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## Two Normal Form Theorems for CSP Programs

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We define two normal forms for *CSP* programs. In the First Normal Form, each process contains only one I/O repetitive command and all its I/O commands appear as guards of this command. In the Second Normal Form, it is moreover required that all guards of this I/O repetitive command are in fact I/O guards. We describe an inductive method which transforms any *CSP* program into an equivalent program in first or second normal form. The involved equivalence notion is discussed. It is shown in particular that no transformation into second normal form can preserve deadlock-freedom.

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## 1. Introduction

One of the best known theorems in Theoretical Computer Science states that every while program is equivalent to a program with one loop only (see e.g. [Ha]). We prove here a similar result for *CSP* programs. *CSP* is the language for distributed programming introduced by Hoare in [Ho1]. We exhibit two normal forms to which every *CSP* program can be brought. A *CSP* program is in a normal form (a **normal program**, in short) if each of its component processes contains only one I/O repetitive command and all its I/O commands appear as guards in this command. There are various reasons why the study of normal programs can be of interest.

**1) Program construction** — In the case of *CSP* programs, in opposition to the case of while programs, several algorithms can be naturally expressed as normal programs. For example, most solutions to the distributed termination problem of Francez [F] are normal programs (see e.g. Francez *et al.* [FRS] and Apt and Richier [AR]). For other algorithms written as normal programs, see e.g. Bougé [B].

**2) Verification** — We found (see [A]) that there is a very simple proof system allowing us to prove correctness of normal programs.

Moreover, Queille and Sifakis [QS] built a system allowing an automatic verification of finite state normal programs. Adding to their system a preprocessor implementing the transformations described in this paper allows us to extend the use of their system to arbitrary finite state *CSP* programs.

**3) Event-driven computing** — In an event-driven concurrent system, local actions are triggered by the occurrence of external events. This type of computing is typical in the case of network protocols. It is often modeled by means of interacting automata (see e.g. Gouda [G]). Normal programs exhibit a structure which makes this view of distributed computing more explicit as each process alternates between communications and resulting local actions.

Equivalence of concurrent programs is a delicate and difficult issue. In the last section of this paper we analyze the notion used in this paper and indicate its limitations.

## 2. Normal forms in CSP

We assume from the reader knowledge of the language of Communicating Sequential Processes (*CSP* for short) as defined by Hoare [Ho1]. We consider here a variant of *CSP* without nested parallelism and where output guards are allowed. We do not consider the Distributed Termination Convention. For simplicity, we omit all kinds of declarations.

A *CSP* program  $P$  is a parallel composition of named processes which operate on disjoint memories:

$$[ P_1 :: S_1 \parallel \dots \parallel P_k :: S_k ] .$$

Each process  $S$  is generated by the following grammar ( $m \geq 1$ ):

$$\begin{aligned} S ::= & \text{skip} \mid \text{cmd} \mid \alpha \mid S_1 ; S_2 \mid \\ & [ \bigcap_{j=1}^m G_j \rightarrow S_j ] \mid * [ \bigcap_{j=1}^m G_j \rightarrow S_j ] \\ G ::= & b \mid b ; \alpha \end{aligned}$$

Here,  $\text{cmd}$  denotes an assignment,  $b$  a Boolean expression and  $\alpha$  an I/O command. If a guard  $G$  is of the form  $b$  then it is called a *purely Boolean guard*, and otherwise an *I/O guard*.  $\text{Bool}(G)$  denotes the Boolean part of a guard. A guard  $G$  is *enabled* when the control is in front of it if  $\text{Bool}(G)$  evaluates to true. In the sequel,  $b_1 ; b_2$  denotes the conjunction  $b_1 \wedge b_2$  of Boolean expressions.

**Definition 1.** A process  $S$  is in *first normal form* if it is either of the form  $S_0$  or

$$S_0 ; * [ \bigcap_{j=1}^m G_j \rightarrow S_j ]$$

where  $m \geq 1$  and none of the  $S_j$ ,  $j = 0, 1, \dots, m$  contains an I/O command.

**Definition 2.** A process  $S$  is in *second normal form* if it is either of the form  $S_0$  or

$$S_0 ; * [ \bigcap_{j=1}^m G_j \rightarrow S_j ]$$

where  $m \geq 1$ , none of the  $S_j$ ,  $j = 0, 1, \dots, m$  contains any I/O command, and moreover all of the  $G_j$ ,  $j = 1, \dots, m$  are I/O guards.

