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UNIQUENESS OF WHITTAKER-MODELS FOR
IRREDUCIBLE OBJECTS IN $Alg(Mp(k))$

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by

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ABSTRACT

We discuss several methods to prove the uniqueness of Whittaker-models for the metaplectic group and relate them to work of S. Gelbart and I. Pyateckii-Shapiro.

Introduction

Throughout this paper k denotes a finite extension of the p-adic field $\mathbf{Q}_p$. $\mathbf{O}$ its ring of integers and $\mathbf{p}$ the maximal ideal of $\mathbf{O}$.

In this paper I will first describe several methods to prove the uniqueness of Whittaker-models for irreducible algebraic representations of the metaplectic group $Mp(k)$ as defined in [5] or [3]. Next I will show that this uniqueness implies that of the models used in [2].

Recall that $Mp(k)$ is a central extension of $Sl(2,k)$ with $\mathbf{T} = \{z \mid z \in \mathbf{C}, |z| = 1\}$. Let $\mathbf{R}$: $Sl(2,k) \rightarrow Mp(k)$ be the section as defined in [5] or [3]. There one can also find the definition of the grouphomomorphism $\mathbf{R}_0$ from a certain open subgroup of $Sl(2,k)$ to $Mp(k)$. The notion of *algebraic* representation for closed subgroups H of $Mp(k)$ was introduced in [3]. It simply requires that any vector in the representation space is H_0-finite, for some open compact subgroup H_0 of H. This category is denoted by $Alg(H)$. There is a natural action of the Hecke-algebra $\mathbf{H}$ of $Mp(k)$ on these objects and they correspond exactly to the non-degenerate $\mathbf{H}$-modules.

Any (σ, V) in $Alg(\mathbf{H})$ can be induced to an algebraic representation of $Mp(k)$. Namely, let $S(\sigma)$ be the space of functions $f : Mp(k) \rightarrow V$ satisfying

(i) $f(hx) = \sigma(h)(f(x))$ for all $h \in H$ and $x \in Mp(k)$

(ii) $f(x \mathbf{R}_0(y)) = f(x)$ for all $x \in Mp(k)$ and all y belonging to some open compact subgroup of $Sl(2,k)$

(iii) Under right translations with elements of $\mathbf{T}, f$ is $\mathbf{T}$-finite and this action is continuous.

The action of $Mp(k)$ on $S(\sigma)$ by means of right translations is denoted by $Ind(\sigma)$. Now, let τ be a non-trivial character of k and (π, E) an object in $Alg(Mp(k))$. Recall that a *Whittaker model* of (π, E) *with respect to* τ is the image of a non-zero element in $Hom_{\mathbf{H}}(E, S(\tau))$. Hence our object will be to show

0.1 Theorem *For all irreducible (π, E) in $Alg(Mp(k))$ we have*

$$dim \ (Hom_{\mathbf{H}}(E, S(\tau))) \leq 1$$

As was shown in [3], this uniqueness for the principal series plays a role in the proof of the functional equation of the Eisenstein series. The example of the even Weil representation shows that a Whittaker-model does not have to exist (see theorem 14.8 in [3]). Recall from [3] that if one denotes for any (σ, V) in $Alg(Mp(k))$

$$\{v \mid v \in V, \text{there exists a } N \in \mathbb{N} \text{ such that } \int_{\mathbf{p}^{-N}} \tau(-x)\pi(\mathbf{R}\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix})(v)dx = 0\}$$

by $V(\tau)$ and $V / V(\tau)$ by V_τ, then theorem (0.1) is equivalent to

$$dim(E_\tau) \leq 1$$

We call (σ, V) in $Alg(Mp(k))$ *genuine* if

$$\sigma(t) = t^{2m+1}Id \quad \text{for all } t \in \mathbf{T}.$$

Non-genuine representations correspond to algebraic representations of $Sl(2,k)$ with a character of $\mathbf{T}$ pasted on it and in that case the theorem is well-known (see e.g [4]). In view of the results in [3] one only has to prove still the genuine quasi-cuspidal case and this is carried out in section 1. The second proof does not require the subdivision in several types of representations and is based on an idea sketched for $Gl(2,k)$ by R. Howe. It forms the content of section 2. Finally in the last section we give the correspondence with [2].

