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H.E. de Swart

Spectral modelling of a potential vorticity equation for a barotropic flow on a beta-plane

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SPECTRAL MODELLING OF A POTENTIAL VORTICITY EQUATION FOR A BAROTROPIC FLOW ON A BETA-PLANE

H.E. DE SWART
Centre for Mathematics and Computer Science, Amsterdam

A quasi-geostrophic potential vorticity equation, including forcing - and dissipation mechanisms, is derived for a barotropic flow over a large scale topography on a β-plane. The model is assumed to describe large scale motions in the atmosphere. Finally, from the vorticity equation a spectral model is constructed.

1980 MATHEMATICS SUBJECT CLASSIFICATION: 34B25, 35A35, 76C15, 76D30, 86A10
KEY WORDS & PHRASES: scale analysis, hydrostatic balance, Rossby number, quasi-geostrophy, beta-plane, barotropic assumption, Ekman boundary layer, orography, spectral model.

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1. INTRODUCTION

It is well known that the atmospheric circulation is driven by the inhomogeneous radiation input of the sun. Globally, there is a radiation surplus in the tropical areas and a radiation deficit near the poles. As can be seen from figure 1 these differences create a meridional temperature gradient in the midlatitudes.

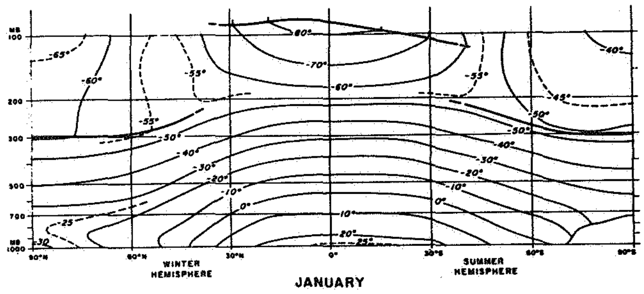

Figure 1. Mean temperature (^{o}C) averaged around latitude circles for January. The heavier lines show approximate tropopauses. From Palmén & Newton (1969).

It is remarkable that the gradient is not present in the tropics. This indicates that the dynamics of the tropical and midlatitude areas have to be treated differently.

In the tropics strong heating of the lower layers of air causes a direct convective thermal circulation: warm air is ascending at the equator; it cools, spreads out and after descending near the 30^{o} latitudes it flows back to the equator (figure 2).

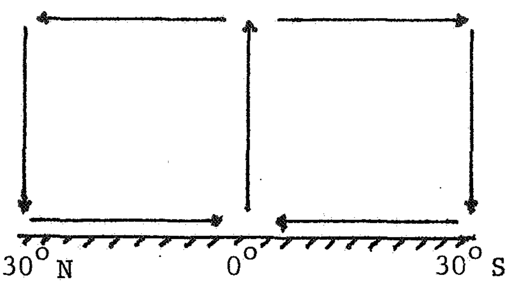

Figure 2. The Hadley circulation.

This is known as the Hadley circulation. Due to the ascending motions at the equator condensation takes place, resulting in cumulus clouds and rainfall. In regions of descending motions the desert circles occur.

The midlatitude dynamics is fundamentally different. The meridional temperature gradient causes slopes of the presssure level, since for an atmosphere satisfying the ideal gas law the height of such a level is proportional to the mean temperature in the layer between that level and the surface of the earth (Holton, 1979). The resulting pressure gradient force tends to move the air polewards. However, globally a balance will be established with the coriolis force, resulting in quasi-horizontal zonal western winds. This so-called geostrophic balance applies to a flow outside the frictional boundary layer, which is situated near the earth's surface.

From a daily weathermap it can immediately be seen that this flow is not zonal symmetric; it has a wavelike structure in which several length scales are present. In the first place we have the planetary waves, with a typical length scale of 10000 km. These semi-permanent structures are forced by the thermal differences between land and oceans and by the large scale topographic variations (called the orography). It appears that this flow is unstable, i.e. small disturbances may increase their amplitudes, and withdraw in this way energy from the basic flow. These disturbances are called the transient eddies; they appear on the weathermap as high- and low pressure cells. These shorter waves (typical length scale 1000 km) propagate much faster (10-to 20 ms^{-1}) than the longer ones (0-to 5ms^{-1}); their positions are rather unpredictable.

Model studies of Frederiksen (1978, 1979, 1980, 1982, 1983 a/b) and Niehaus (1980) demonstrate that the positions of the planetary waves largely determine the development of the transient eddies. There is also a reversal effect, as argued by e.g. Gall et.al. (1979), Sanders & Gyakum (1980), Opsteegh & Vernekar (1982) and Hoskins et.al. (1983), i.e. the transient eddies are capable of forcing and altering the planetary waves.

Apart from thermal and orographic forcing, next to forcing due to the transient eddies, it is suggested by e.g. Charney & Devore (1979) that the large scale flow is also driven by interactions between mutual ultralong wave components, which have different length scales. This means that the dynamics are essentially nonlinear, and, as a consequence, it follows that the flow must have several equilibrium states.

The aim of this study is to construct a simple nonlinear model of the planetary scale atmospheric circulation at midlatitudes, which has multiple equilibria. Starting-point is a quasi-geostrophic potential vorticity equation for a barotropic atmosphere on a β-plane (Charney, 1973). In sections 2 and 3 we give a derivation of this equation, following Pedlosky (1979). This result is analyzed in section 4 by means of a spectral method, in which the streamfunction describing the flow is developed in a Fourier series. The result is an infinite number of nonlinear ordinary differential equations. By truncation of the series the model reduces to a finite set of equations, as is for instance presented by Charney & Devore (1979). Such a system will by analyzed in a subsequent report.

2. CONSTRUCTION OF A QUASI-GEOSTROPHIC MODEL

In this section a simple nonlinear model of the large scale atmospheric flow will be formulated. It will include the most relevant physical processes. We will make some assumptions which make the model less suitable for describing planetary waves. However, our primary aim is to get insight in the basic nonlinear properties of atmospheric systems.

In general, the state of the fluid is determined by its velocity components (3), density, temperature, pressure and specific humidity, the latter being the mass of water vapour per unit mass of moist air. Consequently, seven equations are needed to describe the fluid. They follow from physical laws: conservation of mass gives the continuity equation, the second law of Newton results in three momentum equations and from thermodynamical considerations, it follows an equation of state and a thermal energy balance. The model is completed by a balance for the specific humidity. A systematic derivation of these basic equations can be found in Gill (1982).

Following the method of Pedlosky (1979), we will, for the moment, only be concerned with the continuity- and momentum equations, being

(2.1) $$\frac{d\rho}{dt} + \rho\vec{\nabla}\cdot\vec{u} = 0,$$

(2.2) $$\frac{d\vec{u}}{dt} + 2\vec{\Omega} \times \vec{u} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g}$$

Here ρ is the density, t is time, $\vec{u} = (u,v,w)$ a threedimensional velocity vector, $\vec{\Omega}$ the angular velocity vector of the earth, p the pressure and $\vec{g}$ the acceleration due to gravity.

