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In the Netherlands a freeway control and signalling system has been installed on several freeways some years ago. One purpose of the system is to improve traffic flow and avoid the development of congestion. In this report the first step towards this aim is set in the development of a traffic model. The proposed model is simulated for various traffic situations and modified to achieve realistic performance.

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Contents

Chapter 1

Introduction

In the Netherlands a freeway control and signalling system [21] has been installed on several freeways some years ago. The system consists of measuring loops embedded in the road surface, matrix signal boards above the road mounted on gantries and computer and communications hardware. See appendix A for a description of the system. The measuring loops are spaced approximately 500 meters apart and allow detection of vehicles passing and measurement of their speed. The measurements are sent to a control centre from where the matrix boards can be controlled. The matrix board gantries are spaced approximately 500 to 1000 meters apart. The boards can show advisory speed signals, lane arrows, a red cross and a road clear signal.

One of the main objectives of the system is to improve traffic flow and prevent congestion. It turns out that when traffic density reaches a value of approximately 25 - 30 veh/km/lane the traffic stream becomes very sensitive to small disturbances. Based on the traffic measurements one may recognize critical situations and try to prevent the occurrence of congestion by showing suitable advisory speed signals to the oncoming traffic.

A solution to this traffic control problem might consist of the following steps :

1. the development of a model of freeway traffic flow that is able to describe instabilities at critical density values and the development of congestion;
2. the derivation of an algorithm based on the model (to be called *filter* henceforth) which recursively estimates the state of traffic from the measurements of the signalling system;
3. the derivation of an optimal, state dependent control strategy for the matrix boards.

To solve the problem we will use techniques available from a branch of mathematics called system and control theory. In addition, in the modelling process we will use notions from the theory of stochastic processes like a counting process and a martingale [2] . This approach allows us to formulate a model in continuous time, in contrast to Cremer [4] for example, who develops a discrete time model.

In this report we will only be concerned with the first step mentioned above : the development of a freeway traffic model. We will heavily rely on earlier research by Van Maarseveen [15] . The second step mentioned, the development of a filter, will be discussed in a subsequent report.

We start in chapter 2 by extensively discussing the traffic model we propose. The stochastic differential equations for the state variables density and mean speed are given as well as the relation between the measured passing times and passing speeds on one hand and the state variables on the other hand.

In chapter 3 results of model simulations are shown and discussed. The model has been simulated under various circumstances : low, moderate and high traffic density. The simulation results have led to modifications of the model. The refined model will be summarised at the end of the chapter.

Chapter 4 contains our conclusions and suggestions and plans for further research.

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Chapter 2

Modelling of freeway traffic flow

In this chapter we present a stochastic model of traffic flow which is continuous in time and discrete with respect to space. We start with some preliminary remarks in section 2.1. In the next two section the stochastic differential equations of the model are derived. The observations or measurements are modelled in section 2.4. We end in section 2.5 by summarising the complete model and making some remarks concerning ramps, boundary conditions etc.

1. Preliminary considerations

The numerous freeway traffic models quoted in literature can be classified in micro- or macroscopic ones. Microscopic models try to describe the behaviour of individual drivers explicitly. In these models it is often assumed that drivers react to a difference in speed in comparison to the driver right in front by accelerating or decelerating. This is generally referred to as car following [8] . The intensity of the reaction may depend on the distance between the two vehicles, perceptual or other factors. Computer programs for the simulation of such models have appeared in the literature, e.g. Wiedemann [25] . A major drawback of this type of model in application to freeway traffic is the amount of computation needed. In dense traffic, 25 veh/km/lane, the simulation of a stretch of freeway of 10 km and 2 lanes would require the solution of a set of 500 differential equations. In this case macroscopic models, which try to describe freeway traffic in terms of aggregate variables, seem to be more appropriate. For a typical macroscopic model 40 differential equations would suffice in the example mentioned above.

The macroscopic model as developed by Payne [20] is the most widely used, see for example Cremer [4] , Papageorgiou [18] , Van Maarseveen [15] . A simplified version of the model was already given by Lighthill and Whitham [14] . Payne derived his model by starting from a general microscopic car following model and applying an aggregation procedure. The model he obtains consists of two partial differential equations for the variables density and speed and may be interpreted as a kind of fluid flow model :

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial q}{\partial x} = 0 \\ \dfrac{\partial v}{\partial t} + v\dfrac{\partial v}{\partial x} = -\dfrac{1}{T}\left[v - v^e(\rho)\right] + \dfrac{\dot{v}^e(\rho)}{2\rho T}\dfrac{\partial \rho}{\partial x} \end{cases} \tag{2.1}$$

where

$\rho(x,t)$: density (veh/km/lane)

$v(x,t)$: speed (km/h)

$q(x,t)$: flow (veh/h/lane)

and

$$q(x,t) = \rho(x,t)v(x,t)$$

Several discretisation schemes have been proposed to solve the equations numerically. For our purpose it seems to be natural to discretise with respect to space such that the freeway is split up into sections with measuring loops at the boundaries of each section. See appendix B for an overview. Then we can define the variables

$\rho_i(t)$: the number of vehicles in section i per kilometer per lane

$v_i(t)$: the mean speed of the vehicles in section i

for all sections i. Our model will then have the following general form :

$$d\rho_i(t) = f_i(\ldots,\rho_i,v_i,\ldots)dt + dz_i(t) \tag{2.2}$$

$$dv_i(t) = g_i(\ldots,\rho_i,v_i,\ldots)dt + dw_i(t) \tag{2.3}$$

where w_i and z_i represent stochastic fluctuations.

The measurements the signalling system provides consist of passing times and passing speeds. Now introduce

$\pi_i(t)$: the number of vehicles that left section i since time t_0

This is a *counting process* for which a large amount of theory has been developed in the past decade [2, 3] . A short overview of the main definitions and results is given in appendix C. According to one of these results, the *Doob-Meyer decomposition* of submartingales, we can write

$$d\pi_i(t) = \lambda_i(t)dt + dm_i(t) \tag{2.4}$$

where

λ_i : the intensity of the process π_i

m_i : the martingale associated with π_i

The decomposition (2.4) holds with respect to the family of σ-algebras $\{ \mathcal{F}_t, t \in \mathbb{R} \}$ where

$\mathcal{F}_t$: the smallest σ-algebra generated by the stochastic processes

$$\{ \rho_i(s),\ v_i(s),\ w_i(s),\ \pi_i(s),\ s \leq t \}$$

Now λ_i can be seen as the conditional mean of the number of jumps of π_i per unit of time. The mean is conditioned upon the past $\{\mathcal{F}_s, s \leq t\}$. Hence λ_i is the mathematical representation of traffic volume. The decomposition (2.4) says that $\lambda_i(t)$ is the momentaneous rate of increase of $\pi_i(t)$ at time t, apart from some stochastic fluctuations represented by $m_i(t)$. See figure 1 in appendix C for an example. In the simplest case $\lambda_i(t)$ will be a constant in which case $\pi_i(t)$ is a Poisson process with parameter $\lambda = \lambda_i$. In general λ_i will depend on time or on other stochastic processes however. In our case λ_i clearly depends on the density and mean speed of the sections i and i+1.

In the next section we will use decomposition (2.4) in the derivation of the differential equation for density.

2. Derivation of the density equation

As mentioned in the previous section our model has the general form (2.2), (2.3). In this section we derive the first equation.

Following the definition of ρ_i and π_i in section 2.1 we find that

$$\rho_i(t) = \rho_i(t_0) + \frac{1}{l_i L_i}[\, \pi_{i-1}(t) - \pi_i(t)\,] \qquad (\, t \geqslant t_0\,)$$

where

l_i : the number of lanes in section i

L_i : the length of section i

This is just the principle of conservation of vehicles : the number of vehicles in a section only changes when vehicles enter or leave. Note that for the moment we assume the absence of any on- and off-ramps. The modelling of these will be discussed in section 2.5.

In differential form the conservation law looks like :

$$d\rho_i(t) = \frac{1}{l_i L_i}[\, d\pi_{i-1}(t) - d\pi_i(t)\,]$$

Now, using the Doob-Meyer decomposition (2.4)

$$d\rho_i(t) = \frac{1}{l_i L_i}[\, \lambda_{i-1}(t) - \lambda_i(t)\,]dt + \frac{1}{l_i L_i}[\, dm_{i-1}(t) - dm_i(t)\,] \tag{2.5}$$

As mentioned in section 2.1 $\lambda_i(t)$ is the intensity or momentaneous traffic volume of the counting process $\pi_i(t)$ at time t.

A common way of modelling $\lambda_i(t)$ is by the approximation

$$\lambda_i(t) = l_i[\, \alpha\rho_i(t) + (1-\alpha)\rho_{i+1}(t)\,][\, \alpha v_i(t) + (1-\alpha)v_{i+1}(t)\,] \tag{2.6}$$

where

$\alpha \in [0,1]$, a weighting factor

This approximation is justified by the fact that in the case of stationary and homogeneous flow the relation is exact. (As ρ_i is measured in veh/km/lane and π_i counts over all lanes the factor l_i appears in (2.6)). In (2.6) $[\, \alpha\rho_i(t) + (1-\alpha)\rho_{i+1}(t)\,]$ is an approximation of the density in the vicinity of the common boundary of section i and section i+1 and analogously $[\, \alpha v_i(t) + (1-\alpha)v_{i+1}(t)\,]$ an approximation of the speed. In general α is taken to be larger than 0.5 to stress the fact that only vehicles upstream of the boundary can really pass.

Substituting (2.6) in (2.5) we finally get

$$\begin{aligned} d\rho_i(t) = \frac{1}{l_i L_i}\Big\{ & l_{i-1}[\, \alpha\rho_{i-1}(t) + (1-\alpha)\rho_i(t)\,][\, \alpha v_{i-1}(t) + (1-\alpha)v_i(t)\,] \\ & - l_i[\, \alpha\rho_i(t) + (1-\alpha)\rho_{i+1}(t)\,][\, \alpha v_i(t) + (1-\alpha)v_{i+1}(t)\,] \Big\} dt \\ & + \frac{1}{l_i L_i}\Big\{ dm_{i-1}(t) - dm_i(t) \Big\} \end{aligned} \tag{2.7}$$

The stochastic differential equation (2.7) describes the time evolution of the density in section i. This evolution depends on the density and mean speed in the section itself and in the neighbouring sections and also on some stochastic fluctuations.

Comparing (2.7) with equation (2.38) of Cremer [4] one notes the following differences :

- our equation is continuous in time whereas Cremer takes a time step of approximately ten seconds.
- the accuracy of the approximation (2.32) of Cremer is influenced by the time step. If this step is too small the approximation will be too rough but it cannot be taken too large either if one wants to model short term phenomena like instabilities. This problem does not occur in our approach as in our view λ_i is the (conditional) *mean* number of vehicles that pass per unit of time where the mean is taken over all realisations.
- in our case the stochastics appear in a natural way : they follow from the character of a counting process. Cremer just adds an artificial Gaussian random variable to his density equation.

3. The equation for the mean speed

We will now describe the second equation of our freeway traffic model. Our equation will be the discrete analogue of the speed equation of Payne [20] .

As one can see from the second equation of (2.1) in section 2.1 three processes are supposed to influence the mean speed of a section :

1. *relaxation* : it is known that there is a tendency of the mean speed to move to an equilibrium value v^e. The value of this equilibrium speed depends on the density : $v^e(\rho)$. The simplest model for this relation was given by Greenshields [9] : a linear decrease of the mean speed with increasing density :

$$v^e(\rho) = v_f(1 - \frac{\rho}{\rho_j})$$

where

v_f : free speed in km/h

ρ_j : jam density in veh/km/lane

We then propose to model the influence of the relaxation process on the mean speed as follows :

$$(dv_i(t))_{rel} = -\frac{1}{T}\left[v_i - v^e(\rho_i) \right]dt$$

where

T : relaxation time in h ;

2. *anticipation* : it is intuitively clear that drivers tend to anticipate changing conditions downstream. When traffic is more dense downstream a driver will slow down beforehand and vice versa. The effect of this microscopic phenomenon on the evolution of mean speed is represented by the anticipation term of the model. Following Payne's derivation we obtain :

$$(dv_i(t))_{ant} = -\frac{\nu}{T(L_i + L_{i+1})}\left[\frac{\rho_{i+1} - \rho_i}{\rho_i + c} \right]dt$$

where

ν : a positive constant of dimension km^2/h

c : a constant of dimension veh/km/lane, a modification of Payne's model due to Cremer [4] ;

3. *convection* : the mean speed in a section is influenced by vehicles entering or leaving the section. From simple calculations it follows that

$$(dv_i(t))_{con} = v_i(t+) - v_i(t-)$$

$$= \frac{1}{l_iL_i\rho_i(t-)-1} \left[v_i(t-) - v^{out}(\pi_i(t-)+1) \right] d\pi_i(t)$$

$$+ \frac{1}{l_iL_i\rho_i(t-)+1} \left[v^{in}(\pi_i(t-)+1) - v_i(t-) \right] d\pi_{i-1}(t)$$

where

$v^{out}(k)$: the speed of the k-th vehicle leaving section i starting from time t_0

$v^{in}(k)$: the speed of the k-th vehicle entering section i starting from time t_0

$t-$: the time just before a vehicle enters or leaves the section

$t+$: the time just after a vehicle enters or leaves the section

Now, as in the case of π_i in section 2.1 we would like to decompose $(v_i(t))_{con}$ in a martingale part and a part which represents the behaviour in the mean. Unfortunately this *semimartingale decomposition* is very complex. Regarding the fact that the convection effect only represents a small microscopic adjustment of the mean speed, some approximation seems acceptable. Instead of considering the continuum of all possible values that the passing speed can take just consider an approximate mean passing speed : $\alpha v_i + (1-\alpha)v_{i+1}$ where α is the same weighting factor as in (2.6). Substituting this in the equation for $(dv_i(t))_{con}$ and using the decomposition (2.4) for $d\pi_i(t)$ we find

$$(dv_i(t))_{con} \approx \frac{l_i[\alpha\rho_i + (1-\alpha)\rho_{i+1}]\,[\alpha v_i + (1-\alpha)v_{i+1}]}{l_iL_i\rho_i(t-)-1} (1-\alpha) [v_i - v_{i+1}]dt$$

$$+ \frac{l_{i-1}[\alpha\rho_{i-1} + (1-\alpha)\rho_i]\,[\alpha v_{i-1} + (1-\alpha)v_i]}{l_iL_i\rho_i(t-)+1} \alpha [v_{i-1} - v_i]dt$$

$$+ \frac{(1-\alpha)[v_i - v_{i+1}]}{l_iL_i\rho_i(t-)-1} dm_i(t) + \frac{\alpha[v_{i-1} - v_i]}{l_iL_i\rho_i(t-)+1} dm_{i-1}(t)$$

where the argument of ρ_{i-1} , v_{i-1} , ρ_i , v_i , ρ_{i+1} , v_{i+1} is $t-$. To simplify matters choose α equal to 1, neglect the +1 and -1 in comparison to $l_iL_i\rho_i(t-)$, set $\frac{\rho_{i-1}}{\rho_i}$ equal to 1 and neglect the martingales :

$$(dv_i(t))_{con} \approx \frac{l_{i-1}}{l_iL_i} v_{i-1}[v_{i-1} - v_i]dt$$

Summarising, the equation for the mean speed looks like :

$$dv_i(t) = (dv_i(t))_{rel} + (dv_i(t))_{ant} + (dv_i(t))_{con} + dw_i(t)$$

$$= -\frac{1}{T} \left[v_i - v^e(\rho_i) \right] dt - \frac{v}{T(L_i+L_{i+1})} \left[\frac{\rho_{i+1} - \rho_i}{\rho_i + c} \right] dt$$

$$+ \frac{l_{i-1}}{l_iL_i} v_{i-1}[v_{i-1} - v_i]dt + dw_i(t) \tag{2.8}$$

where $w_i(t)$ is a Brownian motion process representing the stochastic fluctuations like acceleration noise.

Comparing (2.8) to equation (2.38) of Cremer one notes that the equations are much alike. There is only a small difference in the convection term due to a different approximation.

4. Observation equations

As was explained in the introductory chapter the signalling system does not provide the exact values of density and mean speed but only registers the passing of vehicles and their speed at the measuring sites. To be able to control the matrix signal boards in an optimal way we need to have a good estimate of the state of traffic at all time instants. Our task then is to develop a filter which estimates traffic density and mean speed from the measurements. This will only be possible if we have a good model for the relation between these measurements and the traffic state. This modelling is discussed in this section. We start by modelling the passing times and in the second part we model the passing speeds.

