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S.A. Smulders

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Modelling and Filtering of Freeway Traffic Flow

S.A. Smulders
Centre for Mathematics and Computer Science
P.O. Box 4079, 1009 AB Amsterdam, The Netherlands

In the Netherlands a freeway control and signalling system has been installed on several freeways some years ago. One purpose of the system is to improve traffic flow and avoid the development of congestion. In this paper the first steps towards this aim are set in the development of a traffic model and of a filter that estimates the state of traffic at every time instant. The proposed model is simulated for various traffic situations and modified to achieve realistic performance. The filter is presented and its performance when applied to simulated traffic data shown. The investigations are currently being continued with the application of the filter to real data.

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1. Introduction

In the Netherlands a freeway control and signalling system [8] has been installed on several freeways some years ago. The system consists of measuring loops embedded in the road surface, matrix signal boards above the road mounted on gantries and computer and communications hardware. The measuring locations are spaced approximately 500 meters apart. At each of these locations there is one pair of loops per lane to allow detection of vehicles passing and measurement of their speed. The measurements are sent to a control centre from where the matrix boards can be controlled. The matrix board gantries are spaced approximately 500 to 1000 meters apart. The boards can display advisory speed signals, lane arrows, a red cross and a road clear signal.

One of the main objectives of the system is to improve traffic flow and prevent congestion. It turns out that when traffic density reaches a value of approximately 25 to 30 veh/km/lane the traffic stream becomes very sensitive to small disturbances. For example, one driver applying his brakes for a short time period may be the cause of congestion. If one would be able to detect these disturbances in time it might be possible to prevent the actual occurrence of congestion by showing suitable advisory speed signals to the oncoming traffic.

A solution to this traffic control problem might consist of the following steps :

1. the development of a model of freeway traffic flow that is able to describe instabilities at critical density values and the development of congestion;
2. the derivation of an algorithm based on the model (to be called *filter* henceforth) which recursively estimates the state of traffic from the measurements of the signalling system;
3. the derivation of an optimal, state dependent control strategy for the matrix boards.

To solve the problem we will use techniques available from a branch of mathematics called system and control theory.

In this paper we will only be concerned with the first two steps : the development of a freeway traffic model and the derivation of a filter.

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2. Freeway traffic model

The model we start with is based on a well-known model described by Payne [6]. This model was used by several other researchers : Cremer [2], Van Maarseveen [3,4], Papageorgiou [5] and Payne himself [7]. For our purpose the freeway is divided into sections of approximately 500 meters long with the measuring loops at the boundaries of each section. For section i we define the variables :

$\rho_i(t)$: the density in section i, the number of vehicles per km per lane

$v_i(t)$: the mean speed of the vehicles in section i at time t

Our initial model then consists of the following stochastic differential equations :

$$d\rho_i(t) = \frac{1}{l_i L_i}\left\{\xi_{i-1}[\ \alpha\rho_{i-1} + (1-\alpha)\rho_i\ ][\ \alpha v_{i-1} + (1-\alpha)v_i\ ]\right.$$
$$\left. - \xi_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ][\ \alpha v_i + (1-\alpha)v_{i+1}\ ]\right\}dt$$
$$+ \frac{1}{l_i L_i}[\ dm_{i-1}(t) - dm_i(t)\ ] \tag{2.1}$$

$$dv_i(t) = -\frac{1}{T}\left[\ v_i - v_{free}(1-\frac{\rho_i}{\rho_{jam}})\right]dt - \frac{\nu}{T(L_i+L_{i+1})}\left[\frac{\rho_{i+1} - \rho_i}{\rho_i + c}\right]dt$$
$$+ \frac{\xi_{i-1}}{l_i L_i}v_{i-1}[\ v_{i-1} - v_i\ ]dt + dw_i(t) \tag{2.2}$$

for $i=1,...,n$ where n is the number of freeway sections considered. In (2.1) the parameters have the following meaning :

l_i : the number of lanes of section i

ξ_i : the minimum of l_i and l_{i+1}

L_i : the length of section i in km

α : weighting factor $\in [0,1]$

In (2.2) the parameters have the following meaning :

T : relaxation time in hours

v_{free} : the free speed, the equilibrium speed at zero density in km/h

ρ_{jam} : the jam density in veh/km/lane, the density at which the equilibrium speed is zero

ν : anticipation factor km^2/h

c : correction constant in veh/km/lane

The stochastic process $m_i(t)$ that appears in the density equation is a *martingale* and accounts for the stochastic character of the jumps in $\rho_i(t)$, whereas the first term on the right-hand side of (2.1) represents the behaviour in the mean. Note that we neglect possible on- and off-ramps for the time being. Comparing (2.1) and (2.2) to Payne's model one notes the resemblance, especially in the speed equation. We will now explain how we arrived at the model (2.1), (2.2).

The derivation of the density equation goes as follows. Introduce the *counting processes*

$\pi_i(t)$: the number of vehicles that left section i starting from time t_0

for i=0,...,n . Note that $\pi_0(t)$ is the number of vehicles that left the imaginary section 0, that is, the number that *entered* section 1. Under weak assumptions about the $\pi_i(t)$-processes, one can show that these allow the following decomposition [1] :

$$d\pi_i(t) = \lambda_i dt + dm_i(t)$$

where λ_i is the *intensity* process of π_i and m_i is the martingale. In words : the evolution in time of $\pi_i(t)$ can be separated in a term describing the increase in the mean and a term describing the stochastic fluctuations around that mean. Now λ_i is the mathematical representation of traffic volume and we may use the approximate relation

$$[\text{volume}] \approx [\text{density}] * [\text{speed}]$$

to obtain

$$\lambda_i = \xi_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ][\ \alpha v_i + (1-\alpha)v_{i+1}\ ] \tag{2.3}$$

Here $[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]$ and $[\ \alpha v_i + (1-\alpha)v_{i+1}\ ]$ are estimates of the density and speed in the vicinity of the common boundary of sections i and i+1. When the number of lanes changes at the boundary we take the minimum. In case of a lane drop the bottleneck effect is stressed in this way. Equation (2.1) now follows from the conservation law

$$L_i l_i\ d\rho_i(t) = [\ d\pi_{i-1}(t) - d\pi_i(t)]$$

The mean speed equation (2.2) is based on Payne's derivation : we see a relaxation term, an anticipation term, a convection term and a noise term. The relaxation term describes the tendency of $v_i(t)$ to move to an equilibrium value v^e which only depends on the density ρ. For the relation between equilibrium speed v^e and traffic density ρ we have taken the simple linear Greenshields model

$$v^e(\rho) = v_{free}[\ 1 - \frac{\rho}{\rho_{jam}}\ ]$$

The anticipation term describes the effect of drivers reacting beforehand to changing conditions downstream. The correction constant c is due to Cremer and was added to prevent the anticipation from becoming infinite at zero density.

