyber 175-750 of SARA (Foundation Academic Computing Centre Amsterdam). This program computed good to very good solutions at reasonable costs.

The computations were organized as follows. For each variant, our general LP matrix generator produced an input file for one combination of capacities. After that, a special procedure handled each case by substituting a combination of capacities in the input file, calling the APEX system, and adding the solution to an output file. Finally, a report generator made manageable surveys of all solutions.

The problems have also been solved for possible future situations involving increased capacities or modifications of the network.

As has been stated before, one had no intention to simply implement the best solution in practice. Many of the variants and situations considered are unrealistic, but this very feature does contribute to the value of the collected results as an aid to decision making. The transportation service is now investigating the possibility of changing to an allocation rule according to the second variant; this would yield substantial savings.

3. Nasty clients

A Dutch firm, primarily engaged in the retail trade, had decided to diversify and had acquired a large number of summer cottages. A client can make a reservation at any one of the firm's branches and is immediately told whether a cottage is still available for the period (s)he is applying for. Only at a later stage is it determined in which cottage each accepted client has to spend the holidays. This procedure led to a couple of questions.

Does there exist a simple rule that indicates whether a client can be accepted? Yes, there does: cottages can be assigned to clients in their desired periods if and only if, at any time, the number of clients is no larger than the number of cottages. How about a method that assigns the accepted clients to a minimum number of cottages? This exists as well: assign the clients to cottages in order of their starting times, giving priority to cottages used before.

As early as 1954, more complicated versions of these problems were solved by G.B. Dantzig and D.R. Fulkerson, founding fathers of operations research, as witnessed by the existence of the Dantzig Prize and the Fulkerson Prize. The questions asked were closely related to our research in machine sequencing and scheduling, so the answers could be given offhand during the first (and, as it turned out, the last) contact with our potential client.

While leaving, a trivial complication crossed his mind: a client can reserve a specific cottage by paying Dfl.25 upon application and is then preassigned. This has a dramatic effect on the problem's computational complexity. So far, we had identical cottages and nonidentical clients; but now the cottages are nonidentical as well. The question whether a client who expresses no preference can be accepted boils down to the following problem: is it possible to pack n given time intervals (the unassigned clients) into m other given time intervals (the idle periods of the cottages)?

This is a beautiful combinatorial problem, but it has not been dealt with previously in the literature. The above necessary and sufficient condition for acceptance remains valid only under the (false) assumption that a client would be willing to move into another cottage now and then. It recently turned out that the problem is solvable in polynomial time for each fixed m and *NP*-complete for arbitrary m. The complexity theorist is delighted by such a classification. However, the polynomial method is not practicable for realistic valies of m, and *NP*-completeness does not imply absolute insolvability. It seems very well possible to develop an algorithm that resolves most cases fairly quickly - although one can always construct an artificial instance that keeps the computer running until holidays are over.

We never heard from our client again: the complications caused by the nasty clients are probably trivial indeed and do not prohibit the application of the existing methods.

Is this an example of a consultation that *failed*? No, it rather is the *reverse* of a consultation. A practitioner saddled us with a problem that, after all, was of no concern to him. Continuing research on this problem is a task of the Mathematical Centre. It is primarily motivated by our professional curiosity, but also by possible practical demands in the future.