Notation

We will use the following abbreviations for certain elements of $Sl(2,k)$

$$u(x) = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}, x \in k \; ; d(a) = \begin{bmatrix} a & 0 \\ 0 & a^{-1} \end{bmatrix}, a \in k^*; w(b) = \begin{bmatrix} 0 & b^{-1} \\ b & 0 \end{bmatrix}, b \in k^*.$$

Unspecified notions and notations are as in [3].

First proof

1.0 In this section (π, E) denotes an irreducible quasi-cuspidal representation in $Alg(Mp(k))$. Recall that

for any (σ,V) in $Alg(H)$ we write $(\tilde{\sigma},\tilde{V})$ for the contragredient representation of (σ,V). The first property of E that we will need is

1.1 Lemma. $dim(E_\tau)=dim(\tilde{E}_{\bar{\tau}})$

Proof: Since E is quasi-cuspidal, we know from corollary 11.4 in [3] that $(\tilde{\pi},\tilde{E})\cong(\pi,E^1)$, where E^1 denotes the space E equiped with the complex-conjugate $\mathbb{C}$-module structure. Hence $Hom_H(E,S(\tau))\cong Hom_H(\tilde{E},S(\bar{\tau}))$.

1.2 Let H and $(\sigma,V)\in Alg(H)$ be as in the introduction. Denote the subspace of $S(\sigma)$, consisting of functions with compact support modulo H, by $\underline{S}(\sigma)$. This is an H-submodule of $S(\sigma)$ and instead of $Ind(\sigma)|\underline{S}(\sigma)$ we simply write $ind(\sigma)$. As in [1] one proves that

$$\widetilde{ind}(\sigma)\cong Ind(\Delta_H^{-1}\tilde{\sigma}),$$

with Δ_H the module of H.

1.3 From this last property and the fact that $\overset{\approx}{E}$ is isomorphic to E we conclude

$$Hom_H(E,S(\tau))\cong Hom_H(\underline{S}(\bar{\tau}),\tilde{E}).$$

Let A be any non-zero element of $Hom_H(\underline{S}(\bar{\tau}),\tilde{E})$. According to Lemma 1.1 there exists a non-zero B in $Hom_H(\underline{S}(\tau),E)$. They determine a non-trivial $Mp(k)$-invariant bilinear form β on $S(\tau)\times\underline{S}(\bar{\tau})$ by

$$\beta(f,g)=<B(f),A(g)> \tag{1.4}$$

Define $P_\tau\in Hom_H(\underline{S}(1),\underline{S}(\tau))$ by

$$P_\tau(f)(m)=\int_k \tau(-x)f(\mathbf{R}(u(x))m)dx.$$

By means of P_τ and $P_{\bar{\tau}}$, we can lift β to a $Mp(k)$-invariant bilinear form α on $\underline{S}(1)\times\underline{S}(1)$. Since the natural pairing between $\underline{S}(1)$ and $S(1)$ is given by

$$(f,g)\rightarrow \int_{Mp(k)} f(x)g(x)dx,$$

α induces a $D\in Hom_H(\underline{S}(1),S(1))$ such that

$$\alpha(f,h)=\int_{Mp(k)} f(x)D(h)(x)dx$$

for all f and h in $\underline{S}(1)$. If we denote the linear form $f\rightarrow D(f)(e)$ by T then this equality becomes

$$\alpha(f,h)=T(\int_{Mp(k)} f(x)ind(1)(x)(h)dx)=T(h\overset{v}{\star}f)$$

since the integral in the second member amounts to a finite sum.

Let $I(-1)$ be the lifting to $Mp(k)$ of the automorphism $Int(\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix})$ of $Sl(2,k)$, as given in [3]. I want to show now that for all f and h in $\underline{S}(1)$

$$\alpha(f,h\circ I(-1))=\alpha(h,f\circ I(-1)) \tag{1.5}$$

By combining this equality with the surjectivity of A, B and P_τ we get

$$f\in Ker(A\circ P_\tau)\Leftrightarrow f\circ I(-1)\in Ker(B\circ P_\tau).$$

Hence all non-zero elements of $Hom_H(\underline{S}(\tau),\tilde{E})$ have the same kernel, in other words

$$dim(Hom_H(\underline{S}(\tau),\tilde{E}))\leqslant 1.$$

1.6 Let ρ be the anti-automorphism $x\rightarrow I(-1)(x^{-1})$. Then equality (1.5) is a consequence of the invariance of T under ρ. Namely

$$\alpha(f,h\circ I(-1))=T((h\circ I(-1)\overset{v}{\star}f)$$

$$=T(\{f\circ I(-1))\star h\}\circ\rho)$$

$$= \alpha(h, fo I(-1))$$

The rest of this section will be devoted to the proof of this invariance.