Specifying the coordinate system, we note that the shape of the earth differs only slightly from a sphere, so it seems useful to introduce spherical coordinates λ,ϕ and r, which are the longitude, latitude and distance to the centre of the earth respectively, as shown in figure 3.

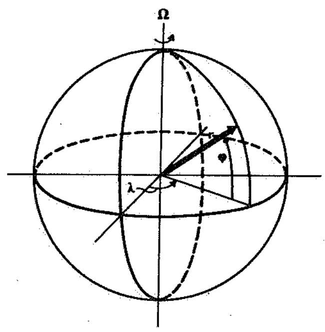

Figure 3. Spherical coordinates for the earth.

The continuity - and momentum equations are

(2.3) $$\frac{d\rho}{dt} + \rho \left\{ \frac{1}{r\cos\phi}\frac{\partial u}{\partial \lambda} - \frac{v}{r}\tan\phi + \frac{1}{r}\frac{\partial v}{\partial \phi} + \frac{\partial w}{\partial r} + \frac{2w}{r} \right\} = 0,$$

(2.4) $$\frac{du}{dt} + \frac{uw}{r} - \frac{uv}{r}\tan\phi - 2\Omega\sin\phi\ v + 2\Omega\cos\phi\ w = -\frac{1}{\rho}\frac{1}{r\cos\phi}\frac{\partial p}{\partial \lambda},$$

(2.5) $$\frac{dv}{dt} + \frac{vw}{r} + \frac{u^2}{r}\tan\phi + 2\Omega\sin\phi\ u = -\frac{1}{\rho r}\frac{\partial p}{\partial \phi},$$

(2.6) $$\frac{dw}{dt} - \frac{u^2+v^2}{r} - 2\Omega\cos\phi\ u = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g,$$

where

(2.7) $$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{u}{r\cos\phi}\frac{\partial}{\partial \lambda} + \frac{v}{r}\frac{\partial}{\partial \phi} + w\frac{\partial}{\partial r}.$$

To this set of equations scale analysis will be applied, as it can not be solved without a priori knowledge about the particular motion to be considered. Scale analysis yields, by means of physical arguments, characteristic scales for the flow. Then the continuity - and momentum equations, written in a nondimensional form, will contain several nondimensional parameters indicating the relative strength of contributions in the dimensional equations. The intention of the method is to find small parameters. Then, by means of standard perturbation techniques, it is possible to derive simplified equations, which are expected to describe the type of motion under consideration. Since this formal procedure cannot be made rigorous, the original assumptions must be verified in the final result to see that no inconsistency occurs.

To apply scale analysis to a large scale motion, we first have to specify its form. Consider a flow, having a characteristic vertical length scale H, a horizontal length scale L, and a velocity scale U. It is assumed that variations of the acceleration due to gravity through the depth of the fluid may be neglected. Large scale motions are supposed to be quasi-horizontal and are significantly influenced by the coriolis force. The first condition requires that H is much smaller than L, the second means that the advective time scale L/U must be much larger than the period $2\pi\Omega^{-1}$ of the rotation of the earth. Thus, it is assumed that

$$(2.8) \qquad \frac{H}{L} \ll 1, \qquad\qquad \frac{U}{2\Omega L} \ll 1.$$

The motion occurs around some central latitude ϕ_0. We introduce new coordinates

$$(2.9) \qquad x = r_0 \cos\phi_0 \, \lambda; \quad y = r_0(\phi-\phi_0); \quad z = r - r_0,$$

where x and y are simple new longitude - and latitude coordinates, and z is the distance to the surface of the earth. The reason for doing this is that, at a later stage, (x,y,z) may be interpreted as carthesian coordinates. By means of (2.9) the derivatives in the continuity - and momentum equations may be rewritten in terms of x,y and z. Making x and y nondimensional with L, z with H and t with L/U, we have that

$$(2.10) \qquad x = L\tilde{x}, \quad y = L\tilde{y}, \quad z = H\tilde{z}; \quad t = L/U\,\tilde{t}.$$

Here the advective time scale L/U is set equal to the timescale T of the local changes. This is not an additional restriction, as for L/U much smaller than T we expect the nonlinear contributions to vanish, while for T much smaller than L/U the time-dependent effects are negligible.

The velocity components are nondimensionalized by the horizontal velocity scale U. The scaling for w follows by geometrical considerations: the slope of a fluid trajectory cannot exceed H/L in the present type of flow, thus the vertical velocity scale is $\frac{H}{L}$ U. Consequently,

$$(2.11) \qquad u = U\tilde{u}, \qquad v = U\tilde{v}, \qquad w = \frac{H}{L} U\tilde{w}.$$

Since we are dealing with small velocities (in the sense of (2.8)), the pressure - and density will differ slightly from their equilibrium values in the absence of motion. In the rest state we have

$$(2.12) \qquad \frac{\partial p_s}{\partial z} = - \rho_s g$$

which is the hydrostatic balance. The variables ρ_s and p_s are only functions of the vertical coordinate z.

Next, we write

$$(2.13) \qquad \begin{aligned} p &= p_s(z) + p'(x,y,z,t), \\ \rho &= \rho_s(z) + \rho'(x,y,z,t). \end{aligned}$$

The scaling for p' follows from the fact that the planetary scale flow is approximately in geostrophic balance, i.e. a balance between the coriolis - and pressure force. Thus from (2.4) and (2.5) we have

$$(2.14) \qquad f_0 U \sim \frac{p'}{\rho_s L},$$

where

$$(2.15) \qquad f_0 = 2\Omega \sin\phi_0$$

is the coriolisparameter at the central latitude ϕ_0, and ~ denotes an order estimate.

Consequently, p' may be scaled by $\rho_s\ f_0$ UL, so

$$(2.16) \qquad p' = \rho_s f_0 UL\tilde{p}.$$

The scale for the density perturbations in (2.13) is found from the hydrostatic balance, as it also applies to the disturbances. This is shown by Holton (1979) by scaling arguments. Thus $\rho'g$ must be of the same order as $\partial p'/\partial z$. Using (2.14), we obtain

$$(2.17) \qquad \rho' \sim \frac{p'}{gH} \sim \frac{\rho_s f_0 UL}{gH} .$$

Hence

$$(2.18) \qquad \rho = \rho_0[1+\varepsilon F\tilde{\rho}],$$

where

$$(2.19) \qquad \varepsilon = \frac{U}{f_0 L} ; \qquad F = \frac{f_0^2 L^2}{gH} .$$

ε is the Rossbynumber. By (2.8) and (2.15), ε is a small parameter as long as the central latitude ϕ_0 is distant from the equator.

