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Dynamic MRI Reconstruction as a Moment Problem

Part I. The Beating Heart: a Problem Formulation

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This paper deals with some mathematical aspects of magnetic resonance imaging (MRI) concerning the beating heart. MRI is used in diagnostic medicine to measure and display the cross section of e.g. a human organ. The field of MRI was disclosed by, among others, Lauterbur and Ernst around 1975, when developing imaging techniques to depict the magnetic behaviour of protons. Since then the image quality has grown steadily, bringing MRI within the reach of clinical practice as a promising method. The development of MRI has not come to an end yet - people are still looking for new methods of measuring the data and reconstructing the images. In this paper some of the basic theory behind magnetic resonance is given. Our special interest is the mathematical theory concerning MRI and we will formulate the ideas and problems in mathematical terms. If one uses MRI to measure and display a so called "dynamic" organ, like the beating heart, the situation is more complex than the case of a static organ. We describe a strategy how a cross section of a beating human heart is measured in practice and how the measurements are arranged before ăn image can be made. This technique is called retrospective synchronization. If the beating heart is measured and displayed with help of this method, artefacts often deteriorate the image quality. Some of these artefacts have a physical cause, while others are caused by the reconstruction algorithm. Perhaps mathematical techniques may be used to improve these algorithms which are currently used in practice. The aim of this paper is not to solve problems, but to give an adequate mathematical formulation of the inversion problem concerning retrospective synchronization.

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0. Introduction.

Magnetic resonance imaging (MRI) is a technique to measure and display the proton density of a cross section of e.g. a human organ, which is used in diagnostic medicin (see [1]). In section 1, it is explained how cross sections of "static" organs are measured by magnetic resonance imaging and how the Fourier transform comes into play. We describe in sections 2 and 3 how the proton density of a beating human heart is measured and depicted in practice, using MRI. This technique, called retrospective synchronization, is described here in detail. In the last section a mathematical problem formulation is given in terms of the Fourier transform.

1. The physics of magnetic resonance imaging.

Magnetic resonance imaging is a technique to measure the proton density of a cross section of an organ and to display a picture of this cross section on a computer screen. The measurements are performed by means of magnetic fields and a radio frequency pulse (rf-pulse), by which the spins of hydrogen atoms in the human body are forced to emit radiation with a unique frequency at each point.

This radiation is measured by the MR-machine, in which (approximately) a signal given by

$$S(t) \simeq \int_{R^2} f(\mathbf{x})\, e^{-i\gamma\, t(\mathbf{G}.\mathbf{x})} d\mathbf{x} \tag{1}$$

is induced. Here $f : \mathbb{R}^2 \to \mathbb{R}$ is the proton density of the measured cross section, t is time, γ is the gyromagnetic ratio and $\mathbf{G} = (G_x, G_y)$ is the xy-component of the gradient vector, as explained in [2]. The notational convention used in this paper is to denote vectors and matrices in bold face. Formula (1) is the Fourier transfom of the protondensity $f : \mathbb{R}^2 \to \mathbb{R}$. In the following we explain how (1) is obtained.

We distinguish the following four types of magnetic fields. The fields described in 1,3 and 4 are parallel to the z-axis and the radio frequency pulse, which is described in 2 lies in the xy-plane.

1. A strong homogeneous field to align the spins in one direction, called the z-direction; this direction is the equilibrium direction of the spins.
2. A radio frequency pulse (rf-pulse), that is, a rotating electromagnetic field in the xy-plane, which is applied for a very short time to push the spins out of equilibrium.
3. The z-component of the gradient vector G_z, by which the cross section is selected, see for example [2]. We will not take this component into consideration, but we always assume to measure the proton density of a particular two dimensional object.

4. The gradient field, $(0, 0, \mathbf{G}.\mathbf{x})^t$, which forces the proton at position $\mathbf{x} = (x, y)$ to resonate with a unique frequency. Here $\mathbf{G} = (G_x, G_y)$ is the xy-component of the gradient vector (G_x, G_y, G_z).

In order to study the effect of magnetic fields on the protons in the selected cross section of a human organ, we consider the magnetization $\mathbf{M}(\mathbf{x}, t)$, which is the sum of the spins of all the particles in the area around $\mathbf{x}$ at time t.