Since α factorizes through P_τ and $P_{\bar\tau}$ and $\underline{S}(1)$ is spanned by elements of the form $f * h$, with f and h in $\underline{S}(1)$, equation 1.4 implies that

$$T(\delta_{R(u(x))} * g * \delta_{R(u(y))}) = \tau(-x - y)T(g) \tag{1.7}$$

for all $g \in \underline{S}(1)$ and $x, y \in k$. If we write $\rho(T)$ for the linear form $f \to T(fo\rho)$, then it is clear that $\rho(T)$ also satisfies (1.7).

Denote the open subset $N(k)\mathbf{R}(w(1))P(k)\mathbf{T}$ by U and write $S(U)$ for

$$\{f \mid f \in \underline{S}(1), \text{support of } f \text{ contained in } U\}$$

For $f \in S(U)$, we define $\nu(f): D(k) \to \mathbb{C}$ by

$$\nu(f)(R(d(b))) = \int\int_{k^2} \tau(x + y)f(\mathbf{R}(u(-x)w(1)d(b)u(-y)))dxdy$$

Now equation (1.7) implies that

$$Ker(\nu) \subseteq Ker(T) \cap Ker(\rho(T)) \cap S(U).$$

Furthermore it is a straightforward verification to show that for every f in $S(U), \nu(f) = \nu(fo\rho)$. Hence we may conclude

$$T \mid S(U) = \rho(T) \mid S(U)$$

Denote $T - \rho(T)$ by $\tilde{T}$. Let $c(g): D(k)\mathbf{T} \to \mathbb{C}$, for $g \in \underline{S}(1)$, be defined by

$$c(g)(t\mathbf{R}(d(b))) = \int_k g(t\mathbf{R}(u(-x)d(b)))\tau(x)dx \tag{1.8}$$

As before, property (1.7) implies that $\tilde{T}(g) = 0$ for all $g \in \underline{S}(1)$ such that $c(g) = 0$. Since (π, E) is irreducible, there exists an $m \in \mathbb{Z}$ such that

$$\pi(t) = t^m Id = \chi_m(t)Id. \quad \text{for all } t \in \mathbf{T}.$$

From the definition of T one deduces directly that $T(g * \chi_{-m}) = T(g)$. Thus $\tilde{T}$ can be regarded as a linear form on $\{c(g) * \chi_{-m} \mid g \in \underline{S}(1)\}$. However formula (1.8) implies that for all $y \in k$

$$c(g * \delta_{\mathbf{R}(u(y))}) * \chi_{-m}(\mathbf{R}(d(b))) = \tau(b^2 y)c(g) * \chi_{-m}(\mathbf{R}(d(b))).$$

By combining (1.7) and (1.9) one deduces that for $c(g) * \chi_{-m}$ with support contained in a sufficiently small neighbourhood of $\mathbf{R}(d(b)), b^2 \neq 1, \tilde{T}(c(g) * \chi_{-m}) = 0$. Hence there are λ and μ in $\mathbb{C}$ such that

$$\tilde{T}(g * \chi_{-m}) = \lambda c(g) * \chi_{-m}(\mathbf{R}(d(-1))) + \mu c(g) * \chi_{-m}(e)$$

for all $g \in \underline{S}(1)$. This equality implies however that $\tilde{T}(go\rho) = \tilde{T}(g)$ for all $g \in \underline{S}(1)$. On the other hand, $\tilde{T}(go\rho) = (\overline{\rho(T)} - T)(g) = -\tilde{T}(g)$. Conclusion $\tilde{T} = 0$.