Application of all these transformations to the continuity and momentum equations yields

$$(2.20) \qquad \varepsilon F \frac{d\tilde{\rho}}{dt} + (1+\varepsilon F\tilde{\rho}) \left\{ \frac{r_0}{r} \frac{\cos\phi_0}{\cos\phi} \frac{\partial\tilde{u}}{\partial\tilde{x}} - \frac{L}{r}\tilde{v}\tan\phi + \frac{r_0}{r}\frac{\partial\tilde{v}}{\partial\tilde{y}} + \frac{\partial\tilde{w}}{\partial\tilde{z}} + 2\frac{H}{r}\tilde{w} + \frac{\tilde{w}}{\rho_s}\frac{\partial\rho_s}{\partial\tilde{z}} \right\} = 0,$$

$$(2.21) \qquad \varepsilon\left\{\frac{d\tilde{u}}{d\tilde{t}} + \frac{L}{r}[\delta\tilde{u}\tilde{w} - \tilde{u}\tilde{v}\tan\phi]\right\} - \tilde{v}\frac{\sin\phi}{\sin\phi_0} + \delta\tilde{w}\frac{\cos\phi}{\sin\phi_0} = -\frac{\cos\phi_0}{\cos\phi}\frac{r_0}{r}\frac{1}{1+\varepsilon F\tilde{\rho}}\frac{\partial\tilde{p}}{\partial\tilde{x}} ,$$

$$(2.22) \qquad \varepsilon\left\{\frac{d\tilde{v}}{d\tilde{t}} + \frac{L}{r}[\delta\tilde{v}\tilde{w} + \tilde{u}^2\tan\phi]\right\} + \tilde{u}\frac{\sin\phi}{\sin\phi_0} = -\frac{r_0}{r}\frac{1}{1+\varepsilon F\tilde{\rho}}\frac{\partial\tilde{p}}{\partial\tilde{y}} ,$$

$$(1+\varepsilon F\tilde{\rho})\left\{\varepsilon\delta^2 \frac{d\tilde{w}}{d\tilde{t}} - \varepsilon\delta \frac{L}{r} (\tilde{u}^2+\tilde{v}^2) - \delta\tilde{u} \frac{\cos\phi}{\sin\phi_0}\right\} = \tag{2.23}$$

$$= - \frac{1}{\rho_s} \frac{\partial}{\partial\tilde{z}} (\rho_s\tilde{p}) - \tilde{\rho} ,$$

where

$$\frac{d}{d\tilde{t}} = \frac{\partial}{\partial\tilde{t}} + \tilde{u} \frac{\cos\phi_0}{\cos\phi} \frac{r_0}{r} \frac{\partial}{\partial\tilde{x}} + \tilde{v} \frac{r_0}{r} \frac{\partial}{\partial\tilde{y}} + \tilde{w} \frac{\partial}{\partial\tilde{z}} , \tag{2.24}$$

and

$$\delta = \frac{H}{L} ; \quad \frac{r}{r_0} = 1 + \delta(\frac{L}{r_0})\tilde{z}. \tag{2.25}$$

To this point no approximations have been made. The equations have been scaled so that the relative order of each term is measured by its nondimensional multiplicative parameter. General solutions of this system will be functions of time and space, and will moreover depend on four parameters, viz. ε,F, $\frac{L}{r_0}$ and δ. In this report we will study a large scale atmospheric flow in midlatitudes for which, apart from ε and δ, also L/r_0 is a small parameter. Since $r_0 \sim 6,4.10^6$ m this means that $L \lesssim 10^6$ m. Consequently, F will be a small parameter, for $f_0 \sim 10^{-4}s^{-1}$, $H \sim 10^4$m, $g \sim 10ms^{-2}$. Although this is not correct for planetary waves, we proceed this analysis, as we are primarily interested in the effect of nonlinearities.

Since L/r_0 is small, it is convenient to make a local expansion of the goniometric functions in the basic equations at the central latitude ϕ_0. Developing these functions we obtain

$$\begin{aligned} \sin\phi &= \sin\phi_0 + \frac{L}{r_0} \tilde{y} \cos\phi_0 +\ldots, \\ \cos\phi &= \cos\phi_0 - \frac{L}{r_0} \tilde{y} \sin\phi_0 +\ldots, \\ \tan\phi &= \tan\phi_0 + \frac{L}{r_0} \tilde{y} \cos^{-2}\phi_0 +\ldots . \end{aligned} \tag{2.26}$$

The basic equations for large scale flow may be analyzed by expanding all relevant quantities in perturbation series of a small parameter, for which we choose the Rossbynumber ε. Thus

$$u = \sum_{n=0}^{\infty} \varepsilon^n \tilde{u}_n, \qquad \tilde{p} = \sum_{n=0}^{\infty} \varepsilon^n \tilde{p}_n,$$

$$(2.27) \qquad \tilde{v} = \sum_{n=0}^{\infty} \varepsilon^n \tilde{v}_n$$

$$\tilde{\rho} = \sum_{n=0}^{\infty} \varepsilon^n \tilde{\rho}_n,$$

$$\tilde{w} = \sum_{n=0}^{\infty} \varepsilon^n \tilde{w}_n,$$

where the coefficients in the series are no longer explicite functions of ε. Substituting (2.26) and (2.27) in the basic equations and collecting terms with the same powers of ε, we obtain in zeroth order

$$(2.28) \qquad \frac{1}{\rho_s} \frac{\partial}{\partial \tilde{z}} (\tilde{w}_0 \rho_s) + \frac{\partial \tilde{u}_0}{\partial \tilde{x}} + \frac{\partial \tilde{v}_0}{\partial \tilde{y}} = 0,$$

$$(2.29) \qquad \tilde{v}_0 = \frac{\partial \tilde{p}_0}{\partial \tilde{x}},$$

$$(2.30) \qquad \tilde{u}_0 = \frac{-\partial \tilde{p}_0}{\partial \tilde{y}},$$

$$(2.31) \qquad \tilde{\rho}_0 = \frac{-1}{\rho_s} \frac{\partial}{\partial \tilde{z}} (\rho_s \tilde{p}_0).$$

Equations (2.29) and (2.30) state that in zeroth order the coriolis force is balanced by the horizontal pressure gradient. This is the well-known geostrophic balance, formulated here in curvilinear coordinates. It immediately follows that this horizontal velocity field is free of divergence, i.e.

$$(2.32) \qquad \frac{\partial \tilde{u}_0}{\partial \tilde{x}} + \frac{\partial \tilde{v}_0}{\partial \tilde{y}} = 0.$$

In the first place this implies that (2.28) reduces to

$$(2.33) \qquad \frac{1}{\rho_s} \frac{\partial}{\partial \tilde{z}} (\tilde{w}_0 \rho_s) = 0.$$

Hence $\tilde{w}_0 \rho_s$ must be independent of $\tilde{z}$. It seems natural to assume that $\tilde{w}_0 \to 0$ for $\tilde{z} \to \infty$, thus by (2.33) it follows

$$(2.34) \qquad \tilde{w}_0 = 0 \qquad \text{for all } \tilde{z}.$$

As a second consequence of (2.32) we can introduce a streamfunction $\tilde{\psi}$, with

$$(2.35) \qquad \tilde{u}_0 = -\frac{\partial\tilde{\psi}}{\partial\tilde{y}}, \qquad \tilde{v}_0 = \frac{\partial\tilde{\psi}}{\partial\tilde{x}} .$$

The geostrophic balance states that once the pressure field $\tilde{p}_0$ is known, the velocity components $\tilde{u}_0$ and $\tilde{v}_0$ are uniquely determined.