The convection term describes the effect of vehicles entering or leaving a section on the section mean speed. This effect can in principle be calculated exactly but the term in (2.2) is the result of some approximations. These approximations are justified by the fact that the convection only plays a minor role at the moderate to high density traffic situations we are mainly interested in.

The noise $w_i(t)$ was added to represent the uncertainty in (2.2) and to model stochastic effects such as acceleration noise.

The model (2.1), (2.2) contains a series of parameters which will have to be assigned realistic values. Previous investigations with the described model by Van Maarseveen [3,4] led to the values as in table 2.1.

parameter	value	unit
α	0.5	
T	0.0044	h
ρ_{jam}	116.0	veh/km/lane
v_{free}	106.0	km/h
ν	138.0	km^2/h
c	10.0	veh/km/lane

TABLE 2.1

In order to be able to solve (2.1), (2.2) we need to specify ρ_0 , v_0 , ρ_{n+1} , v_{n+1}. That is, we need to specify *boundary conditions*. Several choices are possible of which we will only mention two. First, one may choose *stationarity conditions* for entrance and/or exit which means that

$$\rho_0(t) = \rho_1(t)$$
$$v_0(t) = v_1(t)$$
$$\rho_{n+1}(t) = \rho_n(t)$$
$$v_{n+1}(t) = v_n(t)$$

by definition, for all t.

Alternatively, we may prescribe the entrance and/or exit intensity by giving two functions of time : $\lambda_0(t)$ and $\lambda_n(t)$.

3. SIMULATION AND MODIFICATION OF THE MODEL

With initial values for the densities ρ_i and mean speeds v_i and after specification of boundary conditions we can solve the equations (2.1) and (2.2) by numerical integration. A FORTRAN computer program was written for this purpose. The stochastics represented by m_i and w_i are simulated following some random number generation procedure. For more details see [9]. We now present the results of some simulation experiments.

Low density traffic

For a two lane freeway stretch of 12 sections of 500 m each we have simulated traffic starting from an initial density of 15 veh/km/lane in each section and the equilibrium speed of 92.2 km/h in each section. The parameters were chosen from table 2.1 and for boundary conditions we chose an entrance intensity of 1383.3 veh/h/lane and stationarity at the exit. The acceleration noises for the section mean speeds were taken to be independent with zero mean and variance 50 km^2/h^4. The simulation resulted in unrealistically large fluctuations of the mean speed : after 2 minutes of simulation it ranges from 0 to 141 km/h over the freeway stretch. See table 3.1 for numerical results.

time (h)	0.0		0.033	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	15.0	92.2	12.0	61.5
2	15.0	92.2	11.0	109.9
3	15.0	92.2	1.0	0.0
4	15.0	92.2	37.0	141.0
5	15.0	92.2	0.0	1.1
6	15.0	92.2	23.0	124.2
7	15.0	92.2	8.0	61.6
8	15.0	92.2	18.0	102.8
9	15.0	92.2	15.0	114.7
10	15.0	92.2	8.0	14.6
11	15.0	92.2	23.0	115.8
12	15.0	92.2	17.0	104.2

TABLE 3.1

Since mean speed is highest where traffic is most dense (compare sections 3 and 4 e.g.) we concluded that the anticipative effect of the model is too strong. We decided to take a smaller value for ν , the anticipation factor :

$$\nu = 40.0 \text{ km}^2/\text{h}$$

This value was found after some experimentation. We will try to get better estimates of parameters in the near future, using real-life traffic data from the signalling system. With the new value of ν we repeated the above simulation. The results are presented in table 3.2.

time (h)	0.0		0.033	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	15.0	92.2	13.0	83.9
2	15.0	92.2	15.0	93.6
3	15.0	92.2	8.0	79.8
4	15.0	92.2	23.0	96.3
5	15.0	92.2	1.0	67.6
6	15.0	92.2	32.0	93.0
7	15.0	92.2	10.0	100.0
8	15.0	92.2	0.0	69.8
9	15.0	92.2	22.0	80.6
10	15.0	92.2	25.0	92.5
11	15.0	92.2	12.0	92.4
12	15.0	92.2	14.0	91.9

TABLE 3.2

We now see a more realistic behaviour of the speed but the density still shows large fluctuations : one of the sections is empty for a period of time whereas an other contains 32 vehicles. This is mainly caused by the weighting of densities and mean speeds in neighbouring sections in (2.3). Taking α equal to 0.5 leads to an intensity at the common section boundary which is too high when the density in the upstream section is low. When $\rho_i = 0$ and $\rho_{i+1} = 32$ veh/km/lane e.g. and $v_i = v_{i+1} = 80$ km/h then $\lambda_i = 2560$ veh/h while no vehicle can really pass the boundary. Taking a larger α we may partly correct this. Repeating the previous simulation with

$$\alpha = 0.85$$

gives the results displayed in table 3.3.

time (h)	0.0		0.033	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	15.0	92.2	13.0	93.1
2	15.0	92.2	12.0	94.1
3	15.0	92.2	12.0	93.3
4	15.0	92.2	15.0	94.4
5	15.0	92.2	12.0	92.0
6	15.0	92.2	13.0	92.8
7	15.0	92.2	11.0	88.9
8	15.0	92.2	19.0	90.2
9	15.0	92.2	18.0	93.9
10	15.0	92.2	15.0	97.1
11	15.0	92.2	5.0	79.5
12	15.0	92.2	21.0	84.7

TABLE 3.3

We now see a reasonable behaviour of both density and mean speed in all sections.