Second proof

2.1 In this section (π, E) denotes an irreducible object in $Alg(Mp(k))$. For each $a \in k^*$ and $W \in Hom_\mathbf{H}(E, S(\tau))$ one constructs as follows $a W_{a^2}$ in $Hom_\mathbf{H}(E, S(\tau_{a^2}))$

$$W_{a^2}(v)(g) = W(v)(R(d(a))g),$$

where $v \in E$ and $g \in Mp(k)$. Hence we may assume that the conductor $\mathbf{p}^{m_0}$ of τ is contained in $\mathbf{O}$. Assume that $dim(E_\tau) \geq 2$. Then there are v_1 and v_2 in E which are linear independent modulo $E(\tau)$. For a sufficiently large $m \in \mathbb{N}$ we have

$$\pi(\mathbf{R}(d(a)))(v_i) = v_i,$$

for all $a \in 1 + \mathbf{p}^{m + m_0 - \nu(2)}$ and $i \in \{1.2\}$. Recall that for $m \gg 0$ one has

$$a^2 \in \{1 + \mathbf{p}^{m + m_0}\} \Leftrightarrow a \in \pm\{1 + p^{m + m_0 - \nu(2)}\}$$

We will assume from now on that m has been chosen so large that both properties hold. For such a m, define $\pi_m: E \to E$ by

$$\pi_m(v) = \int_{\mathbf{p}^{-m}} \tau(-x)\pi(\mathbf{R}(u(x)))(v)dx,$$

where $\int_{\mathbf{p}^{-m}} dx = 1$. Now the following points are clear

(2.2) (i) $\pi_m(v_1)$ and $\pi_m(v_2)$ are linear independent

(ii) $\pi(\mathbf{R}(d(a)u(x)))\pi_m(v_i) = \tau(x)\pi_m(v_i)$ for all $a \in \{1 + \mathbf{p}^{m+m_o-\nu(2)}\}, x \in \mathbf{p}^{-m}$ and $i \in \{1,2\}$.

I write P_m for the group $\{\mathbf{R}(d(a)u(x)) | x \in \mathbf{p}^{-m}, a \in 1 + \mathbf{p}^{m+m_0-\nu(2)}\}$ and τ_m for its character $\mathbf{R}(d(a)u(x)) \to \tau(x)$. Clearly $\pi_m = \pi(\tau_m)$ and $\pi_m(E)$ is a $\tau_m \star H \star \tau_m$ - module. Since E is irreducible we have moreover

2.3 Lemma $\pi_m(E)$ is an irreducible $\tau_m \star H \star \tau_m$ - module

Proof: For every non-zero v in $\pi_m(E), \pi(H)(v) = E$ and hence $\pi_m(\pi(H)(v)) = \pi(\tau_m \star H \star \tau_m)(v) = \pi_m(E)$.

2.4 If we can prove that $\tau_m \star H \star \tau_m$ is commutative then non-degenerate irreducible $\tau_m \star H \star \tau_m$ - modules are one-dimensional and lemma 2.3 furnishes a contradiction with (2.2). In other words, $\dim(E_\tau) \leqslant 1$.

2.5 The commutativity of $\tau_m \star H \star \tau_m$ will be a consequence of its invariance under a suitable chosen anti-automorphism of $Mp(k)$. Let $\sigma \to Mp(k) \to Mp(k)$ be defined by

$$\sigma(t\,\mathbf{R}\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = t\,\mathbf{R}\begin{bmatrix} d & b \\ c & a \end{bmatrix}$$

Using proposition 1.11 from [3], one verifies that σ is an anti-automorphism of $Mp(k)$. Now we are left to prove

2.6 Theorem All $f \in \tau_m \star H \star \tau_m$ are invariant under $\sigma, i.e. f \circ \sigma = f$

Proof Let f belong to $\tau_m \star H \star \tau_m$. Then it is clearly sufficient to show that for each g in $Mp(k)$ one of the two following situations occurs

(α) $f(g) = 0$

(β) There is a $g^1 \in P_m g P_m$ such that $\sigma(g^1) = g^1$.

First we take g of the form $t\,\mathbf{R}(u(x)d(b))$. For each $z \in \mathbf{p}^{-m}$ such that $b^2 z \in \mathbf{p}^{-m}$ there must hold

$$\tau(z)f(t\,\mathbf{R}(u(x)d(b))) = \tau(zb^2)f(t\,\mathbf{R}(u(x)d(b))).$$

Hence, if we can find a z such that $\tau(zb^2) \neq \tau(z)$, then case (α) applies. It is a straightforward verification to show that this happens only if $b^2 \notin 1 + \mathbf{p}^{m+m_0}$. If $b^2 \in 1 + \mathbf{p}^{m+m_0}$, then $b \in \pm\{1 + \mathbf{p}^{m+m_0-\nu(2)}\}$ and we can choose as a representative of $P_m g P_m, t\,\mathbf{R}(u(x))$ or $t\,\mathbf{R}(u(x)d(-1))$, which are both invariant under σ. Next we take g of the form $t\,\mathbf{R}(u(x)w(b)u(y))$. Since