The three magnetic fields described in (1), (2) and (4), are here denoted, for computational convenience, as one magnetic field which is position and time dependent,

$$\mathbf{B}(\mathbf{x}, t) = \mathbf{B_0} + \Delta\mathbf{B}(\mathbf{x}) + \mathbf{B_1}(t). \tag{2}$$

Here $\mathbf{B_0}$ is the homogeneous field parallel to the z-axis, $\mathbf{B_1}(t)$ is the rf-pulse which depends on the Larmor frequency $\omega_L = \gamma B_0$ and $\Delta\mathbf{B}$ is the gradient field,

$$\mathbf{B_0} = \begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix},$$

$$\Delta\mathbf{B}(\mathbf{x}) = \begin{pmatrix} 0 \\ 0 \\ \mathbf{G}.\mathbf{x} \end{pmatrix}, \quad \mathbf{B_1}(t) = \begin{pmatrix} B_1 \cos\omega_L t \\ -B_1 \sin\omega_L t \\ 0 \end{pmatrix}.$$

The magnetization $\mathbf{M}(\mathbf{x}, t)$ satisfies the Bloch equation,

$$\frac{\partial \mathbf{M}(\mathbf{x}, t)}{\partial t} = \gamma \mathbf{M}(\mathbf{x}, t) \times \mathbf{B}(\mathbf{x}, t) + \begin{pmatrix} -M_x(\mathbf{x}, t)/T_2(\mathbf{x}) \\ -M_y(\mathbf{x}, t)/T_2(\mathbf{x}) \\ (M_0 - M_z(\mathbf{x}, t))/T_1(\mathbf{x}, t) \end{pmatrix}, \tag{3}$$

where $T_1(\mathbf{x})$ and $T_2(\mathbf{x})$ are relaxation times and the equilibrium magnetization is

$$\mathbf{M_0} = \begin{pmatrix} 0 \\ 0 \\ M_0 \end{pmatrix}.$$

The relaxation times T_1 and T_2 represent the effect of the relaxation processes. T_1 is the longitudinal or spin-lattice relaxation time which governs the evolution of M_z towards its equilibrium value M_0; T_2 is the transverse or spin-spin relaxation time which governs the evolution of the magnitude of the transverse magnetization (M_x, M_y) towards its equilibrium value of zero; in general T_1 is much bigger than T_2.

Dropping the variables $\mathbf{x}$ and t, we rewrite the equation (3) as (cf. [1])

$$\frac{\partial \mathbf{M}}{\partial t} = \mathbf{Q}\mathbf{M} + \mathbf{M_0}/T_1, \tag{4}$$

where

$$\mathbf{Q} = \begin{pmatrix} -1/T_2 & \omega & \gamma B_1 \sin\omega_L t \\ -\omega & -1/T_2 & \gamma B_1 \cos\omega_L t \\ -\gamma B_1 \sin\omega_L t & -\gamma B_1 \cos\omega_L t & -1/T_1 \end{pmatrix}.$$

Here $\omega = \gamma(B_0 + \mathbf{G}.\mathbf{x})$.

Now consider (4) in a coordinate frame that rotates with the Larmor frequency ω_L around the z-axis. If we introduce the variable $\tilde{\mathbf{M}} = \mathbf{R}\mathbf{M}$, where the rotation matrix $\mathbf{R}$ is given by

$$\mathbf{R} = \begin{pmatrix} \cos\omega_L t & -\sin\omega_L t & 0 \\ \sin\omega_L t & \cos\omega_L t & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

the Bloch equation (4) reduces to

$$\frac{\partial \tilde{\mathbf{M}}}{\partial t} = \Lambda \tilde{\mathbf{M}} + \mathbf{M}_0/T_1. \tag{5}$$

Here

$$\Lambda = \begin{pmatrix} -1/T_2 & \Delta\omega & 0 \\ -\Delta\omega & -1/T_2 & \omega_1 \\ 0 & -\omega_1 & -1/T_1 \end{pmatrix},$$

and $\omega_1 = \gamma B_1, \Delta\omega = \mathbf{G}.\mathbf{x}$. The tilde which is written above a variable indicates that this variable is transformed to the rotating coordinate frame.