Moderate density traffic
In practice it turns out that when traffic density reaches a level of approximately 25 to 30 veh/km/lane the traffic stream becomes unstable. We would like the model to show unstable behaviour at this density level also. We have investigated this by simulating the model again for a two lane freeway stretch of 12 sections of 500 m each, starting from an initial density of 20 veh/km/lane in all sections and the equilibrium speed of 87.7 km/h. The entrance intensity was taken to be 2000 veh/h/lane which corresponds to a density of approximately 24 veh/km/lane in equilibrium. At the exit we have taken a stationarity condition. The parameters of the model were taken the same as in the last simulation.

Although density reaches a value of approximately 30 veh/km/lane for a considerable period of time the traffic stream remains stable, there are no serious disturbances. Stability analysis of the model confirms this result : the model turns out to be stable for densities up to 60 veh/km/lane. It is known that the equilibrium relation between speed and density plays a major role in the stability properties of the model (Papageorgiou [5]). The change from stable to unstable flow occurs at about that value of the density for which the equilibrium flow reaches its maximum value. In our case :

$$v^e(\rho) = 106.0\,[1 - \frac{\rho}{116.0}] \qquad (3.1)$$

and so

$$\lambda^e_{max} = 3074 \text{ veh/h/lane} \quad \text{at} \quad \rho = 58 \text{ veh/km/lane}$$

We conclude that to obtain unstable behaviour at a realistic density value we need to change the relation $v^e(\rho)$.

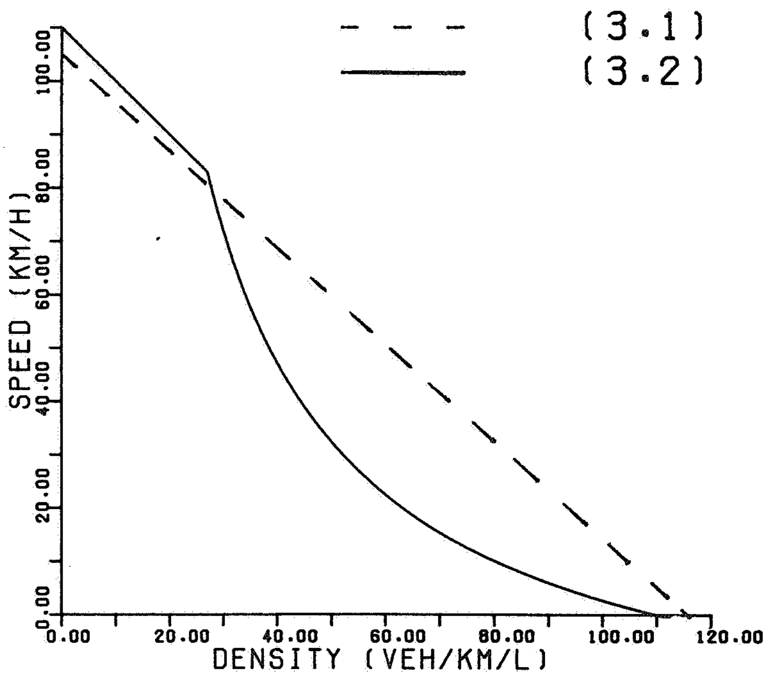

FIGURE 3.1

For the time being we will take a simple modification of (3.1) which suits our purposes :

$$v^e(\rho) = \begin{cases} v_{free}\,[1-\dfrac{\rho}{\rho_{jam}}]\,, & 0 \leqslant \rho \leqslant \rho_{crit} \\ d\,[\dfrac{1}{\rho} - \dfrac{1}{\rho_{jam}}]\,, & \rho_{crit} \leqslant \rho \leqslant \rho_{jam} \end{cases} \tag{3.2}$$

where d is chosen such that v^e is continuous at ρ_{crit} . We will take $\rho_{jam} = 110.0$ veh/km/lane, $v_{free} = 110$ km/h and $\rho_{crit} = 27.0$ veh/km/lane which leads to $d = 2970.0$ 1/h. The above relation is linear in the low density region and decreases fast enough for densities above ρ_{crit} to prevent λ^e from having a maximum which is too large :

$$\lambda^e_{\max} = 2241 \text{ veh/h/lane} \quad \text{at} \quad \rho = 27 \text{ veh/km/lane}$$

See figure 3.1 for a comparison between the two relations.

Repeating the previous simulation with the new $v^e(\rho)$ relation we found that the model is now able to describe instabilities. See figures 3.2 and 3.3. There is a region of low speed (60 km/h) which moves downstream with a speed of approximately 35 km/h. Stability analysis shows the model to be stable for densities up to 30 veh/km/lane. For higher density values the model is unstable.

