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M. Hušek, J. de Vries

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M. Hušek, J. de Vries

Centre for Mathematics and Computer Science
P.O. Box 4079, 1009 AB Amsterdam, The Netherlands

Junghenn, generalizing results of De Leeuw and Glicksberg and of Berglund and Milnes, has shown that the almost periodic (AP) compactification of an arbitrary cartesian product of semitopological semigroups with identity is (canonically) isomorphic to the corresponding product of the AP compactifications of the factors. He also showed that the analog of this result holds for the strongly almost periodic (SAP) case. The proofs are leaning heavily on the characterizations of these compactifications in terms of algebras of functions. In this note we give an "intrinsic" proof, using only the objects and morphisms of the categories at hand.

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1. Introduction

Notation and terminology will be as in [1] except for some minor modifications. All semigroups under consideration are assumed to have an identity. Thus, **STSgp** is the category whose objects are the semitopological semigroups with identity and whose morphisms are the continuous identity preserving homomorphisms. By **TopSgp** (resp. **TopGrp**) we denote the full subcategory of **STSgp** having as objects all topological semigroups with identity (resp. all topological groups), while **CSTSgp**, **CTopSgp** and **CTopGrp** denote the full subcategories of all compact Hausdorff objects in these categories. As is pointed out in [1], it is a straightforward consequence of general results from category theory that all inclusion functors between these categories have left adjoints. In particular, the following reflectors exist:

STSgp → **CTopGrp** : F^{SAP}
STSgp → **CTopSgp** : F^{AP}
STSgp → **CSTSgp** : F^{WAP}

(Here our notation deviates from [1], where M, A and W are used for F^{SAP}, F^{AP} and F^{WAP}, respectively.). If F is any one of these reflectors, then for each object S of **STSgp** there is an essentially unique "universal arrow", the reflection into the corresponding subcategory,

$$S \xrightarrow{\eta_S} FS$$

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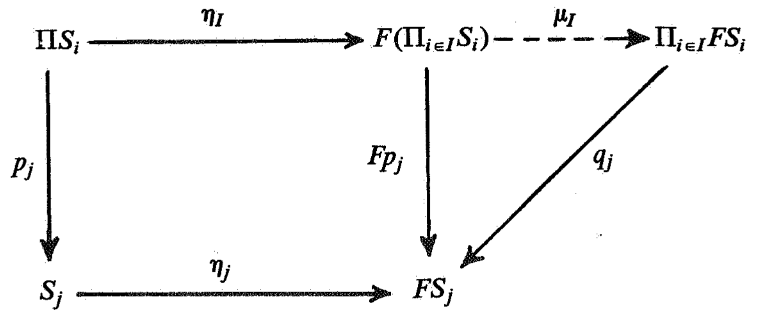
which is, in all cases, a morphism with dense range.

We shall consider two additional functors, namely, F^{LUC} and F^{LMC}. These can also be obtained as reflectors, but it is easier to describe them by means of the corresponding universal arrows $\eta_S : S \to FS$ (S an object of **STSgp**). Here FS is a compact Hausdorff *right* topological semigroup (i.e., all right translations $\xi \mapsto \xi\xi' : FS \mapsto FS$ for $\xi' \in FS$ are continuous), $\eta_S : S \to FS$ is a continuous homomorphism with dense range such that the mapping $(s, \xi) \mapsto \eta_S(s)\xi : S \times FS \to FS$ is continuous (in the case $F = F^{LUC}$) or separately continuous (in the case $F = F^{LMC}$), and η_S is universal for this type of homomorphisms (so we use the characterizations given in Theorems III. 5.5 and III. 4.5 of [1] as definitions).

The following remarks apply to each of the functors mentioned above. If $\{S_i : i \in I\}$ is a set of objects in **STSgp**, then there exists a unique morphism $\mu_I : F(\Pi_{i \in I} S_i) \to \Pi_{i \in I} FS_i$, completing the following commutative diagram for each $j \in I$:



Here $\Pi_{i \in I} S_i$ and $\Pi_{i \in I} FS_i$ denote cartesian products (with coordinate-wise semigroup operations and ordinary product topology; this are just the products in the corresponding categories), the p_j and q denote canonical projections, η_I stands for $\eta_{\Pi S_i}$ and η_j for η_{S_j}. The question is: when is η_I an isomorphism? If μ_I is an isomorphism for all (finite) products, then F is said to *preserve* all (finite) products.

