Centrum voor Wiskunde en Informatica
Centre for Mathematics and Computer Science

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Quantifiers

Computer Science/Department of Software Technology Report CS-R9038 August

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Quantifiers

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This paper gives an overview of the treatment of the semantics of quantified expressions in natural language.

Note: The paper will appear in the *Encyclopedia of Language and Linguistics*, Pergamon Press and Aberdeen University Press.

1985 Mathematics Subject Classification: 68S10.
CR Categories: H.2.3, I.2.4, I.2.7.
Keywords and Phrases: semantics of natural language, query languages, knowledge representation languages.

1 Introduction

In formal semantics it is the commonly held view that the ultimate goal of natural language semantics is to provide an account of *the process of drawing inferences* in natural language. As quantifiers play a very important rôle in this process, quantification is a topic of central interest in semantics.

After a summary of Aristotle's and Frege's theories of quantification, the relational perspective on quantifiers that forms the basis of most current accounts of quantifiers in natural language is sketched. General conditions on quantifier relations are given, and several kinds of representations for quantifiers are discussed. Next, the linguistic interest of relational properties such as symmetry and various forms of monotonicity is illustrated, and the connections between quantifier theory and automata theory are sketched. Finally, presuppositional quantifiers, partial quantifiers, adverbial quantifiers and the phenomenon of quantifier branching are briefly discussed.

2 Aristotle on Quantification

The first systematic account of quantification was given by Aristotle in his theory of the syllogism. Aristotle studied the following inferential pattern:

$$\frac{\begin{array}{c}\text{Quantifier}_1\ \text{Nbar}_1\ \text{VP}_1\\ \text{Quantifier}_2\ \text{Nbar}_2\ \text{VP}_2\end{array}}{\text{Quantifier}_3\ \text{Nbar}_3\ \text{VP}_3}$$

Example (the valid syllogism BARBARA):

$$\frac{\begin{array}{c}\text{All A are B}\\ \text{All B are C}\end{array}}{\text{All A are C}}$$

Syllogistic theory focusses on the quantifiers in the so called *Square of Opposition*: see figure (1). The quantifiers in the square express relations between a first and a second argument, where both arguments denote sets of entities

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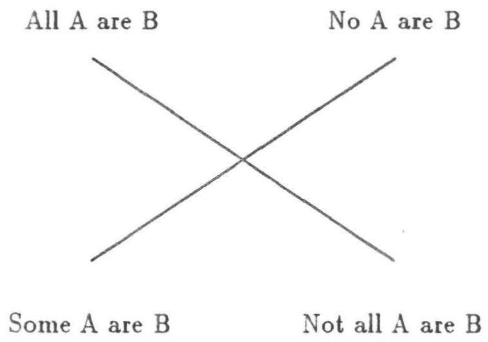
Figure 1: The Square of Opposition



taken from some domain of discourse, a set of entities which is assumed in the background as the subject matter of the discourse.

The quantified expressions in the square are related across the diagonals by external (sentential) negation, and across the horizontal edges by internal (or verb phrase) negation. It follows that the relation across the vertical edges of the square is that of internal plus external negation; this is the relation of so-called *quantifier duality*. Because Aristotle assumes that the domain of discourse is non-empty, the two quantified expressions on the top edge of the square cannot both be false; these expressions are called *contraries*. For the same reason, the two quantified expressions on the bottom edge cannot both be false: they are so-called *subcontraries*. Next, Aristotle interprets his quantifiers with *existential import*: *All A are B* is taken to imply that there are *A*, and similarly for the other quantifiers. Under this assumption, the quantified expressions at the top edge of the square imply those immediately below them.

For those who care for some more terminology: the universal affirmative quantifier *all* implies the individual affirmative *some* and the universal negative *no* implies the individual negative *not all*. The universal and individual affirmative quantifiers are said to be of types **A** and **I** respectively, from Latin **A***ff***I***rmo*, the universal and individual negative quantifiers of type **E** and **O**, from Latin *N***E***g***O**. The Medieval mnemonics for the Aristotelean syllogisms derive from these abbreviations: *Barbara* is the name of the syllogism with two universal affirmative premisses and a universal affirmative conclusion, and so on.

Impressive though it is, Aristotle's theory of quantification has two grave logical defects:

1. Quantifier combinations are not treated; only one quantifier per sentence is allowed.

2. 'Non-standard quantifiers' such as *most*, *half of*, *at least five*, ...are not covered.

A minor additional flaw is the assumption of existential presupposition. In mathematical reasoning, and sometimes also in everyday reasoning, one wants to be able to assert universally quantified statements without the bother of first having to provide existence proofs.