Thus the only difference between the first and second normal form is in the condition imposed on the guards  $G_i$ . A *CSP* program is in *first (resp. second) normal form* if all its component processes are.

## 3. The transformations

### 3.1. First normal form

We now describe a procedure  $NF_1$  which transforms each process  $S$  into a process  $S' = NF_1(S)$  in first normal form. We proceed by induction on the structure of  $S$ . We assume an infinite set  $Z$  of fresh Boolean variables  $z_1, z_2, \dots$ . We omit indices when no confusion can arise.

**Base case**

When  $S$  is `skip` or an atomic assignment command  $cmd$ ,  $S$  is already in first normal form and we put  $NF_1(S) = S$ .

**I/O command**

Suppose  $S$  is an I/O command  $\alpha$ . We select a fresh variable  $z$  from  $Z$ . We define  $NF_1(S)$  to be

$$z := \text{true}; * [ z; \alpha \rightarrow z := \text{false} ] .$$

**Sequential composition**

Suppose  $S$  is of the form  $S_1 ; S_2$ . By induction, each  $S_i$ ,  $i = 1, 2$ , has been transformed using a set  $Z_i$  of fresh variables into the process

$$NF_1(S_i) = \text{Init}_i ; * [ \prod_{j=1}^{m_i} G_j^i \rightarrow S_j^i ]$$

in first normal form. We can assume that sets  $Z_1$  and  $Z_2$  are disjoint. Let  $z_1$  and  $z_2$  be two variables of  $Z \setminus Z_1 \cup Z_2$ . Then we define  $NF_1(S)$  as follows:

$$\begin{aligned} & \text{Init}_1 ; z_1 := \text{true}; z_2 := \text{false}; \\ & * [ \prod_{j=1}^{m_1} z_1 ; G_j^1 \rightarrow S_j^1 ; \text{TEST} \\ & \quad \prod_{j=1}^{m_2} z_2 ; G_j^2 \rightarrow S_j^2 ] \end{aligned}$$

where  $\text{TEST}$  stands for

$$\begin{aligned} & [ \prod_{j=1}^{m_1} \text{Bool}(G_j^1) \rightarrow \text{skip} \\ & \quad \prod_{j=1}^{m_1} \neg \text{Bool}(G_j^1) \rightarrow z_1 := \text{false}; z_2 := \text{true}; \text{Init}_2 ] \end{aligned}$$

Intuitively,  $z_1$  is true when the control is still in  $S_1$  and  $z_2$  is true when the control is in  $S_2$ .

**Repetitive command**

Suppose  $S$  is of the form  $*[ \bigsqcup_{i=1}^m H_i \rightarrow R_i ]$ . By induction, each  $R_i$ ,  $i = 1, \dots, m$ , has been transformed using a set  $Z_i$  of fresh variables into the process

$$NF_1(R_i) = \text{Init}_i; * [ \bigsqcup_{j=1}^{m_i} G_j^i \rightarrow S_j^i ]$$

in first normal form. We can assume without loss of generality that sets  $Z_i$  are pairwise disjoint. Let  $\text{turn}_i$ ,  $i = 1, \dots, m$  be fresh variables of  $Z \setminus \bigcup_{i=1}^m Z_i$ . Then we define  $NF_1(S)$  as follows:

$$\begin{aligned} & \text{turn}_1 := \text{false}; \dots; \text{turn}_m := \text{false}; \\ & * [ \bigsqcup_{i=1}^m \bigwedge_{i=1}^m \neg \text{turn}_i; H_i \rightarrow \text{turn}_i := \text{true}; \text{Init}_i; \text{TEST}_i; \\ & \quad \bigsqcup_{\substack{i=1, \dots, m \\ j=1, \dots, m_i}} \text{turn}_i; G_j^i \rightarrow S_j^i; \text{TEST}_i ] \end{aligned}$$

where  $\text{TEST}_i$  stands for

$$\begin{aligned} & [ \bigsqcup_{k=1}^{m_i} \text{Bool}(G_k^i) \rightarrow \text{skip} \\ & \quad \bigsqcup_{k=1}^{m_i} \bigwedge \neg \text{Bool}(G_k^i) \rightarrow \text{turn}_i := \text{false} ]. \end{aligned}$$

Intuitively,  $\text{turn}_i$  holds when the control is inside subprogram  $R_i$ . Then,  $\text{TEST}_i$  tests whether  $R_i$  is terminated, and  $\text{turn}_i$  is reset to false if it is the case.