The unique solution of (5) with initial value $\tilde{\mathbf{M}}(0)$ is

$$\tilde{\mathbf{M}}(t) = e^{\Lambda t}\, \tilde{\mathbf{M}}(0) + \Lambda^{-1}[e^{\Lambda t} - \mathbf{Id}]\mathbf{M}_0/T_1, \tag{6}$$

where $\mathbf{Id}$ is the identity matrix. The inverse of Λ is

$$\Lambda^{-1} = \frac{1}{\det(\Lambda)} \begin{pmatrix} \frac{1}{T_1 T_2} + {\omega_1}^2 & \frac{\Delta\omega}{T_1} & (\Delta\omega)\omega_1 \\ -\frac{\Delta\omega}{T_1} & \frac{1}{T_1 T_2} & \frac{\omega_1}{T_2} \\ (\Delta\omega)\omega_1 & -\frac{\omega_1}{T_2} & \frac{1}{{T_2}^2} + (\Delta\omega)^2 \end{pmatrix},$$

with $\det(\Lambda) = -(\frac{1}{T_1 {T_2}^2} + \frac{\omega_1^2}{T_2} + \frac{(\Delta\omega)^2}{T_1})$.

It is convenient to decompose Λ as the sum of the two matrices $\mathbf{T}$ and $\mathbf{F}$,

$$\mathbf{T} = \begin{pmatrix} -1/T_2 & & \\ & -1/T_2 & \\ & & -1/T_1 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} 0 & \Delta\omega & 0 \\ -\Delta\omega & 0 & \omega_1 \\ 0 & -\omega_1 & 0 \end{pmatrix}.$$

We consider the effect of the three magnetic fields on the magnetization $\mathbf{M}$.

1. The homogeneous field $\mathbf{B_0}$ is applied, so $\omega = \omega_{I.} \neq 0$, $\omega_1 = 0$ and $\Delta\omega = 0$; hence $\mathbf{\Lambda} = \mathbf{T}$ and (6) reduces to

$$\tilde{\mathbf{M}}(t) = e^{\mathbf{T}t}\tilde{\mathbf{M}}(0) + (1 - e^{-t/T_1})\mathbf{M}_0. \tag{7}$$

If t is large, then $\tilde{\mathbf{M}}(t) \simeq \mathbf{M}_0$, parallel to the z-axis, i.e. $\tilde{\mathbf{M}}$ is approximately in its equilibrium if t is large.

2. The rf-pulse is a strong field which is applied for a very short time, while the gradient field is zero, so

$$\mathbf{F} = \begin{pmatrix} 0 & & \\ & 0 & \omega_1 \\ & -\omega_1 & 0 \end{pmatrix}$$

and

$$\mathbf{\Lambda}^{-1}[e^{\mathbf{\Lambda}t} - Id]\mathbf{M}_0/T_1 \simeq 0$$

and $\mathbf{\Lambda} \simeq \mathbf{F}$. Suppose $\tilde{\mathbf{M}}(0) = \mathbf{M}_0$, then (6) becomes

$$\tilde{\mathbf{M}}(t) \simeq e^{-\mathbf{F}t}\mathbf{M}_0, \tag{8}$$

which is a rotation around the $\tilde{x}$-axis with frequenty $\omega_1 = \gamma B_1$. Applying the rf-pulse for a time period of $t_{\omega_1} = \frac{1}{2}\pi/\omega_1$, we obtain the so called 90^o pulse, which results in the following state for the magnetization,

$$\tilde{\mathbf{M}}(t_{\omega_1}) = \begin{pmatrix} 0 \\ M_0 \\ 0 \end{pmatrix}.$$

3. In this case the gradient field ($\Delta\mathbf{B}$) is considered, which is applied after the rf-pulse. The matrix element $\Delta\omega \neq 0$, but $\omega_1 = 0$ (because the rf-pulse is off). Then equation (6) becomes

$$\tilde{\mathbf{M}}(t + t_{\omega_1}) = e^{\mathbf{F}t + \mathbf{T}t}\tilde{\mathbf{M}}(t_{\omega_1}) + (1 - e^{-t/T_1})\mathbf{M}_0. \tag{9}$$

After some time the magnetization has returned to equilibrium:

$$\tilde{\mathbf{M}}(t) \simeq \begin{pmatrix} 0 \\ 0 \\ M_0 \end{pmatrix}.$$

Note that the magnetization at position $\mathbf{x}$ depends on the frequency $\omega_{I.} + \Delta\omega$ ($\Delta\omega = \gamma\mathbf{G}.\mathbf{x}$).