The zeroth order system consists of a set diagnostic relationships, from which the time evolution of the pressure field cannot be calculated. However, this is necessary in order to obtain a new velocity field. The problem marked here is called the problem of the geostrophic degeneracy, implying that the zeroth order system is not sufficient to describe the evolution of motions in the fluid. Hence the first order equations have to be considered as well. After substitution of (2.32) and (2.34) they read

$$(2.36) \qquad \frac{\partial\tilde{u}_1}{\partial\tilde{x}} + \frac{\partial\tilde{v}_1}{\partial\tilde{z}} + \frac{\partial\tilde{w}_1}{\partial\tilde{z}} + \frac{L}{r_0\varepsilon}\tan\phi_0\frac{\partial\tilde{u}_0}{\partial\tilde{x}} - \frac{L}{r_0\varepsilon}\tan\phi_0\tilde{v}_0 + \frac{\tilde{w}_1}{\rho_s}\frac{\partial\rho_s}{\partial\tilde{z}} = 0,$$

$$(2.37) \qquad \frac{\partial\tilde{u}_0}{\partial\tilde{t}} + \tilde{u}_0\frac{\partial\tilde{u}_0}{\partial\tilde{x}} + \tilde{v}_0\frac{\partial\tilde{u}_0}{\partial\tilde{y}} - \tilde{v}_1 - \tilde{v}_0\left(\frac{L}{r_0\varepsilon}\right)\tilde{y}\,\mathrm{cotg}\phi_0 = -\frac{\partial\tilde{p}_1}{\partial\tilde{x}} - \frac{L\tilde{y}}{r_0\varepsilon}\tan\phi_0\frac{\partial\tilde{p}_0}{\partial\tilde{x}},$$

$$(2.38) \qquad \frac{\partial\tilde{v}_0}{\partial\tilde{t}} + \tilde{u}_0\frac{\partial\tilde{v}_0}{\partial\tilde{x}} + \tilde{v}_0\frac{\partial\tilde{v}_0}{\partial\tilde{y}} + \tilde{u}_1 + \tilde{u}_0\left(\frac{L}{r_0\varepsilon}\right)\tilde{y}\,\mathrm{cotg}\phi_0 = -\frac{\partial\tilde{p}_1}{\partial\tilde{y}},$$

$$(2.39) \qquad \tilde{\rho}_1 = -\frac{1}{\rho_s}\frac{\partial}{\partial\tilde{z}}(\rho_s\tilde{p}_1).$$

The first order velocity field is not free of divergence, thus the equations describe small departures from the geostrophic balance. The model is then called quasi-geostrophic.

The contributions to the left hand side of the horizontal momentum equations, which are proportional to $\tilde{y}$, are due to the variation of the coriolis parameter with latitude, for

$$(2.40) \qquad \beta_0 = (\vec{\nabla}f)_{\phi=\phi_0} = \left(\frac{1}{r_0}\frac{df}{d\phi}\right)_{\phi=\phi_0} = \frac{2\Omega}{r_0}\cos\phi_0 .$$

The constant β_0 may be interpreted as the gradient of the planetary vorticity f_0 at latitude ϕ_0. It has the dimension of $\ell^{-1}t^{-1}$. The only way to write β_0 in a nondimensional form, using our scalings, is to multiply the quantity by L^2/U. Thus

$$(2.41) \qquad \tilde{\beta} = \frac{\beta_0 L^2}{U}$$

is a measure for the variation of the coriolis parameter with latitude in nondimensional coordinates. Using (2.19) and (2.40) we have that

$$(2.42) \qquad \tilde{\beta} = \frac{L}{r_0 \varepsilon} \operatorname{cotg}\phi_0 \ ,$$

which is substituted in the momentum equations.

The first order system can be analysed by a vorticity equation, that is derived from the two momentum equations. We differentiate (2.38) with respect to $\tilde{x}$, (2.37) with respect to $\tilde{y}$, and subtract the resulting equations. Using (2.32) and (2.42) we have

$$(2.43) \qquad \begin{aligned} \frac{\partial \tilde{\zeta}_0}{\partial \tilde{t}} + \tilde{u}_0 \frac{\partial \tilde{\zeta}_0}{\partial \tilde{x}} + \tilde{v}_0 \frac{\partial \tilde{\zeta}_0}{\partial \tilde{y}} + \tilde{\beta}\tilde{v}_0 &= \frac{L}{r_0 \varepsilon} \tan\phi_0 \frac{\partial \tilde{p}_0}{\partial \tilde{x}} + \\ &+ \frac{L}{r_0 \varepsilon} \tilde{y} \tan\phi_0 \frac{\partial^2 \tilde{p}_0}{\partial \tilde{x} \partial \tilde{y}} - \left(\frac{\partial \tilde{u}_1}{\partial \tilde{x}} + \frac{\partial \tilde{v}_1}{\partial \tilde{y}}\right), \end{aligned}$$

where

$$(2.44) \qquad \tilde{\zeta}_0 = \frac{\partial \tilde{v}_0}{\partial \tilde{x}} - \frac{\partial \tilde{u}_0}{\partial \tilde{y}}$$

is the zeroth order relative vorticity. The first order divergences may be eliminated by the continuity equation (2.36). By application of (2.29) and (2.30), the equation in a dimensional form finally reads

$$(2.45) \qquad \frac{\partial \zeta_0}{\partial t} + u_0 \frac{\partial \zeta_0}{\partial x} + v_0 \frac{\partial \zeta_0}{\partial y} + \beta_0 v_0 = f_0\left[\frac{\partial w_1}{\partial z} + \frac{w_1}{\rho_s} \frac{\partial \rho_s}{\partial z}\right].$$

Note that $\tilde{w}_1 = Lw_1/\varepsilon HU$ as $w_1 = O(\varepsilon)$!

The second term on the right hand side is the contribution due to compressibility.