**Alternative command**

Suppose  $S$  is of the form  $[ \bigsqcup_{i=1}^m H_i \rightarrow R_i ]$ . By induction, each  $R_i$ ,  $i = 1, \dots, m$ , has been transformed using a set  $Z_i$  of fresh variables into the process

$$NF_1(R_i) = \text{Init}_i; * [ \bigsqcup_{j=1}^{m_i} G_j^i \rightarrow S_j^i ]$$

in first normal form. Using a new variable  $z$  from  $Z \setminus \bigcup_{i=1}^m Z_i$ , we first transform  $S$  into the following process  $S'$ :

$$\begin{aligned} & z := \text{true}; \\ & [ \bigsqcup_{i=1}^m \text{Bool}(H_i) \rightarrow \text{skip} ]; \\ & * [ \bigsqcup_{i=1}^m z; H_i \rightarrow R_i; z := \text{false} ] \end{aligned}$$

Those two processes are related as follows. Suppose first that  $S$  fails. This occurs when all conditions  $\text{Bool}(H_i)$  evaluate to false initially. Then  $S'$  fails much in the same way. If now  $S$  does not fail then at least one of those conditions evaluates to true. In  $S'$ , the alternative command boils down then to skip. In the repetitive command, the conditions are evaluated again, and yield the same results as before, because processes operate on disjoint memories. At least one of them is thus guaranteed to evaluate to true,  $S'$  does not fail either, and behaves subsequently like  $S$ .  $NF_1(S)$  is result of applying the transformation  $NF_1$  to process  $S'$ .

This concludes the presentation of the transformation  $NF_1$

**Property 1.** For each process  $S$ ,  $NF_1(S)$  is a process in first normal form. The only atomic commands in  $NF_1(S)$  in which variables from  $Z$  appear are of the form  $z := \text{true}$  or  $z := \text{false}$ .

### 3.2. Second normal form

We now describe a procedure  $NF_2$  which transforms each process  $S$  in first normal form into a process  $NF_2(S)$  in second normal form. A process  $S$  in first normal form whose all external guards contain an I/O command (or which contains no I/O command) is already in second normal form and we put  $NF_2(S) = S$ . Otherwise, it can be written as

$$\begin{array}{l} \text{Init;} \\ * [ \bigvee_{i=1}^m G_i \rightarrow S_i \\ \quad \bigvee_{j=1}^n H_j \rightarrow T_j ] \end{array}$$

with  $m > 0$  and  $n > 0$ , where all guards  $G_i$  are purely Boolean, and all guards  $H_j$  do contain an I/O command. Let now  $CHOOSE$  be the following command:

$$\begin{array}{l} \text{turn}_1 := \text{false}; \dots; \text{turn}_n := \text{false}; \\ * [ \bigvee_{i=1}^m \bigwedge_{k=1}^n \neg \text{turn}_k; G_i \rightarrow S_i \\ \quad \bigvee_{j=1}^n \bigwedge_{k=1}^n \neg \text{turn}_k; \text{Bool}(H_j) \rightarrow \text{turn}_j := \text{true} ]. \end{array}$$

The execution of  $CHOOSE$  consists of some iterations of the repetitive command

$$* [ \bigvee_{i=1}^m G_i \rightarrow S_i ]$$

which contains no I/O command followed by the selection of an I/O guard  $H_j$ , provided its Boolean part  $\text{Bool}(H_j)$  evaluates to true. We then define  $NF_2(S)$  to be the following process

$$\begin{array}{l} \text{Init}; \text{CHOOSE}; \\ * [ \bigvee_{j=1}^n \text{turn}_j; H_j \rightarrow T_j; \text{CHOOSE} ]. \end{array}$$

Observe that  $\text{Bool}(H_j)$  is evaluated twice, once within  $CHOOSE$  and then again within  $H_j$ . Both evaluations return necessarily the same result because processes operate on disjoint memories.

**Property 2.** For each process  $S$  in first normal form,  $NF_2(S)$  is in second normal form. The only atomic commands in  $NF_2(S)$  in which variables from  $Z$  appear are of the form  $z := \text{true}$  and  $z := \text{false}$ .