The receiver coil of the MR-machine measures the magnetization $\mathbf{M}(t + t_{\omega_1})$, (that can be obtained by transforming formula (9) to the nonrotating coordinate frame) before it has returned to equilibrium and an output signal $S(t)$ is generated. In [2] it is explained how in practice the magnetization $\mathbf{M}(t + t_{\omega_1})$ induces the signal $S(t)$ in the receiver coils of the MR-machine,

$$S(t) = const \int_{R^2} e^{-t/T_2} M_0(\mathbf{x}) e^{-i\Delta\omega t} d\mathbf{x}.$$

Now let $f(\mathbf{x}) = M_0(\mathbf{x})$. With $\Delta\omega = \gamma \mathbf{G}.\mathbf{x}$, we have

$$S(t) = const \int_{R^2} e^{-t/T_2} f(\mathbf{x}) e^{-i\gamma \mathbf{G}.\mathbf{x}t} d\mathbf{x}. \tag{10}$$

Note that it is not taken into account that the relaxation time T_2 depends on the position $\mathbf{x}$.

Because the time period during which the measurements are made is much smaller than T_2, we have $e^{-t/T_2} \simeq 1$ so (10) simplifies to

$$S(t) \simeq const \int f(x,y) e^{-i\gamma t(G_x x + G_y y)} dx dy,$$

writing $\kappa := \gamma G_x t, \lambda := \gamma G_y t$, we recognize $S(t)$ as the Fourier transform of f at the frequency (κ, λ), denoted as $\widehat{f}(\kappa, \lambda)$. In the following all frequency parameters will be denoted by Greek letters.

2. Acquisition method.

In practice it is only possible to find the Fourier transform $\widehat{f}$ of a function f at a finite number of frequencies; to be somewhat more specific, $\kappa = 0, \ldots, 255; \lambda = 0, \ldots, 255$. In [2] it is explained how to choose the gradient fields G_x and G_y to measure $\widehat{f}$ at the values $\kappa = 0, \ldots, 255$, for λ fixed. We assume that this sequence can be measured instantaneously (in practice this may take from 2 up to 10 *msec*). One such sequence of measurements $\{\widehat{f}(\kappa, \lambda)\}_{\kappa=0,\ldots,255}$, for fixed λ is called a *profile* .

Previously we described how the measurements are made in the case of a static object, i.e. we considered a function $f : I\!R^2 \to I\!R$ which only depends on the position varable $\mathbf{x}$, and not on the time.

If we want to use MRI to measure and display cross sections of "dynamic" organs, e.g. the heart, then we have to consider a function that does not only depend on the variable $\mathbf{x}$, but also on the time T. So in the following we want to consider a function $F(\mathbf{x}, T)$ which we can think of as the proton density of a cross section of the beating heart. The reason why the function and the time are denoted by capitals will become clear later.

Before describing the acquisition method which is used in practice for measuring the Fourier transform of the proton density of a beating heart, we first give some terminology.

R-pulse is the electric pulse in the heart that marks the beginning of a heartbeat. It is recorded by means of an ECG, simultaneously with the measuremements.

RR-interval is the duration (in seconds) between two consecutive R-pulses.

Unit RR-interval is an RR-interval of one unit time length, say one second, which will be used as a reference interval, we call this interval J.

Heart phase is a phase in the periodic movement of the heart.

We explain an acquisition method wich we will call retrospective synchronization, as described in [3] under the name retrospective gating. We first introduce a function $f: \mathbb{R}^2 \times J \to \mathbb{R}$ which we define as the *standard heartbeat.* Here J is the unit RR-interval. We assume that the heart, during each heartbeat, is a rescaled copy of the function f in time; this rescaling should be based on a biological model of the movement of the heart. In order to give an example, we assume the rescaling to be linear. Suppose the k-th R-pulse is measured at the time r_k, for $k = 1, 2, \ldots$. We assume that the proton density in a cross section of the beating heart, $F(\mathbf{x}, T)$, is given by