4.1 Modelling of passing times. In the previous sections we already introduced the counting processes $\pi_i(t)$ ($i=1,...,n$) which count the number of vehicles that pass section boundaries from time t_0 on. We also gave a decomposition of these processes

$$d\pi_i(t) = \lambda_i(...)dt + dm_i(t) \tag{2.9}$$

which describes the dynamics of the process as the sum of a mean evolution term and a stochastic term. For the intensity process λ_i we used the approximate relation

$$\lambda_i(...) = l_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]\ [\ \alpha v_i + (1-\alpha)v_{i+1}\ ] \tag{2.10}$$

If there would be no measuring errors (2.9) and (2.10) together would satisfy our purposes : there is a clear relation between the counting observations and the traffic state variables. Unfortunately there are measuring errors mainly due to vehicles changing lanes at the measuring sites : we may *miss* a vehicle passing or have a *false count*. In order to model these errors introduce

$n_i(t)$: the number of vehicles that were *observed* leaving section i after t_0

$e_i^m(t)$: the number of vehicles we missed leaving section i after t_0

$e_i^f(t)$: the number of false counts at the common boundary of section i and section i+1 after t_0

Then

$$n_i(t) = \pi_i(t) + e_i^f(t) - e_i^m(t) \tag{2.11}$$

Next we need to make some assumptions about the nature of the error processes : how often do they occur, do they depend on traffic volume etc. In absence of any detailed information assume

$$de_i^f(t) = \epsilon_i^f\ \lambda_i dt + dr_i^f(t) \tag{2.12}$$

$$de_i^m(t) = \epsilon_i^m\ \lambda_i dt + dr_i^m(t) \tag{2.13}$$

where

$\epsilon_i^f\ ,\ \epsilon_i^m \in [0,1]$

$r_i^f(t)\ ,\ r_i^m(t)$ martingales

Hence a fixed fraction of the vehicles that pass are supposed to be missed and there is a fixed fraction of false counts. These fractions ϵ_i^m , ϵ_i^f will in general be much smaller than one. Together with (2.9) and (2.10) we now have as our observation equation :

$$dn_i(t) = (1+\epsilon_i^f-\epsilon_i^m)\ [\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]\ [\ \alpha v_i + (1-\alpha)v_{i+1}\ ]dt + d[\ m_i + r_i^f - r_i^m\ ](t) \tag{2.14}$$

4.2 Modelling of passing speeds. A direct way of modelling the passing speeds would be to introduce the *jump* or *mark accumulator* processes

$$y_i(t) = \sum_{j=1}^{\pi_i(t)} v(j) \tag{2.15}$$

where

$v(j)$: the passing speed of the j-th vehicle

Neglect possible measurement errors for the moment. Note that from the process $y_i(t)$ we can deduce both passing times and passing speeds.

It turns out that a filter based on the observation processes (2.15) is too complicated to implement : it requires the integration over all possible passing speeds at every time instant (integro - differential equation). We therefore discretise the domain of passing speeds in m classes :

$$V^1 \; , \; V^2 \; , \; \ldots \; , \; V^m$$

where

$$V^j = [\, v^{j-1} \; , \; v^j \,) \qquad (j=1,\ldots,m)$$

Now introduce the counting processes

$\pi_i^j(t)$: the number of vehicles that left section i after t_0 with passing speed in class V^j

for i=0,...,n and j=1,...,m .
Again, the processes $\pi_i^j(t)$ contain the information of passing times :

$$\pi_i(t) = \sum_{j=1}^{m} \pi_i^j(t)$$

Applying the Doob-Meyer decomposition for submartingales as mentioned in section 2.1 to π_i^j we get

$$d\pi_i^j(t) = \lambda_i^j dt + dm_i^j(t)$$

where λ_i^j is the intensity and m_i^j is the martingale. As before λ_i^j depends on the traffic density and mean speed, for

$$\lambda_i = \sum_{j=1}^{m} \lambda_i^j = l_i \, [\, \alpha\rho_i + (1-\alpha)\rho_{i+1} \,] \, [\, \alpha v_i + (1-\alpha) v_{i+1} \,]$$

The problem now is how to distribute the total intensity λ_i over the speed classes. In studying this problem we came up with two possible solutions :

1. Instead of considering as the traffic state $\{ \rho_i \, , \, v_i \}$ consider

   $\rho_i^j(t)$: the number of vehicles in section i per km per lane that at time t drive with a speed that falls in class V^j

   Instead of the general model (2.2), (2.3) we then have

   $$d\rho_i^j(t) = f_i{}^j(\ldots)dt + dz_i^j(t) \tag{2.16}$$

   for i=1,...,n and j=1,...,m .
   The modelling of the intensity is easy :

   $$\lambda_i^j = l_i \, [\, \alpha\rho_i^j + (1-\alpha)\rho_{i+1}^j \,] \, (\, v^{j-1} + v^j \,)/2$$

   in analogy with (2.6) . The speed $(v^{j-1} + v^j)/2$ is taken to be the representative of class V^j.

But how to find a reasonable expression for $f_i^{\ j}$ in (2.16) ? Note that in fact we are trying to model the dynamic behaviour of a discretised version of the speed distribution. For

$$\frac{\rho_i^j}{\sum_{j=1}^{m} \rho_i^j}$$

is the fraction of vehicles driving in class V^j at time t. Attempts to model this behaviour have been made, mainly by Prigogine [16] . At the moment there does not seem to be a satisfactory model of this type. The original model by Prigogine has some serious defects and improved versions turn out to be much more complicated [1, 19] . Hardly any practical investigation with these models has been reported up to date [7] .

Summarising we may state that introduction of the variables ρ_i^j complicates matters considerably whereas to the benefits there is much doubt. Therefore we will not proceed in this way but opt for the next approach.

2. Introduce

$$\gamma_i^j \ : \ \text{the fraction of vehicles that leave section i with passing speed in class } V^j \qquad (2.17)$$

It seems reasonable to assume that these fractions only depend on mean speed and density :

$$\gamma_i^j = \gamma_i^j(\rho_i, \rho_{i+1}, v_i, v_{i+1}) \qquad (2.18)$$

Then we may write

$$\lambda_i^j = \gamma_i^j \ \lambda_i$$

Note that again we are making assumptions about the behaviour of the speed distribution (see appendix D). Whether the assumption (2.18) holds, even under stationary conditions, has to be checked by analysing real-life traffic data. This is what we plan to do in the near future. The analysis will hopefully give us some clues as to the nature of the relation between the form of the distribution and the traffic density and mean speed. We suspect the distribution of passing speeds per lane to be approximately normal and that the distribution aggregated over all lanes may have several peaks : one for each lane. Furthermore the variance may depend on the density.

As a temporary solution one might take the passing speeds to have a logistics distribution which approximates the normal distribution but has the advantage of having an explicit mathematical expression. See appendix D.

Because of the disadvantages of approach 1 we will opt for 2. We then have

$$d\pi_i^j(t) = \gamma_i^j \ l_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]\ [\ \alpha v_i + (1-\alpha)v_{i+1}\ ]dt + dm_i^j(t) \qquad (2.19)$$

Note that as γ_i^j sums to 1 we have

$$\sum_{j=1}^{m} \lambda_i^j = l_i[\ \alpha\rho_i^j + (1-\alpha)\rho_{i+1}^j\ ]\ [\ \alpha v_i^j + (1-\alpha)v_{i+1}^j\ ] = \lambda_i$$

As noted before : the processes π_i^j contain the information of passing times and so (2.19) is not an extra observation equation next to (2.14) but (2.19) *replaces* (2.14).

To conclude, some words regarding measurement errors. There are two types of errors : counting errors and speed measurement errors. Following the derivation in section 2.4.1 we find

$$dn_i^j(t) = (1+\epsilon_i^{f^j} - \epsilon_i^{m^j})\ \gamma_i^j\lambda_i dt + d[\ m_i^j + r_i^{f^j} + r_i^{m^j}\ ](t)$$

where

$n_i^j(t)$: the number of vehicles that were *observed* leaving section i with speed in class V^j

$$\sum_{j=1}^{m} \epsilon_i^{f^j} = \epsilon_i^f$$

$$\sum_{j=1}^{m} \epsilon_i^{m^j} = \epsilon_i^m$$

As to the speed measurement errors we will assume that these are small enough in comparison to the width of the speed classes to justify neglection.

5. SUMMARY

In the previous five sections we have presented a model that we are planning to use for estimation and control of freeway traffic. In this section we summarise the model. Due to results from simulation studies the model will be modified in the next chapter. The revised model will be summarised in section 3.4.

MODEL

$$d\rho_i(t) = \frac{1}{l_i L_i}\Big\{ l_{i-1}[\,\alpha\rho_{i-1} + (1-\alpha)\rho_i\,][\,\alpha v_{i-1} + (1-\alpha)v_i\,]$$

$$- l_i[\,\alpha\rho_i + (1-\alpha)\rho_{i+1}\,][\,\alpha v_i + (1-\alpha)v_{i+1}\,]\Big\}dt$$

$$+ \frac{1}{l_i L_i}[\,dm_{i-1}(t) - dm_i(t)\,]$$

$$dv_i(t) = -\frac{1}{T}\left[v_i - v_f(1-\frac{\rho_i}{\rho_j}) \right]dt - \frac{\nu}{T(L_i+L_{i+1})}\left[\frac{\rho_{i+1} - \rho_i}{\rho_i + c} \right]dt$$

$$+ \frac{l_{i-1}}{l_i L_i} v_{i-1}[\,v_{i-1} - v_i\,]dt + dw_i(t)$$

for i=1,...,n .

OBSERVATIONS

$$dn_i^j(t) = (1+\epsilon_i^{f^j} - \epsilon_i^{m^j})\,\gamma_i{}^j(\rho_i,\rho_{i+1},v_i,v_{i+1})\, l_i[\,\alpha\rho_i + (1-\alpha)\rho_{i+1}\,][\,\alpha v_i + (1-\alpha)v_{i+1}\,]dt$$

$$+ d[\,m_i^j + r_i^{f^j} + r_i^{m^j}\,](t)$$

for i=0,...,n and j=1,...,m .

EXPLANATION of variables and parameters

$\rho_i(t)$: density, the number of vehicles in section i per km per lane

$v_i(t)$: the mean speed of the vehicles in section i (km/h)

$n_i^j(t)$: the number of vehicles that left section i with speed in class V^j starting from t_0

$\epsilon_i^{f^j}$: the fraction of false counts in class V^j at the common boundary of section i and section i+1

$\epsilon_i^{m^j}$: the fraction of vehicles with speed in class j that were missed leaving section i

$m_i(t)$: the martingale associated with $n_i(t) = \sum_{j=1}^{m} n_i^j(t)$

$m_i^j(t)$: the martingale associated with $n_i^j(t)$

$r_i^{f^j}(t)$, $r_i^{m^j}(t)$: martingales associated with the counting error processes

$w_i(t)$: Brownian motion process

n : the number of sections

m : the number of speed classes

V^j : the j^{th} speed class : [v^{j-1}, v^j)

γ_i^j : the fraction of vehicles that leave section i with speed in class V^j

l_i : the number of lanes of section i

L_i : the length of section i (km)

α : weighting factor $\in [0,1]$

T : relaxation time (h)

v_f : free speed, equilibrium mean speed at density zero (km/h)

ρ_j : jam density, at which equilibrium speed is zero (veh/km/lane)

ν : anticipation factor (km^2/h)

c : correction constant (veh/km/lane)

IDENTIFICATION of parameters

In an earlier investigation by Van Maarseveen [15] the parameters T , ρ_j , v_f , ν , c were identified using real traffic data from the signalling system. Only the passing times were used in the estimation, the speed measurements were neglected. The weighting parameter α was chosen beforehand to be equal to 0.5. The counting errors and acceleration noise were assumed to be zero. The estimation procedure resulted in the following parameter values :

parameter	value	unit
T	0.0044	h
ρ_j	116.0	veh/km/lane
v_f	106.0	km/h
ν	138.0	km^2/h
c	10.0	veh/km/lane

TABLE 2.1

BOUNDARY CONDITIONS

In the model the variables ρ_0 , v_0 , ρ_{n+1} , v_{n+1} are not defined. Suitable boundary conditions at the entrance and at the exit of the freeway stretch have to be chosen to get a closed set of equations. As noted by Cremer [4] this is not an easy task. In some traffic situations the values at the boundary will be determined by the internal state of the system while in other situations the state is determined by the boundary values. In stationary traffic flow e.g. the entrance values ρ_0 , v_0 (or to be more clear : the entrance intensity) determine(s) the evolution of the density and mean speed of all the other sections including the imaginary $(n+1)^{th}$ one. When there is a bottleneck downstream however, the roles are interchanged : the values at the exit determine the values in all other sections.

We will now discuss several possible choices of boundary conditions.

1. *prescribed intensity*

$$\text{entrance}: \quad \rho_0 = [\, \frac{\lambda_0}{v_1} - (1-\alpha)\rho_1 \,]/\alpha$$

$$v_0 = v_1$$

$$\text{exit}: \quad \rho_{n+1} = \rho_n$$

$$v_{n+1} = [\, \frac{\lambda_n}{\rho_n} - \alpha v_n \,]/(1-\alpha)$$

where $\lambda_0(t)$ and $\lambda_n(t)$ are given as functions of time.

2. *stationarity*

$$\text{entrance}: \quad \rho_0 = \rho_1$$

$$v_0 = v_1$$

$$\text{exit}: \quad \rho_{n+1} = \rho_n$$

$$v_{n+1} = v_n$$

3. *extrapolation*

$$\text{entrance}: \quad \rho_0 - \rho_1 = \epsilon(\rho_1 - \rho_2)$$

$$v_0 - v_1 = \epsilon(v_1 - v_2)$$

$$\text{exit}: \quad \rho_{n+1} - \rho_n = \delta(\rho_n - \rho_{n-1})$$

$$v_{n+1} - v_n = \delta(v_n - v_{n-1})$$

where ϵ , $\delta \in [0,1]$

We have experimented with the first two possibilities and found that in general prescribing the entrance intensity and choosing a stationarity condition at the exit gave satisfactory results. Choosing a stationarity condition at the entrance led to unrealistically large fluctuations in the densities. Prescribing the exit intensity led to large fluctuations in the density of the last section which then influenced the whole freeway stretch. We have not yet experimented with extrapolation conditions.

ON- and OFF-RAMPS

Up until now we have not considered the possibility of on- and off-ramps. These can be modelled quite easily if the intensities on the ramps are measured. If they are not the situation is more complicated : we will not discuss that situation here.

Suppose that at the beginning of section i there is an on-ramp with intensity $r_i^{on}(t)$ and at the end there is an off-ramp with intensity $r_i^{off}(t)$. Then only the density equation of our model has to be changed :

$$d\rho_i(t) = \ldots\ldots + \frac{1}{L_i l_i} \left(dr_i^{on}(t) - dr_i^{off}(t) \right) \tag{2.20}$$

Chapter 3

Simulation of traffic flow

In the previous chapter we developed a macroscopic model of freeway traffic flow. The model consists of a set of stochastic differential equations for the state variables density and mean speed which were defined for each freeway section. In addition to these equations there are differential equations for the observations : passing times and speeds.

Once initial values are chosen for density and mean speed the set of equations can be solved numerically, simulating the stochastics by some random number generation procedure. The behaviour of the model and its validity can thus be studied. If the model does not show a realistic behaviour it may be adjusted. In this way we may refine our model. This is what we plan to do in this chapter.

A computer program was written in FORTRAN version 5 (which complies ANSI FORTRAN 77) and run on a CDC Cyber 750 mainframe under operating system NOS/BE. The integration of the differential equations is simply done by Euler's method, justified by the fact that in our case the error is of order h^2 instead of the usual h^1 (where h is the integration step) : [22] . The counting processes are generated using the so called *thinning* method developed by Lewis & Shedler [13, 17] . This method allows us to generate *exact* realisations whereas other methods are approximative. See appendix E for a detailed explanation of this method.

In section 3.1 we start by simulation of low density traffic. The results lead to a change of the anticipation strength of the model. In section 3.2 moderate density flow is simulated. The result lead us to modify the equilibrium relation between speed and density. In section 3.3 we simulate high density traffic and change the entire anticipation term of the model. Finally, in section 3.4 the new model is summarised.

1. Low density traffic

To get a first impression of the validity of the model we start by simulating traffic flow at low density : approximately 15 veh/km/lane. We consider stationary flow by taking the entrance intensity constant in time, equal to the equilibrium value of 1383.3 veh/h/lane. The freeway stretch consists of 12 sections of 500 m each and has 2 lanes. The model parameters are chosen according to Van Maarseveen (Table 2.1). The simulation results are presented in appendix H.1.