Equation (2.45) is called the quasi-geostrophic vorticity equation. Although x and y are curvilinear coordinates, the form is exactly a vorticity equation for a carthesian plane, tangential to the earth at the central latitude ϕ_0, in which the variation of the coriolis parameter with latitude is linear, i.e. β_0 is a constant. This is the so-called β-plane approximation. It means that the sole effect of the earth's sphericity on the zeroth order fields is entirely due to the β-term. See figure 4.

Still the theory is not complete of this point. By means of (2.35), u_0, v_0 and ζ_0 can be related to the streamfunction. To close the system the first order velocities must also be related to the streamfunction. This will be done in the next section.

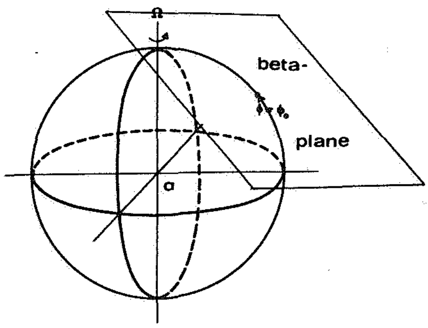

Figure 4. Visualisation of the β-plane.

3. DETERMINATION OF VERTICAL VELOCITIES FOR A BAROTROPIC ATMOSPHERE

For obtaining one equation, describing the large scale atmospheric motions in midlatitudes, the right hand side of (2.45) has to be expressed in terms of the streamfunction. In general we therefore need the thermal energy balance, together with an equation of state and a balance for the specific humidity. However, the analysis becomes rather complicated in that case. To avoid this, we will omit the influence of watervapour, and make also the barotropic assumption.

The thermodynamics of the atmosphere is specified in terms of a few state variables, such as pressure, density and temperature.
Normally, two of them are independent, so for instance the density is a function of pressure and temperature. This is called a baroclinic state. Making the barotropic assumption means that there is only one independent state variable, for instance the pressure, so that density is a function of pressure only. This function may then be interpreted as an equation of state. Temperature has become a passive variable, so the thermal energy

balance is irrelevant.

The barotropic assumption has far-reaching consequences. Since density- and pressure surfaces coincide, the ideal gas law guarantees that on these surfaces no temperature advection by the geostrophic wind is possible. Furthermore, the geostrophic wind becomes independent of height, which means that the dynamics at every height level is the same. Actually, the system consists of one layer.

Returning to the quasi-geostrophic equation, we note that the vertical velocities can only be modelled by adding new terms to the equations, or by adjustment of the boundary conditions. We will study three different mechanisms inducing vertical velocities, viz. frictional dissipation, forcing of the flow and the orography. Our derivation is a slight extension of the method of Pedlosky (1979), i.e. here we will not restrict ourselves to a homogeneous fluid, having a constant density, but we also include compressibility effects.

Frictional effects are caused by the random motion of fluid particles. The dissipation acts upon the small scales. Nevertheless, it influences the large scale flow, since there is a continuous energy transfer from the larger- to the smaller scales, called the cascade process. The way to include viscous dissipation in our large scale model is to parametrize the frictional effects by the large scale quantities. The procedure is that the velocity field $\vec{u}$ is split up in a large scale part $\langle\vec{u}\rangle$ and a part $\vec{u}'$, due to the small scale transient eddies, so

(3.1) $$\vec{u} = \langle\vec{u}\rangle + \vec{u}'.$$

By definition $\langle\vec{u}'\rangle = 0$, where $\langle\ \rangle$ denotes a time average over a sufficiently large time, eliminating the small scale effects. The average time must also be sufficiently small with regard to the time scale of the mean flow.

Substituting (3.1) in the momentum equations (2.2) and averaging, we obtain

(3.2) $$\frac{\partial\langle\vec{u}\rangle}{\partial t} + \langle\vec{u}\rangle.\vec{\nabla}\langle\vec{u}\rangle + \langle\vec{u}'.\vec{\nabla}\vec{u}'\rangle = -\frac{1}{\rho}\vec{\nabla}p + \vec{g}.$$

The new term $\langle\vec{u}'.\vec{\nabla}\vec{u}'\rangle$ is parametrized in terms of the large scale flow by setting

$$\langle \vec{u}'.\nabla\vec{u}'\rangle = -K_H\nabla_h^2\langle\vec{u}\rangle - K_V\frac{\partial^2}{\partial z^2}\langle\vec{u}\rangle, \tag{3.3}$$

which states that the turbulent frictional forces are modelled in the same way as molecular viscous forces. The constants K_H and K_V may be interpreted as horizontal- and vertical turbulent viscosity coefficients, respectively.

The continuity- and momentum equations, including friction terms, read in their nondimensional forms:

$$\begin{aligned}\varepsilon F\frac{d\tilde{\rho}}{d\tilde{t}} + (1+\varepsilon F\tilde{\rho})\Big\{&\frac{r_0}{r}\frac{\cos\phi_0}{\cos\phi}\frac{\partial\tilde{u}}{\partial\tilde{x}} - \frac{L}{r}\tilde{v}\tan\phi + \frac{r_0}{r}\frac{\partial\tilde{v}}{\partial\tilde{y}} + \\ &+\frac{\partial\tilde{w}}{\partial\tilde{z}} + 2\frac{H}{r}\tilde{w} + \frac{\tilde{w}}{\rho_s}\frac{\partial\rho_s}{\partial\tilde{z}}\Big\} = 0,\end{aligned} \tag{3.4}$$

$$\begin{aligned}\varepsilon\Big\{\frac{d\tilde{u}}{d\tilde{t}} + \frac{L}{r}[\delta\tilde{u}\tilde{w}-\tilde{u}\tilde{v}\tan\phi]\Big\} - \tilde{v}\frac{\sin\phi}{\sin\phi_0} + \delta\tilde{w}\frac{\cos\phi}{\sin\phi_0} &= \\ = -\frac{\cos\phi_0}{\cos\phi}\frac{r_0}{r}\frac{1}{1+\varepsilon F\tilde{\rho}}\frac{\partial\tilde{p}}{\partial\tilde{x}} + \tfrac{1}{2}E_H\Big(\frac{r_0}{r}\Big)^2\frac{\cos^2\phi_0}{\cos^2\phi}\frac{\partial^2\tilde{u}}{\partial\tilde{x}^2} &+ \\ + \tfrac{1}{2}E_H\Big(\frac{r_0}{r}\Big)^2\frac{\partial^2\tilde{u}}{\partial\tilde{y}^2} + \tfrac{1}{2}E_V\frac{\partial^2\tilde{u}}{\partial\tilde{z}^2}&,\end{aligned} \tag{3.5}$$