3 Frege's Standard Quantifiers

Gottlob Frege's theory of quantification is basically what we now call first order predicate logic (see the articles on FREGE and FIRST ORDER LOGIC). It is based on the introduction of *individual variables* bound by the quantifiers $\forall$ ('for all') and $\exists$ ('there exists'), and it removes the first of the two defects of the Aristotelian theory. Quantifiers with their associated variables can combine with arbitrarily complex predicate logical formulas to form new predicate logical formulas, so a formula may contain an arbitrary number of quantifiers.

The quantifiers $\forall$ and $\exists$ are called the standard quantifiers. These two quantifiers are interdefinable with the help of negation: *Something stinks* means the same as *It is not the case that it holds for every x that x does not stink*, and *Everything is fine* means the same as *It is not the case that there is a thing x with the property that x is not fine.* More formally: $\exists x Ax$ is true if and only if $-\forall x - Ax$ is true, and $\forall x Ax$ is true if and only if $-\exists x - Ax$ is true.

If one conveniently forgets about the existential presuppositions, the Aristotelian quantifiers from the Square of Opposition can be expressed in terms of the Fregean standard quantifiers, as follows (with $\rightsquigarrow$ for 'translates as') :

All A are B $\rightsquigarrow \forall x(Ax \rightarrow Bx)$.

Some A is/are B $\rightsquigarrow \exists x(Ax \wedge Bx)$.

No A is B $\rightsquigarrow \forall x(Ax \rightarrow -Bx)$.

Not all A are B $\rightsquigarrow -\forall x(Ax \rightarrow Bx)$.

Note that $\forall x(Ax \wedge Bx)$ means something stronger than *All A are B*, for this formula can be paraphrased as: *Everything is both A and B.* Also, $\exists x(Ax \rightarrow Bx)$ means something much weaker than *Some A are B*, for it can be paraphrased as: *Something is B if it is A*, and this is already true provided there is at least one thing in the domain of discourse which lacks property A. Another thing to note is that different ways of rendering an Aristotelian quantifier in Frege's logic may be equivalent. Another possible translation for *No A is B* is $-\exists x(Ax \wedge Bx)$. There is nothing to choose between the two translations because in every situation where the first one is true the second one is true as well, and vice versa.

Using standard quantifiers and the identity sign it is also possible to express numerical constraints like *At least 2 A are B*:

$$\exists x \exists y(x \neq y \wedge Ax \wedge Ay \wedge Bx \wedge By).$$

It is not difficult to see that all disjunctions and conjunctions of quantifiers of the forms '*at least* n' and '*at most* m' can be expressed in terms of standard quantifiers and the identity sign.

To illustrate the claim that first order logic has no difficulty with quantifier combinations, consider the translation of example (1).

(1) *Every prince sang a ballad.*
$\forall x(Px \rightarrow \exists y(By \wedge Sxy))$.

Observe that the translation does not contain phrases corresponding to the noun phrases *every prince* or *a ballad.* Given a natural language sentence and its translation into first order logic, it is impossible to pinpoint the subexpression in the translation that gives the meaning of a particular noun phrase in the original. In the translation into first order logic, the noun phrases have been syntactically eliminated, so to speak.

The next example, (2), with the two possible translations listed below it, can serve to illustrate a few further points.

(2) *One ballad was sung by every prince.*
$\exists y(By \wedge \forall x(Px \rightarrow Sxy))$.
$\forall x(Px \rightarrow \exists y(By \wedge Sxy))$.

The fact that both translations are appropriate for (2) shows that the sentence is ambiguous. It also shows that translating into first order logic can be used to disambiguate natural language sentences. In such cases one says that the different translations express different *readings* of the original sentence.

Note that no systematic procedure for arriving at the translations was given. In fact, logic textbooks teach the art of translating from natural language into first order logic by listing examples, until the reader has got the knack of it. The translations presuppose that the reader does already have a full grasp of what the sentences under consideration mean. It follows that an *ad hoc* process of translating from natural language to a logical representation language like first order logic cannot count as an explication of what natural language expressions mean.

In the antediluvian era of natural language semantics, the time when first order predicate logic was still considered as the one and only tool for semantic analysis, quantified noun phrases were commonly regarded as systematically misleading expressions. Their natural language syntax did not correspond to their logic, for in natural language they were separate constituents, but they evaporated during the process of translation into first order logic.

The Fregean view on quantifiers is a vast improvement over the Aristotelean view. Three areas with scope for further improvement remain: (i) finding logical representation languages permitting the preservation of noun phrases as separate constituents, (ii) finding procedures for translating from natural language to logical representations that are not *ad hoc*, and (iii) finding ways to treat non standard quantifiers such as *most*, preferably in a uniform framework with standard quantifiers.

