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THE EQUICONTINUOUS STRUCTURE RELATION AND EXTENSION OF FUNCTIONS DEFINED ON G-SPACES

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The equicontinuous structure relation and extension of functions defined on G-spaces *)

by

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ABSTRACT

In this paper we consider the question of when the equicontinuous structure relation of a subflow is the restriction of the equicontinuous structure relation of the whole flow. Some necessary and sufficient conditions are given, one in terms of almost periodic functions on the flow, and another one in terms of injective objects in the category of all compact Hausdorff G-spaces.

KEY WORDS & PHRASES: G-*space, flow, equicontinuous structure relation, almost periodic functions, extensor, injective object, compact convex set*

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*) This report will be submitted for publication elsewhere.

1. INTRODUCTION

In abstract topological dynamics, and in particular, in the study of minimal flows, much attention is paid to the equicontinuous structure relation; cf. for instance [8; 4.20] or [9]. The following question has not, as far as we know, been considered earlier (mainly, because it makes no sense in the context of minimal flows): how is the equicontinuous structure relation of a subflow related to the equicontinuous structure relation of the whole flow? We shall not give a complete answer to this problem. Instead, we shall characterize the subflows which behave nicely in this respect, and this characterization will be in terms of certain extension properties. To this end, we shall first describe some relevant facts about injective objects and extensors in the category TOP^G of all G-spaces for a given topological group G. Important in this respect are what we will call MC G-spaces (see 1.13 below). In section 2 we collect some material about equicontinuous G-spaces and finally, in section 3, the equicontinuous structure relation is considered. The main result (Theorem 3.8) has been announced in [21]. Unless stated otherwise, the letter G always denotes an arbitrary (but fixed) topological group.

1.1. We shall first define the category TOP^G we are working in. A G-*space* (or: topological transformation group with acting group G) is a pair $\langle X,\pi\rangle$ where X is a topological space and $\pi: G \times X \to X$ is a continuous mapping (called the *action* of G on X) such that

(i) $\pi(e,x) = x$ for all $x \in X$ (e is the unit element of G);
(ii) $\pi(s,\pi(t,x)) = \pi(st,x)$ for all $x \in X$ and $s,t \in G$.

Often we shall use the following notation: if $x \in X$ and $t \in G$ then

$$\pi^t x := \pi(t,x) =: \pi_x(t).$$

It follows from continuity of π and the axioms (i) and (ii) that, for every $t \in G$, $\pi^t: X \to X$ is a homeomorphism with inverse $\pi^{t^{-1}}$ (in fact, $\pi^e = id_X$ and $\pi^s \circ \pi^t = \pi^{st}$). Moreover, π_x is a continuous mapping from G into X.

The G-spaces are the objects of TOP^G. We now define the morphisms in this category. If $\langle X,\pi\rangle$ and $\langle Y,\sigma\rangle$ are G-spaces, then a mapping $\phi: X \to Y$ is called *equivariant* whenever $\phi\circ\pi^t = \sigma^t\circ\phi$ for all $t \in G$. A *continuous* equivariant mapping will be called a *morphism of G-spaces*: these are the morphisms in TOP^G. It is clear, that in this way a genuine category is defined (e.g., composition in the category is just composition of mappings). For a detailed treatment of the category TOP^G, see [17].

1.2. EXAMPLES. The following G-spaces and morphisms will be needed in the sequel.

1. Let ω: $G \times G \to G$ denote the multiplication mapping. Clearly, $\langle G,\omega\rangle$ is a G-space, and if $\langle X,\pi\rangle$ is an arbitrary G-space, then for every $x \in X$ the mapping π_x: $G \to X$ is a morphism of G-spaces from $\langle G,\omega\rangle$ to $\langle X,\pi\rangle$.

2. If X is a topological space, then $C_c(G,X)$ will denote the space of all continuous mappings from G into X endowed with the compact-open topology. Define ρ: $G \times C_c(G,X) \to C_c(G,X)$ by $\rho^t f(s) := f(st)$ for $f \in C_c(G,X)$ and $s,t \in G$ (right translations). If G is locally compact, then ρ is continuous [17;2.1.3] and ρ is an action of G on $C_c(G,X)$. If $\langle X,\pi\rangle$ is a G-space, then the mapping $\underline{\pi}$: $x \mapsto \pi_x$: $X \to C_c(G,X)$ is a morphism of G-spaces from $\langle X,\pi\rangle$ to $\langle C_c(G,X),\rho\rangle$ (it is even an equivariant embedding; [17;2.1.13]).

3. If $\langle X,\pi\rangle$ is a G-space and A is an *invariant* subset of X (that is, $\pi^t A = A$ for every $t \in G$), then $\langle A,\pi|_{G\times A}\rangle$ is a G-space and the embedding mapping of A into X is a morphism of G-spaces from $\langle X,\pi\rangle$ into $\langle A,\pi|_{G\times A}\rangle$. In this (and every similar) case we shall denote the action of G on A simply by π, and we shall say that the G-space $\langle A,\pi\rangle$ is a *sub-G-space* of $\langle X,\pi\rangle$. Thus, the phrase "i:$\langle A,\pi\rangle \to \langle X,\pi\rangle$ is an equivariant embedding" shall always mean that A is an invariant subset of X and that i is the embedding mapping.

1.3. An *injective object* for a morphism ϕ: $A \to X$ in an arbitrary category $\mathcal{C}$ is an object K in $\mathcal{C}$ such that for every morphism f: $A \to K$ in $\mathcal{C}$ there exists a (not necessarily unique) morphism f': $X \to K$ in $\mathcal{C}$ such that $f = f'\circ\phi$ (i.e. f' is an "extension" of f over ϕ).

$$\begin{tikzcd}
 & K & \\
A \arrow[ur, "f"] \arrow[rr, "\phi"'] & & X \arrow[ul, dashed, "f'"']
\end{tikzcd}$$

If ϕ is a monomorphism, then an injective object for ϕ will also be called an *extensor* for ϕ. If K is simultaneously injective (resp. an extensor) for every morphism ϕ from a class M of morphisms (resp. monomorphisms) in $\mathcal{C}$, then K is called *injective* (resp. an *extensor*) *for* M.