$$\begin{aligned}\varepsilon\Big\{\frac{d\tilde{v}}{d\tilde{t}} + \frac{L}{r}[\delta\tilde{v}\tilde{w}+\tilde{u}^2\tan\phi]\Big\} + \tilde{u}\frac{\sin\phi}{\sin\phi_0} &= \\ = -\frac{r_0}{r}\frac{1}{1+\varepsilon F\tilde{\rho}}\frac{\partial\tilde{p}}{\partial\tilde{y}} + \tfrac{1}{2}E_H\Big(\frac{r_0}{r}\Big)^2\frac{\cos^2\phi_0}{\cos^2\phi}\frac{\partial^2\tilde{v}}{\partial\tilde{x}^2} + \tfrac{1}{2}E_H\Big(\frac{r_0}{r}\Big)^2\frac{\partial^2\tilde{v}}{\partial\tilde{y}^2} &+ \\ + \tfrac{1}{2}E_V\frac{\partial^2\tilde{v}}{\partial\tilde{z}^2}&,\end{aligned} \tag{3.6}$$

$$\begin{aligned}(1+\varepsilon F\tilde{\rho})\Big\{\varepsilon\delta^2\frac{d\tilde{w}}{d\tilde{t}} - \varepsilon\delta\frac{L}{r}(\tilde{u}^2+\tilde{v}^2) - \delta\tilde{u}\frac{\cos\phi}{\sin\phi_0}\Big\} &= \\ = -\frac{1}{\rho_s}\frac{\partial}{\partial\tilde{z}}(\rho_s\tilde{p})-\tilde{\rho}+\tfrac{1}{2}E_H\Big(\frac{r_0}{r}\Big)^2\frac{\cos^2\phi_0}{\cos^2\phi}\frac{\partial^2\tilde{w}}{\partial\tilde{x}^2} + \tfrac{1}{2}E_H\Big(\frac{r_0}{r}\Big)^2\frac{\partial^2\tilde{w}}{\partial\tilde{y}^2} &+ \\ + \tfrac{1}{2}E_V\frac{\partial^2\tilde{w}}{\partial\tilde{z}^2}&.\end{aligned} \tag{3.7}$$

The new parameters are the horizontal - and vertical Ekman number, defined by

$$(3.8) \qquad E_H = \frac{2K_H}{f_0 L^2}, \qquad E_V = \frac{2K_V}{f_0 H^2},$$

which are small ($E_H \sim E_V \ll 1$). These equations are equivalent to the system (2.20)-(2.23), except that here second order derivatives appear. Since the atmosphere is bounded in vertical direction, we have to specify two boundary conditions for each of the horizontal velocity components. They read

$$(3.9) \qquad \begin{aligned} &\tilde{u} = 0, \quad \tilde{v} = 0, \quad \tilde{w} = 0 \quad \text{at} \quad \tilde{z} = 0, \\ &\tilde{u} = \tilde{u}_g, \quad \tilde{v} = \tilde{v}_g, \quad \tilde{w} = 0 \quad \text{at} \quad \tilde{z} = 1, \end{aligned}$$

where $\tilde{u}_g$ and $\tilde{v}_g$ denote the geostrophic velocity components. Thus friction causes the horizontal velocities to be zero at the rigid wall $\tilde{z} = 0$. The coefficients of the highest derivatives of (3.4) - (3.7) are small. However, these terms may not be neglected in zeroth order, otherwise the solutions cannot satisfy all boundary conditions. The equations can be solved by asymptotic methods, see Eckhaus (1979). A so-called outer solution is constructed by substituting regular expansions for $\tilde{u},\tilde{v},\tilde{w},\tilde{p}$ and $\tilde{\rho}$, in the same way as is done in (2.27). It then appears that the zeroth order balance (2.28) - (2.31) remains unchanged, i.e. outside the frictional boundary layer the geostrophic balance is not affected by the presence of friction.

Since the outer solution does not satisfy all boundary conditions, we construct an inner solution which is valid in the boundary layer near $\tilde{z} = 0$. We stretch the dynamics by introducing a local coordinate

$$(3.10) \qquad \xi = \frac{\tilde{z}}{E_V^n},$$

where n is an as yet unknown positive constant. Transforming (3.4) - (3.7) to $(\tilde{x},\tilde{y},\xi)$ coordinates, it follows that all contributions remain unchanged, except the frictional terms and the first term on the right hand side of (3.7), as long as the operator $\frac{d}{d\tilde{t}}$ is interpreted as

(3.11) $$\frac{d}{d\tilde{t}} = \frac{\partial}{\partial\tilde{t}} + \tilde{u}\,\frac{\cos\phi_0}{\cos\phi}\,\frac{r_0}{r}\,\frac{\partial}{\partial\tilde{x}} + \tilde{v}\,\frac{r_0}{r}\,\frac{\partial}{\partial\tilde{y}} + \frac{\tilde{w}}{E_v^n}\,\frac{\partial}{\partial\xi}\,.$$

The frictional terms become

(3.12) $$\tfrac{1}{2}\,E_v^{1-2n}\left(\frac{\partial^2\tilde{u}}{\partial\xi^2},\ \frac{\partial^2\tilde{v}}{\partial\xi^2},\ \frac{\partial^2\tilde{w}}{\partial\xi^2}\right).$$

It is argued that in the boundary layer these contributions have to be of the same order as the coriolis - and pressure forces. This is satisfied if we put in (3.12)

(3.13) $$1 - 2n = 0, \qquad \text{or} \qquad n = \tfrac{1}{2}.$$

Hence $E_v^{\frac{1}{2}}$ is a measure for the width of the boundary layer. Next, transforming

(3.14) $$\tilde{w} = E_v^{\frac{1}{2}}\,\tilde{W},$$

and substituting regular expansions for $\tilde{u},\tilde{v},\tilde{W},\tilde{p}$ and $\tilde{\rho}$, we obtain in zeroth order

(3.15) $$\frac{1}{\rho_s}\,\frac{\partial}{\partial\xi}\,[\tilde{W}_0\rho_s] + \frac{\partial\tilde{u}_0}{\partial\tilde{x}} + \frac{\partial\tilde{v}_0}{\partial\tilde{y}} = 0,$$

(3.16) $$-\,\tilde{v}_0 = -\,\frac{\partial\tilde{p}_0}{\partial\tilde{x}} + \tfrac{1}{2}\,\frac{\partial^2\tilde{u}_0}{\partial\xi^2}\,,$$

(3.17) $$\tilde{u}_0 = -\,\frac{\partial\tilde{p}_0}{\partial\tilde{y}} + \tfrac{1}{2}\,\frac{\partial^2\tilde{v}_0}{\partial\xi^2}$$

(3.18) $$-\,\frac{1}{\rho_s}\,\frac{\partial}{\partial\xi}\,(\rho_s\tilde{p}_0) = 0.$$