There are some very large fluctuations in the mean speed at time 0.033 h : the speed ranges from 0 to 141 km/h over the freeway stretch. Furthermore, it seems that in the sections where traffic is most dense speed is highest : compare sections 3 and 4 e.g. This does not seem to be very realistic. In case of stationary flow one would not expect such large fluctuations in the *mean* speed to occur and in dense regions one would expect traffic to flow at a lower speed than in less dense regions. It is the anticipation term that forces the speed to behave in the described unrealistic way. Anticipation forces drivers to accelerate when downstream traffic is less dense. Apparently in our model this effect is too strong.

In identifying the parameters of the model Van Maarseveen used a data set which did not contain enough information to estimate the anticipation factor accurately [15] , pp. 177 - 178. Identification of discrete time models by Grewal & Payne [10] and Cremer & Papageorgiou [5] led to much smaller anticipation factors than Van Maarseveen's. We therefore take a lower value for this factor. At the moment our choice is a bit arbitrary, we just choose a value that gives reasonable results. In the future we will use an identification technique to obtain a better estimate. We now repeat the previous simulation with $\nu = 40.0$ km^2/h and all other parameter values unchanged. See appendix H.2 for the results.

This time the speed is behaving smoothly and the differences between the sections are not as large as before. The density however shows very large fluctuations, one of the sections is empty for a period of time whereas an other contains 32 vehicles. Although large fluctuations are to be expected the ones above seem to be too extreme. This unrealistic behaviour is mainly caused by the fact that when a section is almost empty but its downstream neighbour is not, the intensity at their common boundary is still relatively high. This intensity is modelled as the product of weighted density and mean speed :

$$l_i[\,\alpha\rho_i(t) + (1-\alpha)\rho_{i+1}(t)\,]\,[\,\alpha v_i(t) + (1-\alpha)v_{i+1}(t)\,]$$

Up until now we took α to be 0.5. By taking a larger value we may increase the influence of the upstream density and thereby decrease the intensity at the boundary when this upstream density is low. We now repeat the previous simulation with $\alpha = 0.85$. See appendix H.3 for the results.

Now the behaviour of density and mean speed is satisfactory. We conclude that our model is now able to describe low density traffic in a reasonable way.

2. MODERATE DENSITY TRAFFIC

Now that we have a model which shows reasonable behaviour at low density we will test its validity at moderate density values : 20 - 30 veh/km/lane. Note that in practice the critical density (at which the traffic stream becomes unstable) is at approximately 25 - 30 veh/km/lane. We again consider a freeway stretch of 12 sections 500 m each, 2 lanes. Parameters are as in the last simulation of the previous section. We start with 20 veh/km/lane in all freeway sections and take the entrance intensity constant in time and equal to 2000 veh/h/lane. In equilibrium this corresponds to a density of approximately 24 veh/km/lane. See appendix H.4 for the results.

As with the one realisation presented in the appendix all realisations show very stable traffic behaviour analogous to the behaviour at low density. Nevertheless the density reaches a value of about 30 veh/km/lane for a considerable period of time. So one would expect the model to show some instabilities in at least some of the realisations. Stability analysis of the model confirms our suspicion that the model can't show instabilities : it is stable for densities up to 60 veh/km/lane. (The analysis consists of linearising the model around an equilibrium solution with $\rho_i = \bar{\rho}$, $v_i = v^e(\bar{\rho})$ for $i=1,\ldots,n$ and examining the eigenvalues of the linear model. See appendix G for more details). Already from basic considerations (Papageorgiou [18] , Whitham [24]) and also from a more complicated one (Kühne [12]) it follows that the equilibrium relation between speed and density plays a major role in the stability properties of the model. The change from stable to unstable flow occurs at about that value of the density for which the equilibrium flow reaches its maximum value.

In our case :

$$v^e(\rho) = 106.0[1-\rho/116.0] \qquad 0 \leq \rho \leq 116 \tag{3.1}$$

and

$$\lambda^e(\rho) = \rho v^e(\rho)$$

so

$$\lambda^e{}_{\max} = 3074 \text{ veh/h/lane} \quad \text{at} \quad \rho = 58 \text{ veh/km/lane}$$

See figure 1 in appendix F.

We now conclude that we will have to change our equilibrium relation $v^e(\rho)$ in order to have unstable behaviour at a realistic density. In fact, the relation we have used until now is the simplest one, already introduced in 1934 by Greenshields [9] . Since then a large amount of literature has appeared concerning the equilibrium relationship between density, speed and intensity : [11] . It turns out to be difficult to choose between the various proposed relations. We will therefore for the time being take a simple modification of (3.1) which suits our purposes :

$$v^e(\rho) = \begin{cases} v_f\,[1-\dfrac{\rho}{\rho_j}] & 0 \leqslant \rho \leqslant \rho_{crit} \\ d\,[\dfrac{1}{\rho} - \dfrac{1}{\rho_j}] & \rho_{crit} \leqslant \rho \leqslant \rho_j \end{cases} \tag{3.2}$$

where d is chosen in such a way that v^e is continuous at ρ_{crit} . We will take $\rho_j = 110.0$ veh/km/lane, $v_f = 110$ km/h and $\rho_{crit} = 27.0$ veh/km/lane, which leads to $d = 2970.0$ 1/h. The above relation is linear in the low density region and decreases fast enough for densities larger than the critical density to prevent λ^e from having a maximum which is too large :

$$\lambda^e(\rho) = \begin{cases} 110.0\,\rho\,[1-\dfrac{\rho}{110.0}] & 0 \leqslant \rho \leqslant 27 \\ 2970.0\,[1-\dfrac{\rho}{110.0}] & 27 \leqslant \rho \leqslant 110 \end{cases}$$

so

$$\lambda^e{}_{\max} = 2241 \text{ veh/h/lane} \quad \text{at} \quad \rho = 27 \text{ veh/km/lane}$$

See figure 2 in appendix F. Stability analysis of the new model shows the region of stability to reach from 0 to 30 veh/km/lane.

Now we will repeat the previous simulation with our new model. See appendix H.5 for the results.

A collapse of the traffic stream occurs in which density increases to 35 veh/km/lane and speed decreases to 60 km/h and which starts in section 1 and then moves downstreams with a speed of approximately 35 km/h. We conclude that the form of the equilibrium relation plays a major role in the stability properties of the model and that our choice leads to realistic behaviour.

3. High density traffic

Having constructed a model that is able to describe traffic flow up to moderate density in a satisfactory way we might stop there and proceed with the development of a filter based on the model or an identification procedure to obtain near optimal values of the model parameters. It may be interesting however to look at an extreme traffic situation : almost complete standstill at maximal density. We consider a freeway stretch of 12 sections 500 m each, 2 lanes and simulate starting from a traffic situation where there is a complete standstill at the end of the stretch while at the beginning traffic is still streaming at high speed. See appendix H.6 for the results.

The results are highly unrealistic : vehicles pile up in sections 7, 8 and 9 and density reaches values up to 216 veh/km/lane. This means that the space available for a vehicle reduces to 4.6 m on the average. The jam density was modelled at 110 veh/km/lane. In the upstream sections there is no effect at all : drivers keep driving at about their equilibrium speed.

In practice one would expect the congested region to grow in the upstream direction and density not to exceed the jam value. It is clear that the anticipation strength in the model is too small.

Recall that we weakened the anticipative effect in section 3.1 in order to avoid unrealistically large speed fluctuations at low densities. It seems however that at high densities anticipation should be strong.

The above results suggest to model anticipation in such a way that the effect increases with increasing density. The term as it is in our present model :

$$(dv_i(t))_{ant} = -\frac{\nu}{(L_i+L_{i+1})T}\left[\frac{\rho_{i+1} - \rho_i}{\rho_i + c}\right]dt$$

has just the opposite effect. If the constant c would not be there (and it isn't in the original Payne model) the anticipation strength would grow without bounds as $\rho_i \rightarrow 0$. Instead we now propose

$$(dv_i(t))_{ant} = -\gamma\,(L_i l_i)^2[\,\beta\rho_i + (1-\beta)\rho_{i+1}\,]\,[\,\rho_{i+1}-\rho_i\,]dt \qquad (3.3)$$

where

γ : a constant of dimension km/h^2

β : a weighting factor $\in [0,1]$

In (3.3)

$$[\,\beta\rho_i + (1-\beta)\rho_{i+1}\,]$$

is an approximation of the prevailing density on the freeway stretch near section i.

If $\beta = 0$ all the emphasis would lie on the density of the downstream section and there would be no anticipation if ρ_i was 0 even if ρ_{i+1} would be as large as 100.0. On the other hand there would be a very strong effect if $\rho_i = 100.0$ and $\rho_{i+1} = 0$. If $\beta = 1.0$ the emphasis would lie on the upstream section and the conclusions would go the other way around. It is clear that we would not like to choose either of the two extremes but some intermediate value.

Investigations with respect to the so called *hysteresis phenomenon* in traffic flow (Treiterer & Myers [23]) suggest an asymmetric driver behaviour. This means that a driver anticipating an *increase* of density is likely to react stronger than when anticipating a *decrease* of density. We may model this effect by taking a value of β larger than 0.5 . We plan to investigate the effect of an asymmetric anticipation term in more detail in the future.[1] For the time being we just choose $\beta = 0.5$. The parameter γ is chosen such that at a density of approximately 20 veh/km/lane the anticipation effect is about as strong as it was in the previous model. This means that the new model shows the same behaviour as the old model for low and moderate densities.

Let us now repeat the simulation 3.6 with our new model. See appendix H.7 for the results.

We now see the desired behaviour : the congested region spreads itself in the upstream direction and density hardly exceeds the jam value. Apart from that we now see a fascinating behaviour in the congested region : *stop-start traffic*. This phenomenon is observed in practice, especially in tunnel flow [6] or upstream of bottlenecks [11] . In how far the amplitude and period of the stop-start waves are in accordance with reality is not yet clear. For the moment we restrict ourselves to noting the ability of the new model to describe this type of behaviour at high densities. As a final remark we note that stability analysis shows that at the critical density of 31 veh/km/lane the new model has a *pair* of purely imaginary eigenvalues, in contrast to the old model which only had one eigenvalue on the imaginary axis of the complex plane : zero. See appendix G.

1. Another suggestion is to take the anticipation strength to be quadratic in $[\,\beta\rho_i + (1-\beta)\rho_{i+1}\,]$. instead of linear as in (3.3). The effect of such a term has still to be investigated.

4. The revised model

The simulation results of the previous three sections have resulted in a revised model of freeway traffic flow. The changes consist of adjustment of the parameter α , the equilibrium speed-density relationship and the anticipation term.

MODEL

$$d\rho_i(t) = \frac{1}{l_i L_i}\Big\{ l_{i-1}[\ \alpha\rho_{i-1} + (1-\alpha)\rho_i\ ][\ \alpha v_{i-1} + (1-\alpha)v_i\ ]$$

$$- l_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ][\ \alpha v_i + (1-\alpha)v_{i+1}\ ]\Big\}dt$$

$$+ \frac{1}{l_i L_i}[\ dm_{i-1}(t) - dm_i(t)\ ]$$

$$dv_i(t) = -\frac{1}{T}(\ v_i - v^e(\rho_i)\ )dt - \gamma(L_i l_i)^2[\ \beta\rho_i + (1-\beta)\rho_{i+1}\ ][\ \rho_{i+1} - \rho_i\ ]dt$$

$$+ \frac{l_{i-1}}{l_i L_i} v_{i-1}[\ v_{i-1} - v_i\ ]dt + dw_i(t)$$

where

$$v^e(\rho) = \begin{cases} v_f[1 - \frac{\rho}{\rho_j}] & 0 \leqslant \rho \leqslant \rho_{crit} \\ d\,[\frac{1}{\rho} - \frac{1}{\rho_j}] & \rho_{crit} \leqslant \rho \leqslant \rho_j \end{cases}$$

for i=1,...,n .

OBSERVATIONS

$$dn_i^j(t) = (1+\epsilon_i^{f^j} - \epsilon_i^{m^j})\ \gamma_i{}^j(\rho_i,\rho_{i+1},v_i,v_{i+1})\ l_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ][\ \alpha v_i + (1-\alpha)v_{i+1}\ ]dt$$

$$+ d[\ m_i^j + r_i^{f^j} + r_i^{m^j}\ ](t)$$

for i=0,...,n and j=1,...,m .

EXPLANATION of variables and parameters

$\rho_i(t)$: density, the number of vehicles in section i per km per lane

$v_i(t)$: the mean speed of the vehicles in section i (km/h)

$n_i^j(t)$: the number of vehicles that left section i with speed in class V^j starting from t_0

$\epsilon_i^{f^j}$: the fraction of false counts in class V^j at the common boundary of section i and section i+1

$\epsilon_i^{m^j}$: the fraction of vehicles with speed in class j that were missed leaving section i

$m_i(t)$: the martingale associated with $n_i(t) = \sum_{j=1}^{m} n_i^j(t)$

$m_i^j(t)$: the martingale associated with $n_i^j(t)$

$r_i^{f^j}(t)$, $r_i^{m^j}(t)$: martingales associated with the counting error processes

$w_i(t)$: Brownian motion process

n : the number of sections

m : the number of speed classes

V^j : the j^{th} speed class : [v^{j-1}, v^j)

γ_i^j : the fraction of vehicles that leave section i with speed in class V^j

l_i : the number of lanes of section i

L_i : the length of section i (km)

α : weighting factor $\in [0,1]$

T : relaxation time (h)

v_f : free speed, equilibrium mean speed at density zero (km/h)

ρ_j : jam density, at which equilibrium speed is zero (veh/km/lane)

ρ_{crit} : the critical density : the density at which the equilibrium intensity reaches its maximum value

d : parameter in $v^e(\rho)$

γ : anticipation factor (km/h^2)

β : weighting factor $\in [0,1]$

IDENTIFICATION of parameters

We have not yet applied an identification procedure to our new model in order to find good estimates of the parameter values. For the moment we have chosen values that gave reasonable model behaviour. These values are as follows :

parameter	value	unit
α	0.85	
T	0.01	h
ρ_j	110.0	veh/km/lane
v_f	110.0	km/h
ρ_{crit}	27.0	veh/km/lane
d	2970.0	1/h
γ	6.5	km/h^2
β	0.5	

TABLE 3.1

BOUNDARY CONDITIONS

See section 2.5

ON- and OFF-RAMPS

See section 2.5

Chapter 4

Conclusions

In this report we presented a model for freeway traffic and with help of simulations we were able to improve its validity. The final model shows reasonable behaviour under various circumstances : low, moderate and high density traffic.

In the simulation experiments parameters were given values for which the model showed realistic behaviour. Identification techniques using real-life traffic data will have to be applied to obtain better estimates and improved model behaviour.

Part of the model is the probability distribution of passing speeds of vehicles at a fixed location along the freeway. Some assumptions about the form of the distribution have been made. These assumptions have to be checked against real-life data.

The anticipation term of the model leads to stop-start behaviour at high densities. The validity of this behaviour also has to be checked by analysing real-life traffic data.

On Dutch freeways there are many on- and off-ramps. The modelling of these when the ramp intensities are not measured was not discussed. This problem will have to be investigated to obtain a practically useful model.

The development of a good model for freeway traffic flow is the first step towards the design of control strategies for the matrix boards of the signalling system. The next step is to develop a filter which recursively estimates the traffic state (the densities and mean speeds in all freeway sections) from the observations. Approximations to the theoretically optimal filter and comparisons among several approximative filters have to be made. This filter design procedure may lead to modification of the traffic model.