For example, Tietze's theorem states that the closed unit interval $[0;1]$ is an extensor in TOP for the class M_{nor} of all closed embeddings into normal spaces (for examples in other categories, see e.g. [10] or [15]). Several generalizations of this result are known (see e.g. [12]), and the following will be used in this paper (it is a form of a result of ARENS'; for the proof, cf. [12; Thm 1]):

1.4. THEOREM. *Let* M_0 *be the class of all closed embeddings in* TOP *for which* $[0;1]$ *is an extensor in* TOP, *and let* K *be a metrizable compact convex subset of a locally convex topological vector space. Then* K *is an extensor in* TOP *for* M_0. □

1.5. REMARK. By Tietze's theorem, $M_{nor} \subseteq M_0$. In addition, the class M_0 contains all embeddings of compact spaces into Tychonov spaces (using Stone-Čech compactifications, this reduces to closed embeddings into compact Hausdorff spaces, a subclass of M_{nor}). In fact, all embeddings of compact spaces into functionally Hausdorff spaces are in M_0 [12;p.366].

1.6. For convenience, a Metrizable Compact Convex subset of a locally convex topological vector space will be called an *MC-set* (it should be MC^2-set, but MC will do). Thus, according to 1.3, every MC-set is an extensor in TOP for M_0. In some of our results below, the metrizability of compact convex sets can be removed by restricting the attention to closed embeddings into metrizable spaces instead of normal spaces, using Dugundji's extension theorem instead of 1.4.

We now return to the category TOP^G. The following result comes from [19;4.1]:

1.7. PROPOSITION. *Assume that* G *is locally compact, and let* K *be injective in* TOP *for some class* M *of morphisms in* TOP. *Then the* G-*space* $\langle C_c(G,K),\rho\rangle$ *is injective in* TOP^G *for the class* M^G *of all those morphisms of* G-*spaces* $\phi: \langle X,\pi\rangle \to \langle Y,\sigma\rangle$ *such that the continuous mapping* $\phi: X \to Y$ (*regarded as morphism in* TOP) *belongs to* M. □

1.8. COROLLARY. *Assume that* G *is locally compact and let* K *be an* MC-*set.*

Then the G*-space* $\langle C_c(G,K),\rho\rangle$ *is an extensor in* TOP^G *for the class* M_0^G *of all closed equivariant embeddings* $i\colon \langle A,\pi\rangle \to \langle X,\pi\rangle$ *such that* $i\colon A \to X$ *belongs to the class* M_0 (cf. 1.4 above).

PROOF. Use 1.4 and 1.7. □

1.9. The existence of an extensor for a large class of equivariant embeddings should be no surprise: the trivial G-space, consisting of a one-point space (with the obvious action of G) is an extensor in TOP^G for every equivariant embedding. More generally, we shall call a G-space $\langle X,\pi\rangle$ *non-trivial* whenevery not all homeomorphisms π^t for $t \in G$ are equal to the identity mapping. So $\langle X,\pi\rangle$ is non-trivial iff not each orbit consists of one point. What we want is, of course, a non-trivial extensor in TOP^G. If K is a non-trivial MC-set, then $C_c(G,K)$ is also non-trivial, but its disadvantage is, that it is too large to have nice properties; in particular, $C_c(G,K)$ is not compact (unless G is discrete). In fact, we want to find a compact Hausdorff G-space which is not trivial and which is an extensor for at least all closed equivariant embedding in $COMP^G$ (this is the full subcategory of TOP^G, determined by all compact Hausdorff G-spaces). For a motivation of this problem, see among others [16]. For the case that G is *compact*, the problem is solved in [3]; see also Section 2 below. The following illustrates, why the extensor itself should be compact.

1.10. PROPOSITION. *Assume, that* G *is locally compact, and let* $\langle K,\alpha\rangle$ *be a compact Hausdorff* G*-space. The following conditions are equivalent:*

(i) $\langle K,\alpha\rangle$ *is an extensor in* TOP^G *for the class of all equivariant embeddings of compact* G*-spaces into functionally Hausdorff* G*-spaces*;

(ii) $\langle K,\alpha\rangle$ *is an absolute retract in* TOP^G *for the class of all functionally Hausdorff spaces.*

(Condition (ii) means, that if $\langle K,\alpha\rangle$ is equivariantly embedded in a functionally Hausdorff G-space $\langle X,\pi\rangle$, then there exists an equivariant retraction of X onto K.)

PROOF. (i) ⇒ (ii): trivial (here it is essential that K is compact).
(ii) ⇒ (i): we apply a standard construction (see for instance [17;7.1.4 and 8.1.4] or [20]) in order to observe, that there exists an equivariant

embedding of $\langle K,\alpha\rangle$ into the G-space $\langle C_c(G,\mathbb{R}^\kappa),\rho\rangle$ for some cardinal number κ. By condition (ii), there exists an equivariant retraction of $\langle C_c(G,\mathbb{R}^\kappa),\rho\rangle$ onto $\langle K,\alpha\rangle$. However, by Proposition 1.7, $\langle C_c(G,\mathbb{R}^\kappa),\rho\rangle$ is an extensor in TOP^G for a class of equivariant embeddings which comprises all embeddings, mentioned in condition (i) (see 1.4). Hence $\langle K,\alpha\rangle$, being an equivariant retraction of $\langle C_c(G,\mathbb{R}^\kappa),\rho\rangle$ has the desired property (i). □

1.11. REMARK. Proposition 1.10 with the additional condition that G is *compact* appears in [3;Thm.3]. Our next result depends on a compactification result, published in [18]. It shows, that for the case that G is locally compact, we may restrict our attention to $COMP^G$ without much loss of generality (see also 2.7 below).