### 3.3. Homogeneous processes

For certain processes however, it is possible to describe a direct transformation which yields a process in second normal form. A process is **homogeneous** if in each repetitive or alternative command either all guards are purely Boolean or all of them contain an I/O command. Observe that a homogeneous process is in first normal form if and only if it is in second normal form. If we can modify procedure  $NF_1$  so as to preserve homogeneity, then it will transform homogeneous processes into processes in *second* normal form. It can be seen that the only part of  $NF_1$  which does not preserve homogeneity is that dealing with a repetitive command  $S$

$$* [ \bigsqcup_{i=1}^m H_i \rightarrow R_i ]$$

whose all guards are purely Boolean. In this case, let *SWITCH* be

$$\begin{aligned} & [ \bigsqcup_{i=1}^m H_i \rightarrow \text{turn}_i := \text{true}; \text{Init}_i \\ & \bigsqcup \bigwedge_{i=1}^m \neg H_i \rightarrow \text{skip} ]. \end{aligned}$$

Then, assuming the notation used in  $NF_1(S)$ , the transformed process is

$$\begin{aligned} & \text{turn}_1 := \text{false}; \dots; \text{turn}_m := \text{false}; \\ & \text{SWITCH}; \\ & * [ \bigsqcup_{\substack{i=1, \dots, m \\ j=1, \dots, m_i}} \text{turn}_i; G_j^i \rightarrow S_j^i; \text{TEST}_i; \\ & \quad [ \text{turn}_i \rightarrow \text{skip} \bigsqcup \neg \text{turn}_i \rightarrow \text{SWITCH} ] ]. \end{aligned}$$

Denote this modified transformation by  $NF'_1$ . Here, variables  $\text{turn}_i$  are used for the same purpose as before. Setting a variable  $\text{turn}_i$  to true can take place in the *SWITCH* command only.

**Property 3.** *For each homogeneous process  $S$ ,  $NF'_1(S)$  is in second normal form. The only atomic commands in  $NF'_1$  in which variables from  $Z$  appear are of the form  $z := \text{true}$  and  $z := \text{false}$ .*

## 4. A notion of equivalence

We now wish to make precise in what sense every process  $S$  is equivalent to the process  $S'$  generated in section 3 by the transformations  $NF_1$ ,  $NF_2$  and  $NF'_1$ . To this purpose, we first associate to each process  $S$  a regular language  $L(S)$ . Intuitively,  $L(S)$  is the set of all uninterpreted possible computations of process  $S$  according to Plotkin's semantics [P].

The language  $L(S)$  is over the alphabet consisting of atomic actions *cmd*, I/O commands  $\alpha$ , Boolean conditions  $b$ , plus two special tokens  $\langle \text{skip} \rangle$  and  $\langle \text{fail} \rangle$  which denote

respectively termination and failure.  $L(S)$  is defined inductively as follows.

$$\begin{aligned}
L(\text{skip}) &= \{\langle \text{skip} \rangle\}, \\
L(\text{cmd}) &= \{\langle \text{cmd} \rangle\}, \\
L(\alpha) &= \{\langle \alpha \rangle\}, \\
L(G) &= \begin{cases} \{\langle b \rangle \langle \alpha \rangle\} & \text{if } G = b; \alpha, \\ \{\langle b \rangle \langle \text{skip} \rangle\} & \text{if } G = b, \end{cases} \\
L(S_1; S_2) &= L(S_1).L(S_2), \\
L([\ G_1 \rightarrow S_1 \ \square \ \dots \ \square \ G_m \rightarrow S_m \ ] ) &= \\
& \quad [(L(G_1).L(S_1)) \cup \dots \cup (L(G_m).L(S_m))] . \{\langle \text{Bool} \rangle \langle \text{fail} \rangle\}, \\
L(*[\ G_1 \rightarrow S_1 \ \square \ \dots \ \square \ G_m \rightarrow S_m \ ] ) &= \\
& \quad [(L(G_1).L(S_1)) \cup \dots \cup (L(G_m).L(S_m))]^* . \{\langle \text{Bool} \rangle \langle \text{skip} \rangle\},
\end{aligned}$$

where  $\text{Bool}$  stands for

$$\neg \text{Bool}(G_1) \wedge \dots \wedge \neg \text{Bool}(G_m).$$

Observe how the appropriate exit conditions are reflected.