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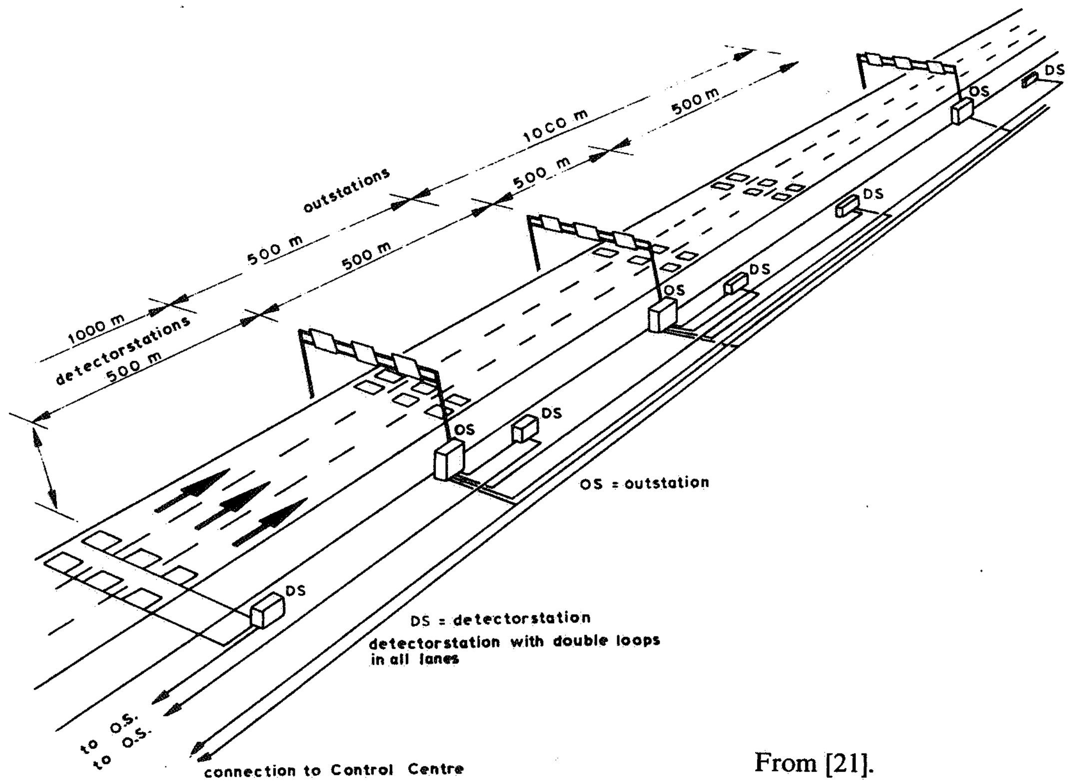
Appendix A

The signalling system



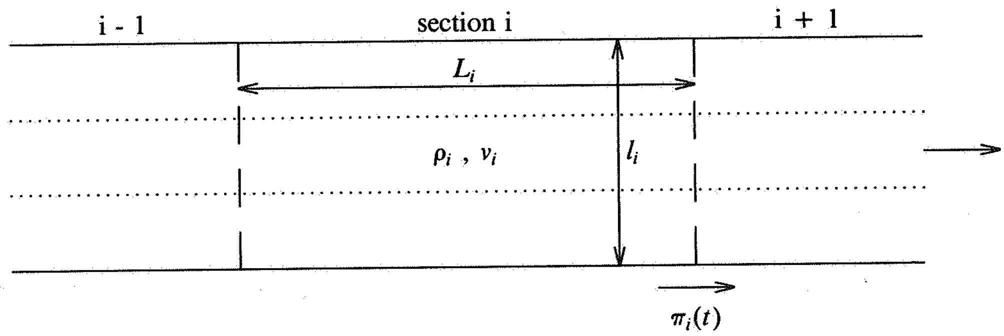
From [21].

FIGURE A.1

Appendix B

The freeway stretch

The freeway is divided into sections in the following way :



where

$\rho_i(t)$: the number of vehicles in section i at time t per length of road per lane (density)

$v_i(t)$: the mean speed of the vehicles in section i at time t

$\pi_i(t)$: the number of vehicles that passed the boundary between section i and i+1 after time t_0 (counting process)

L_i : the length of section i

l_i : the number of lanes of section i

Appendix C

Counting processes and martingales

We only give a few definitions and theorems, for a thorough review of the theory see e.g. [2,3] .

We assume to be given a *probability space* $(\Omega,\mathfrak{F},P)$ where Ω is the set of elementary events, $\mathfrak{F}$ the σ-algebra of events and P a probability measure on $\mathfrak{F}$. Next we introduce the *family* of σ-algebras $\{ \mathfrak{F}_t , t \geq t_0 \}$ which we assume to satisfy the following conditions

(1) $\mathfrak{F}_t \subset \mathfrak{F}$ for all $t_0 \leq t$

(2) $\mathfrak{F}_s \subset \mathfrak{F}_t$ for all $t_0 \leq s \leq t$

(3) $\bigcap_{s>t} \mathfrak{F}_s = \mathfrak{F}_t$ for all $t_0 \leq t$

(4) $\mathfrak{F}_{t_0}$ contains all sets with P-measure 0

The σ-algebra $\mathfrak{F}_t$ can be thought of as being generated by one or more stochastic processes up to and including t. The second condition is the essential one : it describes the increase of information with time.

We now come to a special kind of stochastic process.

DEFINITION C.1 *The process* $\{m_t , t \geq t_0\}$ *is called a* martingale *with respect to the family of* σ*-algebras* $\{\mathfrak{F}_t , t \geq t_0\}$ *if it satisfies the following conditions*

(1) m_t is $\mathfrak{F}_t$-measurable for all $t_0 \leq t$

(2) $m_{t_0} = 0$ except possibly for a set of P-measure zero

(3) $E[|m_t|] < \infty$ for all $t_0 \leq t$

(4) $E[m_t | \mathfrak{F}_s] = m_s$ except possibly for a set of P-measure zero, for all $t_0 \leq t$

The first condition states that at all time instants all the information about m_t has to be contained in $\mathfrak{F}_t$. In practice this will mean that $\{m_t , t \geq t_0\}$ is (one of) the process(es) that generates $\{\mathfrak{F}_t , t \geq t_0\}$. m_t is said to be *adapted* to the family $\{\mathfrak{F}_t , t \geq t_0\}$. The second condition is not a necessary one, it is just a convention. The fourth condition states the martingale condition as such. Conditioned upon the information we have at time s we expect the process to be constant in the mean over all realisations. We do not expect the process to increase or decrease. Specifically, from (4) it follows that $E[m_t] = E[m_{t_0}]$ for all $t \geqslant t_0$. We can also define *sub-* and *supermartingales* by replacing the equality sign in (4) by $\geqslant$ and $\leqslant$ respectively.

Let us now introduce a counting process.

DEFINITION C.2 *The process* $\{n_t\ ,\ t \geqslant t_0\}$ *is called a* counting process *if it satisfies the following conditions*

(1) $n_{t_0} = 0$ except possibly for a set of P-measure zero

(2) the sample paths are piecewise constant and right continuous except possibly for a set of P-measure zero

(3) the jumps are unit jumps

A well-known counting process is the *Poisson process* where the intervals between jump times are independent and exponentially distributed with constant parameter λ.

Clearly, a counting process is an increasing process and under the assumption of a finite number jumps in any finite time interval it is submartingale with respect to any family to which it is adapted.

THEOREM C.3 *Let* $\{n_t\ ,\ t \geqslant t_0\}$ *be a counting process and* $\{\mathcal{F}_t\ ,\ t \geqslant t_0\}$ *a family of* σ*-algebras. If*

(1) $n_t < \infty$ except possibly for a set of P-measure zero, for all t $\geqslant t_0$

(2) n_t is $\mathcal{F}_t$-measurable for all t $\geqslant t_0$

then $\{n_t\ ,\ t \geqslant t_0\}$ *is a submartingale with respect to the family* $\{\mathcal{F}_t\ ,\ t \geqslant t_0\}$.

Under some technical conditions, not to be quoted here, submartingales can be uniquely decomposed into two processes with special characteristics. This is the so called *Doob-Meyer decomposition.* Counting processes satisfy the conditions and thus we have the following theorem :

THEOREM C.4 (Doob-Meyer decomposition) *Let* $\{n_t\ ,\ t \geqslant t_0\}$ *be a counting process which is a submartingale with respect to the family* $\{\mathcal{F}_t\ ,\ t \geqslant t_0\}$. *Then there exists a (right continuous)* $\mathcal{F}_t$*-martingale* $\{m_t\ ,\ t \geqslant t_0\}$ *and a* $\mathcal{F}_t$*-predictable increasing process* $\{a_t\ ,\ t \geqslant t_0\}$ *such that*

$$n_t = a_t + m_t \qquad \text{for all } t \geqslant t_0$$

The predictability of the process $\{a_t\ ,\ t \geqslant t_0\}$ is a mathematical concept which in practice means that the process is adapted and left continuous or the limit of left continuous adapted processes.

In many cases the process $\{a_t\ ,\ t \geqslant t_0\}$ turns out to be absolutely continuous which means that it can be written as

$$a_t = \int_{t_0}^{t} \lambda_s ds \qquad \text{for all } t \geqslant t_0$$

where $\{\lambda_t\ ,\ t \geqslant t_0\}$ is a nonnegative process which is $\mathcal{F}_t$-measurable for all t $\geqslant t_0$. So we may write the decomposition as

$$n_t = \int_{t_0}^{t} \lambda_s ds + m_t$$

or

$$dn_t = \lambda_t dt + dm_t$$

The process $\{\lambda_t\ ,\ t \geqslant t_0\}$ is called the *rate* or *intensity* process of $\{n_t\ ,\ t \geqslant t_0\}$. For an illustration of the decomposition see figure C.1 where we have plotted a realisation of a counting process with constant intensity λ. The dashed line represents the behaviour of the counting process in the mean.

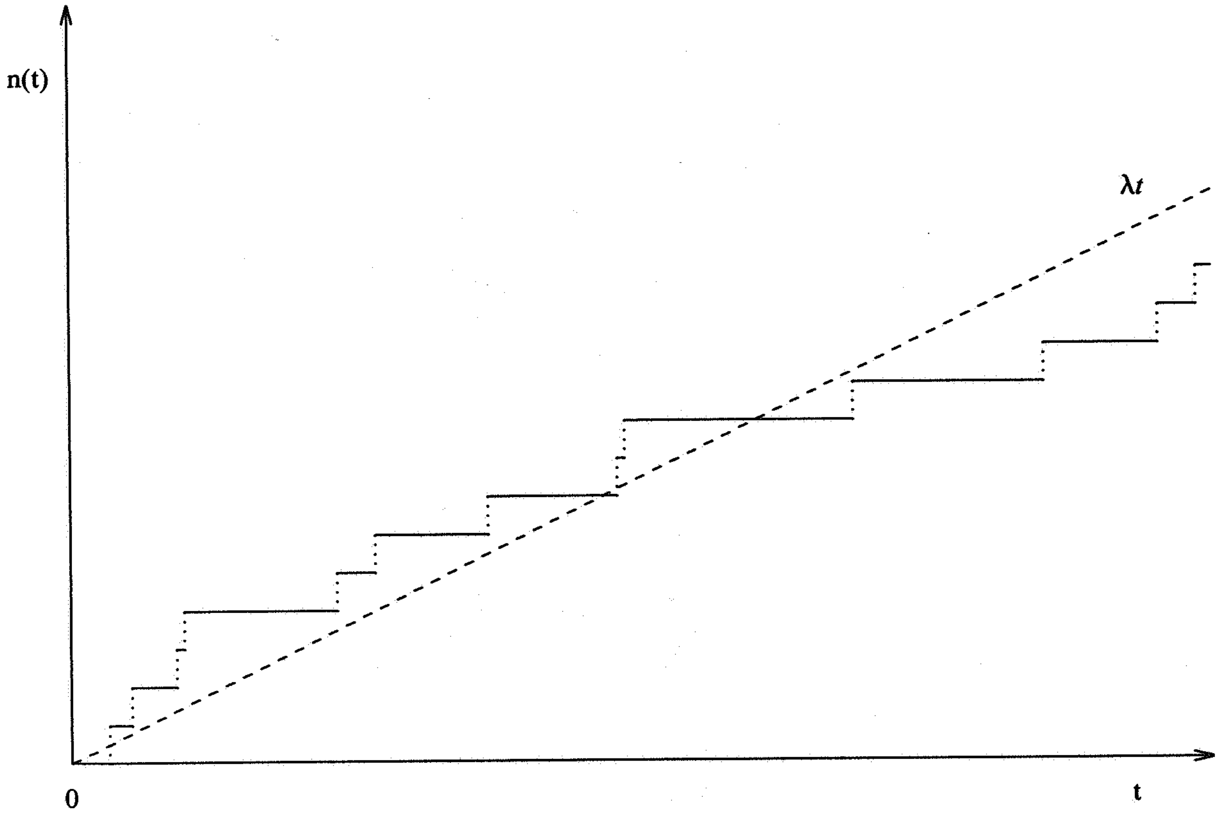


FIGURE C.1

Appendix D

The speed distribution

One may assume that the passing speed of a vehicle at a fixed location along the freeway is a stochastic variable with an absolutely continuous distribution function

$$F(v) = \int_0^v f(u)du$$

Introducing the *speed classes* V^1 , V^2 , ... , V^m with

$$V^j = [v^{j-1}, v^j) \qquad j = 1,\dots,m$$

we may compute the *fractions* γ^j in each class

$$\gamma^j = \int_{v^{j-1}}^{v^j} f(u)du$$

γ^j is the expected fraction of cars that pass the location with speed in class V^j. See figure D.1.

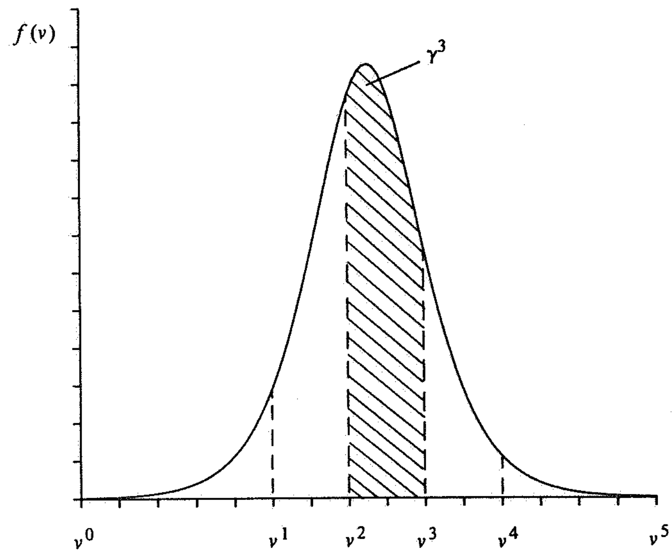


FIGURE D.1

Whenever $v^0 > -\infty$ and $F(v^0) > 0$ or $v^m < \infty$ and $F(v^m) > 0$ we would like to add a slight correction

$$\gamma^j = \int_{v^{j-1}}^{v^j} f(u)du + \frac{1}{m}[F(v^0) - F(v^m)]$$

in order to achieve

$$\sum_{j=1}^{m} \gamma^j = 1$$

In our freeway model we assume the probability density $f(v)$ to depend on traffic density $\bar{\rho}$ and mean speed $\bar{v}$ only : $f^{\bar{\rho},\bar{v}}(v)$. If we measure the passing speed at the boundary between two sections i and i+1 we will take $\bar{\rho} = [\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]$ and $\bar{v} = [\ \alpha v_i + (1-\alpha)v_{i+1}\ ]$. Two possible examples are in figure D.2.

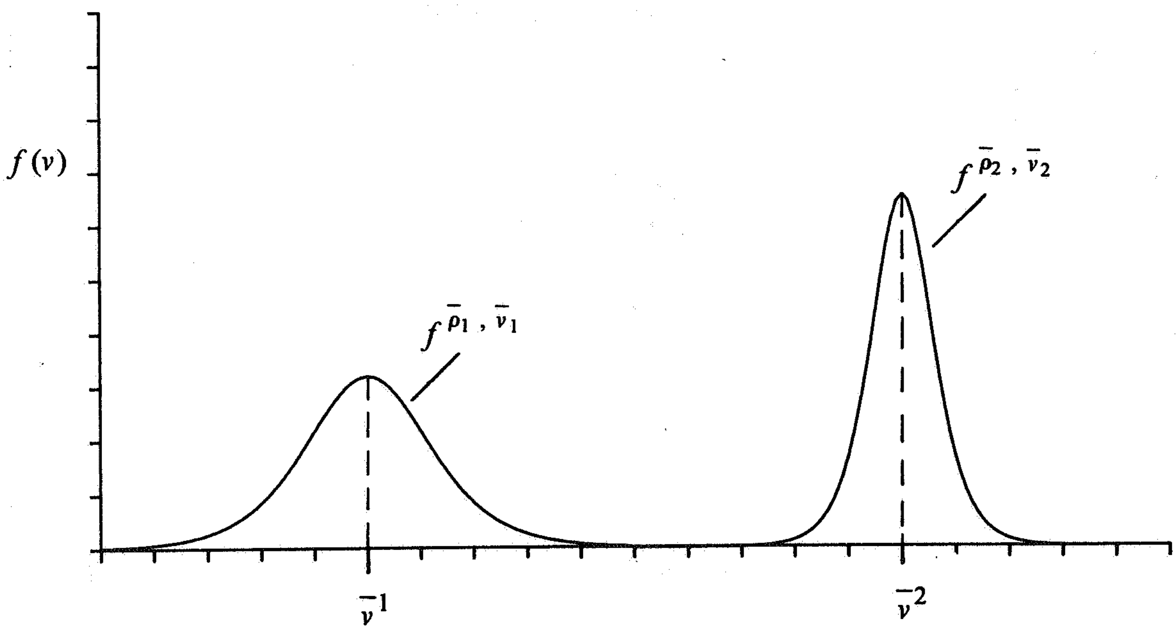


FIGURE D.2

For the distribution one might take the logistics distribution for example :

$$F^{\mu,\sigma}(v) = \frac{1}{1 + e^{\frac{-\pi(v-\mu)}{\sigma\sqrt{3}}}} \qquad v \in (-\infty, \infty)$$

where

μ : mean of the distribution

σ : standard deviation of the distribution

Appendix E

Simulation of counting processes

We will discuss in short a method of simulating a counting process as described in [13] and [17] .