$$F(\mathbf{x}, T) := f(\mathbf{x}, \frac{T - r_k}{r_{k+1} - r_k}), \tag{11}$$

where $T \in [r_k, r_{k+1})$. This so called dynamic case is more complex than the static case; in the dynamic case we measure the Fourier transform of a function F at a certain time T_i, $\widehat{F}(\kappa, \lambda, T_i)$, and the profile that is measured for fixed λ, at the time T_i, is denoted as $\{\widehat{F}(\kappa, \lambda, T_i\}_{\kappa=0..255}$. In order to reconstruct the function F at time T_i with the Fourier inversion formula, we should measure the 256 profiles (i.e. for $\lambda = 0, \ldots, 255$) at the time T_i. In practice one cannot measure fast enough with MRI to obtain all these profiles at one heartphase, but they are obtained during several RR-intervals. The aim of MRI in the dynamic case is not to give a real time reconstruction of the beating heart during different RR-intervals, but to reconstruct the standard heartbeat. That is, we have to translate our measurements in terms of the function f. In the case of linear rescaling this can be done as in formula (11), if $T_i \in [r_k, r_{k+1})$ then we define

$$t_i := \frac{T_i - r_k}{r_{k+1} - r_k} \tag{12}$$

and $\widehat{f}(\kappa, \lambda, t_i) := \widehat{F}(\kappa, \lambda, T_i)$. The variable t_i lies in the unit heart interval. The rescaling from T_i to t_i is called projection on to the unit heart interval.

The profiles are obtained as follows. We fix λ and we measure the profile $\{\widehat{F}(\kappa, \lambda, T_1)\}_{\kappa=0..255}$, briefly denoted as $\{\widehat{F}(\kappa, \lambda, T_1)\}$, at time T_1. After some time (this may be from 10 up to 200 *msec*) we again measure a profile for λ, at time T_2, etc; the time at which the measurements take place is recorded. After we have obtained a fixed number of profiles (in practice this may be up to 80) the value of λ is increased. If an R-pulse has occurred, it is registered, so that, afterwards, the measured profile can be assigned to the corresponding heartphase. For an example how the data are obtained, see Figure 1.

3. Reconstruction method.

In this section we explain how the reconstruction is done in practice. In order to do this we use Figures 1 and 2. In the example in Figure 1 the value of λ is increased after seven profiles are measured. It is illustrated here that the profiles $\{\widehat{F}(\kappa, \lambda, T_i)\}$ for $\lambda = 1, \ldots, 7$, are measured during

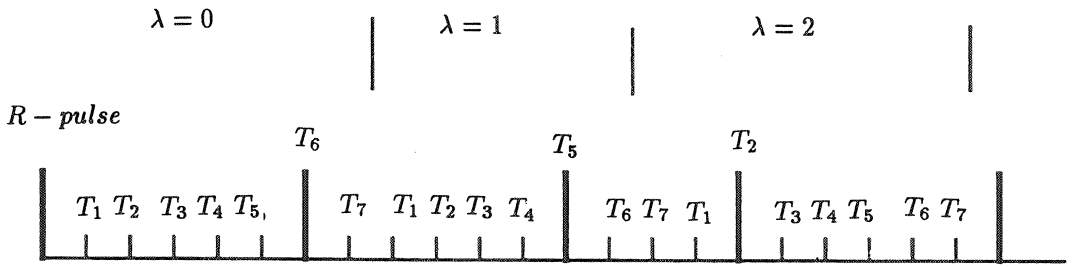

Figure 1. An example of how the measurements are taken.

different RR-intervals. Since two consecutive intervals are not necessarily of equal duration, we have to project the data on to the unit heart interval as follows.

1. It is computed how long the heartbeat (i.e. the RR-interval) was lin which the measurement under consideration occurred. The time of a measurement, relative to the unit RR-interval is computed, e.g. in the case of linear rescaling by (12), and the data are projected on to this interval. For example (cf. Figure 2) for $\lambda = 0$ we have measured the consecutive profiles $\{\widehat{F}(\kappa, \lambda, T_1)\}$, $\{\widehat{F}(\kappa, \lambda, T_2)\}$, $\{\widehat{F}(\kappa, \lambda, T_3)\}$, $\{\widehat{F}(\kappa, \lambda, T_4)\}$, $\{\widehat{F}(\kappa, \lambda, T_5)\}$, $\{\widehat{F}(\kappa, \lambda, T_6)\}$ and $\{\widehat{F}(\kappa, \lambda, T_7)\}$. After projection on the unit RR-interval (Figure 2) these profiles become reordered as $\{\widehat{f}(\kappa, \lambda, t_2)\}$, $\{\widehat{f}(\kappa, \lambda, t_4)\}$, $\{\widehat{f}(\kappa, \lambda, t_5)\}$, $\{\widehat{f}(\kappa, \lambda, t_6)\}$, $\{\widehat{f}(\kappa, \lambda, t_7)\}$, $\{\widehat{f}(\kappa, \lambda, t_1)\}$, $\{\widehat{f}(\kappa, \lambda, t_3)\}$, respectively. We remark that the t_i's depend on the value of λ, in the sense that other values for λ will give rise to another arrangement on the unit RR-interval. To express this dependence we will denote the time as $t_i(\lambda)$; in the above case this should be $t_i(0)$, for $i = 1, \ldots, 7$.

