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PROJECTIVE MONADS AND EXTENSIONS OF FUNCTORS

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Projective monads and extensions of functors

by

Jiří Vinárek *)

ABSTRACT

We present a class of monads with the property that any functor has an extension to the corresponding Kleisli category. These topics are connected with fuzzy-theories used for a categorical description of fuzzy-automata.

KEY WORDS & PHRASES: *set-functor, hom-functor, projective monad, Kleisli category, distributive laws, free semigroup, free monoid*

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0. INTRODUCTION

Category theory can be used to describe efficiently constructions in automata theory. It has led to a great development of studying automata using categorical methods (see e.g. [Ad], [AdTr], [ArM], [BuH], [G], [Th], [Tr1], [Tr2], etc.). Besides deterministic automata, also nondeterministic and fuzzy-automata are studied.

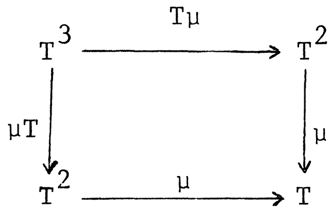
Whereas a deterministic automaton has a state-transition function $Q \times I \to Q$ (Q is a set of states, I- a set of inputs) a nondeterministic one has a state-transition function $Q \times I \to PQ$ (PQ is the powet-set of states) assigning to each state a set of possible successors. Of course, there is a canonical embedding of a set Q into its power-set PQ and there is also a natural transformation $P^2 \to P$. This situation was generalized by the definition of monad and its Kleisli category which is entirely coextensive with fuzzy-theories as is proved in [M].

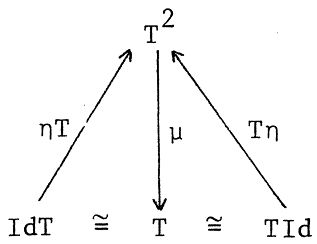
In [ArM], M.A. Arbib and E.G. Manes studied a problem when a functor $F: \underline{C} \to \underline{C}$ could be extended to the Kleisli category of a given monad. They found a sufficient and necessary condition for existence of such an extension. Their condition is analogous to the Beck distributive laws between monads (see [Be]). Therefore, the term "distributive laws" is used for these diagrams as well. It is, however, sometimes quite difficult to decide whether there exist distributive laws for a given monad and a given functor, even for some very natural monads. Such a very natural monad is the monad corresponding to the variety of monoids (i.e. semigroups with units). As was proved in [V1], this monad does not satisfy distributive laws with respect to the very "simple" functor Hom (2,-) assigning to each set X its square $X \times X$. M.A. Arbib and E.G. Manes proved in [ArM] that set-functors $- \times X$ have extensions to the Kleisli category of any monad. On the other hand, we present a class of monads (called projective monads) with the property that any functor has an extension to the corresponding Kleisli category. This general result is applied to the category of free semigroups. We also present a survey of results connected with extensions of functors to the category of free monoids.

1. PRELIMINARIES

First, recall some definitions and notations:

1.1. Let $\underline{C}$ be a category, $T: \underline{C} \to \underline{C}$ a functor, $Id: \underline{C} \to \underline{C}$ the identity functor, $\eta: Id \to T$, $\mu: T^2 \to T$ natural transformations. $\underline{T} = (T,\eta,\mu)$ is called a *monad* iff the following diagrams commute:



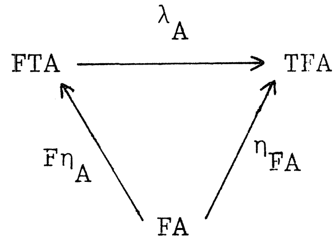


1.2. The *Kleisli category* $\underline{C}(\underline{T})$ is defined as follows: the class of its objects is obj $\underline{C}(\underline{T}) = \{X(\underline{T}) \mid X \in \text{obj}\underline{C}\}$; $f(\underline{T}): X(\underline{T}) \to Y(\underline{T})$ is a morphism in $\underline{C}(\underline{T})$ if $f: X \to TY$ is a morphism of $\underline{C}$. Given $f(\underline{T}): X(\underline{T}) \to Y(\underline{T})$, $g(\underline{T}): Y(\underline{T}) \to Z(\underline{T})$, the corresponding composition is $g(\underline{T}) * f(\underline{T}) = (\mu_z \circ Tg \circ f)(\underline{T})$.