4 The Relational Perspective

In the relational perspective on quantifiers, first proposed in (Mostowski 1957), a quantifier is viewed as a two-place relation on the power set of a domain of discourse E satisfying certain requirements. The power set of a set E, notation $\mathcal{P}(E)$, is the set of all subsets of E. A two-place relation on $\mathcal{P}(E)$ is a set of pairs of subsets of E. The relational perspective on quantification is implicit in Montague grammar (see the article MONTAGUE). It was first systematically applied to natural language analysis in Barwise & Cooper (1981). Below it will be shown that the relational view can be used to remedy the defects of both the Aristotelian and the Fregean theory. It covers non standard quantifiers, it allows quantifier combinations of arbitrary complexity, it does not syntactically eliminate quantified noun phrases, and it can be used as one of the ingredients in a *non ad hoc* translation procedure from natural language to a language of logical representations.

We will start by demonstrating that in the modern relational perspective the suggestion of misleading form disappears. Two simple example sentences will illustrate that a representation language with generalized quantifier expressions (expressions denoting two place relations between sets) and a notation for lambda abstraction (see the article LAMBDA OPERATOR) is eminently suited for the compositional analysis of natural language sentences with quantified noun phrases. First consider example (3).

(3) *Every woman smiled.*

This sentence is composed of a noun phrase *every woman*, composed in turn of a determiner *every* and a noun *woman*, and a verb phrase *smiled*. The determiner *every* translates into an expression **every** denoting a function from properties to a function from properties to truth values. More precisely, **every** denotes the function mapping property **P** to (the characteristic function of) the set of all properties having **P** as a subset. The noun *woman* translates into $\lambda x.Wx$, the verb phrase *smiled* into $\lambda y.Sy$, the noun phrase *every woman* into $\mathbf{every}(\lambda x.Wx)$, and, finally, the whole sentence into the expression $(\mathbf{every}(\lambda x.Wx))(\lambda y.Sy)$. The reader is urged to check that this expression yields true in case the property of being a woman is included in the property of smiling, false otherwise.

To see how quantifier combinations are dealt with compositionally, consider example (4).

(4) *Every mermaid hummed a song.*

The trick is finding the right translation for the transitive verb. This turns out to be the lambda expression $\lambda\mathbf{X}\lambda y.\mathbf{X}(\lambda z.Hyz)$, where **X** is a variable over noun phrase type expressions. The verb translation is of the right type to take the object noun phrase translation as its argument; this gives translation (5) for the verb phrase, which reduces to (6).

(5) $\lambda\mathbf{X}\lambda y.\mathbf{X}(\lambda z.Hyz)(\mathbf{a}(\lambda u.Su))$.

(6) $\lambda y.(\mathbf{a}(\lambda u.Su))(\lambda z.Hyz)$.

Here **a** denotes the function which maps every property **P** to (the characteristic function of) the set of all properties having a non-empty overlap with **P**. Feeding (6) as argument to the expression $\mathbf{every}(\lambda x.Mx)$, the translation of the subject, one gets (7) as translation for the whole sentence.

(7) $(\mathbf{every}(\lambda x.Mx))(\lambda y.(\mathbf{a}(\lambda u.Su))(\lambda z.Hyz))$.

This translation can still be simplified somewhat, by writing M and S for the property denoting expressions $\lambda x.Mx$ and $\lambda u.Su$.

(8) $(\mathbf{every}(M))(\lambda y.(\mathbf{a}(S))(\lambda z.Hyz))$.

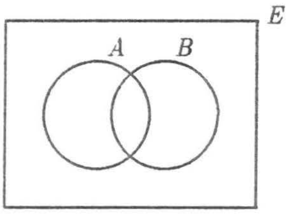
The compositional semantic analysis of natural language sentences involving quantifiers is the reverse of the process of compositional synthesis demonstrated here.

In the remainder of this article, quantifiers will be studied from a relational point of view. For purposes of presentation, attention will be largely limited to noun phrase quantifiers (but see the section on adverbial quantifiers below). Moreover, only discrete quantifiers will be treated, and extending the account to continuous quantifiers, as in *Some milk was spilt* or *Two hundred kilogrammes of hashish were discovered* will be left to the reader: these cases involve issues in the semantics of measurement that are irrelevant to our main issue (see the article MEASUREMENT PHRASES). Finally, intensional phenomena, as in *All fake millionnaires are cunning* and *Some alleged geniuses are conceited* will be ignored.

5 Conditions on Quantifier Relations

All men walk is true in a given model if and only if the relation of inclusion holds between the set of men in the model and the set of walkers in the model. Abstracting from the domain of discourse, we can say that determiner interpretations (henceforth: determiners) pick out binary relations on sets of individuals, on arbitrary universes (or: domains of discourse) E. Notation: $D_E AB$. We call A the *restriction* of the quantifier and B its *body*. If $D_E EAB$ is the translation of a simple sentence consisting of a quantified noun phrase with an intransitive verb phrase then the noun denotation is the restriction and the verb phrase denotation the body. See figure (2) for a graphical representation.