Usually, reflectors do not preserve products (cf. [4] for many examples). In [6] it is shown (generalizing earlier results of others) that F^{AP} and F^{SAP} preserve all products, and an example is cited which shows that F^{WAP} doesn't preserve all finite products. In [4] we obtained these properties of F^{AP} and F^{SAP} as consequences of more general results in certain concrete categories, but it seems worthwhile to write down straightforward proofs for F^{AP} and F^{SAP}, the more so as our proofs are very elementary and make no use of function algebras whatsoever. Also, our proof covers all special cases about F^{WAP} and F^{LUC} dealt with in [6].

2. Finite products

Proposition. *The reflectors F^{AP} and F^{SAP} preserve all finite products.*

Proof. Let F be F^{AP} or F^{SAP} and consider two objects S_1 and S_2 in **STSgp**. Let e_1 and e_2 be the identities in S_1 and S_2, respectively, and

$$\alpha_1 : x \mapsto (x, e_2) : S_1 \to S_1 \times S_2 ; \alpha_2 : x \mapsto (e_1, x) : S_2 \to S_1 \times S_2$$

the canonical embeddings. Other notation is as in Section 1, but we shall write μ for $\mu_{\{1,2\}}$ and η for $\eta_{\{1,2\}}$.

For $\xi \in F(S_1 \times S_2)$ one has, by the definition of $\mu, \mu(\xi) = (Fp_1(\xi), Fp_2(\xi))$. Putting $\xi = \eta(x_1, x_2)$ with $x_i \in S_i$, one sees immediately that $\mu(\eta(x_1, x_2)) = (\eta_1(x_1), \eta_2(x_2))$, so $\mu \circ \eta = \eta_1 \times \eta_2$. It follows that the range of μ contains the subset $\eta_1[S_1] \times \eta_2[S_2]$, which is dense in $FS_1 \times FS_2$. Since the range of μ is compact, it follows that μ is a surjection. Now it is sufficient to show that μ is an injection (for then μ, going from a compact to a Hausdorff space, is a homeomorphism, hence an isomorphism in the category under consideration). To this end, define the mapping $\Phi : FS_1 \times FS_2 \to F(S_1 \times S_2)$ by

$$\Phi(\xi_1, \xi_2) := F\alpha_1(\xi_1) \cdot F\alpha_2(\xi_2),\ (\xi_1, \xi_2) \in FS_1 \times FS_2,$$

where the dot denotes the multiplication in the semigroup $F(S_1 \times S_2)$ (actually, Φ will turns out to be inverse to μ). In order to show that μ is injective, it is sufficient to prove that $\Phi \circ \mu$ is the identity map on $F(S_1 \times S_2)$. Taking into account the observation above that $\mu \circ \eta = \eta_1 \times \eta_2$, and the observation that $\Phi(\eta_1(x_1), \eta_2(x_2)) = \eta(x_1, e_2) \cdot \eta(e_1, x_2) = \eta(x_1, x_2)$ for $(x_1, x_2) \in S_1 \times S_2$, one sees immediately that

$$(\Phi \circ \mu) \circ \eta = \Phi \circ (\eta_1 \times \eta_2) = \eta = id_{F(S_1 \times S_2)} \circ \eta .$$

Since multiplication in $F(S_1 \times S_2)$ is continuous, it follows that Φ, hence $\Phi \circ \mu$, is continuous. As η has a dense range, this implies that $\Phi \circ \mu = id_{F(S_1 \times S_2)}$. This completes the proof that F preserves all products of two factors. A simple induction procedure shows that F preserves all finite products. □

Remarks. 1. In the proof above (i.e., the case of a product of two factors) one needs only that e_1 is a right identity in S_1 and that e_2 is a left identity in S_2 ;cf. [2] and [6].