The simulation of a homogeneous counting process (Poisson process) with constant intensity λ is simple : it can be done by independently drawing from an exponential distribution with expectation $1/\lambda$. The numbers drawn are the lengths of the intervals between successive jumps of the counting process.

The counting processes in our model have the following form :

$$d\pi_i(t) = l_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ][\ \alpha v_i + (1-\alpha)v_{i+1}\ ] + dm_i(t)$$

where $\rho_i(t)$, $v_i(t)$ are stochastic processes described by the differential equations (2.7),(2.8). The process $\rho_i(t)$ is dependent on the past of the counting processes π_{i-1} and π_i : the passage of vehicles over section boundaries. This means that π_i depends on its own past! Counting processes of which the intensity depends on the past of the process itself are called *self exciting processes.* Counting processes of which the intensity only depends on other stochastic processes are called *doubly stochastic processes* and when the intensity is a deterministic function of time the process is called *nonhomogeneous.*

For the simulation of nonhomogeneous counting processes several procedures have been proposed. For a short overview see Lewis and Shedler [13] . Lewis and Shedler themselves propose a procedure which suits our purposes well. It comes down to the thinning of a realisation of a process with higher (and simpler) intensity. The procedure is based on the following theorem.

THEOREM E.1 [Lewis & Shedler] *If* t_1 , t_2 , ... , t_n *are the jump times of the nonhomogeneous counting process* $\{n_t\ ,\ t \geq t_0\}$ *in the interval* $(t_0\ ,\ \bar{t}]$ *and every jump time* t_i *is deleted with probability*

$$1 - \frac{\bar{\lambda}(t_i)}{\lambda(t_i)}$$

where

$$\bar{\lambda}(t) \leq \lambda(t) \qquad \text{on } (t_0\ ,\ \bar{t}]$$

$$\lambda(t) : \text{the intensity of } n_t$$

then the remaining times are the jump times of a realisation on $(t_0\ ,\ \bar{t}]$ *of the nonhomogeneous process* $\{\bar{n}(t)\ ,\ t \geq t_0\}$ *with intensity* $\{\bar{\lambda}(t)\ ,\ t \geq t_0\}$.

The theorem might be applied as follows in the simulation of a nonhomogeneous process $\{\bar{n}_t\ ,\ t \geq t_0\}$ with intensity $\{\bar{\lambda}_t\ ,\ t \geq t_0\}$:

1. choose λ large enough for $\bar{\lambda}(t) \leq \lambda$ to hold on the interval to the next jump of $\{n_t\ ,\ t \geq t_0\}$
2. generate the first jump of the homogeneous process $\{n_t\ ,\ t \geq t_0\}$ with intensity λ
3. accept the jump as a jump of $\bar{n}_t$ with probability $\bar{\lambda}(t)/\lambda$
4. repeat the steps 1 ... 4

In general, when there is no known upper bound for $\bar{\lambda}(t)$, one cannot be sure that $\bar{\lambda}(t) \leq \lambda$ will hold on the interval to the next jump of n_t. The length of this interval depends on λ. So one has to estimate λ. The estimate should not be too large for that would lead to many rejections in step 3 ($\lambda >> \bar{\lambda}$), which is inefficient. The estimate should not be too small either for if it turns out that $\bar{\lambda} > \lambda$ somewhere in the interval one has to repeat this step until $\bar{\lambda} \leq \lambda$ does hold.

The process $\{n_t\ ,\ t \geq t_0\}$ can simply be simulated following the procedure mentioned at the beginning of this appendix.

The procedure described above can be applied to doubly stochastic and self exciting counting processes in an analogous way. The justification for this was given by Ogata [17] and will not be repeated here.

Appendix F

Plots of equilibrium speed/intensity-density relations

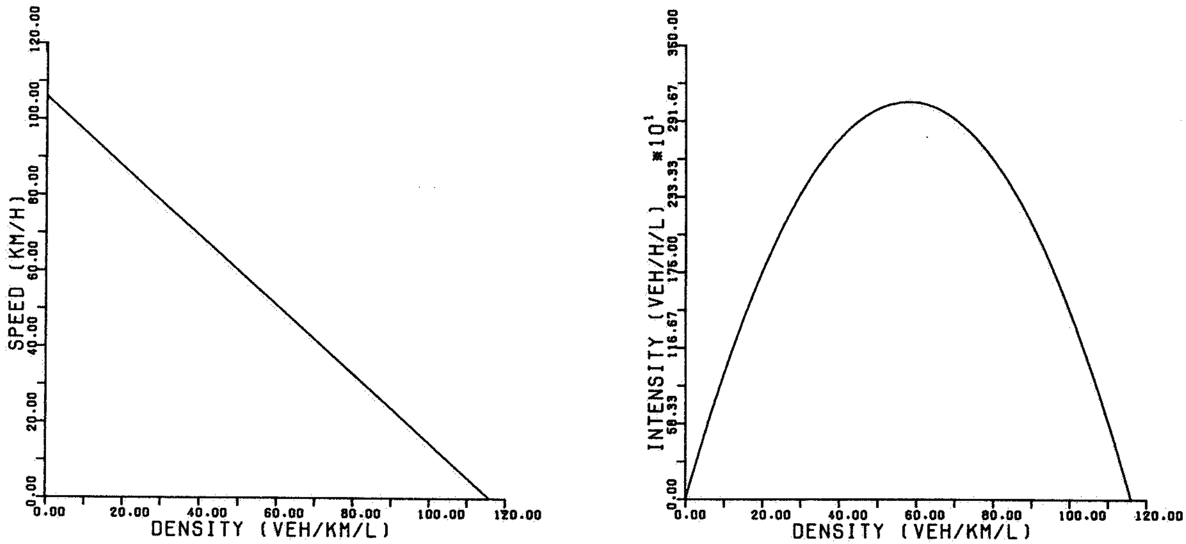


FIGURE F.1

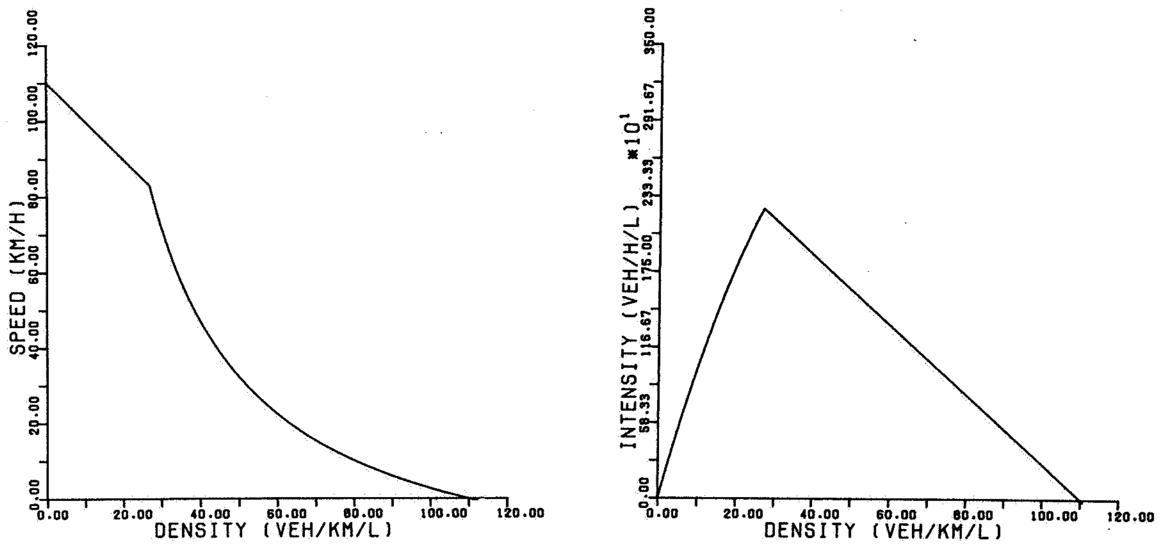

SPEED (KM/H)
DENSITY (VEH/KM/L)
INTENSITY (VEH/H/L) *10^1
DENSITY (VEH/KM/L)

FIGURE F.2

Appendix G

Model stability analysis

Investigation of the stability of a dynamic system (a set of differential equations) is a basic technique. After computing the stationary points of the system one linearises the model around each of these points and computes the eigenvalues of the linearised system. This gives a first order approximation of the behaviour of the original nonlinear system in the neighbourhood of the stationary point. If one or more eigenvalues have a positive real part the system is said to be unstable, if all the eigenvalues have negative real parts the system is stable. (This holds for continuous time systems. In the discrete time case the criterion is whether the modulus of the eigenvalue is smaller than one or not).

Our system (2.7),(2.8) when neglecting the stochastic terms has a continuum of stationary points : all those situations where $\rho_1 = ... = \rho_n = \overline{\rho}$ and $v_1 = ... = v_n = v^e(\overline{\rho})$ for all possible $\overline{\rho}$. (Stationarity implies that the boundary conditions are $\rho_0 = \rho_1$, $v_0 = v_1$ and $\rho_{n+1} = \rho_n$, $v_{n+1} = v_n$). So one can investigate the stability of the system for a series of values of $\overline{\rho}$ and see if and when the system changes from stable to unstable behaviour or vice versa. We will now give the results of the stability analysis referred to in sections 3.2 and 3.3.

First we investigate the original model of section 2.5 with the parameters of Van Maarseveen except that $\alpha = 0.85$ and $\nu = 40.0$ km^2/h. Table G.1 gives all the eigenvalues with real part larger than -100.0 when the system is linearised around $\overline{\rho} = 20.0$, 60.0 and 80.0 veh/km/lane respectively. As one can see the model changes from stable to unstable behaviour at $\overline{\rho} \approx 60.0$ veh/km/lane.

The zero eigenvalue appears because of the fact that our model has a continuum of stationary points. For, if one takes a step from $\overline{\rho}$ to $\overline{\rho} + \delta\overline{\rho}$ for all ρ_i and a step from $v^e(\overline{\rho})$ to $v^e(\overline{\rho} + \delta\overline{\rho})$ for all v_i then one arrives at another stationary point.

Next the model with the nonlinear equilibrium relation (3.2) between density and speed is analysed for $\overline{\rho} = 20.0$, 30.0 and 40.0 veh/km/lane : Table G.2. The change from stable to unstable behaviour now occurs at $\overline{\rho} \approx 30.0$ veh/km/lane.

Finally the stability of the model with the new anticipation term is analysed for $\overline{\rho} = 20.0$, 31.0 and 40.0 veh/km/lane : Table G.3. The change from stable to unstable behaviour again occurs at about 30 veh/km/lane but now there is a conjugate pair of unstable eigenvalues, which indicates oscillatory behaviour near the critical density.

$\bar{\rho}$ = 20.0 veh/km/lane	$\bar{\rho}$ = 60.0 veh/km/lane	$\bar{\rho}$ = 80.0 veh/km/lane
0.0 + 0.0 i	0.0 + 0.0 i	9.4 ± 16.4 i
-71.8 + 0.0 i	-0.4 + 0.0 i	2.0 ± 7.8 i
-81.7 + 0.0 i	-5.2 + 0.0 i	0.0 + 0.0 i
-97.6 + 0.0 i	-15.2 + 0.0 i	-1.6 + 0.0 i
-100.0 + 0.0 i	-29.9 + 0.0 i	-19.4 + 0.0 i
	-48.8 + 0.0 i	-34.7 + 0.0 i
	-69.1 + 0.0 i	-51.2 + 0.0 i
	-90.7 + 0.0 i	-65.2 + 0.0 i
	-100.0 + 0.0 i	-77.5 + 0.0 i
		-85.1 + 0.0 i
		-100.0 + 0.0 i

TABLE G.1

$\bar{\rho}$ = 20.0 veh/km/lane	$\bar{\rho}$ = 30.0 veh/km/lane	$\bar{\rho}$ = 40.0 veh/km/lane
0.0 + 0.0 i	0.4 + 0.0 i	6.8 ± 3.2 i
-74.1 + 0.0 i	0.0 + 0.0 i	2.6 + 0.0 i
-84.1 + 0.0 i	-4.9 + 0.0 i	0.0 + 0.0 i
-99.9 + 0.0 i	-14.9 + 0.0 i	-8.9 + 0.0 i
-100.0 + 0.0 i	-29.1 + 0.0 i	-23.8 + 0.0 i
	-46.8 + 0.0 i	-40.8 + 0.0 i
	-66.3 + 0.0 i	-58.5 + 0.0 i
	-86.0 + 0.0 i	-75.2 + 0.0 i
	-99.3 ± 8.7 i	-89.2 + 0.0 i
	-100.0 + 0.0 i	-99.1 + 0.0 i
		-100.0 + 0.0 i

TABLE G.2

$\bar{\rho}$ = 20.0 veh/km/lane	$\bar{\rho}$ = 31.0 veh/km/lane	$\bar{\rho}$ = 40.0 veh/km/lane
0.0 + 0.0 i	0.4 ± 113.6 i	66.6 ± 76.2 i
-63.5 ± 121.8 i	0.0 + 0.0 i	60.6 ± 67.2 i
-68.5 ± 110.2 i	-6.5 ± 102.7 i	51.6 ± 52.7 i
-75.9 ± 91.5 i	-17.0 ± 85.0 i	41.4 ± 33.6 i
-84.5 ± 66.2 i	-29.1 ± 61.3 i	31.9 ± 11.3 i
-91.9 ± 35.2 i	-39.8 ± 32.4 i	0.5 + 0.0 i
-95.0 + 0.0 i	-44.5 + 0.0 i	0.0 + 0.0 i
-100.0 + 0.0 i	-100.0 + 0.0 i	-100.0 + 0.0 i

TABLE G.3

Appendix H

Simulation results

H.1 Low density traffic : 1

configuration : number of sections : 12
section length : 500 m
number of lanes : 2

parameters : $\alpha = 0.5$
$T = 0.0044$ h
$v_f = 106.0$ km/h
$\rho_j = 116.0$ veh/km/lane
$\nu = 138.0$ km^2/h
$c = 10.0$ veh/km/lane

speed distribution : type : logistic
mean : weighted mean of section speeds
variance : 12.0 km^2/h^2
number of speed classes : 1
class 1 : [0 , 120)

acceleration noise : mean : 0 km/h^2
variance : 50 km^2/h^4

counting errors : $\epsilon_i^f = 0$
$\epsilon_i^m = 0$

boundary conditions : $\lambda_0 = 1383.3$ veh/h/lane
$\rho_{13}(t) = \rho_{12}(t)$
$v_{13}(t) = v_{12}(t)$

results :

time (h)	0.0		0.033	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	15.0	92.2	12.0	61.5
2	15.0	92.2	11.0	109.9
3	15.0	92.2	1.0	0.0
4	15.0	92.2	37.0	141.0
5	15.0	92.2	0.0	1.1
6	15.0	92.2	23.0	124.2
7	15.0	92.2	8.0	61.6
8	15.0	92.2	18.0	102.8
9	15.0	92.2	15.0	114.7
10	15.0	92.2	8.0	14.6
11	15.0	92.2	23.0	115.8
12	15.0	92.2	17.0	104.2