2. If we want to display the heart at several phases, the unit RR-interval is divided into several parts (see Figure 2, for the case that we want to dispay the heart at four phases). All profiles on the unit RR-interval between phase 1 and phase 2 are, in practice, considered to be measured at phase 1. All measurements between phase 2 and phase 3 are considered to be measured at phase 2, etc. If several measurements belong to phase n, then the average of these is assigned to phase n. In the case of the example in Figure 3 we see that for phase 1 and phase 4 the profiles are missing. In order to reconstruct the image of the heart at e.g. phase 1, the profiles belonging to $\lambda = 0, .., 255$ are collected, as far as they were measured. In the case of Figure 3 we miss profiles for the phases 1 and 4, for

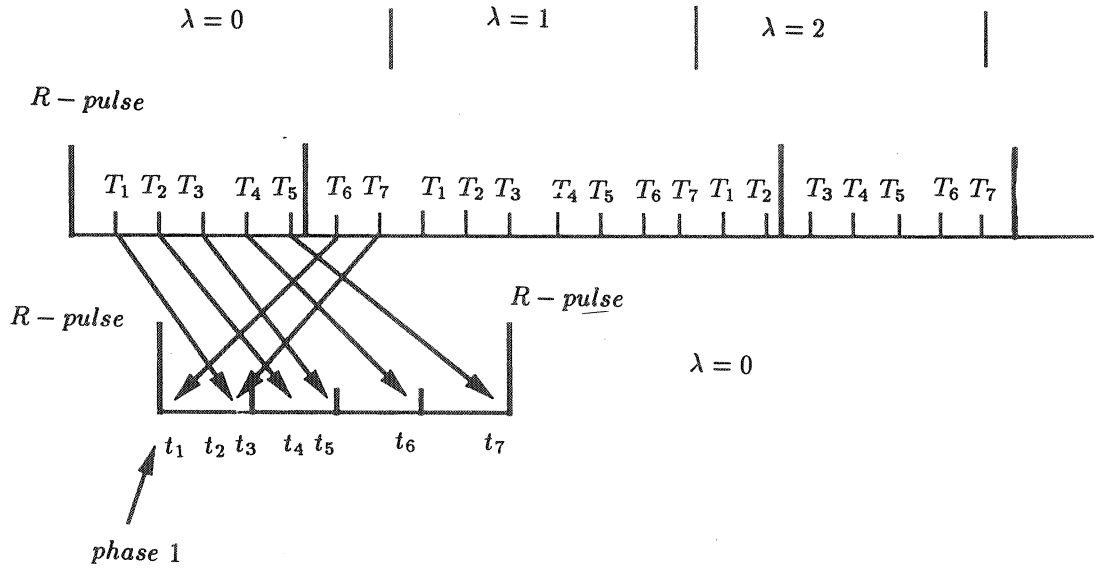

Figure 2. The measurements are projected on to a unit heart interval.

$\lambda = 1$.

In practice it may be the case that there are missing profiles for several values of λ. To perform a Fourier inversion, the missing values are set to zero in current practice and the Fourier inversion formula is applied.

We want to improve the reconstruction algorithm which is described in this section. In order to do this, a problem definition is given in mathematical terms, in the next section.

4. Towards a mathematical problem definition.

In the previous section (section 3) it is described that missing profiles are set to the value zero, before performing a Fourier inversion. In Figure 4 the artefacts, which are caused by making profiles zero, are illustrated. From this figure we see that it probably isn't a good idea to set the missing values to zero.