1.12. PROPOSITION. *Assume that* G *is locally compact, and let* $\langle K,\alpha\rangle$ *be a (not necessarily compact Hausdorff)* G*-space. The following conditions are equivalent:*

(i) $\langle K,\alpha\rangle$ *is an extensor in* TOP^G *for the class of all equivariant embeddings of compact* G*-spaces into Tychonov* G*-spaces.*

(ii) $\langle K,\alpha\rangle$ *is an extensor in* TOP^G *for the class of all closed equivariant embeddings into compact Hausdorff* G*-spaces (i.e. closed equivariant embeddings in* $COMP^G$*).*

PROOF. (i) ⇒ (ii) is trivial, and (ii) ⇒ (i) follows obviously from the fact that every Tychonov G-space can equivariantly be embedded in a compact Hausdorff G-space [18]; for this result, local compactness of G is needed. (Compare this argument with the second statement in 1.5.) □

1.13. The problem whether $COMP^G$ contains a non-trivial extensor for all closed equivariant embeddings has an obvious solution in case G is discrete: apply Corollary 1.8 and observe, that $C_c(G,K)$ is compact in this case. Also in the case that G is compact there is a solution, essentially due to GLEASON; see Section 2 below.

We need one more definition: an *MC* G*-space* is a G-space $\langle K,\alpha\rangle$ with K an MC-set and with action α such, that for every $t \in G$ the homeomorphism $\alpha^t\colon K \to K$ is an *affine* mapping (i.e. $\alpha^t(ax+(1-a)y) = a\alpha^t(x)+(1-a)\alpha^t(y)$ for all $x,y \in K$ and $0 \le a \le 1$). Actually, we may assume that α^t is the restriction of an invertible *linear* mapping in the ambient topological vector space, as the following lemma shows (provided G is locally compact):

1.14. LEMMA. *Assume that* G *is locally compact and let* $\langle K,\alpha\rangle$ *be an MC* G-*space. Then there exists an equivariant embedding* $\Phi: \langle K,\alpha\rangle \to \langle E,\bar{\alpha}\rangle$ *such that* E *is a locally convex topological vector space,* $\bar{\alpha}$ *is a continuous action of* G *on* E *such that* $\bar{\alpha}^t$ *is linear for every* $t \in G$ *and, finally,* Φ *is affine.*

(So in particular, $\Phi[K]$ is an invariant MC subset of E and $\langle\Phi[K],\bar{\alpha}\rangle$ is an MC G-space, affinely isomorphic to $\langle K,\alpha\rangle$ as a G-space.)

PROOF. Suppose K is given as an MC-subset of the locally convex topological vector space F. Now apply the construction, referred to in the proof of Proposition 1.10 ((ii) ⇒ (i)), with $\mathbb{R}^K$ replaced by F. In fact, we obtain the equivariant embedding $\Phi: x \mapsto \alpha_x: \langle K,\alpha\rangle \to \langle C_c(G,F),\rho\rangle$. It is easily checked, that this Φ is affine. Moreover, $E := C_c(G,F)$ is a locally convex topological vector space, and $\bar{\alpha} := \rho$ is a continuous action (G is locally compact; cf.1.2(2)) such that each $\bar{\alpha}^t$ is linear. □

1.15. REMARK. A similar proof works for a semigroup of continuous affine mappings. In particular, by embedding K in a larger vector space, any single continuous affine mapping $\phi: K \to K$ may assumed to be the restriction of a continuous linear mapping (replace G by $\mathbb{N}$ and let $\mathbb{N}$ act on K by $n.x := = \phi^n(x)$ for $n \in \mathbb{N}$ and $x \in K$).

If in 1.14 the group G is sigma-compact and the ambient space F of K is metrizable, then E may also be assumed to be metrizable (indeed, $C_c(G,F)$ is metrizable). Similarly, if F is a Hilbert space, then E may also assumed to be a Hilbert space (in that case, a different construction has to be used; cf. [17;8.2.10]).

We close this section with a lemma concerning the ubiquity of non-trivial MC G-spaces:

1.16. LEMMA. *Every compact metrizable* G-*space* $\langle X,\pi\rangle$ *can equivariantly be embedded in an MC* G-*space.*

PROOF. (Cf. [20],3.9). The space $M_1(X)$ of all probability measures is a compact convex subset of the dual space $C(X)^*$ of $C(X)$, endowed with the

w^*-topology. Since X is a compact metric space, $M_1(X)$ is metrizable as well. Moreover, the action of G on X induces linear mappings $\bar{\alpha}^t\colon C(X)^* \to C(X)^*$ which are continuous with respect to the w^*-topology, and which leave $M_1(X)$ invariant. Note also, that $\bar{\alpha}^e$ is the identity mapping of $C(X)^*$, and that $\bar{\alpha}^{st} = \bar{\alpha}^s \circ \bar{\alpha}^t$ for all $s,t \in G$. The restrictions of these mappings to $M_1(X)$ define a continuous mapping $\bar{\alpha}\colon G \times M_1(X) \to M_1(X)$, namely, by the rule $\bar{\alpha}(t,\mu) = \bar{\alpha}^t\mu$ for $t \in G$, $\mu \in M_1(X)$. So $\langle M_1(X), \bar{\alpha}\rangle$ is a G-space, and since $M_1(X)$ is an MC-set in $C(X)^*$, we have an MC G-space. Finaly, the natural embedding $x \mapsto \delta_x$ (= Dirac measure at x) provides an equivariant embedding of X into $M_1(X)$. □

1.17. REMARK. In the case of a sigma-compact, locally compact group G, an alternative proof can be given, using [17; 8.2.4] (embed X in $C_c(G,\mathbb{R}^{\aleph_0})$ = =: E and observe that E is metrizable with a complete metric) and [5'] Chap.I,§4,no 1 (the closed convex hull of a compact subset in a complete locally convex topological vector space is compact). For a related result, cf.[2].

2. EQUICONTINUOUS G-SPACES

Unless stated otherwise, G is an arbitrary topological group.