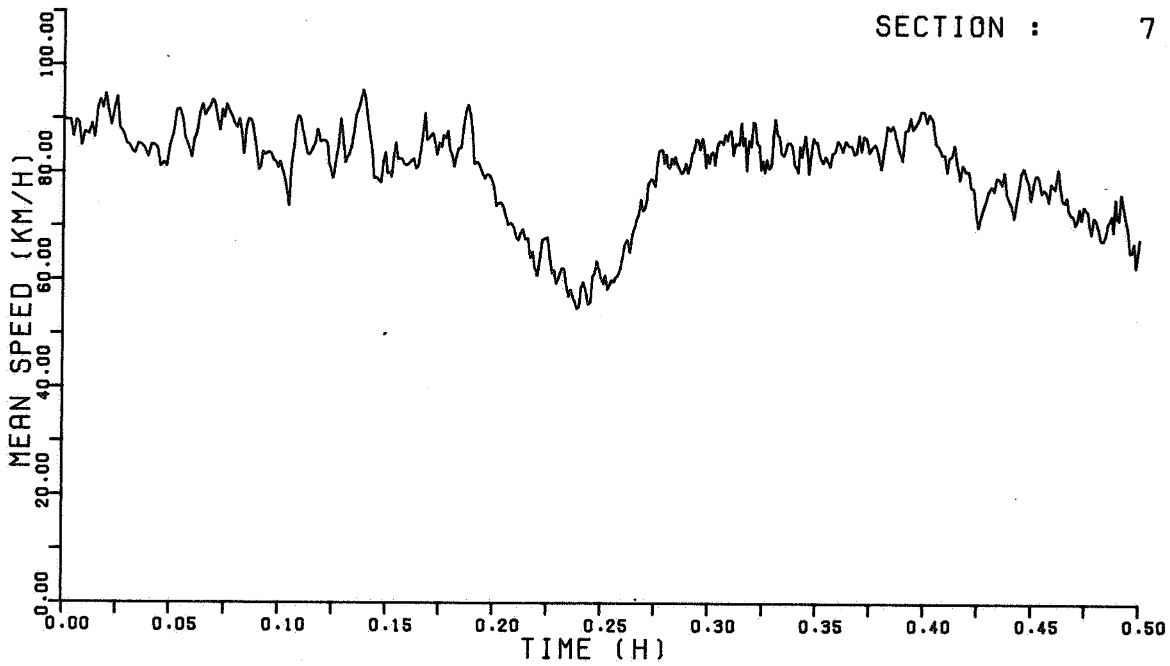

FIGURE 3.2

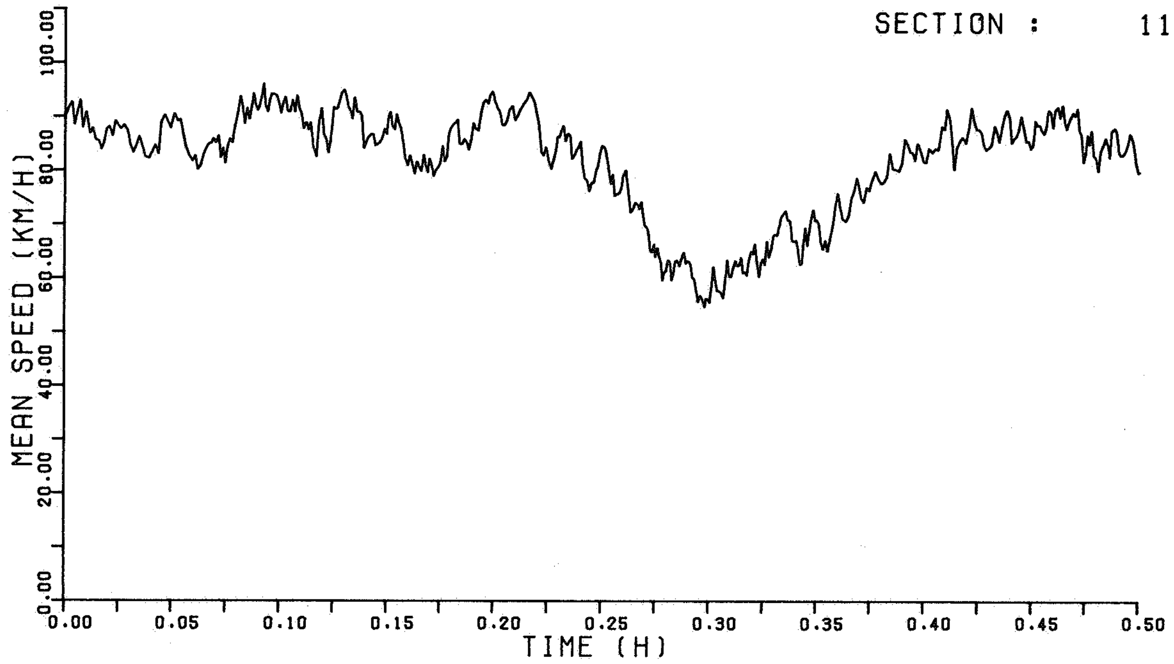

FIGURE 3.3

High density traffic
To conclude our experiments we now simulate the model for a situation where traffic is almost standing still at the end of the freeway stretch while density is moderate at the beginning of the stretch. We again take a two lane freeway of 12 sections, 500 m each. The initial densities and speeds are given in table 3.4. For boundary conditions we take an entrance intensity of 1800 veh/h/lane and stationarity at the exit. The parameters are as in the last simulation (using the new α and ν values and the new $v^e(\rho)$ relationship).

The results are highly unrealistic : vehicles pile up in sections 7, 8 and 9 and density reaches values up to 216 veh/km/lane. Remember that the jam density was modelled at 110 veh/km/lane. A density of 216 veh/km/lane means that the space available for a vehicle is 4.6 m on the average. In the upstream sections drivers keep driving at approximately their equilibrium speed. See table 3.4 for the results.

In practice one would expect the congested region to grow in the upstream direction and density not to exceed the jam value. Clearly the effect of the anticipation term in the model is too small. Recall that we weakened the anticipative effect in an earlier simulation to avoid unrealistically large speed fluctuations at low densities. It seems however, that at high densities anticipation should be strong.

time (h)	0.0		0.12	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	20.0	90.0	22.0	89.2
2	20.0	90.0	19.0	90.6
3	20.0	90.0	14.0	90.5
4	20.0	90.0	15.0	88.6
5	20.0	90.0	25.0	79.2
6	30.0	72.0	41.0	14.1
7	60.0	22.5	216.0	0.0
8	100.0	2.7	208.0	2.3
9	100.0	2.7	132.0	5.4
10	100.0	2.7	94.0	8.9
11	90.0	6.0	82.0	7.0
12	90.0	6.0	88.0	7.0

TABLE 3.4

The results in table 3.4 suggest to model anticipation in such a way that the effect increases with increasing density. The term in our present model :

$$(dv_i(t))_{ant} = -\frac{v}{(L_i + L_{i+1})T} \left[\frac{\rho_{i+1} - \rho_i}{\rho_i + c} \right] dt$$

has just the opposite effect. If the constant c would not be there the anticipation strength would grow without bounds as $\rho_i \to 0$. Instead we now propose

$$(dv_i(t))_{ant} = -\gamma \, (L_i l_i)^2 [\, \beta\rho_i + (1-\beta)\rho_{i+1} \,] [\, \rho_{i+1} - \rho_i \,] dt \tag{3.3}$$

where

γ : a constant of dimension km/h^2

β : a weighting factor $\in [0,1]$

In (3.3)

$$[\, \beta\rho_i + (1-\beta)\rho_{i+1} \,]$$

is an approximation of the prevailing density on the freeway stretch near the common boundary of sections i and i+1.