To obtain the desired equivalence, we partially interpret the computations by evaluating the commands and conditions associated with auxiliary variables. In the processes generated by the transformations of section 3, they can be of the following type exclusively:

$$\begin{aligned}
z &:= \text{true}, \\
z &:= \text{false}, \\
B(z_1, \dots, z_m, b_1, \dots, b_n)
\end{aligned}$$

where  $B$  is some Boolean combination of its arguments. When evaluating the condition, variables  $z_i$ 's are substituted with their current value, *true* or *false*. The condition is said to be *unsatisfiable* if the resulting formula is equivalent to *false* as a formula of the predicate calculus with variables  $b_j$ 's. The condition is *satisfiable* if it is not satisfiable. Then we exclude contradictory computations, i.e. those which violate the rule that the selected Boolean conditions are all satisfiable. Finally, we erase all *skip*'s and assignments  $z := \text{true}$  and  $z := \text{false}$  to auxiliary variables. Also, we merge adjacent Boolean formulas into their Boolean conjunction, and reduce the resulting formula to some normal form (say, a conjunction of disjunctions for definiteness). Tautologies are then erased. Let  $L'(S)$  be the resulting language. It is a language over the alphabet consisting of atomic actions *cmd*, I/O commands  $\alpha$ , Boolean formula  $B(b_1, \dots, b_n)$ , and token  $\langle \text{fail} \rangle$ . We say that two processes  $S_1$  and  $S_2$  are *equivalent* with respect to a set  $Z$  of auxiliary variables if

$$L'(S_1) = L'(S_2).$$

This equivalence can be best understood with a example. Consider the processes

$$S_1 = \alpha$$

and

$$S_2 = z := \text{true}; * [ z; \alpha \rightarrow z := \text{false} ] .$$

Then

$$L(S_1) = L'(S_1) = \{ \langle \alpha \rangle \}$$

On the other hand,

$$L(S_2) = \langle z := \text{true} \rangle \langle z \rangle \langle \alpha \rangle \langle z := \text{false} \rangle^* \langle \neg z \rangle \langle \text{skip} \rangle .$$

After the interpretation of actions related to the variable  $z$  we obtain the following set of words

$$\begin{aligned} & \{ \langle z := \text{true} \rangle \langle \text{true} \rangle \langle \alpha \rangle \langle z := \text{false} \rangle \langle \neg \text{false} \rangle \langle \text{skip} \rangle, \\ & \langle z := \text{true} \rangle \langle \text{true} \rangle \langle \alpha \rangle \langle z := \text{false} \rangle \langle \langle \text{false} \rangle \langle \alpha \rangle \langle z := \text{false} \rangle \rangle^* \langle \neg \text{false} \rangle \langle \text{skip} \rangle \} . \end{aligned}$$

Here, only the computation

$$\langle z := \text{true} \rangle \langle \text{true} \rangle \langle \alpha \rangle \langle z := \text{false} \rangle \langle \neg \text{false} \rangle \langle \text{skip} \rangle$$

is not contradictory. Deleting from it all assignments to the variables of  $Z$  and  $\text{skip}$ 's, reducing sequences of adjacent Boolean formula to their normal form and erasing tautologies, we get  $\langle \alpha \rangle$  as desired.

We have the following theorems whose tedious but straightforward proofs are omitted.

**Theorem 1.** *Both  $NF_1(S)$  and  $NF'_1(S)$  are equivalent to  $S$  with respect to set  $Z$  of auxiliary variables.*

**Theorem 2.**  *$NF_2(S)$  is equivalent to  $S$  with respect to set  $Z$  of auxiliary variables.*

These equivalences are on the level of processes considered in isolation. The following theorem states some of its semantic consequences. By a *state* we mean a function assigning values to each of the variables. We consider  $\perp$  as a special state indicating divergence. Given a CSP program  $P$ , we define its meaning  $\mathcal{M}[P]$  by putting

$$\begin{aligned} \mathcal{M}[P](\sigma) = & \\ & \{ \tau, \tau \text{ is the final state of a properly terminating computation starting in state } \sigma \} \\ & \cup \{ \perp, \text{ there exists a diverging computation of } P \text{ starting in state } \sigma \} . \end{aligned}$$

For two sets  $\Sigma_1$  and  $\Sigma_2$ , and a set of variables  $Z$  we put

$$\Sigma_1 = \Sigma_2 \text{ mod } Z \text{ iff } \{ \sigma \setminus Z, \sigma \in \Sigma_1 \} = \{ \sigma \setminus Z, \sigma \in \Sigma_2 \}$$

where  $\sigma \setminus Z$  is the restriction of  $\sigma$  to the variables not in  $Z$ . We now say that two programs CSP  $P_1$  and  $P_2$  are *equivalent modulo  $Z$*  if for all states  $\sigma$

$$\mathcal{M}[P_1](\sigma) = \mathcal{M}[P_2](\sigma) \text{ mod } Z .$$

Note that this equivalence definition does not take into account possible deadlocks. We can finally state the appropriate theorem.