2.1. LEMMA (GLEASON). *Let* H *be a compact topological group and let* $\langle K,\alpha\rangle$ *be an MC* H*-space. Then* $\langle K,\alpha\rangle$ *is an extensor in* TOP^H *for the class* M_0^H (cf. 1.8 and 1.4 for the definition).

PROOF. For the case that K is finite-dimensional, see for example [14] (but use Theorem 1.4 instead of Tietze's theorem). Exactly the same proof works for infinite dimensional MC-sets, taking into account [5;§1.2, the Corollary of Proposition 5]. (For these proofs it is necessary that the mappings α^t ($t \in H$) commute with a K-valued integral (with respect to Haar measure) on H. We could find no reference to justify this for continuous affine mappings; however, by Lemma 1.14 we need to justify it only for restrictions of continuous *linear* mappings, and for that case it is well-known; see e.g.[5;§1.1, Proposition 1].) □

2.2. <u>REMARK</u>. A version of this lemma is included in [3]; since we are interested only in compact extensors we do not bother about weakening the compactness hypothesis of K.

2.3. Recall (see e.g. [1]), that the *Bohr compactification* ψ: $G \to bG$ of G is a compact Hausdorff topological group bG, together with a continuous homomorphism ψ of G onto a dense subgroup of bG which has the following universal property: if ϕ: $G \to H$ is any continuous homomorphism of G into a compact Hausdorff topological group H, then there exists a unique continuous homomorphism ϕ': $bG \to H$ such that $\phi = \phi' \circ \psi$. It is well-known and easy to prove, that this definition coincides with the definition in [11; 26.11] for the case that G is a locally compact abelian group: in that case bG can be realized as $(G^\wedge)_d^\wedge$ (here $(G^\wedge)_d$ is the group $G^\wedge$, the character group of G, endowed with the discrete topology), and ψ: $G \to bG$ can be realized as the mapping $t \mapsto \delta_t : G \to (G^\wedge)_d^\wedge$, where $\delta_t(\chi) = \chi(t)$ for $\chi \in G^\wedge$ and $t \in G$. In particular, ψ: $G \to bG$ is injective in this case. So locally compact abelian group are examples of so-called "maximally almost periodic" groups. The other extreme are the so-called "*minimally almost periodic*" groups: topological groups G for which the Bohr-compactification bG is trivial (i.e. bG is a one-point group). This latter class of groups is characterized by the fact that their homomorphic images in compact Hausdorff groups are all trivial; in particular, they have no non-trivial, finite dimensional, unitary representations. An example is the group $SL(2,\mathbb{R})$ (also $SL(2,\mathbb{C})$) with its usual topology or with the discrete topology (cf.[11;22.22h]).

2.4. <u>LEMMA</u>. *Let* $\langle X,\pi\rangle$ *be an equicontinuous compact Hausdorff* G-*space. Then there exists an action* $\tilde{\pi}$ *of* bG *on* X *such that*

$$\pi(t,x) = \tilde{\pi}(\psi(t),x) \text{ for all } (t,x) \in G \times X,$$

that is, the action of G *on* X *can be extended to an action of the compact Hausdorff group* bG.

<u>PROOF</u>. By [8;4.5] or [6;Chap.10], the closure E(X) of the family $\{\pi^t : t \in G\}$ in X^X is a compact Hausdorff topological group such that δ: $(\xi,x) \mapsto \xi(x)$: $E(X) \times X \to X$ is a continuous action of E(X) on X. By the

universal property of the Bohr compactification there exists a continuous homomorphism $\phi'\colon bG \to E(X)$ such that $\phi'(\psi(t)) = \pi^t$ for every $t \in G$. Now put

$$\tilde{\pi}(\tau,x) := \delta(\phi'(\tau),x) \quad \text{for} \quad (\tau,x) \in bG \times X.$$

Then $\tilde{\pi}$ is a continuous action of bG on X, having the desired property. □

2.5. REMARK. A similar result holds for equicontinuous G-spaces $\langle X,\pi\rangle$ such that X is a Tychonov space and for every $x \in X$ the *orbit closure* $\overline{Gx}(:=\overline{\{tx : t \in G\}})$ is compact. Indeed, the proof of Theorem 7 in [7] shows, that also in this case E(X) is a compact Hausdorff topological group of continuous maps. Since E(X) is also equicontinuous on X, it follows that $\delta\colon (\xi,x) \mapsto \xi(x)\colon E(X) \times X \to X$ is a continuous action of E(X) on X. Hence the proof for this case can be completed as in the lemma above.

2.6. THEOREM. *Let* $\langle K,\alpha\rangle$ *be an equicontinuous MC* G*-space. Then* $\langle K,\alpha\rangle$ *is an extensor in* COMP^G *for the class of all closed equivariant embeddings* $i\colon \langle A,\pi\rangle \to \langle X,\pi\rangle$ *with* $\langle X,\pi\rangle$ *an equicontinuous compact Hausdorff* G*-space.*

PROOF. By 2.4, $\langle K,\alpha\rangle$, $\langle A,\pi\rangle$ and $\langle X,\pi\rangle$ may be considered as bG-spaces, and it is easily seen, that *continuous* mappings between these spaces are G-equivariant iff they are bG-equivariant ($\psi[G]$ is dense in bG). Now the theorem follows from 2.1. □

2.7. REMARK. Using 2.5 instead of 2.4, we obtain a slightly more general result: every equicontinuous MC G-space $\langle K,\alpha\rangle$ is an extensor in TOP^G for the class of all equivariant embeddings $i\colon \langle A,\pi\rangle \to \langle X,\pi\rangle$ such that A is compact and $\langle X,\pi\rangle$ is an equicontinuous Tychonov G-space in which all orbit closures are compact (we could also use lemma 2.13 below). Note, that this statement is related to 2.6 in the same way as (i) to (ii) in 1.12 above.