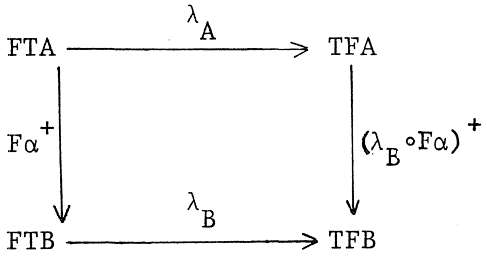
1.3. a) Denote Sem $=(S,i,s)$ the monad which assigns to each set A the semigroup of words created by A, i.e. the free semigroup over A. ($SA=\{a_1a_2\dots a_n \mid n\in\{1,2,\dots\},\ a_i \in A \text{ for } i = 1,2,\dots,n\}$, $i_A(a) = a$, $s_A((a_{11}\dots a_{1k(1)})\dots(a_{n1}\dots a_{nk(n)})) = a_{11}\dots a_{1k(1)}a_{21}\dots a_{n1}\dots a_{nk(n)}$ whenever $k(1),\dots,k(n)\geq 1$.)

b) Denote Mon $= (M,e,m)$ the monad which assigns to each set the monoid of words created by A, i.e. the free monoid over A. ($MA=SA\cup\{\Lambda\}$ where Λ is the empty word, $e_A(a) = a$, $m_A((a_{11}\dots a_{1k(1)})\dots(a_{n1}\dots a_{nk(n)})) = a_{11}\dots a_{1k(1)}a_{21}\dots a_{n1}\dots a_{nk(n)}$ whenever $k(1),\dots, k(n)\geq 0$.)

1.4. DEFINITION (Arbib-Manes, [ArM]). Let $\underline{C}$ be a category, $F: \underline{C} \to \underline{C}$ a functor, (T,η,μ) a monad, F is said to satisfy *distributive laws* over (T,η,μ) if to each object A of $\underline{C}$ a morphism λ_A: FTA $\to$ TFA is assigned such that the following two diagrams commute for any A and $\alpha: A \to TB$:

(1) 

(the first distributive law)

(2) 

where $\alpha^{+} = \mu_B \circ T\alpha$.

1.5. REMARK. A functor F can be extended to the Kleisli category $\underline{C}((T,\eta,\mu))$ iff it satisfies distributive laws.

PROOF. is given in [ArM].

1.6. a) Hom(A,-) denotes a functor which assigns to each set X the set of functors $\{f:A\to X\}$.

b) A *symmetric hom-functor* Sym(A,-) is a factorfunctor Hom(A,-)/~ where $f \sim g$ whenever there exists a bijection $b: A \to A$ such that $fb = g$.

1.7. REMARK. Symmetric hom-functors are a special case of tree-group varietors, investigated by V. Trnková and J. Adámek (see [TrAd]) which are the only superfinitary varietors for which the Kleene theorem holds. (A survey of results connected with varietors is given in [AdTr].)

2. DISTRIBUTIVE LAWS AS NATURAL TRANSFORMATIONS

In this section we present an equivalent characterization of distributive laws using the language of natural transformations between functors.

<u>2.1. THEOREM.</u> *A functor* $F\colon \underline{C} \to \underline{C}$ *satisfies distributive laws over a monad* $\underline{T} = (T,\eta,\mu)$ *iff there exists a natural transformation* $\lambda\colon FT \to TF$ *such that*

(i) $\lambda \circ F\eta = \eta F$

(ii) $\mu F \circ T\lambda \circ \lambda T = \lambda \circ F\mu$

<u>PROOF.</u>

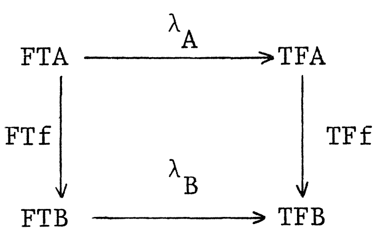
1. Suppose that F satisfies distributive laws over $\underline{T}$, and let $\{\lambda_A\colon FTA \to TFA \mid A \in \text{obj } \underline{C}\}$ be the system of morphisms from 1.4. Let $f\colon A \to B$ be a morphism in $\underline{C}$. According to 1.1 and (1), (2) from 1.4 we have