2. The proposition above is valid for any reflector F of **STSgp** into a dense-reflective subcategory of **CTopSgp**: we only needed that the η's have dense range, and that multiplication in $F(S_1 \times S_2)$ is simultaneously continuous. Thus, F might be the reflector of **STSgp** into the subcategory of 0-dimensional compact Hausdorff topological semigroup (or groups).

3. In the above proof, continuity of the multiplication in $F(S_1 \times S_2)$ is used only to guarantee that the mapping Φ is continuous. Actually, one needs only *continuity of the restriction to* $F\alpha_1[FS_1] \times F\alpha_2[FS_2]$ *of the multiplication map* $(\xi_1, \xi_2) \mapsto \xi_1 \xi_2$. Continuity of this restriction, however, can easily be obtained in

some additional special cases, so that for those special cases Φ is continuous and products are preserved as well.

(a)$F=F^{WAP}$ and FS_1 is algebraically a group; then for every object S_2 in **STSgp**,$F^{WAP}(S_1 \times S_2)=F^{WAP}S_1 \times F^{WAP}S_2$. Indeed, $F\alpha_1$, being a section to Fp_1, is an isomorphic embedding, hence $F\alpha_1[FS_1]$ is a closed subgroup of the compact Hausdorff semitopological semigroup $F(S_1 \times S_2)$. So Ellis' joint continuity theorem (e.g., as formulated in [7], II. 4.4) implies that the multiplication in $F(S_1 \times S_2)$ is jointly continuous on $F\alpha_1[FS_1] \times F(S_1 \times S_2)$. Hence Φ is continuous, which implies the desired result. Note, that this situation occurs when S_1 is a dense subsemigroup of a compact Hausdorff topological group G : in that case $F^{WAP}S_1=G$ with $\eta_1:S_1 \to G$ the inclusion mapping (this follows from [1], III. 15.7, where it is proved using function algebras; however, we can prove this quite easily by elementary means). This covers the special case mentioned in Corollary 5 of [6].

(b) $F=F^{LUC}$ and S_1 is an object of **CTopSgp**; then for every object S_2 of **STSgp**,$F^{LUC}(S_1 \times S_2)=S_1 \times F^{LUC}S_2=F^{LUC}S_1 \times F^{LUC}S_2$ (the equality $S_1=F^{LUC}S_1$ is trivial for a compact Hausdorff topological semigroup). Indeed, in this case the mapping

$$((s_1,s_2),\xi) \mapsto \eta(s_1,s_2)\xi : (S_1 \times S_2) \times F(S_1 \times S_2) \to F(S_1 \times S_2)$$

is continuous. Put here $s_2=e_2$ and take into account that by assumption $\eta_1:S_1 \to FS_1$ is an isomorphism. Since $\eta(s_1,e_2)=F\alpha_1(\eta_1(s_1))$ for all $s_1 \in S_1$, it follows that the multiplication mapping of $F(S_1 \times S_2)$ is jointly continuous on $F\alpha_1[FS_1] \times F(S_1 \times S_2)$. This implies the desired result. (Compare this with Corollary 3 of [6].).

(c) $F=F^{LMC}$ and S_1 is an object of **CTopGrp**. Then for every object S_2 of **STSgp**,$F^{LMC}(S_1 \times S_2)=S_1 \times F^{LMC}S_2=F^{LMC}S_1 \times F^{LMC}S_2$ (it is obvious that for any semitopological semigroup T one has $F^{LMC}T=T$; this is certainly valid for $T=S_1$). To prove this, first observe that, similar as in (b) above, the multiplication mapping in the right topological semigroup $F(S_1 \times S_2)$ is separately continuous on $F\alpha_1[FS_1] \times F(S_1 \times S_2)$. By the Ellis-Lawson Theorem (cf. [7], II. 4.3), the multiplication is jointly continuous on this set. This implies the desired results (which is, in fact, Theorem 2.6 of [2]).