2.8. EXAMPLE. Let A_G denote the space of all continuous real valued functions on bG, endowed with the topology of uniform convergence, i.e. the topology, induced by the supremum norm on bG; in fact, $A_G = C_c(bG,\mathbb{R})$ (we use the symbol A_G in order to indicate the fact, that this space is in a natural way isometrically isomorphic with the space of almost periodic functions on G). According to Example 1.2(2) there is a continuous action ρ of bG on A_G. Since $\psi: G \to bG$ is a continuous homomorphism, this induces an action $\tilde{\rho}$ of G on bG, as follows:

$$\tilde{\rho}(t,f) := \rho(\psi(t),f) \quad \text{for} \quad (t,f) \in G \times A_G$$

(in particular, $\tilde{\rho}^t f(\xi) = f(\xi\psi(t))$ for $\xi \in bG$). In this way, a G-space $\langle A_G, \tilde{\rho}\rangle$ is defined. Since $\psi[G]$ is dense in bG, it is easily seen that for every $f \in A_G$ the orbit closure $X_f := \overline{\{\tilde{\rho}^t f : t \in G\}}$ equals the compact set $\rho_f[bG] = \{\rho^\tau f : \tau \in bG\}$ (continuous image of the compact group bG). Moreover, the action of G on A_G is isometric, hence equicontinuous.

We state two consequences of this (cf.2.7):

(i) If $\langle K,\alpha\rangle$ is an equicontinuous MC G-space, then $\langle K,\alpha\rangle$ is an extensor in TOP^G for the class of all equivariant embeddings of compact G-spaces into $\langle A_G, \tilde{\rho}\rangle$.

(ii) If K is a compact convex invariant subset of A_G, then there exists an equivariant continuous retraction of A_G onto K (indeed, $\langle K, \tilde{\rho}\rangle$ is an equicontinuous MC G-space).

In connection with these observations, it is useful to note, that for every compact invariant subset X of A_G the closed convex hull $\overline{co}\, X$ is also invariant and compact (use [5';Chap.I,§4,no 1]), so $\langle \overline{co} X, \tilde{\rho}\rangle$ is an equicontinuous MC G-space. In particular, we can take for X the orbit-closure of some $f \in A_G$. Clearly, X is non-trivial iff f is a non-constant function. Since there exist non-constant continuous real-valued functions on bG iff bG is non-trivial, this proves (i) ⇒ (ii) in the following proposition:

2.9. PROPOSITION. *The following assumptions about* G *are mutually equivalent*:

(i) bG *is non-trivial*;

(ii) *There exists a non-trivial equicontinuous MC* G-*space* $\langle K,\alpha\rangle$;

(iii) *There exists a non-trivial equicontinuous compact Hausdorff* G-*space*;

(iv) *There exists a non-trivial equicontinuous Tychonov* G-*space with compact orbit closures*.

PROOF. (i) ⇒ (ii): see the remarks above. The implications (ii) ⇒ (iii) and (iii) ⇒ (iv) are obviously valid. To prove (iv) ⇒ (i), observe that each non-trivial equicontinuous Tychonov G-space $\langle X,\pi\rangle$ with compact orbit closures can be seen as a bG-space (use 2.5), and it is almost obvious, that the closures of the G-orbits are just equal to the bG-orbits. Hence not all bG-orbits in X consist of one point, and therefore bG must contain more than one point. □

2.10. REMARK. Notice, that compact orbit-closures in an equicontinuous G-space are *minimal* [8; 4.4 and 2.5]. So the statements in 2.9 are also equivalent with:

(v) There exists a non-trivial equicontinuous minimal compact Hausdorff G-space.

It is in accordance with this, that every equicontinuous minimal compact Hausdorff space can, up to isomorphism, be obtained as bG/H for some closed subgroup H of bG.

The following result shows that the collection of equicontinuous MC G-spaces plays the role of the unit interval in topology:

2.11. PROPOSITION. *Let* $\langle X,\pi\rangle$ *be an equicontinuous compact Hausdorff* G*-space. Then the morphisms of* G*-spaces from* $\langle X,\pi\rangle$ *into equicontinuous MC* G*-spaces separate points and closed subsets of* X.

PROOF. By considering X as a bG-space, for every $f \in C(X)$ we have a continuous and equivariant mapping $\tilde{f}: x \mapsto f\circ\pi_x : X \to C_c(bG,\mathbb{R}) = A_G$. By an observation made in 2.8, the set $K_f := \overline{\mathrm{co}}\, \tilde{f}[X]$ is an invariant MC-subset of A_G. Thus we have (consider the bG-spaces as G-spaces) a morphism of G-spaces $\tilde{f}: \langle X,\pi\rangle \to \langle K_f,\tilde{\rho}\rangle$, where $\langle K_f,\tilde{\rho}\rangle$ is an equicontinuous MC G-space. If F is a closed subset of X and $x_0 \in X \sim F$, then there exists $f \in C(X)$ such that $f[F] = \{0\}$ and $f(x_0) = 1$, and

$$\|\tilde{f}(x_0)-\tilde{f}(x)\| \geq |\tilde{f}(x_0)(e)-\tilde{f}(x)(e)| = |f(x_0)-f(x)| = 1$$

for all $x \in F$, hence $\tilde{f}(x_0) \notin \overline{\tilde{f}[F]}$. □

2.12. REMARK. If $\langle X,\pi\rangle$ is a *metrizable* equicontinuous compact Hausdorff G-space, then it follows from 2.11 that there exists a countable collection of morphisms of G-spaces $f_i: \langle X,\pi\rangle \to \langle K_i,\alpha_i\rangle$ separating points and closed subsets of X, where each $\langle K_i,\alpha_i\rangle$ is an equicontinuous MC G-space.