$$\lambda_B \circ FTf = \lambda_B \circ F\mu_B \circ FT\eta_B \circ FTf = \lambda_B \circ F((\eta_B \circ f)^+) =$$

$$= (\lambda_B \circ F(\eta_B \circ f))^+ \circ \lambda_A = \mu_{FB} \circ T\lambda_B \circ TF\eta_B \circ TFf \circ \lambda_A =$$

$$= \mu_{FB} \circ T\eta_{FB} \circ TFf \circ \lambda_A = TFf \circ \lambda_A.$$

Hence, the diagram



commutes for any morphism $f\colon A \to B$ and λ is a natural transformation.

2. Denote by id_{TA} the identity map on TA. By the definition of $^+$-operation, there is $\mu_A = id^+_{TA}$ and $\mu_{FA} \circ T\lambda_A = (\lambda_A \circ Fid_{TA})^+$. By (2), we have

$$\mu_{FA} \circ T\lambda_A \circ \lambda_{TA} = (\lambda_A \circ Fid_{TA})^+ \circ \lambda_{TA} = \lambda_A \circ Fid_{TA}^+ = \lambda_A \circ F\mu_A .$$

Hence, (ii) is satisfied.

3. On the other hand, suppose that λ: FT $\to$ TF is a natural transformation satisfying (i) and (ii). Since (i) is the first distributive law, it remains to check only the second distributive law. Let α: A $\to$ TB be a mapping. By (ii), there is $\lambda_B \circ F\mu_B = \mu_{FB} \circ T\lambda_B \circ \lambda_{TB}$. Since λ is a natural transformation, there is $\lambda_{TB} \circ FT\alpha = TF\alpha \circ \lambda_A$. Therefore,

$$(\lambda_B \; F\alpha)^+ \circ \lambda_A = \mu_{FB} \circ T\lambda_B \circ TF\alpha \circ \lambda_A = \mu_{FB} \circ T\lambda_B \circ \lambda_{TB} \circ FT\alpha =$$

$$= \lambda_B \circ F\mu_B \circ FT\alpha = \lambda_B \circ F\alpha^+ .$$

Hence, (2) is satisfied. □

2.2. COROLLARY. *Let* $\underline{C}$ *be a category,* F: $\underline{C} \to \underline{C}$, T: $\underline{C} \to \underline{C}$ *be functors,* $\underline{T}$ = (T,η,μ) *a monad. Then* F *has an extension to* $\underline{C}(\underline{T})$ *iff there exists a natural transformation* λ: FT $\to$ TF *satisfying (i) and (ii) for Theorem 2.1.*

3. PROJECTIVE MONADS

In this section we introduce the concept of projective monad and prove that every functor has an extension to the Kleisli category of a projective monad.

3.1. DEFINITION. A nonad $\underline{T}$ = (T,η,μ) is called *projective* if there exists a natural transformation π: T $\to$ Id such that $\pi \circ \eta$ is an identity transformation and $\pi \circ \mu = \pi \circ \pi T$. (We call such a transformation π a projection.)

3.2. REMARKS. 1. For any projective monad (T,η,μ) there is $T\emptyset = \emptyset$.

2. Since π is a natural transformation there is $\pi \circ T\pi = \pi \circ \pi T$.

3.3. EXAMPLE. Sem (see 1.3) is a projective monad.

PROOF. Define a natural transformation $p\colon T \to Id$ by $p_A(a_1 \dots a_n) = a_1$. For any set A and $a, a_{11}, \dots, a_{1k(1)}, \dots, a_{n1}, \dots, a_{nk(n)} \in A$ there is:

$$p_A i_A(a) = p_A(a) = a$$

$$p_A s_A((a_{11} \dots a_{1k(1)}) \dots (a_{n1} \dots a_{nk(n)})) = $$
$$= p_A(a_{11} \dots a_{1k(1)} \dots a_{n1} \dots s_{nk(n)}) = a_{11},$$

$$p_A p_{SA}((a_{11} \dots a_{1k(1)}) \dots (a_{n1} \dots a_{nk(n)})) = p_A(a_{11} \dots a_{1k(1)}) = a_{11}.$$