(d) $F=F^{LUC}$ and S_1 is a dense subsemigroup of a compact topological Hausdorff group G. Then for every object S_2 of **STSgp**,$F^{LUC}(S_1 \times S_2)=G \times F^{LUC}S_2=F^{LUC}S_1 \times F^{LUC}S_2$; here $F^{LUC}S_1=G$ with $\eta_1:S_1 \to G$ the inclusion mapping. To prove this, first observe that $F^{LUC}S_1=G$: this follows from [1], III. 15.4, but an elementary proof, not using function algebras, is possible. Now similar as in case (b) one sees that the multiplication mapping of $F(S_1 \times S_2)$ is jointly continuous on the set $F\alpha_1[\eta_1 S_1] \times F(S_1 \times S_2)$. The following lemma then shows that it is continuous on $F\alpha_1[G] \times F(S_1 \times S_2)$, which is sufficient for the continuity of Φ. Note, that this implies the special case, mentioned in Corollary 4 of [6].

LEMMA. *Let T be a compact Hausdorff right topological semigroup, and let T_0 be a subsemigroup such that $H:=\overline{T}_0$ is a topological group. If the multiplication* *mapping of T is jointly continuous on $T_0 \times T$, then it is also jointly continuous on $H \times T$.*

PROOF. By the Ellis-Lawson Theorem it would be sufficient to show that the multiplication mapping is separately continuous on $H \times T$, but it requires almost no additional effort to prove joint continuity directly. So let $h \in H, t \in T$ and let W be a closed nbd (= neighbourhood) of ht in T. Since $ht = e.ht$ with e (the identity of T) in T_0, there are a nbd U of e in T_0 and an open nbd V of ht in T such that $UV \subseteq W$. So for every $s \in V, Us \subseteq W$, hence $\overline{Us} \subseteq \overline{W} = W$ by continuity of right translations. Thus,

$$\overline{U}.V \subseteq W. \tag{1}$$

Now observe that $U = U' \cap T_0$ for some nbd U' of e in H. Since T_0 is dense in H, it follows that $\overline{U} = \overline{U' \cap T_0} = \overline{U'}$. Replacing U by U', we may and shall assume henceforth that the set U in formula (1) is a nbd of e in H rather than a nbd of e in T_0. Next, recall that V is a nbd of ht in T. There is a nbd U_1 of h in H such that $U_1 t \subseteq V$ and, in addition, there is a nbd U_2 of e in H such that $U_1 \supseteq U_2 h$ and, moreover, $U_2 = U_2^{-1}, U_2^2 \subseteq U$. So by (1), $U_2 U_2 V \subseteq W$. Select any $s \in U_2 h \cap T_0 (\neq \varnothing$ because T_0 is dense in H). Then $hs^{-1} \in U_2$ (inverse taken in H), hence

$$U_2 h.s^{-1} V \subseteq U_2.U_2.V \subseteq W \tag{2}$$

Here $U_2 h$ is a nbd of h in H. Also, by the choice of U_1 and s we have $t \in s^{-1} V$. As the mapping $\tau \mapsto s\tau : T \to T$ is a bijection (with inverse $\tau \mapsto s^{-1}\tau$) and since it is continuous (for $s \in T_0$), the inverse mapping is continuous as well. In particular, $s^{-1} V$ is an open subset of T, hence a nbd of t. So (2) is just what we want. □

REMARKS (continued). 4. The following shows that F^{WAP} doesn't preserve all finite products (cf. also [2], p. 171, and [5]; we believe our arguments to be much simpler). Let S be a *commutative* topological semigroup with identity. Then the multiplication mapping $\omega : S \times S \to S$ is a morphism in **TopSgp**, so it "extends" uniquely to a morphism