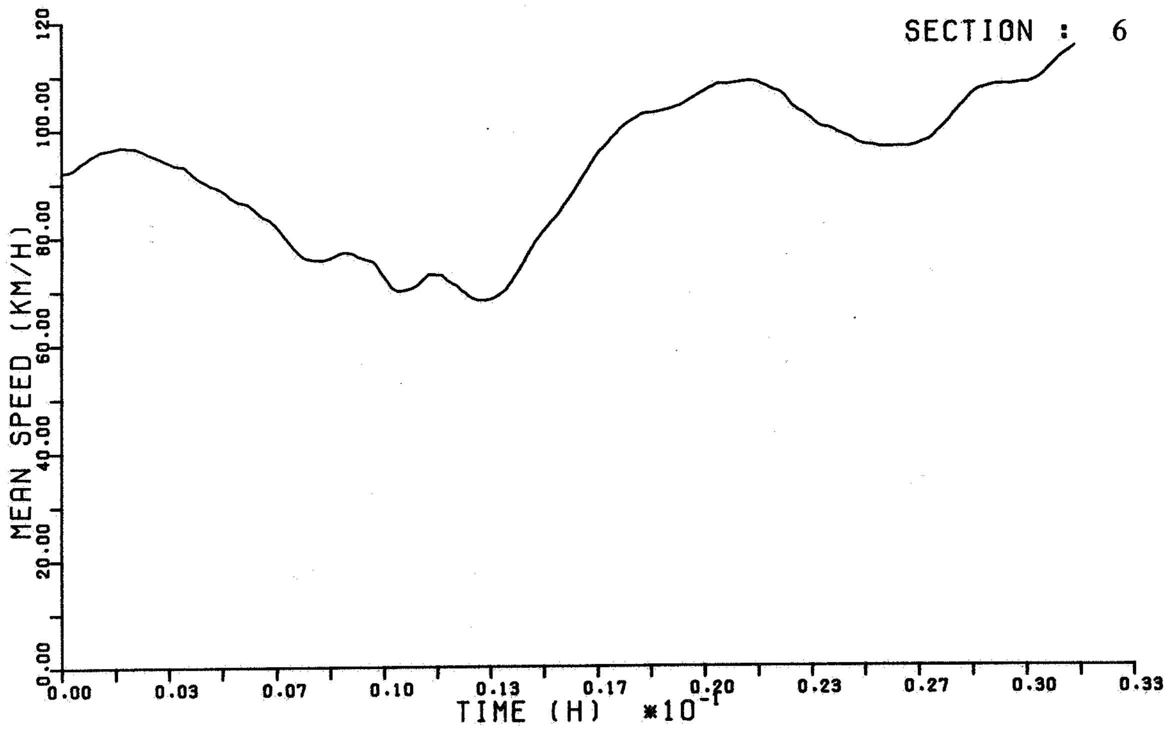


FIGURE H.1

H.2 LOW DENSITY TRAFFIC : 2

configuration : number of sections : 12
section length : 500 m
number of lanes : 2

parameters : $\alpha = 0.5$
$T = 0.0044$ h
$v_f = 106.0$ km/h
$\rho_j = 116.0$ veh/km/lane
$\nu = 40.0$ km^2/h
$c = 10.0$ veh/km/lane

speed distribution : type : logistic
mean : weighted mean of section speeds
variance : 12.0 km^2/h^2
number of speed classes : 1
class 1 : [0 , 120)

acceleration noise : mean : 0 km/h^2
variance : 50 km^2/h^4

counting errors : $\epsilon_i^f = 0$

$\epsilon_i^m = 0$

boundary conditions : $\lambda_0 = 1383.3$ veh/h/lane

$\rho_{13}(t) = \rho_{12}(t)$

$v_{13}(t) = v_{12}(t)$

results :

time (h)	0.0		0.033	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	15.0	92.2	13.0	83.9
2	15.0	92.2	15.0	93.6
3	15.0	92.2	8.0	79.8
4	15.0	92.2	23.0	96.3
5	15.0	92.2	1.0	67.6
6	15.0	92.2	32.0	93.0
7	15.0	92.2	10.0	100.0
8	15.0	92.2	0.0	69.8
9	15.0	92.2	22.0	80.6
10	15.0	92.2	25.0	92.5
11	15.0	92.2	12.0	92.4
12	15.0	92.2	14.0	91.9

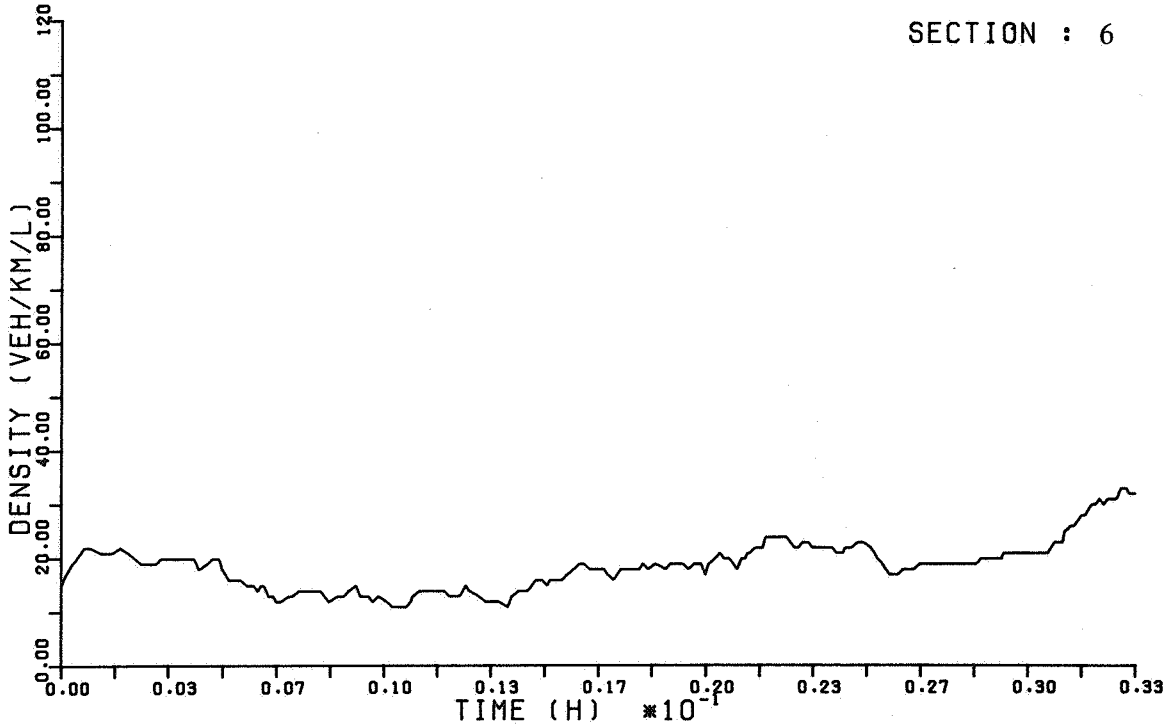


FIGURE H.2

H.3 Low density traffic : 3

configuration : number of sections : 12

section length : 500 m

number of lanes : 2

parameters : $\alpha = 0.85$

$T = 0.0044$ h

$v_f = 106.0$ km/h

$\rho_j = 116.0$ veh/km/lane

$\nu = 40.0$ km^2/h

$c = 10.0$ veh/km/lane

speed distribution : type : logistic

mean : weighted mean of section speeds

variance : 12.0 km^2/h^2

number of speed classes : 1

class 1 : [0 , 120)

acceleration noise : mean : 0 km/h^2

variance : 50 km^2/h^4

counting errors : $\epsilon_i^f = 0$

$\epsilon_i^m = 0$

boundary conditions : $\lambda_0 = 1383.3$ veh/h/lane

$\rho_{13}(t) = \rho_{12}(t)$

$v_{13}(t) = v_{12}(t)$

results :

time (h)	0.0		0.033	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	15.0	92.2	13.0	93.1
2	15.0	92.2	12.0	94.1
3	15.0	92.2	12.0	93.3
4	15.0	92.2	15.0	94.4
5	15.0	92.2	12.0	92.0
6	15.0	92.2	13.0	92.8
7	15.0	92.2	11.0	88.9
8	15.0	92.2	19.0	90.2
9	15.0	92.2	18.0	93.9
10	15.0	92.2	15.0	97.1
11	15.0	92.2	5.0	79.5
12	15.0	92.2	21.0	84.7

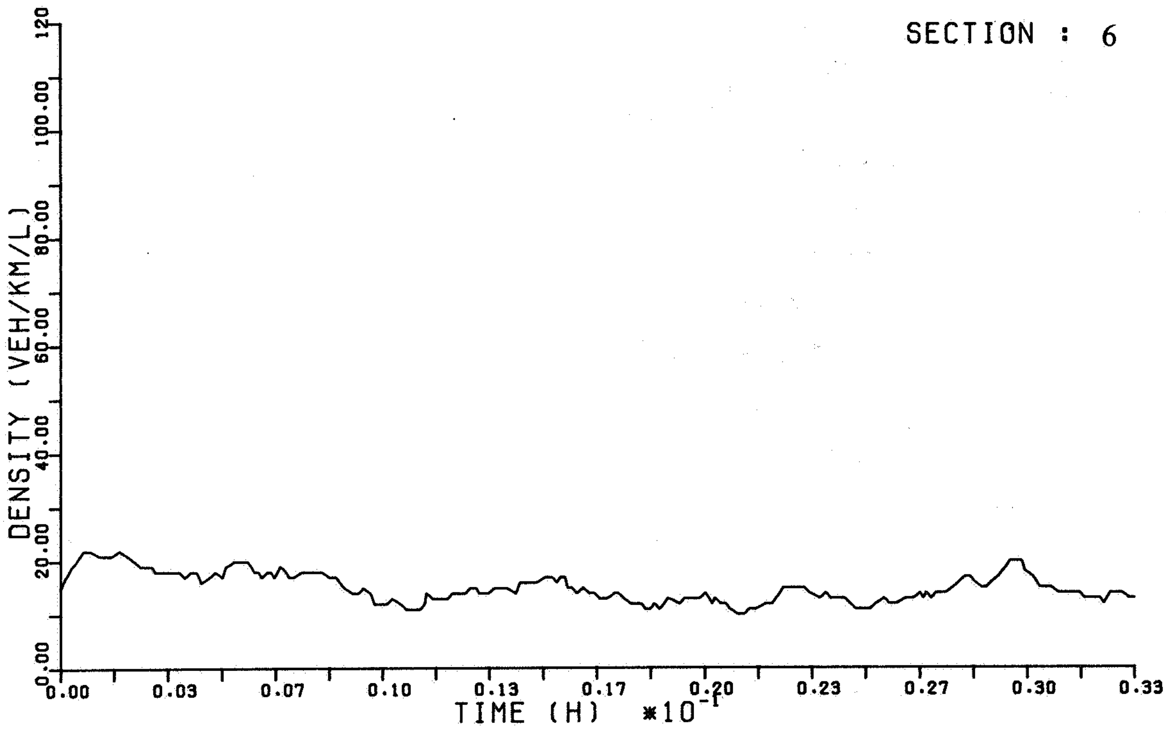


FIGURE H.3

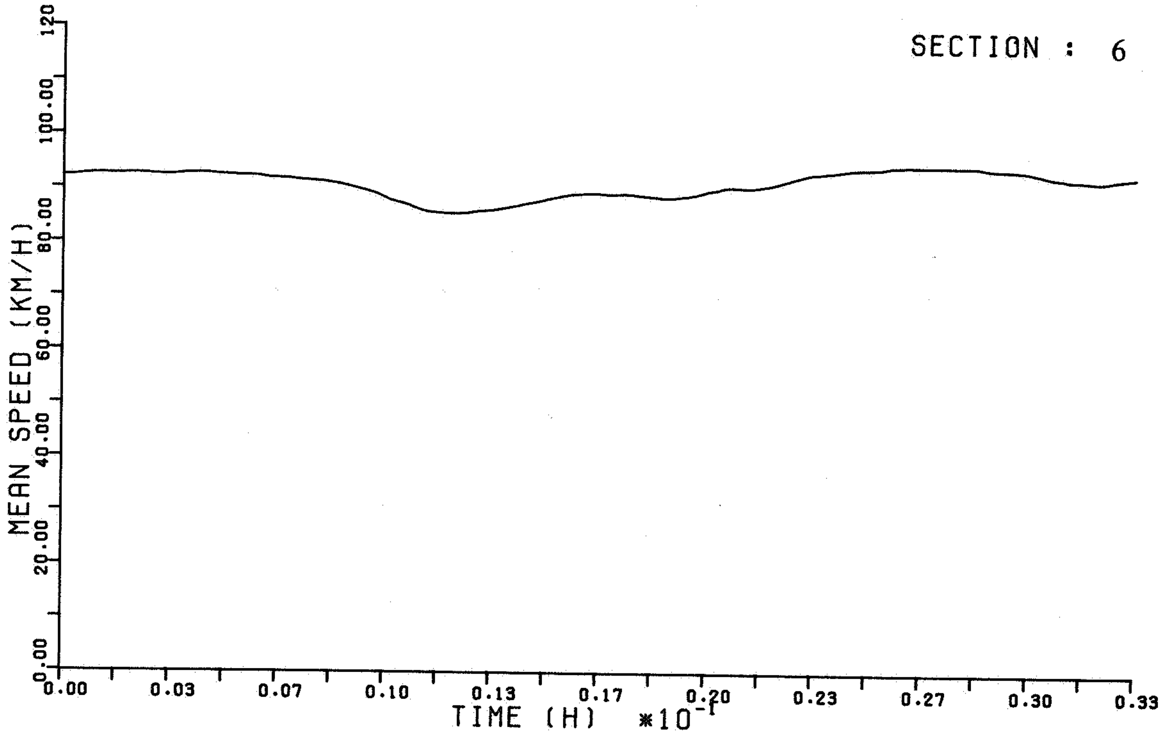


FIGURE H.4

H.4 MODERATE DENSITY TRAFFIC : 1

configuration : number of sections : 12
section length : 500 m
number of lanes : 2

parameters : $\alpha = 0.85$
$T = 0.01$ h
$v_f = 106.0$ km/h
$\rho_j = 116.0$ veh/km/lane
$\nu = 40.0$ km^2/h
$c = 10.0$ veh/km/lane

speed distribution : type : logistic
mean : weighted mean of section speeds
variance : 12.0 km^2/h^2
number of speed classes : 1
class 1 : [0 , 120)

acceleration noise : mean : 0 km/h^2
variance : 50 km^2/h^4

counting errors : $\epsilon_i^f = 0$
$\epsilon_i^m = 0$

boundary conditions : $\lambda_0 = 2000.0$ veh/h/lane
$\rho_{13}(t) = \rho_{12}(t)$
$v_{13}(t) = v_{12}(t)$

results :

time (h)	0.0		0.5	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	20.0	87.7	39.0	82.4
2	20.0	87.7	24.0	83.4
3	20.0	87.7	19.0	79.7
4	20.0	87.7	21.0	81.6
5	20.0	87.7	29.0	75.9
6	20.0	87.7	26.0	74.2
7	20.0	87.7	32.0	80.7
8	20.0	87.7	29.0	81.7
9	20.0	87.7	14.0	79.6
10	20.0	87.7	18.0	80.5
11	20.0	87.7	27.0	87.1
12	20.0	87.7	18.0	90.4