Hence, p is a projection. □

3.4. EXAMPLE. Let n be a positive integer; $(Hom(n,-), e^{(n)}, m^{(n)})$ is a monad of n-tuples defined as follows:

$$e_A^{(n)}(a) = (\underbrace{a, \dots, a}_{n}),$$

$$m_A^{(n)}((a_{11}, \dots, a_{1n}), \dots, (a_{n1}, \dots, a_{nn})) = (a_{11}, a_{22}, \dots, a_{nn}).$$

Define

$$p^{(n)}\colon Hom(n,-) \to Id \quad \text{by} \quad p_A^{(n)}(a_1, \dots, a_n) = a_1.$$

For any set A and $a, a_{11}, \dots, a_{1n}, \dots, a_{n1}, \dots, a_{nn} \in A$ there is

$$p_A^{(n)} e_A^{(n)}(a) = p_A^{(n)}(\underbrace{a, \dots, a}_{n}) = a,$$

$$p_A^{(n)} m_A^{(n)}((a_{11}, \dots, a_{1n}), \dots, (a_{n1}, \dots, a_{nn})) =$$
$$= p_A^{(n)}(a_{11}, a_{22}, \dots, a_{nn}) = a_{11},$$

$$p_A^{(n)} p_{Hom(n,A)}^{(n)}((a_{11}, \dots, a_{1n}), \dots, (a_{n1}, \dots, a_{nn})) =$$
$$= p_A^{(n)}(a_{11}, \dots, a_{1n}) = a_{11}.$$

Hence, $p^{(n)}$ is a projection. □

3.5. THEOREM. *Let* $\underline{C}$ *be a category,* $T\colon \underline{C} \to \underline{C}$ *a functor,* $\underline{T} = (T,\eta,\mu)$ *a projective monad (with the projection* π*). Then an arbitrary functor* $F\colon \underline{C} \to \underline{C}$ *satisfies distributive laws over* $\underline{T}$*.*

PROOF. Define a natural transformation $\lambda\colon FT \to TF$ by $\lambda = \eta F \circ F\pi$. Since $\underline{T}$ is projective there is $\lambda \circ F\eta = \eta F \circ F\pi \circ F\eta = \eta F$ and (i) from 2.1 holds. Since $\underline{T}$ is a monad there is

$$\mu F \circ T\lambda \circ \lambda T = \mu F \circ T\eta F \circ TF\pi \circ \eta FT \circ F\pi T = TF\pi \circ \eta FT \circ F\pi T.$$

Since η is a natural transformation, there is

$$TF\pi \circ \eta FT \circ F\pi T = \eta F \circ F\pi \circ F\pi T = \eta F \circ F\pi \circ F\mu = \lambda \circ F\mu$$

and (ii) from 2.1 holds as well.

According to 2.1, F satisfies distributive laws over $\underline{T}$. □

3.6. THEOREM. *Let* $\underline{C}$ *be a category,* $T\colon \underline{C} \to \underline{C}$ *a functor,* $\underline{T} = (T,\eta,\mu)$ *a projective monad. Then an arbitrary functor* $F\colon \underline{C} \to \underline{C}$ *has an extension to* $\underline{C}(\underline{T})$.

PROOF. follows from 1.5 and 3.5. □

3.7. COROLLARY. *An arbitrary set-functor has an extension to the Kleisli category over Sem, i.e. to the category of free semigroups.*

PROOF. follows from 3.3 and 3.6. □

4. CATEGORY OF FREE MONOIDS

While the previous section has solved the question of extensions of set-functors to the category of free semigroups completely and positively, the problem of extensions of set-functors to the category of free monoids seems to be more difficult. In this section, we summarize the known results in this area.

Throughout this section, N denotes the set of all positive integers, FM denotes the category of free monoids (i.e. the Kleisli category over Mon), $X \neq \emptyset$ will be a set and $f: X \to N$ a mapping.

4.1. THEOREM. *If* $f(x) = 1$ *for any* $x \in X$ *then* $\coprod_{x\in X} \mathrm{Hom}(f(x),-) \cong$ $\cong - \times X$ *has an extension to* FM.

PROOF. is given in [ArM]. □

4.2. THEOREM. *If* f *is bounded,* $\max\{f(x) \mid x \in X\} \geq 2$ *then* $\coprod_{x\in X} \mathrm{Hom}(f(x),-)$ *has no extension to* FM.