We choose β equal to 0.5 and γ to 6.5 km/h^2 for the time being. The simulation with this modified model gives far more realistic results than the previous model. See table 3.5 and figures 3.4 and 3.5. We now see the desired behaviour : the congested region spreads itself in the upstream direction and density hardly exceeds the jam value. We now also see a fascinating behaviour in the congested region : *stop-start traffic*. Whether or not the amplitude and period of the stop-start waves are in accordance with reality is not yet clear. We restrict ourselves to noting the ability of the new model to describe this type of behaviour at high densities.

We conclude this section by noting that the modified model shows satisfactory behaviour at various traffic situations.

time (h)	0.0		0.333	
section	ρ veh/km/lane	v km/h	ρ veh/km/lane	v km/h
1	20.0	90.0	95.0	0.0
2	20.0	90.0	100.0	29.8
3	20.0	90.0	113.0	25.9
4	20.0	90.0	94.0	9.1
5	20.0	90.0	97.0	11.9
6	30.0	72.0	97.0	4.1
7	60.0	22.5	103.0	9.9
8	100.0	2.7	100.0	17.5
9	100.0	2.7	95.0	0.0
10	100.0	2.7	97.0	6.3
11	90.0	6.0	98.0	2.6
12	90.0	6.0	102.0	4.0