$$\tilde{\omega} := F^{WAP}(\omega) : F^{WAP}(S \times S) \to F^{WAP} S.$$

Now assume that $F^{WAP}(S \times S) = F^{WAP} S \times F^{WAP} S$ (canonically). Then it is easy to see that $\tilde{\omega}$ coincides with the multiplication mapping of $F^{WAP} S$ (which maps $F^{WAP} S \times F^{WAP} S$ into $F^{WAP} S$) on the dense image of $S \times S$. Hence, by a straightforward continuity argument, $\tilde{\omega}$ coincides with this multiplication map on all of $F^{WAP} S \times F^{WAP} S$, and since $\tilde{\omega}$ is jointly continuous it follows that $F^{WAP} S$ is an object in **CTopSgp** rather than **CSTSgp**. Stated otherwise, $F^{WAP} S = F^{AP} S$. Many examples are known where this equality is violated, so those examples must have $F^{WAP}(S \times S) \neq F^{WAP} S \times F^{WAP} S$. In order to keep within the philosophy of this paper, we present an elementary argument (not using (weakly) almost periodic functions) showing that $F^{WAP} S \neq F^{AP} S$ for every non-compact locally compact Hausdorff topological group S. To this end, observe that for such S the one-point compactification $S^*:=S\cup\{\infty\}$ is an object in **CSTSgp** (put $\xi.\infty=\infty.\xi=\infty$ for all $\xi\in S^*$). So the embedding $j:S\to S^*$ factorises over the universal arrow $\eta_S:S\to F^{WAP}S$ as $j=\tilde{j}\circ\eta_S$, with $\tilde{j}:F^{WAP}S\to S^*$ a *surjective* morphism. Now suppose that $F^{WAP}S=F^{AP}S$. It is an elementary fact that in the present situation $F^{AP}S$ is a group (even a topological group: by [3], A. 1.5, a compact topological semigroup with a dense group in it is a topological group). So if $\xi\in F^{AP}S$ is such that $\tilde{j}(\xi)=\infty$, then $j(e)=\tilde{j}(\xi.\xi^{-1})=\infty.j(\xi^{-1})=\infty$, which is not the case because $j(e)=e\in S$. Hence $F^{WAP}S\neq F^{AP}S$.

5. The argument in 4 above can be modified so as to show that in 3(a) above the condition that $F^{WAP}S_1$ is a compact topological group cannot be weakened to the condition that S_1 is a compact semitopological semigroup, not even if S_2 is a locally compact topological group. For let S be a commutative semitopological semigroup which is, algebraically, a group. Put $\tilde{S}:=F^{WAP}S$, with canonical mapping $\eta:S\to\tilde{S}$. By the Ellis-Lawson theorem (cf. [7], Theorem II. 4.3), the mapping $w:(\xi,s)\mapsto\eta(s)\xi:\tilde{S}\times S\to\tilde{S}$ is continuous. Since S is commutative, w is a morphism in **STSgp**, so it "extends" so a morphism $\tilde{w}:=F^{WAP}w:F^{WAP}(\tilde{S}\times S)\to F^{WAP}\tilde{S}=\tilde{S}$. Now again, the assumption that $F^{WAP}(\tilde{S}\times S)=F^{WAP}\tilde{S}\times F^{WAP}S=\tilde{S}\times\tilde{S}$ would lead to the conclusion that $\tilde{w}:\tilde{S}\times\tilde{S}\to\tilde{S}$ is the multiplication mapping of $\tilde{S}$, which would be continuous. This would mean that $F^{WAP}S=F^{AP}S$, which is certainly not true if S is a non-compact locally compact topological group.

3. Infinite products

Theorem. *The reflectors F^{AP} and F^{SAP} preserve all products*

Proof. Consider a set $\{S_i;i\in I\}$ of objects in **STSgp**. Then for each non-empty subset J of I one has the following diagram

$$
\begin{array}{ccccc}
\Pi_{i\in I}S_i & \xrightarrow{\ \eta_I\ } & F(\Pi_{i\in I}S_i) & \xrightarrow{\ \mu_I\ } & \Pi_{i\in I}FS_i \\
p_J\downarrow\ \uparrow\alpha_J & & Fp_J\downarrow\ \uparrow F\alpha_J & & \downarrow q_J \\
\Pi_{i\in J}S_i & \xrightarrow{\ \eta_J\ } & F(\Pi_{i\in J}S_i) & \xrightarrow{\ \mu_J\ } & \Pi_{i\in J}FS_i
\end{array}
$$

Here p_J and q_J are projections and α_J is the canonical embedding $(x)_{i\in J}\mapsto(\bar{x}_i)_{i\in I}$ with $\bar{x}_i=x_i$ if $i\in J$ and $\bar{x}_i=e_i$ (the identity of S_i) otherwise. As in the proof of the proposition in Section 2 it is sufficient to show that μ_I is injective (having a dense range, μ_I is surjective). For this proof it will be convenient to introduce the following notation: $w_J := \alpha_J \circ p_J$ and $\rho_J := Fw_J = F\alpha_J \circ Fp_J$. In addition, $\mathcal{F}$ will denote the directed (under $\supseteq$) set of all non-empty finite subsets of I. CLAIM: for every $\xi \in F(\Pi_{i\in I}S_i)$ the net $\{\rho_J(\xi)\}_{J\in\mathcal{F}}$ converges to ξ.