PROOF. is given [V1]. □

4.3. THEOREM. *If* f *is a mapping onto* $N \setminus \{1\}$ *then* $F = \coprod_{x\in X} \mathrm{Hom}(f(x),-)$ *has an extension to* FM.

PROOF. Let A be a set, $x \in X$, $f(x) = n > 1$. Define $\lambda_{Ax}: \mathrm{Hom}(f(x),MA) \to MFA$ by

$$\lambda_{Ax}(a_{11}\dots a_{1k(1)}, a_{21}\dots a_{2k(2)}, \dots, a_{n1}\dots a_{nk(n)}) = w$$

where

$$w = (a_{11}, a_{21}, a_{22}, \dots, a_{2k(2)}, \dots, a_{n1}, \dots, a_{nk(n)})(a_{12}, a_{21}, a_{22}, \dots, a_{2k(2)}, \dots, a_{n1}, \dots, a_{nk(n)}) \dots (a_{1k(1)}, a_{21}, a_{22}, \dots, a_{n1}, \dots, a_{nk(n)})$$

if $k(1) \cdot (k(2)+\dots+k(n)) > 0$, $w = \Lambda$ otherwise.

Then for any $x \in X$, $v \in \mathrm{Hom}(f(x), MA)$, put $\lambda_A(v) = \lambda_{Ax}(v)$. We have to check conditions (1), (2) from 1.4.

a) There is

$$\lambda_A F(e_A)\underbrace{(a_1,\dots,a_n)}_{\in FA} = \lambda_A\underbrace{(a_1,\dots,a_n)}_{\in FMA} = \underbrace{(a_1,\dots,a_n)}_{\in MFA} =$$
$$= e_{FA}(a_1,\dots,a_n).$$

b)

$$\begin{array}{ccc} FMA & \xrightarrow{\lambda_A} & MFA \\ {\scriptstyle F\alpha^+}\downarrow & & \downarrow{\scriptstyle (\lambda_B \circ F\alpha)^+} \\ FMB & \xrightarrow{\lambda_B} & MFB \end{array}$$

commutes for any $\alpha: A \to MB$ because

$$(\lambda_B \circ F\alpha)^+ \lambda_A (a_{11} \ldots a_{1k(1)}, a_{21} \ldots a_{2k(2)}, \ldots, a_{n1} \ldots a_{nk(n)}) =$$

$$= (\lambda_B \circ F\alpha)^+ ((a_{11}, a_{21}, \ldots, a_{2k(2)}, \ldots, a_{n1}, \ldots, a_{nk(n)}) (a_{12},$$

$$a_{21}, \ldots, a_{2k(2)}, \ldots, a_{n1}, \ldots, a_{nk(n)}) \ldots (a_{1k(1)}, a_{21},$$

$$\ldots, a_{2k(2)}, \ldots, a_{n1}, \ldots, a_{nk(n)})) = \mu_{FB} \circ M(\lambda_B)(F\alpha(a_{11},$$

$$a_{21}, \ldots, a_{2k(2)}, \ldots, a_{n1}, \ldots, a_{nk(n)}),\ F\alpha(a_{12}, a_{21}, \ldots,$$

$$a_{2k(2)}, \ldots, a_{n1}, \ldots, a_{nk(n)}), \ldots, F\alpha(a_{1k(1)}, a_{21}, \ldots,$$

$$a_{2k(2)}, \ldots, a_{n1}, \ldots, a_{nk(n)})) =$$

$$= \mu_{FB} \circ M(\lambda_B)((b_{111} \ldots b_{11r(1,1)}\ b_{211} \ldots b_{21r(2,1)}, \ldots,$$

$$b_{nk(n)1} \ldots b_{nk(n)r(n,k(n))})\ (b_{121} \ldots b_{12r(1,2)},$$

$$b_{211} \ldots b_{21r(2,1)}, \ldots, b_{nk(n)1} \ldots b_{nk(n)r(n,k(n))}) \ldots$$

$$\ldots (b_{1k(1)1} \ldots b_{1k(1)r(1,k(1))}, b_{211} \ldots b_{21r(2,1)}, \ldots,$$