TABLE 3.5

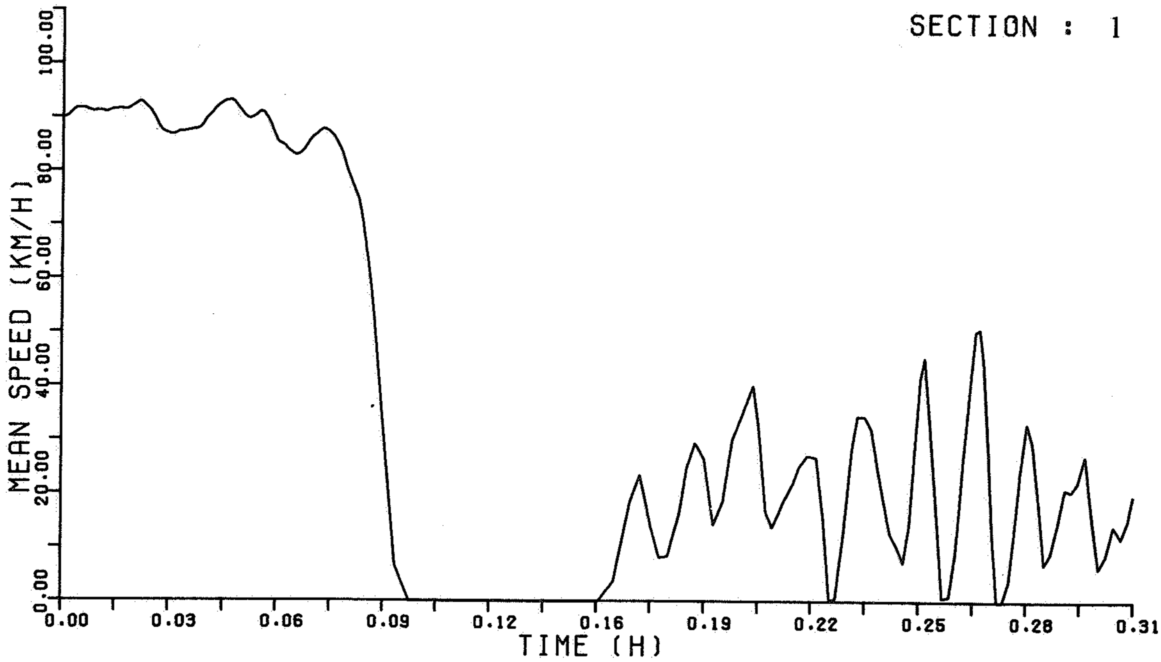

FIGURE 3.4

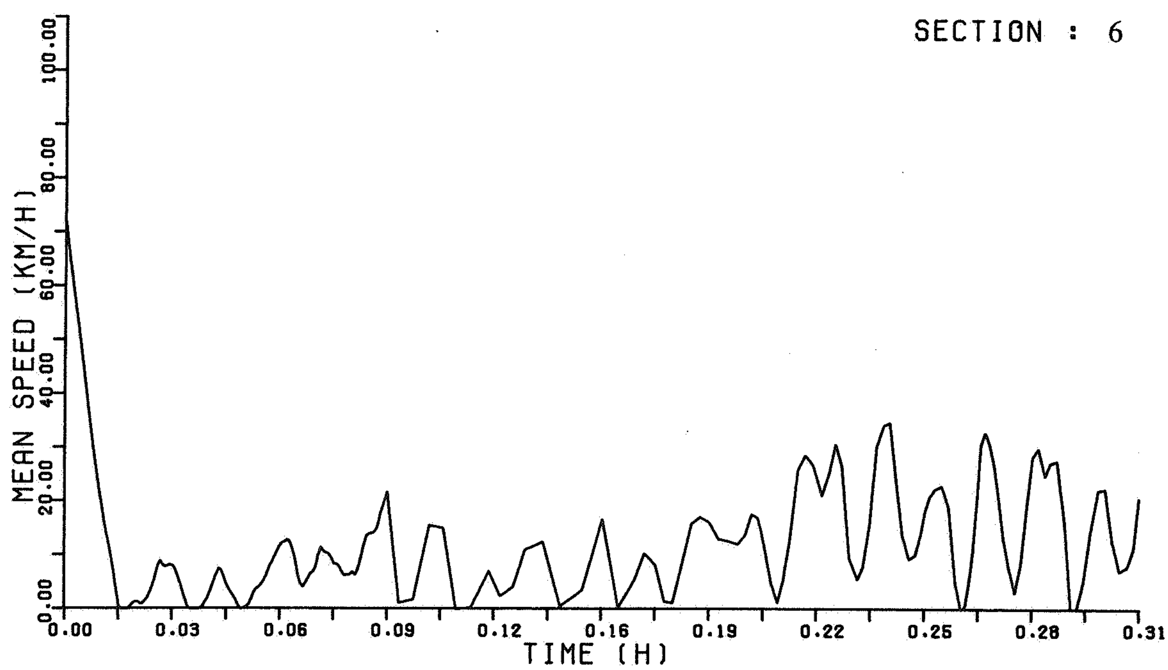

FIGURE 3.5

4. MEASUREMENTS

Our revised model of traffic flow shows satisfactory behaviour at traffic situations ranging from low to high density. If the actual values of the density and mean speed in all sections would be available at every moment the model would allow us to generate predictions over short term periods (e.g. 10 minutes). Based on these predictions a control strategy could be developed for the signals of the freeway control system.

Unfortunately, the system does not provide the exact values of density and mean speed. The only information we receive are the passing times of vehicles at measuring sites and their speed. These measurements clearly contain information on the prevailing densities and mean speeds, but these cannot be computed exactly. We will have to estimate the density and mean speed from the measurements.

An algorithm for this will be developed in the next section. In this section we will give the mathematical relation between the measurements and the state variables density and mean speed.

In section 2 we introduced the counting processes $\pi_i(t)$ (i=0,...,n) which counted the number of vehicles leaving section i starting from time t_0. We found the approximate relation

$$d\pi_i(t) = \xi_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]\ [\ \alpha v_i + (1-\alpha)v_{i+1}\ ]dt + dm_i(t) \tag{4.1}$$

Now the processes $\pi_i(t)$ contain all the information on passing times of vehicles at measuring sites. Therefore (4.1) gives the relation between these measurements and the state variables we are looking for. Based on (4.1) an estimation algorithm was developed by Van Maarseveen [3,4].

However, we do not only have passing times, the system also provides passing speeds. We want to use this information in our estimation procedure too. It turns out that to obtain a practically useful algorithm we will have to discretise the speed space in a finite number of classes. We therefore introduce the *speed classes*

$$V^j \qquad (j = 1,...,m)$$

where m is the number of speed classes. Next introduce the counting processes

π_i^j : the number of vehicles that left section i after time t_0 with speed in class V^j

Instead of the single counting process $\pi_i(t)$ we now have the m counting processes $\pi_i^j(t)$ at each measuring site i. Clearly, some information is lost in the discretisation, but by choosing a large number of classes this effect can be reduced. The actual choice of the number of classes and their location will be shortly discussed in section 5 and be studied in more detail in the near future.

Under some weak assumptions about the π_i^j processes we can derive the decomposition

$$d\pi_i^j(t) = \lambda_i^j dt + dm_i^j(t)$$

just as in (2.3). Now, from

$$\pi_i = \sum_{j=1}^{m} \pi_i^j$$

it follows that

$$\sum_{j=1}^{m} \lambda_i^j = \lambda_i = \xi_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]\,[\ \alpha v_i + (1-\alpha)v_{i+1}\ ]$$

But how to distribute the total intensity λ_i over the individual intensities λ_i^j ? This will be done by assuming that a *fraction* γ_i^j of the vehicles that pass site i will pass with speed in class V^j. This fraction is allowed to depend on traffic density and mean speed :

$$\gamma_i^j(\rho_i,\rho_{i+1},v_i,v_{i+1})$$

We choose these fractions such that $\sum_{j=1}^{m} \gamma_i^j = 1$ always holds. Then

$$\lambda_i^j = \gamma_i^j\,\lambda_i$$

and

$$d\pi_i^j(t) = \gamma_i^j(\rho_i,\rho_{i+1},v_i,v_{i+1})\,\xi_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]\,[\ \alpha v_i + (1-\alpha)v_{i+1}\ ]dt + dm_i^j(t)$$

We now have a relation between the measured passing times and speeds and the traffic state variables : density and mean speed.

There is a last point which has to be taken into account : measuring errors. There are two types of error : counting errors and speed measurement errors. As to the latter we will assume that the speed classes are chosen large enough to justify neglection of these errors.

There are two types of counting errors : we may miss a vehicle passing or falsely count a vehicle when none is passing. A reasonable assumption seems to be that a fixed fraction of all vehicles that pass site i are missed and also that there is fixed fraction of false counts. If we define

n_i^j : the number of vehicles that are *measured* passing site i from time t_0 with speed in class V^j

then

$$dn_i^j(t) = (1 + \epsilon_i^{f} - \epsilon_i^{m^j})\,\gamma_i^j\,\xi_i[\ \alpha\rho_i + (1-\alpha)\rho_{i+1}\ ]\,[\ \alpha v_i + (1-\alpha)v_{i+1}\ ]dt + [dm_i^j(t) + dr_i^{f}(t) - dr_i^{m^j}(t)] \tag{4.2}$$

where

$\epsilon_i^{f^j}$: the fraction of false counts at site i in class V^j

$\epsilon_i^{m^j}$: the fraction of missed vehicles at site i in class V^j

$r_i^{f^j}$, $r_i^{m^j}$: the martingales associated with the error processes

Equations (4.2) for i=0,...,n and j=1,...,m are the measurement equations and will be used in the development of the filter in the next section.

5. Traffic state estimation : filtering

We now develop the algorithm for the estimation of the section densities and mean speeds from the measured passing times and speeds.

To simplify notation introduce the *state vector*

$$X_t = [\ \rho_1(t)\ ,\ v_1(t)\ ,\ \ldots\ ,\ \rho_n(t)\ ,\ v_n(t)\ ]^T$$

and the *measurement vector*

$$N_t = [\ n_0^1(t)\ ,\ n_0^2(t)\ ,\ \ldots\ ,\ n_n^m(t)\ ]^T$$

Our model and measurement equations can then be summarised as

$$dX_t = F(X_t)dt + dZ_t \tag{5.1}$$

$$dN_t = H(X_t)dt + dM_t \tag{5.2}$$

where F(.) and H(.) and Z and M follow from (2.1), (2.2) and (4.2).