In section 3 we described how data were projected on to a unit RR-interval and how measurements between phase n and phase $(n+1)$ were assigned to phase n. We do not want to introduce the error of

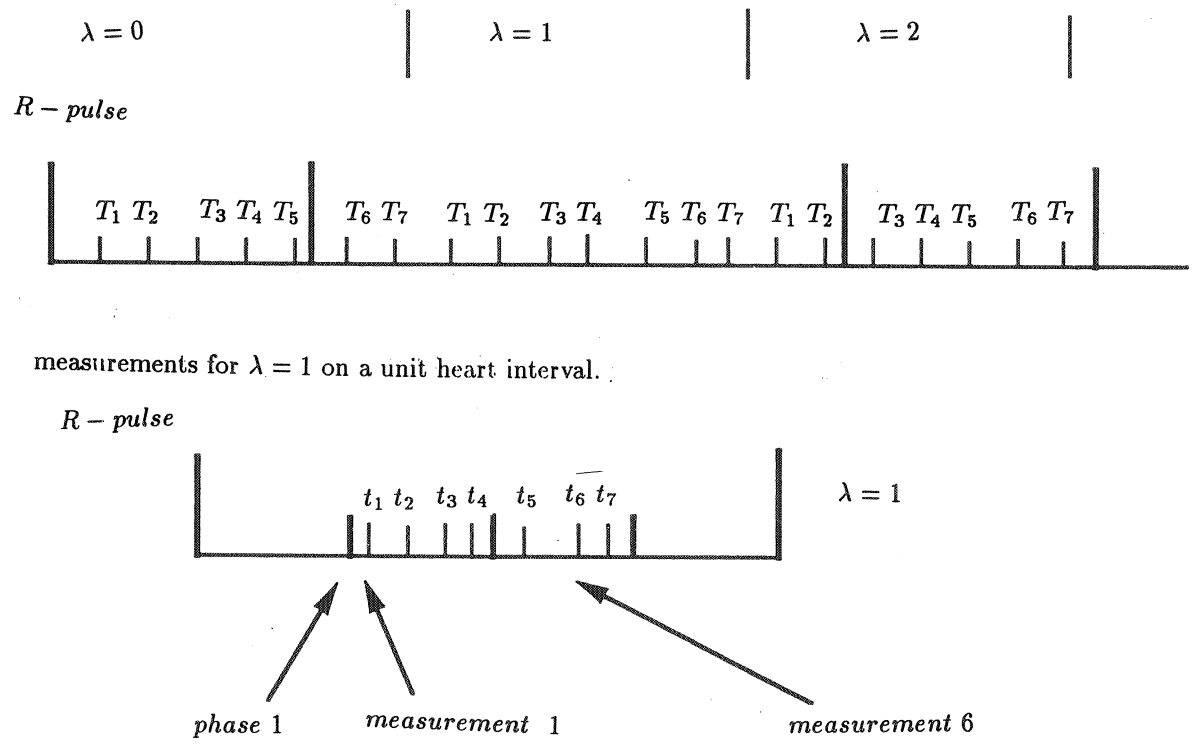

Figure 3. Missing data at phase 0 and phase four.

assigning measurements to other timepoints than they were measured at. Instead, we want to use interpolation of the measurements at the given timepoints, to obtain values at the desired phase. This will be the approach in Part II.

Before giving a problem definition, we first introduce some notation. Let $D \subset I\!R^2$ be the unit square $D = [-\pi, \pi]^2$ and J is the unit RR-interval. Suppose the object to be measured has support in this interval D. The function $f : D \times J \ni (\mathbf{x}, t) \rightarrow I\!R$ can be thought of as a two dimensional cross section of a standard beating heart. The Fourier transform of f, taken with respect to the variable $\mathbf{x} = (x, y)$, is defined by

$$\widehat{f}(\kappa, \lambda, t) := \frac{1}{2\pi} \int_{[-\pi,\pi]} \int_{[-\pi,\pi]} f(x, y, t) e^{-i(\kappa x + \lambda y)} dx dy.$$

The profile $\{\widehat{f}(\kappa, \lambda, t_i(\lambda))\}_{\kappa=0..255}$ is measured, for fixed λ, at the rescaled time $t_i(\lambda)$, for $i = 1, \ldots, I$. We now formulate the problem. Suppose we