From this, injectivity of μ_I follows easily: if ξ_1, ξ_2 are in $F(\Pi_{i\in I}S_i)$ and $\xi_1 \neq \xi_2$, then these points have disjoint nbds, and the claim implies that there is $J \in \mathcal{F}$ with $\rho_J\xi_1 \neq \rho_J\xi_2$. But then $Fp_J(\xi_1) \neq Fp_J(\xi_2)$, and as μ_J is injective by the main result of Section 2, this implies that $\mu_I(\xi_1) \neq \mu_I(\xi_2)$.

It remains to prove the claim. Assume the contrary: there exists a point ξ_0 in $F(\Pi_{i\in I}S_i)$ which has a nbd U such that the subset

$$\mathcal{F}_1 := \{J \in \mathcal{F} : \rho_J(\xi_0) \notin U\}$$

of $\mathcal{F}$ is cofinal in $\mathcal{F}$. By compactness, the net $\{\rho_J\xi_0\}_{J\in\mathcal{F}_1}$ has an accumulation point ζ_0. Then $\zeta_0 \notin U$ and ζ_0 has a nbd V such that $\xi_0 \notin V$. Since multiplication in $F(\Pi_{i\in I}S_i)$ is simultaneously continuous, the equality $\zeta_0 = \zeta_0 \cdot e$ (where e is the identity in $F(\Pi_{i\in I}S_i)$) implies that there are nbds V' and V_e of ζ_0 and e, respectively, such that $V'.V_e \subseteq V$; replacing V_e by a smaller nbd of e whose closure is contained in V_e (regularity of the topology) shows that one may assume that $V'.\overline{V}_e \subseteq V$. Note, that by the choice of $\zeta_0, \mathcal{F}_2 := \{J \in \mathcal{F}_1 : \rho_J(\zeta_0) \in V'\}$ is cofinal in $\mathcal{F}_1$, hence in $\mathcal{F}$.

By continuity of η_I, there is a nbd W of $(e_i)_{i\in I}$ in $\Pi_{i\in I}S_i$ such that $\eta_I[W] \subseteq V_e$. Let J be a finite subset of I determining a basic nbd of $(e_i)_{i\in I}$ included in W. Then $w_{I\setminus J}(x) \in W$ for all $x \in \Pi_{i\in I}S_i$. Since this J can be replaced by any larger member of $\mathcal{F}$ and $\mathcal{F}_2$ is cofinal in $\mathcal{F}_1$ we may assume that $J \in \mathcal{F}_2$, so that

$$\rho_{I\setminus J}(\eta_I(x)) = \eta_I(w_{I\setminus J}(x)) \in \eta_I[W] \subseteq V_e$$

for all $x \in \Pi_{i\in I}S_i$. Stated otherwise, $\rho_{I\setminus J}$ maps the dense (!) range of η_I into V_e. Hence $\rho_{I\setminus J}(\xi) \in \overline{V}_e$ for all $\xi \in F(\Pi_{i\in I}S_i)$. Next, notice that $x = w_J(x).w_{I\setminus J}(x)$ for all $x \in \Pi_{i\in I}S_i$, whence $\xi = \rho_J(\xi)\cdot\rho_{I\setminus J}(\xi)$ for all ξ in the range of η_I. By a continuity argument, this equality holds for all $\xi \in F(\Pi_{i\in I}S_i)$. Taking into account that $J \in \mathcal{F}_2$, this implies in particular that

$$\xi_0 = \rho_J(\xi_0)\cdot\rho_{I\setminus J}(\xi_0) \in V'\cdot\overline{V}_e \subseteq V.$$

This contradicts the choice of V. □

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M. Hušek
Math. Inst. of the Charles University
Sokolovská 83
Prague 8
Czechoslovakia

J. de Vries
CWI
Kruislaan 413
1098 SJ Amsterdam
the Netherlands