For the estimation of the state X from the measurements N techniques have been developed in the area of system and control theory. Well-known is the Kalman filter for the case where F and H are linear and Z and M are Brownian motion processes. For the case of counting type measurements theory has been developed also (Brémaud [1]). We will only give a very brief account here.

As can be easily shown, the estimator of X_t that minimizes the estimation error variance is given by

$$\hat{X}_t = E[\ X_t\ |\ \mathcal{F}_t^N\ ]$$

It is the mean of X_t conditioned upon the measurements. $\mathcal{F}_t^N$ is the σ-algebra generated by $\{N_s\ ,\ s \leq t\}$ and represents the information contained in the measurements up to and including time t.

For $\hat{X}_t$ the following differential equation can be derived :

$$d\hat{X}_t = E[\ F(X_t)\ |\ \mathcal{F}_t^N\ ]dt + \Phi_t(\ dN_t - E[\ H(X_t)\ |\ \mathcal{F}_t^N\ ]dt\ ) \tag{5.3}$$

Here Φ_t is called the *gain matrix* which satisfies a complicated equation that we will not present here. In (5.3) the first term on the right-hand side shows that the filter follows the model to which the second term is a correction, based on the measurements.

Equation (5.3) gives the exact evolution of $\hat{X}_t$ in time. Unfortunately $\hat{X}_t$ cannot be computed from (5.3) in general. For the evaluation of $E[\ F(X_t)\ |\ \mathcal{F}_t^N\ ]$ the computation of the entire conditional probability distribution of X_t is needed, which is practically impossible. The same holds for other terms in the equations for $\hat{X}_t$ and Φ_t. We therefore have to resort to approximations.

The usual approach is to develop Taylor series of the nonlinear functions like F and H around the estimated state $\hat{X}_t$ and neglect higher order terms. Depending on whether one only takes first or second order terms into account one speaks of a first or second order filter. We now give the equations of the *second order filter* :

$$d\hat{X}_t = \hat{F}(\hat{X}_t)dt + \Phi_t[\, dN_t - \hat{H}(\hat{X}_t)dt \,] \tag{5.4}$$

$$\Phi_t = \left\{\hat{P}_t^x H'(\hat{X}_t)^T \mathrm{diag}^{-1}[\hat{H}(\hat{X}_t)] + \begin{bmatrix} A \\ O \end{bmatrix}\right\}_{t-} \tag{5.5}$$

$$d\hat{P}_t^x = \left\{\hat{P}_t^x H'(\hat{X}_t)^T + H'(\hat{X}_t)\hat{P}_t^x + \begin{bmatrix} A\,\mathrm{diag}[\hat{L}(\hat{X}_t)]A^T & O \\ O & \Sigma \end{bmatrix} - \Phi_t \mathrm{diag}[\hat{H}(\hat{X}_t)]\Phi_t^T\right\}dt \tag{5.6}$$

In (5.5) A is a constant matrix of a special structure which accounts for the +1 or -1 jumps of the estimated densities when vehicles enter or leave a section. In (5.6) L(.) is the vector of intensities :

$$L(.) = [\, \lambda_0^1(.) \,,\, \lambda_0^2(.) \,,\, \dots \,,\, \lambda_n^m(.) \,]^T$$

$\hat{P}_t^x$ is an approximation of the matrix of conditional error covariances

$$E[\, (X_t - \hat{X}_t)(X_t - \hat{X}_t)^T \mid \mathcal{F}_t^N \,]$$

In order to compute first or second order approximations to the optimal estimator of X_t it is necessary to compute (an approximation of) the conditional error covariance matrix as well. This is why equation (5.6) appears. Σ is the covariance matrix of the Brownian motion processes w_i.

In the equations above $\hat{F}$ is given by the following

$$\hat{F}_i(.) = F_i(.) + \frac{1}{2}\sum_j\sum_k \frac{\partial^2 F_i(.)}{\partial x_j \partial x_k} (\hat{P}_t^x)_{jk}$$

$\hat{H}$ and $\hat{L}$ are given by analogous expressions.

Using equations (5.4), (5.5) and (5.6) we can now compute approximately optimal estimates of the state of traffic from the measured passing times and speeds. A FORTRAN computer program was written for the numerical integration of the differential equations and was tested on simulated traffic data. Equation (5.6) was not solved directly but by means of a so called square root method to improve numerical stability. The testing consists of simulating our traffic model as in section 3 and feeding the corresponding passing times and speeds into the filter via equation (5.4). The estimated densities and mean speeds can be compared with the simulated ones to get an idea of the estimation accuracy. We now present two examples of such tests.

We have simulated the traffic model for a two lane freeway stretch of three sections, 500 m each, starting from an initial density of 20 veh/km/lane and the equilibrium speed of 90 km/h in each section. The entrance intensity was chosen to be 1800 veh/h/lane and at the exit we have taken a stationarity condition. There were no counting errors and the acceleration noise was taken to be negligibly small. As to the speed measurements : we have first taken one speed class, which means that we only used the passing time information in the estimation procedure. Next we repeated the test taking *four* speed classes : [0,80) , [80,85) , [85,90) , [90,150). In both tests the filter was started in the right state but with a large initial error variance : $\hat{P}_t^x$ was taken to be a diagonal matrix with diagonal elements all equal to 100. For the results see figures 5.1 and 5.2. Note that the continuous line represents the simulated data and the dashed line the estimated data.