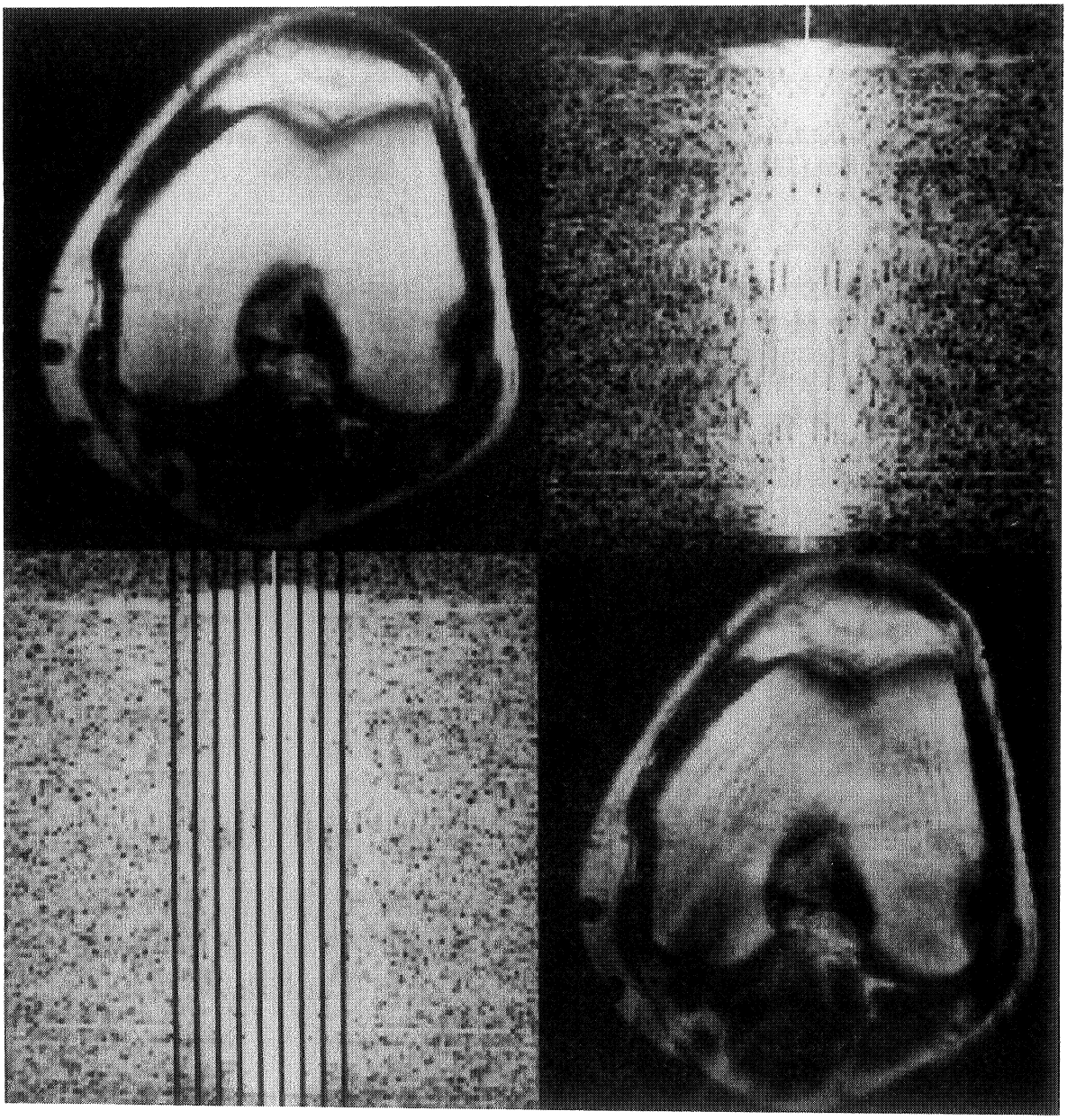

Figure 4. The effect of setting profiles to zero.

a The upper left picture is the original (a cross section of a knee).

b The upper right picture is the modulus of its Fourier spectrum.

c The lower left is the same as (b), but with a number of lines (profiles) set to zero.

d The lower right is the Fourier inverse of (c).

have measured

$$g_{\kappa,\lambda,i} := \widehat{f}(\kappa, \lambda, t_i(\lambda)), \tag{13}$$

for $\kappa, \lambda = 0, ..., 255$ and $i = 1, ..., I$. Find a function $f : D \times J \rightarrow \mathbb{R}$ such that

$$\widehat{f}(\kappa, \lambda, t_i(\lambda)) = g_{\kappa,\lambda,i}, \tag{14}$$

for $\kappa, \lambda = 0, ..., 255$, and $i = 1, ..., I$. A possible Hilbert space setting for this problem (14) is discussed in part II.

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