**Theorem 3.** *Let  $S_1$  and  $S_2$  be two equivalent processes with respect to a set  $Z$  of auxiliary variables. Let*

$$C = [ Q_1 :: T_1 \parallel \dots \parallel Q_k :: [ ] \parallel \dots \parallel Q_n :: T_n ]$$

*be a context, and let  $P_i = C[S_i]$ ,  $i = 1, 2$ , be the CSP programs obtained by plugging process  $S_i$  into the context  $C$ . Then  $P_1$  and  $P_2$  are equivalent modulo  $Z$ .*

Thus, up to deadlock,  $P_1$  and  $P_2$  exhibit the same functional behaviors.

## 5. Discussion

The equivalence relation introduced in section 4 seems at first sight very strong as it is basically a syntactic equivalence. However, this equivalence is concerned only with some form of traces (in the sense of [Ho2]) of computations. Semantically, it assures only theorem 3. In particular, it does not capture all relevant semantic properties naturally associated with concurrent programs, like deadlock freedom.

Indeed, consider two processes  $S$  and  $S'$  where

$$\begin{aligned} S &= [ \text{true}; Q?x \rightarrow \text{skip} \\ &\quad \square \text{true}; Q!x \rightarrow \text{skip} ], \\ S' &= [ \text{true} \rightarrow Q?x; \text{skip} \\ &\quad \square \text{true} \rightarrow Q!x; \text{skip} ]. \end{aligned}$$

Then  $S$  and  $S'$  are equivalent in the sense of section 4. However, the program

$$[ P :: S \parallel Q :: P?y ]$$

cannot deadlock whereas the program

$$[ P :: S' \parallel Q :: P?y ]$$

can. Thus, plugging equivalent processes in the same context, here

$$[ P :: [ ] \parallel Q :: P?y ],$$

can yield two programs which behave differently. Currently, we look for a simple, stronger form of equivalence for which theorem 1 holds and moreover such that substituting in any context a process by a an equivalent one preserves at least deadlock freedom.

We can however prove that theorem 2 cannot be strengthened so that deadlock freedom is preserved in the above sense. This follows from the following theorem.

**Theorem 4.** *Let*

$$[ P :: S \parallel Q :: T ]$$

*be a program in second normal form. Suppose that it admits two properly terminating computations,  $C_1$  with some communication, and  $C_2$  without any communication. Then it admits a deadlocked computation.*

**Proof** Construct the deadlocked computation as follows. First take all steps carried out by  $P$  in  $C_1$  until the I/O command selected for its first communication is reached. Then append to it all step carried out by  $Q$  in  $C_2$ . In the resulting computation,  $Q$  properly terminates whereas  $P$  reaches an entry to a repetitive command with all guards containing an I/O guard. Thus a deadlock arises (observe that this would not necessarily hold if the Distributed Termination Convention of CSP were used).  $\square$

Indeed, take the program  $[ P::S \parallel Q::T ]$  with

$$S = [ Q!x \rightarrow \text{skip} \square \text{true} \rightarrow \text{skip} ]$$

and

$$T = [ P?y \rightarrow \text{skip} \square \text{true} \rightarrow \text{skip} ].$$

Consider now an arbitrary pair  $S', T'$  of processes in second normal form such that  $S'$  is equivalent to  $S$  and  $T'$  to  $T$ . Then the program  $[ P::S' \parallel Q::T' ]$  satisfies the conditions of theorem 4. So it admits a deadlocked computation. But  $[ P::S \parallel Q::T ]$  is deadlock free.

This shows that first normal form cannot be reduced to second normal form when also deadlock freedom is to be preserved. This can be interpreted as a statement that use of non-homogeneous guards strictly increases the expressive power of CSP.

## Note

First version of this paper appeared as Apt and Clermont ([AC]). After having written the present version, we learned of a related work by Zöbel ([Z]). Zöbel proposes transformations similar to ours, but does not elaborate on the underlying notion of equivalence.

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