$$\mathbf{R}(d(c))g\,\mathbf{R}(d(a)) = t\,\mathbf{R}(u(xc^2)w(bc^{-1}a)u(ya^{-2}))$$

we are in case (β) if there is a $c \in 1 + \mathbf{p}^{m+m_0-\nu(2)}$ such that $c^2 x = y$ modulo $\mathbf{p}^{-m}$. Assume now that this is not the case. Applying the same formula with $a = c$ we get

$$\tau((c^2-1)(x - c^{-2}y))f(g) = f(g)$$

for all $c \in 1 + \mathbf{p}^{m+m_0-\nu(2)}$ such that $(c^2-1)x$ and $(c^2-1)y \in \mathbf{p}^{-m}$. By symmetry we may assume that $|x| \geqslant |y|$. If $x \in \mathbf{p}^{-m}$ then we can choose the representative $t\,\mathbf{R}(w(b))$ and we are in case (β). Hence we assume that $x \notin \mathbf{p}^{-m}$ If $|x| > |y|$, we can pick out a c such that $\nu((1-c^2)x) = m_0 - 1$ and $\tau((c^2-1)x) \neq 1$. Then $\tau((c^2-1)(x - c^{-2}y)) = \tau((c^2-1)x)$ and we are in case (α). For $|x| = |y|$, we consider first the case $\nu(x) \geqslant -2m - m_0$. In that case there are no restrictions and moreover $\tau((c^2-1)(x - c^{-2}y)) = \tau((c^2-1)(x - y))$. Hence if $(c^2-1)(x - y) \in \mathbf{p}^{m_0}$, for all $c \in \{1 + \mathbf{p}^{m_0+m-\nu(2)}\}$ then $x - y \in \mathbf{p}^{-m}$ and that is in contradiction with our assumptions. Finally if $\nu(x) < -2m - m_0$ then one chooses c such that $c = 1 + \alpha t$, with $\alpha \in \mathbf{O}^*$ and $\nu(tx) = -m$. Then $\tau((c^2-1)x - c^{-2}y) = \tau(\alpha tx(1 - yx^{-1}))$ and if this is equal to one for all $\alpha \in \mathbf{O}^*$ the element $1 - yx^{-1}$ has to belong to $1 + \mathbf{p}^{m_0-\nu(tx)}$ and this contradicts again our assumptions.

The connection with [2]

3.1 Using the formulas on page 11 of [3], one checks that $a \to I(a)$ is a grouphomomorphism of k^* to $Aut(Mp(k))$. Let $\underline{M}$ be the semi-direct product of k^* and $Mp(k)$ defined by it and write $\underline{a}$ for (a,e) in

$\underline{M}$, with $a \in k^{*}$. Analogously to [3] one defines for representations of closed subgroups of $\underline{M}$ the notions of algebraic, irreducible etc. The group $\underline{M}$ is isomorphic to a central extension of $Gl(2,\overline{k})$ by $\mathbf{T}$. The two-fold covering of $Gl(2,k)$ used in [2] can be embedded into $\underline{M}$ as a closed subgroup $\underline{G}$. Each irreducible algebraic representation of $\underline{G}$ can be extended to one of $\underline{M}$ by simply pasting on a suitable character of $\mathbf{T}$. Thus their notion of genuine corresponds to ours. Since the special Whittaker models in [2] only concern genuine representations, we will consider from now on, without furher mentioning only genuine representations.