The induced mapping $f\colon X \to \Pi_{i=1}^{\infty} K_i =: K$ is an embedding and is equivariant with respect to the coordinate-wise action of G on K:

$$\alpha^t(x_1,x_2,\ldots) := (\alpha_1^t(x_1),\alpha_2^t(x_2),\ldots) \text{ for } t \in G,(x_1,x_2,\ldots) \in K.$$

A straightforward argument shows, that the G-space $\langle K,\alpha\rangle$ is equicontinuous and that it is, in fact, an equicontinuous MC G-space (the countability of the collection $\{K_i\}$ is only used in order to assure that K is metrizable: an uncountable product of equicontinuous MC G-spaces is still an equicontinuous G-space $\langle K,\alpha\rangle$ with K compact and convex and each α^t affine!). Thus, *every metrizable equicontinuous compact Hausdorff* G-*space can equivariantly be embedded in an equicontinuous MC* G-*space*.
(This result could also be derived from Lemma 1.16 by considering all G-spaces under consideration as bG-spaces and observing that the action of G, induced on an MC bG-space is equicontinuous.)

The following result could be used for a generalization of Proposition 1.12: see 2.7. It has some interest in its own (see [21]).

2.13. PROPOSITION. *Every equicontinuous Tychonov* G-*space* $\langle X,\pi\rangle$ *with compact orbit closures can equivariantly be embedded in an equicontinuous compact Hausdorff* G-*space* $\langle\tilde{X},\tilde{\pi}\rangle$.

PROOF. By 2.5, we may consider $\langle X,\pi\rangle$ as a bG-space. By the results of [18], $\langle X,\pi\rangle$ can equivariantly be embedded in a compact Hausdorff bG-space $\langle\tilde{X},\tilde{\pi}\rangle$. Now consider $\tilde{X}$ as a G-space and observe that on $\tilde{X}$ the action of bG, hence the induced action of G, is equicontinuous. □

2.14. REMARK. If $\langle X,\pi\rangle$ is as in 2.13, then we may assume that $\tilde{X}$ has the same weight as X: $w(\tilde{X}) = w(X)$. This follows immediately from [18; Proposition 2.10] because we consider bG-spaces, and bG has countable Lindelöf degree. A similar reasoning shows, that also the *maximal* G-*compactification* $\beta_G\langle X,\pi\rangle$ is equicontinuous.

3. E-ADMISSIBLE SUBSETS

Again, we assume that, unless stated otherwise, G is an arbitrary topological group.

3.1. The following construction is standard in Topological Dynamics: see [8;4.20]. Let $\langle X,\pi\rangle$ be a compact Hausdorff G-space, and let $\mathcal{U}$ denote the (unique) uniformity for X. With coordinate wise action, G also acts on $X \times X$, and since each $\alpha \in \mathcal{U}$ is a subset of $X \times X$, the expression $G\alpha := \{(tx,ty) \mid t \in G \;\&\; (x,y) \in \alpha\}$ makes sense. Let

$$Q_X := \bigcap \{\overline{G\alpha} \mid \alpha \in \mathcal{U}\}.$$

Then Q_X is a closed invariant non-empty subset of $X \times X$, and in general Q_X is not an equivalence relation. Let E_X be the smallest closed invariant subset of $X \times X$ which is an equivalence relation and which contains Q_X. Then there exists a unique continuous action $\pi^{\#}$ of G on the quotient space X/E_X which makes the quotient mapping

$$q_X\colon X \to X/E_X =: X^{\#}$$

equivariant. It can be shown, that $\langle X^{\#},\pi^{\#}\rangle$ is an *equicontinuous* compact Hausdorff G-space, which is characterized by the following "universal" property: if $\phi\colon \langle X,\pi\rangle \to \langle Y,\sigma\rangle$ is a morphism of G-spaces, and $\langle Y,\sigma\rangle$ is an equicontinuous compact Hausdorff G-space, then ϕ factorizes over q_X, i.e. there exists a (unique) morphism of G-spaces $\phi^{\#}\colon \langle X^{\#},\pi^{\#}\rangle \to \langle Y,\sigma\rangle$ such that $\phi = \phi^{\#}\circ q_X$. This is the reason, that $q_X\colon \langle X,\pi\rangle \to \langle X^{\#},\pi^{\#}\rangle$ is called the *maximal equicontinuous factor of* $\langle X,\pi\rangle$ (cf. also [17;4.4.8]). It follows easily from the "universal property" of the maximal equicontinuous factor, that this construction is functorial. That is, if $\phi\colon \langle X,\pi\rangle \to \langle Y,\sigma\rangle$ is a morphism in COMP^G, then there is a unique morphism $\phi^{\#}\colon \langle X^{\#},\pi^{\#}\rangle \to \langle Y^{\#},\sigma^{\#}\rangle$ which is induced by ϕ in such a way that $\phi^{\#}\circ q_X = q_Y\circ\phi$.

3.2. Let $\langle X,\pi\rangle$ be a compact Hausdorff G-space. A closed invariant subset A of X will be called E-*admissible* whenever $E_A = E_X \cap (A\times A)$. Equivalently, if $i\colon \langle A,\pi|_{G\times A}\rangle \to \langle X,\pi\rangle$ is a closed equivariant embedding, then A is an E-admissible subset (and i is called an E-admissible embedding) iff the morphism of G-spaces $i^{\#}\colon \langle A^{\#},(\pi|_{G\times A})^{\#}\rangle \to \langle X^{\#},\pi^{\#}\rangle$, induced by i, is injective (hence a topological embedding: $A^{\#}$ and $X^{\#}$ are compact Hausdorff spaces).

(N.B. Here our usual notation ($\langle A,\pi\rangle$ instead of $\langle A,\pi|_{G\times A}\rangle$) would be misleading, for $(\pi|_{G\times A})^{\#}$ need not be the same as $\pi^{\#}|_{G\times A^{\#}}$. It is the same iff $i^{\#}$ is an embedding.)