$$b_{nk(n)1} \ldots b_{nk(n)r(n,k(n))})) =$$

$$= (b_{111}, b_{211}, b_{212}, \ldots, b_{21r(2,1)}, \ldots, b_{nk(n)1}, \ldots,$$

$$b_{nk(n)r(n,k(n))})(b_{112}, b_{211}, b_{212}, \ldots, b_{21r(2,1)}, \ldots,$$

$$b_{nk(n)1}, \ldots, b_{nk(n)r(n,k(n))}) \ldots (b_{11r(1,1)}, b_{211},$$

$$b_{212},\dots,b_{21r(2,1)},\dots,b_{nk(n)1},\dots,b_{nk(n)r(n,k(n))})\dots$$

$$\dots(b_{1k(1)r(1,k(1))},b_{211},b_{212},\dots,b_{21r(2,1)},$$

$$\dots,b_{nk(n)1},\dots,b_{nk(n)r(n,k(n))})$$

where $\alpha(a_{ij}) = b_{ij1}\dots b_{ijr(i,j)}$, and

$$\lambda_B\ F\alpha^+(a_{11}\dots a_{1k(1)},\dots,a_{n1}\dots a_{nk(n)}) =$$

$$= \lambda_B(b_{111}b_{112}\dots b_{1k(1)r(1,k(1))},$$

$$b_{211}b_{212}\dots b_{2k(2)r(2,k(2))},\dots,$$

$$b_{n11}b_{n12}\dots b_{nk(n)r(n,k(n))}) =$$

$$= (b_{111},b_{211},b_{212},\dots,b_{21r(2,1)},\dots,$$

$$b_{nk(n)1},\dots,b_{nk(n)r(n,k(n))})$$

$$(b_{112},b_{211},b_{212},\dots,b_{21r(2,1)},\dots,$$

$$b_{nk(n)1},\dots,b_{nk(n)r(n,k(n))})\dots$$

$$\dots(b_{11r(1,1)},b_{211},b_{212},\dots,b_{21r(2,1)},\dots,$$

$$b_{nk(n)1},\dots,b_{nk(n)r(n,k(n))})\dots$$

$$\dots(b_{1k(1)r(1,k(1))},b_{211},b_{212},\dots,$$

$$b_{21r(2,1)},\dots,b_{nk(n)1},\dots,b_{nk(n)r(n,k(n))}) =$$

$$= (\lambda_B \circ F\alpha)^+\ \lambda_A(a_{11}\dots a_{1k(1)},\dots,a_{n1}\dots a_{nk(n)}).$$

Obviously,

$$(\lambda_B \circ F\alpha)^+ \ \lambda_A(a_{11}\dots a_{1k(1)},\Lambda,\dots,\Lambda) = \Lambda =$$

$$= \lambda_B F\alpha^+(a_{11}\dots a_{1k(1)},\Lambda,\dots,\Lambda),$$

$$(\lambda_B \circ F\alpha)^+ \ \lambda_A(\Lambda,a_{21}\dots a_{2k(2)},\dots,a_{n1}\dots a_{nk(n)}) = \Lambda =$$

$$= \lambda_B F\alpha^+(\Lambda,a_{21}\dots a_{2k(2)},\dots,a_{n1}\dots a_{nk(n)}),$$

$$(\lambda_B \circ F\alpha)^+ \ \lambda_A(\Lambda,\dots,\Lambda) = \Lambda = \lambda_B F\alpha^+(\Lambda,\dots,\Lambda).$$

This finishes the proof. □

4.4. THEOREM. *If* f *is a mapping onto* N *then* $\coprod_{x\in X}$ Hom(f(x),-) *has an extension to* FM.

PROOF. Denote $Y = \{x \in X \mid f(x) = 1\}$, $Z = \{x \in X \mid f(x) > 1\}$. By Theorem 4.1, $\coprod_{x\in Y}$ Hom(f(x),-) has an extension to FM. By Theorem 4.3, $\coprod_{x\in Z}$ Hom(f(x),-) has an extension to FM. Hence, also $\coprod_{x\in X}$ Hom(f(x),-) has an extension to FM. □

4.5. THEOREM. $\coprod_{x\in X}$ Sym(f(x),-) *has an extension to* FM *if either* f *is a maping onto* N, *or* $f(x) = 1$ *for any* $x \in X$.

PROOF. is given in [V2].

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