3.2 Let (π, V) be an irreducible object in $Alg(Mp(k))$. We will indicate now how one can construct starting with (π, V), an irreducible object in $Alg(\underline{M})$. Let M_2 be $\{a^2 g \mid a \in k^{*}, g \in Mp(k)\}$.
Note that the center of M_2 is equal to $\{a^2 \underline{R}(d(a^{-1}))t \mid t \in \mathbf{T}$ and $a \in k^{*}\}$. Take any quasi-character χ of k^{*} such that $\chi(-1) = 1$. Since for all $a \in k^{*}$ and $g \in Mp(k)$

$$\pi(R(d(a^{-1}))gR(d(a))) = \pi(I(a^{-2})(g))$$

we can define a representation π_{χ} of M_2 by

$$\pi_{\chi}(\underline{a}^2 g) = \chi(a)\pi(R(d(a))g)$$

Moreover all irreducible objects in $Alg(M_2)$ are obtained in this way. By "inducing" π_{χ} to $\underline{M}$ we get the representation $Ind(\pi_{\chi})$ of $\underline{M}$, by right translations in the space

$$\mathbf{S}(\pi_{\chi}) = \{f \mid f : \underline{M} \to V, f(mg) = \pi_{\chi}(m)(f(g)), \text{for all } m \in M_2 \text{ and } g \in \underline{M}\}.$$

Let $\{t_i\}_{i \in I}$ be a set of representatives of $k^{*}/_{(k^{\cdot})^2}$. From 4.6 in [3] one sees that

$$\pi_{\chi} \circ Int(\underline{t_i})(\underline{a}^2 R(d(a^{-1}))) = \chi(a)(a, t_i) Id$$

Hence the $\pi_{\chi} \circ Int(t_i), i \in I$, are mutually inequivalent. Since moreover $Ind(\pi_{\chi}) \mid M_2 \cong \underset{i \in I}{\oplus} \pi_{\chi} \circ Int(t_i)$, we may conclude that $Ind(\pi_{\chi})$ is irreducible. Conversely, let (σ, E) be an irreducible representation of $\underline{M}$. Take any irreducible M_2 - submodule (ρ, V) of E. For every $i \in I, M_2$ acts on $\sigma(t_i)(V)$ according to $\rho \circ Int(\underline{t_i}^{-1})$. Consequently, $E = \underset{i \in I}{\oplus} \sigma(t_i)(V)$ and (σ, E) is isomorphic to $(Ind(\rho), \mathbf{S}(\rho))$. In other words we have shown the following

3.3 Proposition: *Any irreducible genuine object (σ, E) in $Alg(\underline{M})$ is isomorphic to some $(Ind(\rho), \mathbf{S}(\rho))$, with (ρ, V) an irreducible genuine object in $Alg(M_2)$.*

3.4 Let E and V be as in 3.3. Then it is clear that

$$E_{\tau} = \underset{i \in I}{\oplus} (\sigma(t_i)(V))_{\tau}.$$

Hence one cannot have the uniqueness of Whittaker-models in the ordinary sense. Note that there is a natural action of $\{a^2 R(d(a^{-1})) \mid a \in k^{*}\}$ on E_{τ}. From the foregoing we know that this group acts according to the-quasi-characters

$$a \to \chi(a)(a, t_i), \quad i \in I.$$

Hence the subspace of E_{τ}, consisting of those elements on which $\{a^2 R(d(a^{-1}))\}$ acts according to a specified character, has dimension one or zero. This is the uniqueness result of [2].

3.5 Remark From the foregoing it will be clear that it would also have been sufficient to show that one has uniqueness for irreducible genuine quasi-cuspidal representations of M_2. For such a representation one has a bilinear form β as in (1.4) and by making use of the method described in §6 of [1] one can prove that

$$\beta(f, g) = \beta(g \circ \rho, f \circ \rho) \tag{3.6}$$

where ρ is the composition of $I(-1)$ and the automorphism $a^2 g \to a^{-2} g$, $g \in Mp(k)$, of M_2. Clearly (3.6) implies the desired result. However, the details would require more space than the methods described here.

References

[1] Bernshtein, I.N & A.V. Zelevinsky, *Representations of the group Gl(n,F)*, where F is a nonarchimedean local field, Russian Mathematical Surveys 31 (1976), pages 1-64.

[2] Gelbart, S & I. Pyateckii-Shapiro, *Automorphic L-functions of half-integral weight*, Proc. Nat. Acad. Sci. U.S.A.75 (1978), pages 1620-1623.

[3] Helminck, G.F, *Eisenstein series on the metaplectic group. An algebraic approach.* Mathematical Centre Tract 161 (1983).

[4] Rodier, F, *Whittaker models for admissible representations of reductive p-adic split groups*, Proceedings of Symposia in Pure Mathematics, vol 26, pages 425-430.

[5] Weil, A., *Sur certains groupes d'opérateurs unitaires*, Acta Mathematica 111 (1964), pages 143-211.