3.3. EXAMPLES. The following characterization of Q_X is very convenient for the determination of Q_X and E_X in concrete examples. If $\langle X,\pi\rangle$ is a compact Hausdorff G-space, then for $(x,y) \in X \times X$ we have: $(x,y) \in Q_X$ iff there are nets $(x_\lambda,y_\lambda)_{\lambda\in\Lambda}$ in $X \times X$ and $(t_\lambda)_{\lambda\in\Lambda}$ in G such that $(x_\lambda,y_\lambda) \rightsquigarrow (x,y)$ in $X \times X$ and $(t_\lambda x_\lambda, t_\lambda y_\lambda) \rightsquigarrow (z,z)$ in $X \times X$ for some point (z,z) on the diagonal of $X \times X$.

1. Let $G := \mathbb{R}$ and let X be the unit disc in the plane. Let the action of $\mathbb{R}$ on X be such that the centre of the disc is an invariant point, the boundary rotates uniformly, and all other points spiral outwards (cf. Fig.1; for an exact description, we refer to [4]). Let A be the boundary of the disc. Then A is a closed invariant subset, and the action of $\mathbb{R}$ on A is equicontinuous. Hence $Q_A = E_A$ = diagonal in $A \times A$, and $A^{\#} = A$. On the other hand, $E_X = X \times X$, so $X^{\#}$ is a one-point space. It is clear, that A is *not* an E-admissible subset of X.
2. Consider the $\mathbb{R}$-space, depicted in Fig.2 below. Each of the one-point invariant subsets A and B is E-admissible, but their union is not E-admissible (indeed, $(A\cup B)^{\#} = A \cup B$ is a two-point space, but A and B are identified with each other in $X^{\#}$).

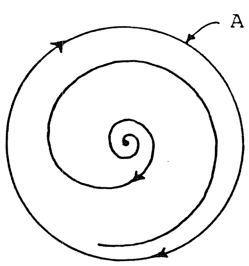

Fig. 1

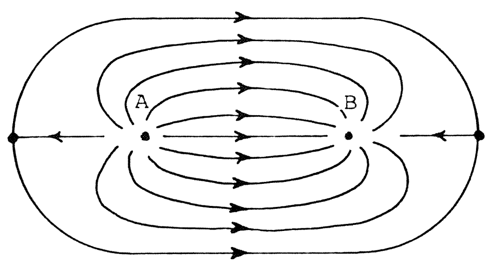

Fig. 2

3. In all cases that $E_A = A \times A$, hence $A^\#$ is trivial, it is clear that A is E-admissible. For conditions, guaranteeing that $A^\#$ is trivial, we refer to [9].
4. If $Q_A = Q_X \cap (A\times A)$ and $E_X = Q_X$ (see e.g. [22]), then also $Q_A = E_A$ and $i^\#$ is an embedding, so we have an E-admissible embedding.

3.4. In Topological Dynamics, the problem to characterize E-admissible sets has not yet been studied explicitly. The following characterization is easily derived from known facts. First, if $\langle X,\pi\rangle$ is a compact Hausdorff G-space, then recall that an element $f \in C(X)$ is called an *almost periodic function* (on X, with respect to the action π) whenever the set $\{f\circ\pi^t\}_{t\in G}$ of "translates" of f is relatively compact with respect to the uniform topology in C(X). Let us denote the set of all almost periodic functions on X by $A\langle X,\pi\rangle$. Then it is well-known, that $\langle X,\pi\rangle$ is equicontinuous iff $A\langle X,\pi\rangle = C(X)$ [8;4.15]. Using this, it is not too difficult to show, that for an arbitrary compact Hausdorff G-space $\langle X,\pi\rangle$ we have (see also [13])

$$A\langle X,\pi\rangle = \{f\circ q_X : f \in C(X^\#)\}.$$

3.6. PROPOSITION. *Let* $\langle X,\pi\rangle$ *be a compact Hausdorff* G-*space and let* A *be a closed invariant subset of* X. *The following conditions are equivalent:*

(i) A *is* E-*admissible*;

(ii) A *is* $A\langle X,\pi\rangle$-*embedded, that is, every almost periodic function on* A *can be extended to an almost periodic function on* X.

PROOF. (i) ⇒ (ii): Let $f: A \to \mathbb{R}$ be almost periodic. By the observation above, f factorizes over q_A, i.e. $f = f'\circ q_A$ with $f' \in C(A^\#)$. Since $A^\#$ is assumed to be a closed subset of $X^\#$, there exists $f'' \in C(X^\#)$ such that $f' = f''|_{A^\#}$ (indeed, $\mathbb{R}$ is an extensor in TOP for all closed embeddings into compact Hausdorff spaces). Now $f''\circ q_X$ is the desired (almost periodic!) extension of f'. See also the following diagram:

$$
\begin{tikzcd}
A \arrow[rr, "i"] \arrow[dd, "q_A"'] \arrow[dr, "f"] & & X \arrow[dd, "q_X"] \\
& \mathbb{R} & \\
A^{\#} \arrow[ur, "f'"] \arrow[rr, "i^{\#}"'] & & X^{\#} \arrow[ul, dashed, "f''"']
\end{tikzcd}
$$

(ii) $\Rightarrow$ (i): Suppose that $i^{\#}$ is not injective: there are points $x_1, x_2 \in A$ such that $q_A(x_1) \neq q_A(x_2)$ and $q_X(x_1) = q_X(x_2)$. Let $f \in C(A^{\#})$ be such that $f(q_A(x_1)) \neq f(q_A(x_2))$, and put $\bar{f} := f \circ q_A$. Then $\bar{f}$ is almost periodic on A, hence $\bar{f} = \bar{\bar{f}}|_A$ for some almost periodic function $\bar{\bar{f}}$ on X. Since $\bar{\bar{f}}(x_1) \neq f(x_2)$ and f factorizes over q_X, we derive, that $q_X(x_1) \neq q_X(x_2)$, contradicting the assumption. □

We come now to another characterization of E-admissible subsets, related to the problem of finding an extensor in COMP^G. First a lemma, which is a consequence of the "universal property" of the maximal equicontinuous factor.