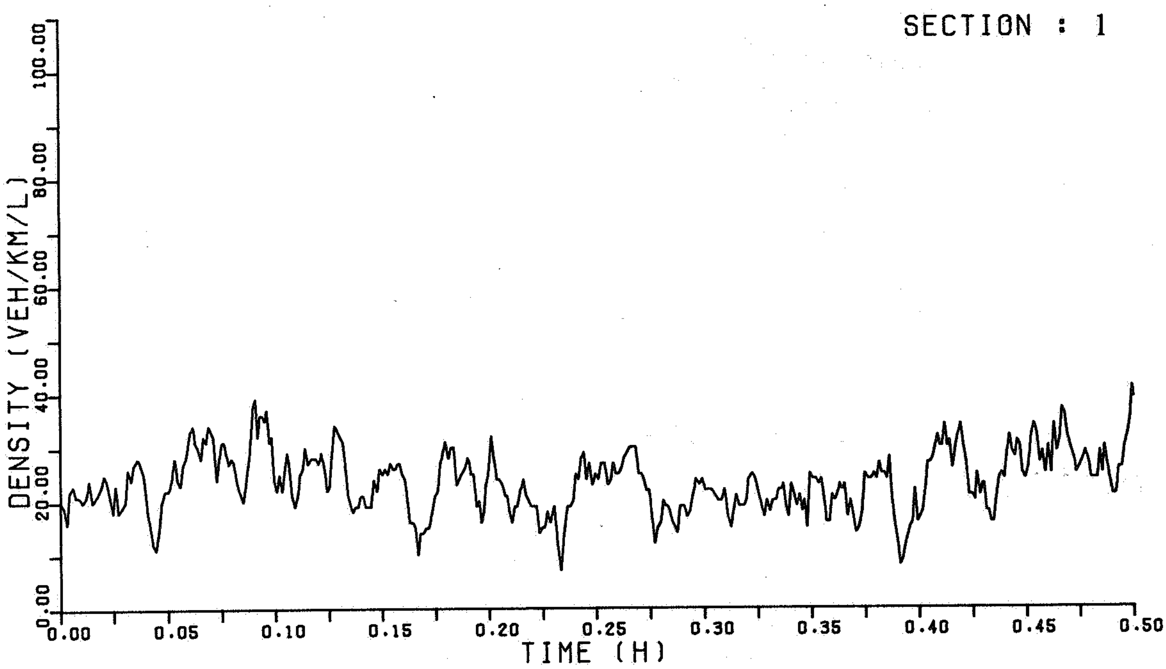


FIGURE H.5

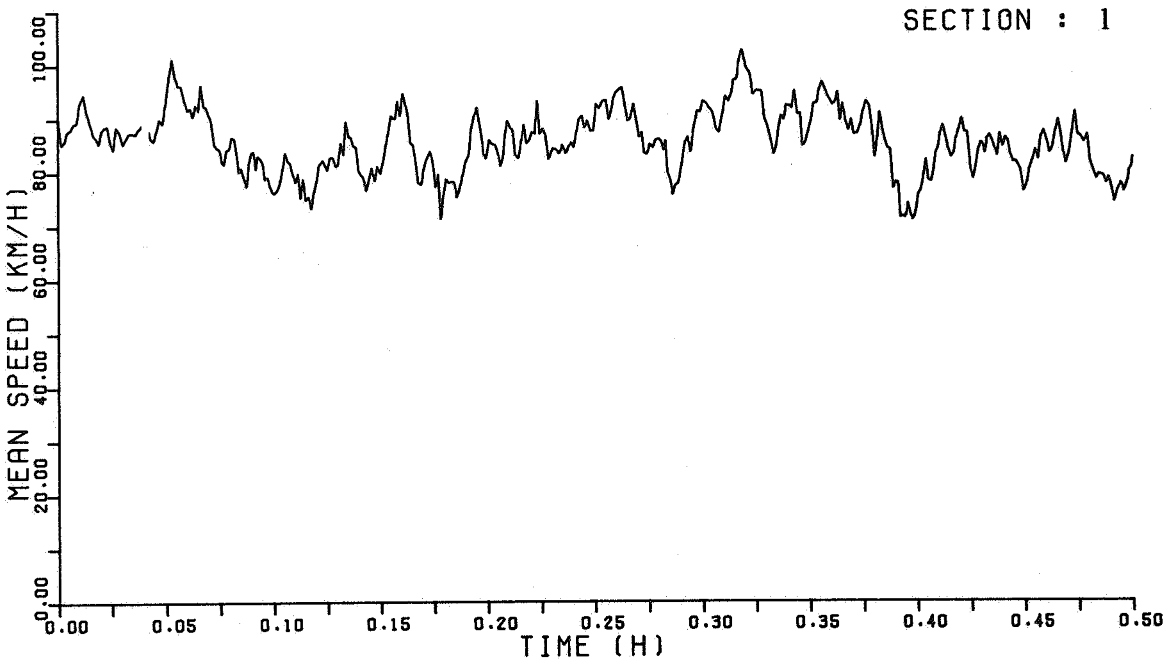


FIGURE H.6

H.5 Moderate density traffic : 2

configuration : number of sections : 12

section length : 500 m

number of lanes : 2

parameters : $\alpha = 0.85$

$T = 0.01$ h

$v_f = 110.0$ km/h

$\rho_j = 110.0$ veh/km/lane

$\rho_{crit} = 27.0$ veh/km/lane

$d = 2970.0$ 1/h

$\nu = 40.0$ km^2/h

$c = 10.0$ veh/km/lane

speed distribution : type : logistic

mean : weighted mean of section speeds

variance : 12.0 km^2/h^2

number of speed classes : 1

class 1 : [0 , 150)

acceleration noise : mean : 0 km/h^2

variance : 50 km^2/h^4

counting errors : $\epsilon_i^f = 0$

$\epsilon_i^m = 0$

boundary conditions : $\lambda_0 = 2000.0$ veh/h/lane

$\rho_{13}(t) = \rho_{12}(t)$

$v_{13}(t) = v_{12}(t)$

results :

time (h)	0.0		0.5	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	20.0	90.0	23.0	80.0
2	20.0	90.0	21.0	79.5
3	20.0	90.0	27.0	80.7
4	20.0	90.0	24.0	84.7
5	20.0	90.0	23.0	76.9
6	20.0	90.0	36.0	72.0
7	20.0	90.0	25.0	67.8
8	20.0	90.0	29.0	64.6
9	20.0	90.0	43.0	64.2
10	20.0	90.0	31.0	71.0
11	20.0	90.0	26.0	80.0
12	20.0	90.0	23.0	83.2

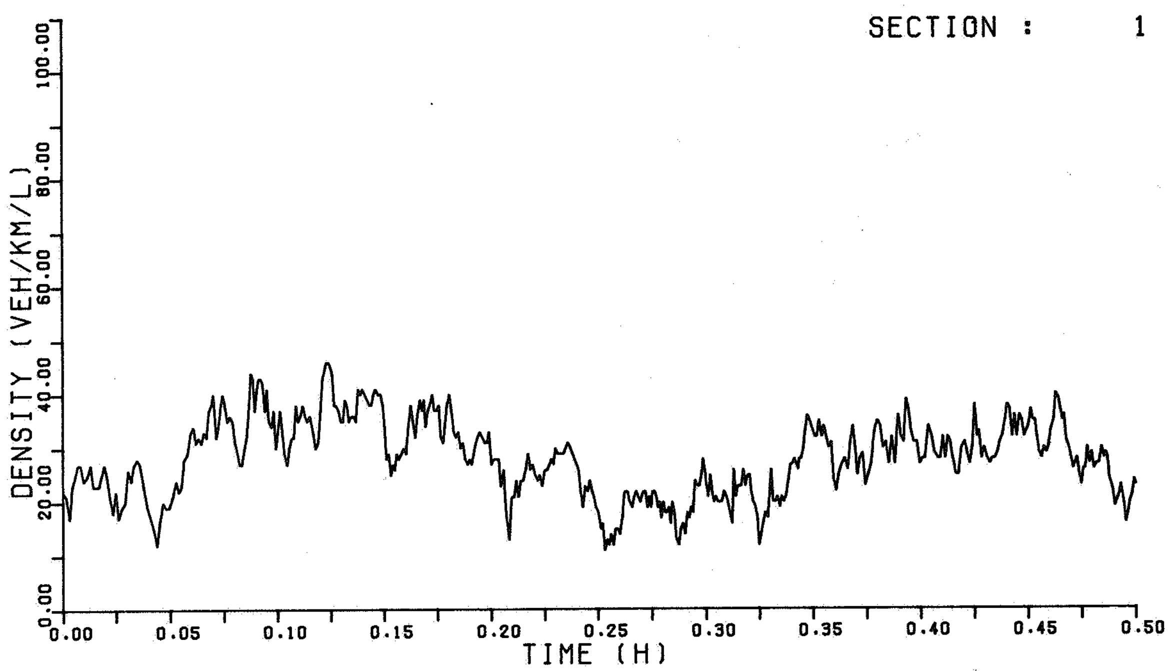


FIGURE H.7

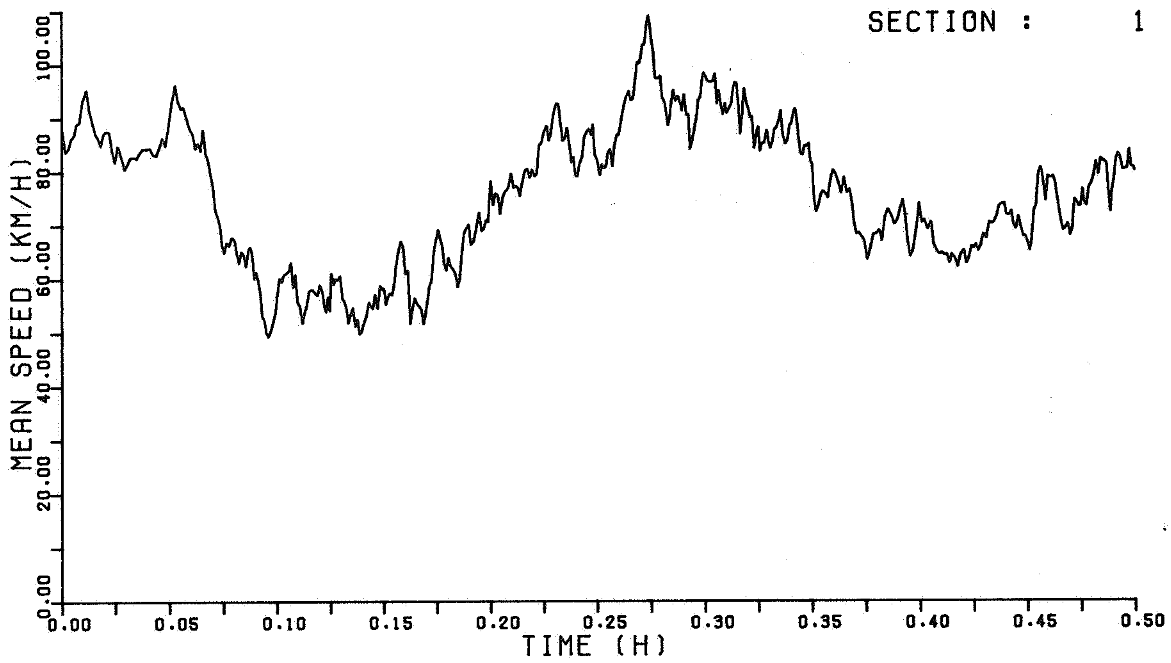


FIGURE H.8

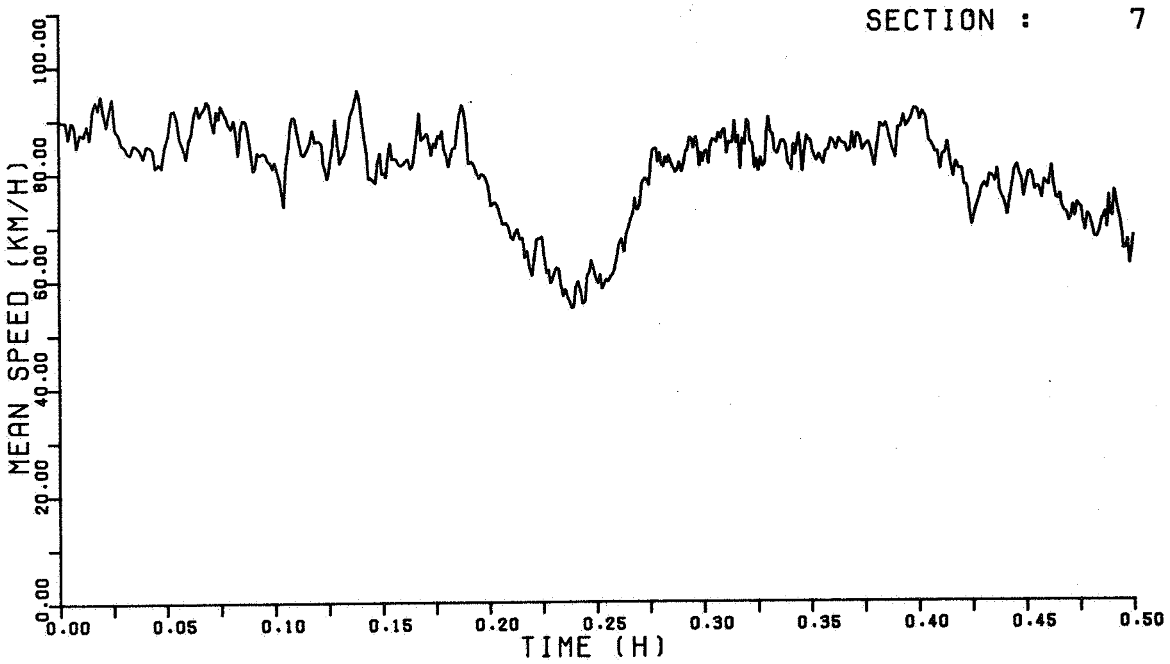


FIGURE H.9

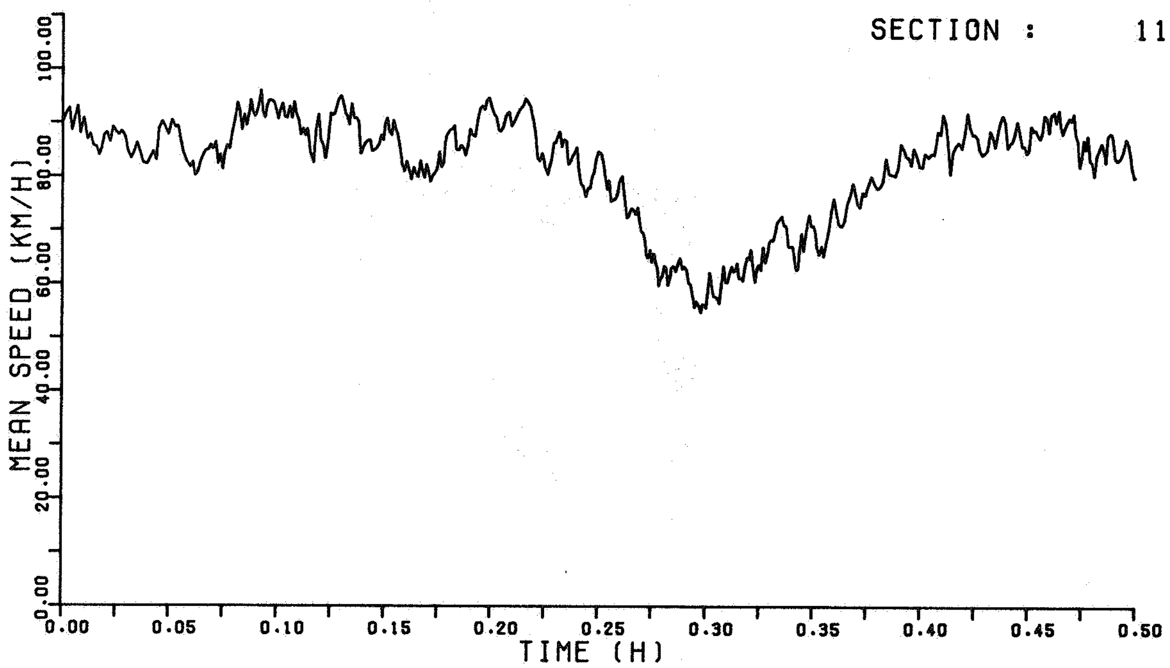


FIGURE H.10

H.6 High density traffic : 1

configuration : number of sections : 12

section length : 500 m

number of lanes : 2

parameters : $\alpha = 0.85$

$T = 0.01$ h

$v_f = 110.0$ km/h

$\rho_j = 110.0$ veh/km/lane

$\rho_{crit} = 27.0$ veh/km/lane

$d = 2970.0$ 1/h

$\nu = 40.0$ km^2/h

$c = 10.0$ veh/km/lane

speed distribution : type : logistic

mean : weighted mean of section speeds

variance : 12.0 km^2/h^2

number of speed classes : 1

class 1 : [0 , 150)

acceleration noise : mean : 0 km/h^2

variance : 0 km^2/h^4

counting errors : $\epsilon_i^f = 0$

$\epsilon_i^m = 0$

boundary conditions : $\lambda_0 = 1800.0$ veh/h/lane

$\rho_{13}(t) = \rho_{12}(t)$

$v_{13}(t) = v_{12}(t)$

results :

time (h)	0.0		0.12	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	20.0	90.0	22.0	89.2
2	20.0	90.0	19.0	90.6
3	20.0	90.0	14.0	90.5
4	20.0	90.0	15.0	88.6
5	20.0	90.0	25.0	79.2
6	30.0	72.0	41.0	14.1
7	60.0	22.5	216.0	0.0
8	100.0	2.7	208.0	2.3
9	100.0	2.7	132.0	5.4
10	100.0	2.7	94.0	8.9
11	90.0	6.0	82.0	7.0
12	90.0	6.0	88.0	7.0