From (3.18) it follows that $\frac{\partial}{\partial\tilde{x}}\,(\rho_s\tilde{p}_0)$ and $\frac{\partial}{\partial\tilde{y}}\,(\rho_s\tilde{p}_0)$ are independent of ξ. Matching the inner- and outer solution, we obtain by applying (2.29) and (2.30):

$$\left.\frac{\partial\tilde{p}_0}{\partial\tilde{x}}\right|_{\substack{\text{inner}\\\text{solution},\ \xi\to\infty}} = \left.\frac{\partial\tilde{p}_0}{\partial\tilde{x}}\right|_{\substack{\text{outer}\\\text{solution},\ \tilde{z}\to 0}} = \tilde{v}_g\ ,$$

(3.19)

$$\left.\frac{\partial\tilde{p}_0}{\partial\tilde{y}}\right|_{\substack{\text{inner}\\\text{solution},\ \xi\to\infty}} = \left.\frac{\partial\tilde{p}_0}{\partial\tilde{y}}\right|_{\substack{\text{outer}\\\text{solution},\ \tilde{z}\to 0}} = -\ \tilde{u}_g\ .$$

With this the momentum equations (3.16) and (3.17) become

$$\frac{1}{2}\,\frac{\partial^2\tilde{u}_0}{\partial\xi^2} + \tilde{v}_0 = \tilde{v}_g, \qquad (3.20)$$

$$\frac{1}{2}\,\frac{\partial^2\tilde{v}_0}{\partial\xi^2} - \tilde{u}_0 = -\ \tilde{u}_g. \qquad (3.21)$$

Conceiving $\tilde{u}_g$ and $\tilde{v}_g$ as prescribed velocities which, due to barotropy, do not depend on ξ, the solutions satisfying the boundary conditions (3.9) read

$$\tilde{u}_0 = \tilde{u}_g[1-e^{-\xi}\cos\xi] - \tilde{v}_g e^{-\xi}\sin\xi\ , \qquad (3.22)$$

$$\tilde{v}_0 = \tilde{v}_g[1-e^{-\xi}\cos\xi] + \tilde{u}_g e^{-\xi}\sin\xi\ . \qquad (3.23)$$

These are the inner solutions of the zeroth order velocity field. They are plotted in figure 5.

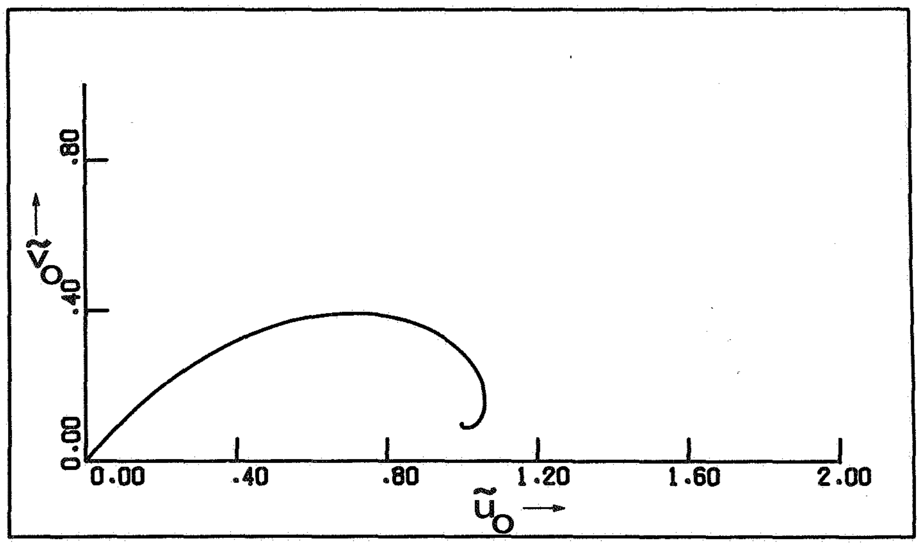

Figure 5. Sketch of the boundary layer solutions for $\tilde{u}_g = 1$ and $\tilde{v}_g = 0,1$.

This is called the Ekmanspiral; for increasing ξ the boundary layer solutions spiral to the geostrophic outer solutions.

The vertical velocity field, in which we are interested, is calculated from (3.15). Using (3.22) and (3.23) we find in a dimensional form

$$(3.24)\qquad \frac{1}{\rho_s}\frac{\partial}{\partial z}[\rho_s w_1] = \left(\frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}\right) e^{-z/\delta_E} \sin(z/\delta_E),$$

where

$$(3.25)\qquad \delta_E = \sqrt{\frac{2K_z}{f_0}}$$

is a measure for the depth of the Ekman boundary layer. Substituting this in the right hand side of (2.44) and integrating the result over the depth of the fluid, we obtain

$$(3.26)\qquad \frac{\partial \zeta_0}{\partial t} + u_0 \frac{\partial \zeta_0}{\partial x} + v_0 \frac{\partial \zeta_0}{\partial y} + \beta_0 v_0 = - \frac{f_0 \delta_E}{2H} \zeta_0.$$

The right hand side is a sink of vorticity. From energy considerations, we have also to include a source of vorticity. Its parametrization is rather arbitrary. For convenience we choose

$$(3.27)\qquad \frac{f_0 \delta_E}{2H} \zeta_0^*,$$

which is added to the right hand side of (3.26).

Finally we have to include the effect of orography. To give an accurate description we study Ekman layers on a sloping surface. This again requires a careful and precise analysis, similar to what we have done in this chapter. We just give the final result, being

$$(3.28)\qquad \frac{\partial \zeta_0}{\partial t} + u_0 \frac{\partial \zeta_0}{\partial x} + v_0 \frac{\partial \zeta_0}{\partial y} + \beta_0 v_0 = - \frac{f_0 \delta_E}{2H} (\zeta_0 - \zeta_0^*) - \frac{f_0}{H} \left[u_0 \frac{\partial h}{\partial x} + v_0 \frac{\partial h}{\partial y}\right],$$

where the function $z = h(x,y)$ describes the orography.
Rewriting in terms of a streamfunction, we find

$$(3.29)\qquad \frac{\partial}{\partial t} \nabla^2 \psi + J(\psi, \nabla^2 \psi + f_0 \frac{h}{H}) + \beta_0 \frac{\partial \psi}{\partial x} = - \frac{f_0 \delta_E}{2H} \nabla^2 (\psi - \psi^*),$$

where

(3.30) $$J(a,b) = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}$$

is the Jacobian of a and b. Equation (3.29) is called a quasi-geostrophic potential vorticity equation on a β-plane for a barotropic atmosphere. This will be analyzed in the next section.