3.7. <u>LEMMA</u>. *Let* $\langle K,\alpha\rangle$ *be an equicontinuous compact Hausdorff* G-*space* (*no further conditions on* K), *and let* $\phi\colon \langle X,\pi\rangle \to \langle Y,\sigma\rangle$ *be a morphism of* G-*spaces, where* X *and* Y *are compact Hausdorff spaces. The following statements are equivalent*:

(i) $\langle K,\alpha\rangle$ *is injective in* COMP^G *for* $\phi\colon \langle X,\pi\rangle \to \langle Y,\sigma\rangle$;

(ii) $\langle K,\alpha\rangle$ *is injective in* COMP^G *for* $\phi^{\#}\colon \langle X^{\#},\pi^{\#}\rangle \to \langle Y^{\#},\sigma^{\#}\rangle$.

<u>PROOF</u>. The straightforward proofs are illustrated by the following diagrams (compare the proof of (i) $\Rightarrow$ (ii) with the corresponding proof in (3.6):

(⇒):

$$\begin{tikzcd}
X \arrow[rr, "\phi"] \arrow[dd, "q_X"'] & & Y \arrow[dd, "q_Y"] \arrow[dl, dashed, "(\psi\circ q_X)'"'] \\
& K & \\
X^{\#} \arrow[ur, "\psi"] \arrow[rr, "\phi^{\#}"'] & & Y^{\#} \arrow[ul, dashed, "(\psi\circ q_X)'^{\#}"]
\end{tikzcd}$$

(ψ extends to $(\psi\circ q_X)'^{\#}$)

(⇐):

$$\begin{tikzcd}
X \arrow[rr, "\phi"] \arrow[dd, "q_X"'] \arrow[dr, "\psi"] & & Y \arrow[dd, "q_Y"] \\
& K & \\
X^{\#} \arrow[ur, dashed, "\psi^{\#}"] \arrow[rr, "\phi^{\#}"'] & & Y^{\#} \arrow[ul, dashed, "\psi^{\#'}"']
\end{tikzcd}$$

(ψ extends to $\psi^{\#'}\circ q_Y$) □

3.8. THEOREM. *Let* $i\colon \langle A,\pi\rangle \to \langle X,\pi\rangle$ *be a closed equivariant embedding, where* $\langle X,\pi\rangle$ *is a compact Hausdorff* G-*space. Then the following conditions are equivalent:*

(i) A *is an* E-*admissible subset of* X;

(ii) *Every equicontinuous MC* G-*space* $\langle K,\alpha\rangle$ *is an extensor in* COMP^G *for the equivariant embedding* i.

PROOF. (i) ⇒ (ii): Combine 3.7 with 2.6, and observe, that $i^{\#}$ is a closed equivariant embedding.

(ii) ⇒ (i): This proof is completely similar to the proof of (ii) ⇒ (i) in proposition 3.6 above (however, in 3.6 we used the fact that continuous real-valued functions on $A^{\#}$ separate the points of $A^{\#}$, but this has to be replaced by an application of proposition 2.11 above). □

3.9. We can reformulate the theorem as follows: let $\langle K,\alpha\rangle$ be an arbitrary equicontinuous MC G-space (for the existence of *non-trivial* such spaces, we refer to 2.9 above). Then $\langle K,\alpha\rangle$ is an extensor in COMP^G for the class of all E-admissible closed equivariant embeddings. Note, that as long as we require the MC G-space $\langle K,\alpha\rangle$ to be equicontinuous this result cannot be improved: if we consider a non-E-admissible closed equivariant embedding j in COMP^G, then some equicontinuous MC G-space $\langle K',\alpha'\rangle$ is not an extensor for j.

3.10. We close this section with a few remarks about the definition of E-admissible subsets of arbitrary Tychonov G-spaces. Of course, we want a definition for this concept such that the analogon of 3.8 remains valid (at least for compact equivariant embeddings into Tychonov G-spaces with compact orbit closures). The crucial question is, of course, how to define $\langle X^{\#},\pi^{\#}\rangle$ for an arbitrary Tychonov G-space. The construction of 3.1 will be worthless as long as we do not know which of the (not necessarily unique!) uniformities for X we have to choose!

A suitable approach would be as follows. Let $\langle X,\pi\rangle$ be a Tychonov G-space, and form its maximal G-compactification $\beta_G X$. (Observe, that there exists a canonical equivariant mapping of X into $\beta_G X$, but this may not be an embedding. Situations, where it is an embedding are mentioned in [21]. See also [18].) Then form for this compact Hausdorff G-space $\beta_G X$ in the way, described in 3.1 above, the maximal equicontinuous factor $(\beta_G X)^{\#}$. Now let $q_X\colon X \to X^{\#}$ be the canonical image of X in $(\beta_G X)^{\#}$. It is easily seen, that this construction is functorial, and now we can define E-admissibility completely similar to 3.2. It is also obvious, that 3.8 is valid in this setting, and we obtain even the analogon of 3.6 by replacing "almost periodic continuous function "by"almost periodic π-uniformly continuous function" (cf. [18] for the definition of π-uniform continuity and its relationship with the maximal G-compactification).

Two comments on the definition of $X^{\#}$. First, only in the case that $\langle X,\pi\rangle$ has *compact orbit closures* we can be sure, that if $\langle X,\pi\rangle$ is equicontinuous, then $X^{\#} = X$ (in that case, X can be considered as a subset of $\beta_G X$, and $\beta_G X$ is equicontinuous, so $(\beta_G X)^{\#} = \beta_G X$; cf.2.14). Second, it is easily seen that the mapping $q_X\colon X \to X^{\#}$ ($\langle X,\pi\rangle$ arbitrary Tychonov) has the following universal property: if $\phi\colon \langle X,\pi\rangle \to \langle Y,\sigma\rangle$ is a morphism of G-spaces and $\langle Y,\sigma\rangle$ is an *equicontinuous* Tychonov G-space with compact orbit closures, then ϕ factorizes over q_X (observe, that $\beta_G Y$ is equicontinuous in this case). This generalizes a result in [13].

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