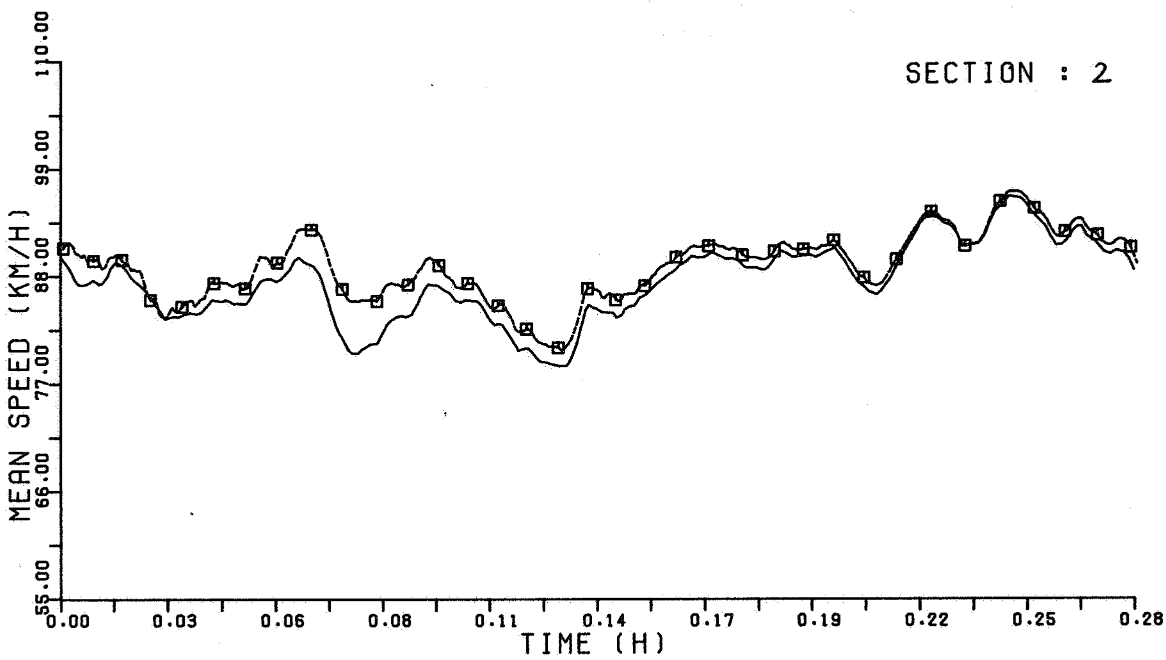

FIGURE 5.1

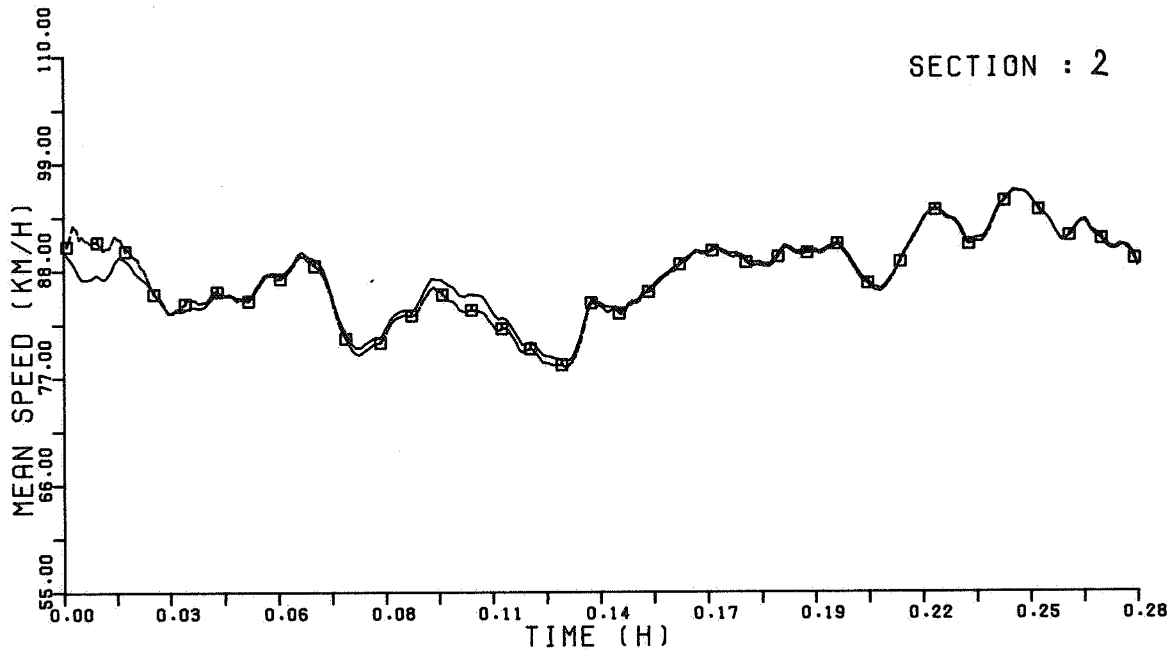

FIGURE 5.2

We see that the use of the passing speed information increases the accuracy of the estimates and that the results with four classes are nearly optimal. The mean estimation error in figures 5.1 and 5.2 was computed to be 3.7 and 1.8 km/h respectively. Further tests with the filter using more than one speed class have shown the estimation error in general to be less than five percent.

6. Conclusions

In this paper we presented a model for freeway traffic and with help of simulations we were able to improve its validity. Based on the model we developed a filter for the estimation of the traffic state from the measurements and showed some results of tests against simulated traffic data. We concluded that the use of passing speed information in the estimation procedure improves the quality of the estimates considerably.

In the near future we plan to test the filter further. We want to compare the quality of the estimates of first and second order filters, investigate filter robustness with respect to modelling errors, investigate filter behaviour when density is at a critical level and apply it to real traffic data provided by the signalling system.

For the latter test we need to have reasonable estimates of the values of the fractions γ_i^j used in the model. Remember that these fractions were allowed to depend on traffic density and mean speed. We plan to estimate the probability distribution of passing speeds from real-life traffic data and compute the γ_i^j's from this distribution. We also plan to use some identification technique to obtain better estimates of the other model parameters.

As to the underlying traffic model we plan to incorporate the possibility of on- and off-ramps. If the ramps contain measuring loops this is no problem. Presently most of the ramps of the signalised freeways do not have ramp loops. We will have to devise some estimation scheme to cope with this problem.

Once a satisfactory filtering algorithm has been developed one may attack various traffic control problems like congestion prevention, ramp control and incident detection. We plan to work on the congestion prevention problem first. This means that we will have to develop control strategies for the signals of the freeway control system.

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