4. SPECTRAL ANALYSIS

The quasi-geostrophic equation (3.29) is a maximal simplified model of the large scale flow occurring at midlatitudes. Although it is a nonlinear equation, it has a free plane wave solution of the form

(4.1) $$\psi = A \exp[i(kx+\ell y-\sigma t+\chi)],$$

where A is the amplitude, $\vec{\kappa} = (k,\ell)$ the wave vector with components k and ℓ, σ the circle frequency and χ the phase angle of the wave. Substitution of (4.1) in (3.29) yields $J(\psi,\nabla^2\psi) = 0$, so that the nonlinear terms cancel. The final result becomes

(4.2) $$\sigma = \frac{1}{\vec{\kappa}^2}\left\{\frac{-f_0}{H}\vec{j}\cdot(\vec{\kappa}\times\vec{\nabla}h)-\beta_0 k\right\},$$

where $\vec{j}$ is unity vector in the vertical. This is the dispersion relation for a so-called Rossby wave. By setting $\vec{\kappa}\times\vec{\nabla}h = 0$ it reduces to a more convenient form. Another limit is the so-called f-plane approximation ($\beta_0 = 0$). Then (4.2) reduces to the dispersion relation of a topographic Rossby wave. Note that there exists a dynamical similarity between the bottom topography variations and the variation of the coriolis parameter with latitude. This is due to the barotropic assumption, as argued by Pedlosky (1979).

The general solution of (3.29) cannot be composed of a linear combination of Rossby waves, since the equation is nonlinear. Only under certain assumptions the nonlinear contributions can be neglected. However, this is not our aim; we wish to study the influence of nonlinear terms upon the final flow pattern. This can be done by developing a spectral model, following the method of Charney & Devore (1979).

The basic step in the spectral approach is the assumption that, in a certain area with proper boundary conditions, ψ, ψ^* and h may be expanded in eigenfunctions of the Laplace operator, thus

$$\psi = \sum_{j=1}^{\infty} \psi_j \phi_j,$$

$$(4.3) \qquad \psi^* = \sum_{j=1}^{\infty} \psi_j^* \phi_j,$$

$$h = \sum_{j=1}^{\infty} h_j \phi_j,$$

where

$$(4.4) \qquad \nabla^2 \phi_j = - \lambda_j^2 \phi_j.$$

The function ϕ_j is an eigenfunction of the Laplace operator, with eigenvalue $- \lambda_j^2$. It is well-known that (4.4) is a Sturm-Liouville problem. In Courant & Hilbert (1953) it is proved that $\lambda_j^2 > 0$ for all j. Furthermore, the eigenfunctions form an orthogonal set; by means of the Gram-Schmidt procedure an orthonormal set of eigenfunctions may be constructed, such that for any eigenfunctions ϕ_k and ϕ_m

$$(4.5) \qquad \overline{\phi_k \phi_m} = \delta_{km}$$

holds, where the bar denotes averaging over the area. The symbol δ_{km} denotes the Kronecker delta, being 1 for k = m and zero oterhwise. Besides the eigenfunctions form a complete set, which means that any, at least partially, continuous function may be expanded in these functions. The latter property justifies (4.3).

The quantities $J(\phi_j,\phi_k)$ and $\frac{\partial \phi_j}{\partial x}$ also have to be expanded in eigenfunctions. Hence we propose

$$J(\phi_j,\phi_k) = \sum_{\ell=1}^{\infty} c_{\ell jk}\phi_\ell,$$

(4.6)

$$\frac{\partial\phi_j}{\partial x} = \sum_{\ell=1}^{\infty} b_{\ell j}\phi_\ell.$$

From the orthonormality relations (4.5) it follows that

$$c_{\ell jk} = \overline{\phi_\ell J(\phi_j,\phi_k)},$$

(4.7)

$$b_{\ell j} = \overline{\phi_\ell \frac{\partial\phi_j}{\partial x}}.$$

Substituting these expansions in the barotropic quasigeostrophic potential vorticity equation, and projecting on component i by multiplying the expression with ϕ_i, we obtain after averaging over the area of interest and applying the orthonormality conditions

$$\lambda_i^2 \frac{d\psi_i}{dt} + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{ijk}\{\lambda_k^2\psi_j\psi_k + \frac{1}{R^2}\psi_j\psi_k - \frac{f_0}{H}h_k\psi_j\} +$$

(4.8)

$$- \beta_0 \sum_{j=1}^{\infty} b_{ij}\psi_j + \frac{f_0\delta_E}{2H}\lambda_i^2(\psi_i-\psi_i^*) = 0.$$

Applying the identity $c_{ijk} = - c_{ikj}$, which immediately follows from (4.9) by noting that the Jacobian operator is antisymmetric, we find that the second term on the left hand side of this equation can be written as

(4.9) $$\tfrac{1}{2}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{ijk}\{(\lambda_k^2-\lambda_j^2)\psi_j\psi_k - \frac{f_0}{H}(h_k\psi_j-h_j\psi_k)\}.$$

The term between brackets is symmetric with regard to the indices j and k, and is zero if j = k. Hence, the summation can be changed into

(4.10) $$\sum_{j=1}^{\infty}\sum_{k=j+1}^{\infty} c_{ijk}\{(\lambda_k^2-\lambda_j^2)\psi_j\psi_k - \frac{f_0}{H}(h_k\psi_j-h_j\psi_k)\}.$$

Then (4.8) becomes

$$\frac{d\psi_i}{dt} = \frac{1}{\lambda_i^2}\left\{\sum_{j=1}^{\infty}\sum_{k=j+1}^{\infty} c_{ijk}\left[(\lambda_j^2-\lambda_k^2)\psi_j\psi_k + \frac{f_0}{H}(h_k\psi_j - h_j\psi_k)\right] + \right.$$

$$\left. + \beta_0 \sum_{j=1}^{\infty} b_{ij}\psi_j - \frac{f_0\delta_E}{2H}\lambda_i^2(\psi_i - \psi_i^*)\right\}, \tag{4.11}$$

which is a system consisting of an infinite number of nonlinear ordinary differential equations. They describe the time evolution of the components in the streamfunction expansion (4.3).

The complete system is difficult to handle. However, since the model only describes the large scale flow, it seems reasonable to take only a few components into account. Thus, truncation of the expansions changes (4.11) into a finite number of nonlinear differential equations. For such a system the steady states, periodic solutions and their stability properties may be computed by analytical and numerical methods. By means of a slow variation of the external control parameters of the model, it is possible to locate in the parameter space bifurcation points, at which two or more solutions branch off. Moreover, we may detect regions, where sudden transitions to other steady states occur.

At this point we remark that it cannot be proved that truncated spectral models (often called low order models) give an accurate description of the large scale flow in midlatitudes. Thus it is not meaningful to ask how the solutions of truncated models are related to those of the original nonlinear partial differential equation. It is expected, however, that there is some similarity between the dynamics of the original system and that of the truncated system.

The next step is to derive a simple low order model and to analyze its properties. This will be done in a subsequent report.

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