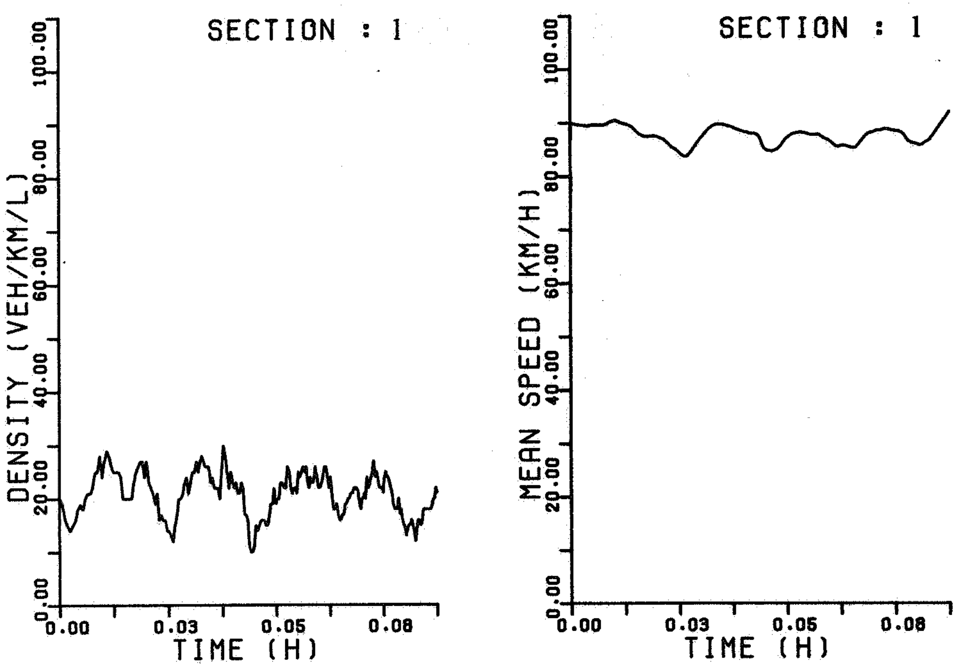


FIGURE H.11

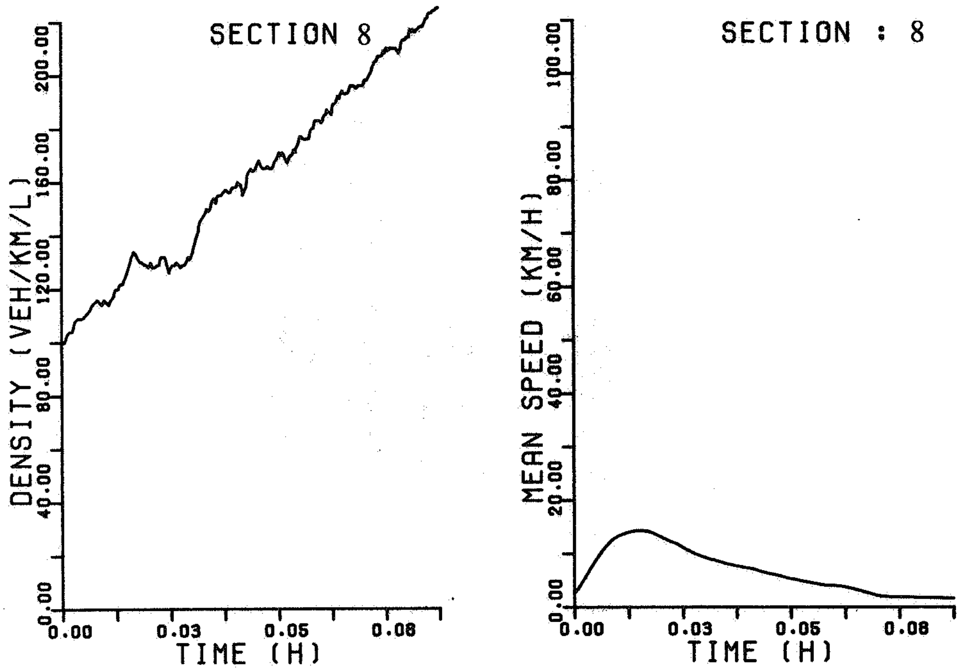


FIGURE H.12

H.7 HIGH DENSITY TRAFFIC : 2

configuration : number of sections : 12

section length : 500 m

number of lanes : 2

parameters : $\alpha = 0.85$

$T = 0.01$ h

$v_f = 110.0$ km/h

$\rho_j = 110.0$ veh/km/lane

$\rho_{crit} = 27.0$ veh/km/lane

$d = 2970.0$ 1/h

$\gamma = 6.5$ km/h^2

$\beta = 0.5$

speed distribution : type : logistic

mean : weighted mean of section speeds

variance : 12.0 km^2/h^2

number of speed classes : 1

class 1 : [0 , 150)

acceleration noise : mean : 0 km/h^2

variance : 0 km^2/h^4

counting errors : $\epsilon_i^f = 0$

$\epsilon_i^m = 0$

boundary conditions : $\lambda_0 = 1800.0$ veh/h/lane

$\rho_{13}(t) = \rho_{12}(t)$

$v_{13}(t) = v_{12}(t)$

results :

time (h)	0.0		0.333	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	20.0	90.0	95.0	0.0
2	20.0	90.0	100.0	29.8
3	20.0	90.0	113.0	25.9
4	20.0	90.0	94.0	9.1
5	20.0	90.0	97.0	11.9
6	30.0	72.0	97.0	4.1
7	60.0	22.5	103.0	9.9
8	100.0	2.7	100.0	17.5
9	100.0	2.7	95.0	0.0
10	100.0	2.7	97.0	6.3
11	90.0	6.0	98.0	2.6
12	90.0	6.0	102.0	4.0

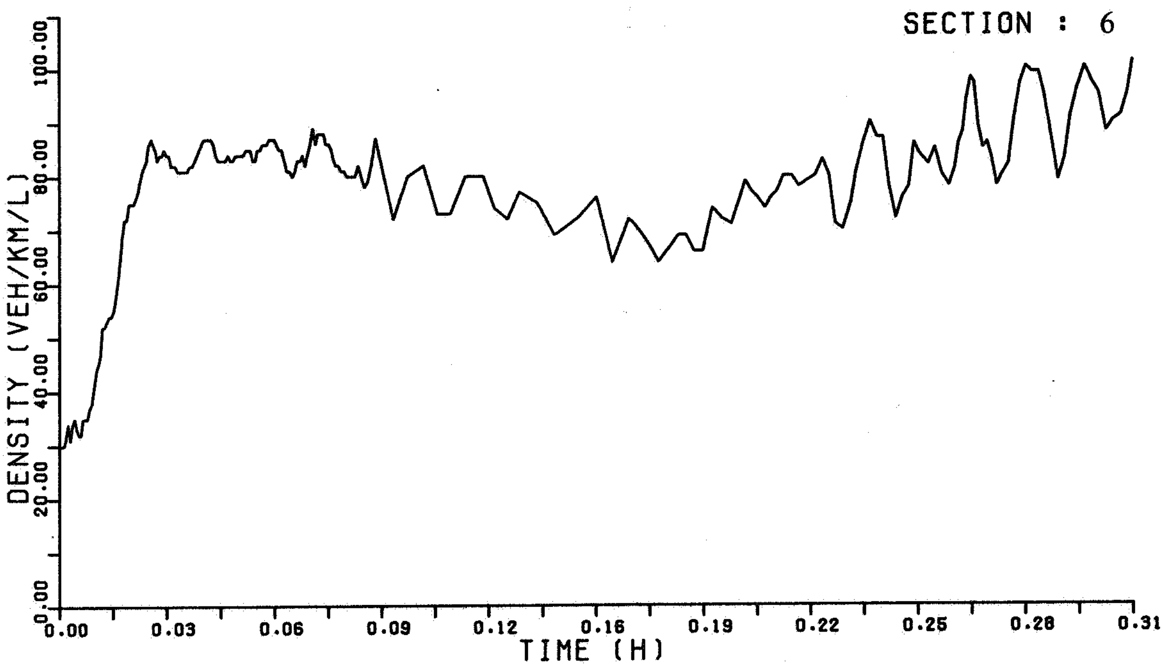


FIGURE H.13

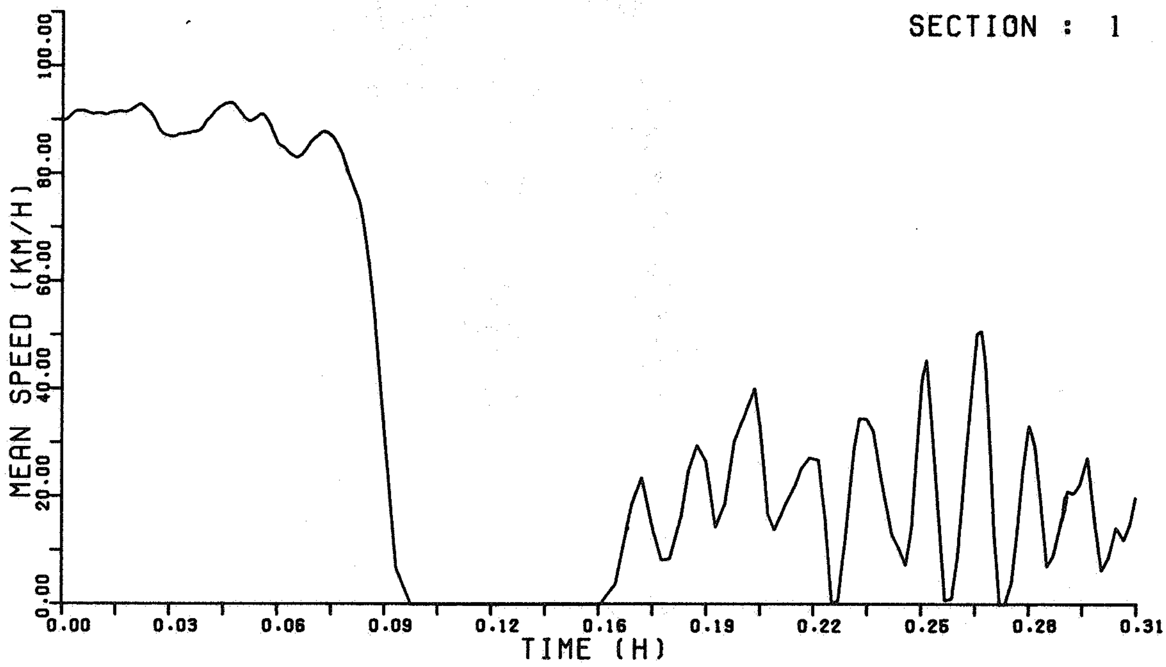


FIGURE H.14

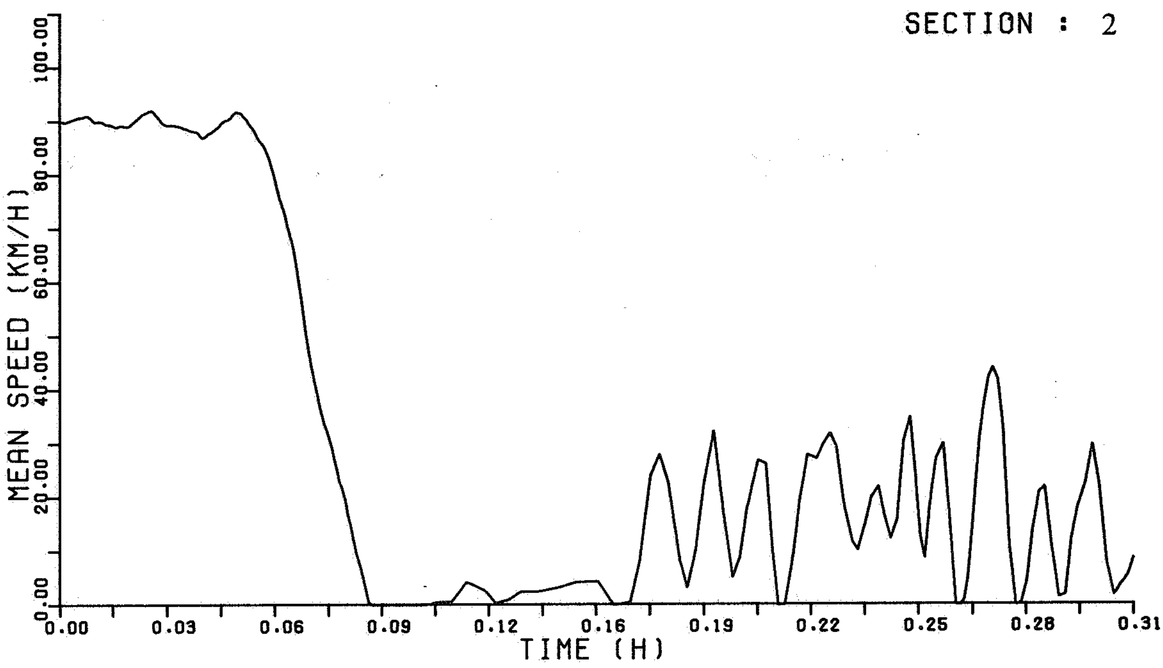


FIGURE H.15

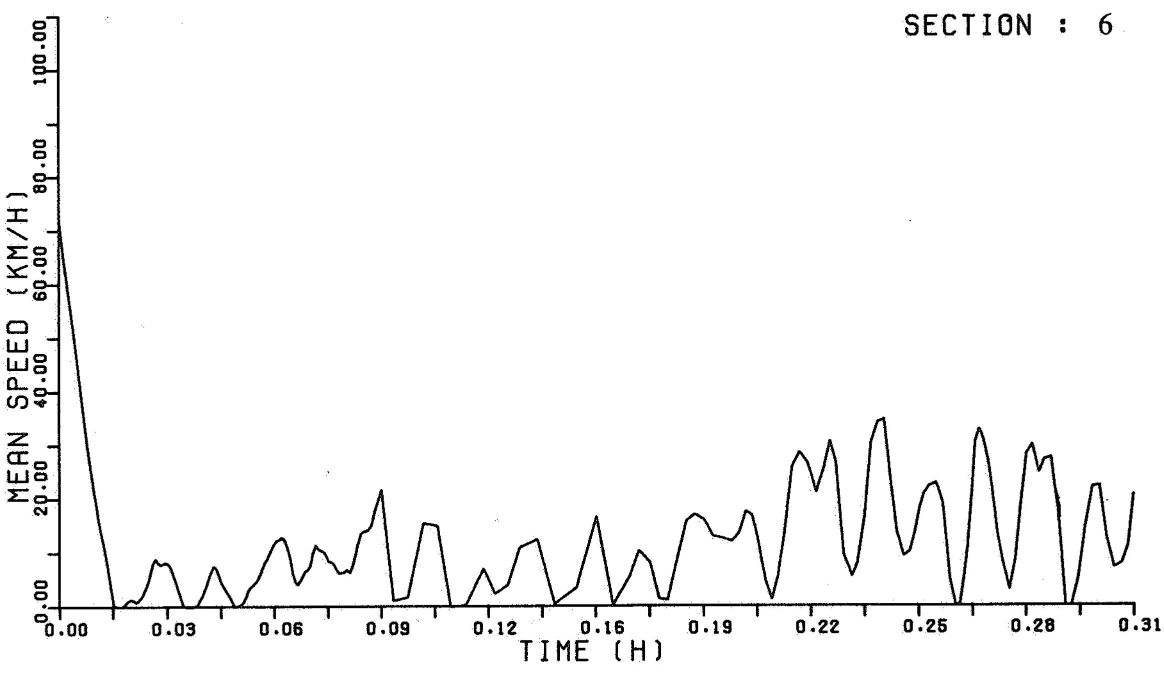


FIGURE H.16

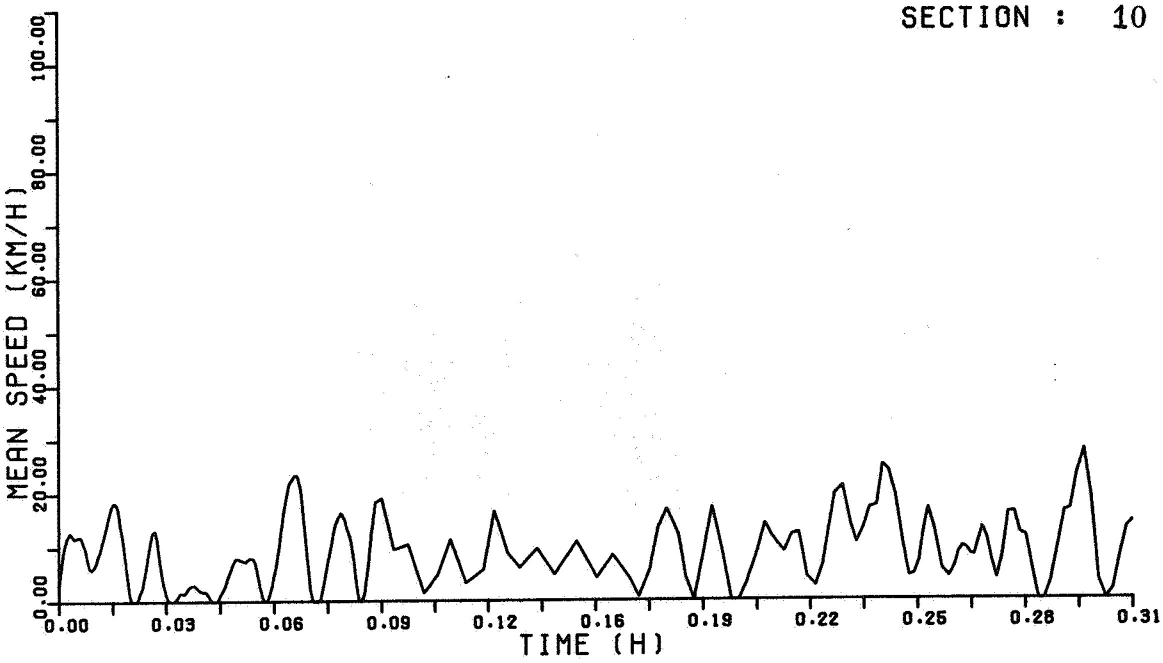

SECTION : 10
MEAN SPEED (KM/H)
0.00
20.00
40.00
60.00
80.00
100.00
0.00
0.03
0.06
0.09
0.12
0.16
0.19
0.22
0.25
0.28
0.31
TIME (H)

FIGURE H.17