Figure 2: Quantifiers as Relations



Not all two-place relations on sets of individuals are quantifier-relations. The first two requirements that quantifiers must meet are general requirements for denotations of determiners: *extension* and *conservativity*, which will be abbreviated as **EXT** and **CONS**, respectively. See the article DETERMINERS for further information. The combined effect of **EXT** and **CONS** boils down to limiting the domain of discourse relevant for the truth or falsity of $D_E AB$ to two sets: the set of things which are A but not B (formally: the set $A - B$), and the set of things which are both A and B (formally: the set $A \cap B$).

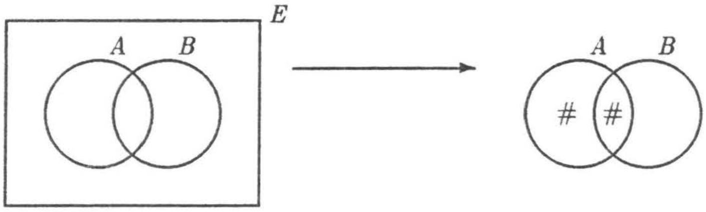
Next, the relational perspective suggests a very natural way of distinguishing between expressions of quantity and other relations. Quantifier relations satisfy the following condition of *isomorphy*, formulated in terms of bijections (see the article BIJECTION).

ISOM If f is a bijection from E to E', then $D_E AB \Rightarrow D_{E'} f[A]f[B]$.

Here $f[A]$, the image of A under f, is the set of all things which are f-values of things in A. **ISOM** expresses that only the *cardinalities* (numbers of elements) of the sets A and B matter, for the image of a set under a bijection is a set with the same number of elements as the original set. If D satisfies **EXT**, **CONS** and **ISOM**, it turns out that the truth of DAB depends only on the cardinal numbers $|A - B|$ and $|A \cap B|$ (respectively, the number of things which are A but not B, and the number of things which are both A and B). See figure (3) for the combined effect of these three conditions. A quantifier simply is a relation Q satisfying **EXT**, **CONS** and **ISOM**.

Some examples will make clearer how the semantic effect of a quantifier QAB can always be described in terms of the properties of the numbers $|A - B|$ and $|A \cap B|$. *All A are B* is true if and only if the number of things which are A and not B is 0. *Some A is B* is true if and only if the number of things that are both A and B is at least 1. *Most A are B* is true if and only if the number of things that are both A and B exceeds the number of things that are A and not B.

Figure 3: The Combined Effect of EXT, CONS, ISOM



6 Numerical Trees

Suppose a quantifier Q has A as a first and B as a second argument. Q can then be characterized as a subset of the *tree of numbers* given in figure (4). The first number in each number pair is $|A - B|$, the second one $|A \cap B|$. Some examples of tree patterns for quantifiers are given in figure (5).

Figure 4: General Format of a Numerical Tree

$\|A\| = 0$						$0,0$					
$\|A\| = 1$					$1,0$		$0,1$				
$\|A\| = 2$				$2,0$		$1,1$		$0,2$			
$\|A\| = 3$			$3,0$		$2,1$		$1,2$		$0,3$		
$\|A\| = 4$		$4,0$		$3,1$		$2,2$		$1,3$		$0,4$	
$\|A\| = 5$	$5,0$		$4,1$		$3,2$		$2,3$		$1,4$		$0,5$
$\vdots$						$\vdots$					

To get used to these representations, one should try and answer some questions about numerical trees, such as the following. What are the tree patterns for *all, some, no* and *not all*? How are these patterns related? What are the tree patterns for *at most three* and *exactly three*? How are the patterns for *at most three, at least three* and *exactly three* related? Which tree operations correspond to taking the negation of a quantifier, the conjunction of two quantifiers, the disjunction of two quantifiers?

7 Logical Representations for Quantifiers

The pairs of cardinals that characterize a quantifier QAB can be used for representation purposes. Every quantifier is defined by means of an arithmetical expression in two variables m and n, where m is the number of elements in $A - B$, n the number of elements in $A \cap B$. Logical forms for quantified expressions can exploit this fact:

- at least two $\leadsto n \geq 2$.
- all $\leadsto m = 0$.
- no $\leadsto n = 0$.

Logical operations on quantifier-determiners can now be handled compositionally, by performing the corresponding logical operations on the arithmetical expressions:

Figure 5: Examples of Numerical Trees

at least three A are B

$$
\begin{array}{ccccccccccc}
 & & & & & - & & & & & \\
 & & & & - & & - & & & & \\
 & & & - & & - & & - & & & \\
 & & - & & - & & - & & + & & \\
 & - & & - & & - & & + & & + & \\
- & & - & & - & & + & & + & & +
\end{array}
$$

less than half of the A are B

$$
\begin{array}{ccccccccccc}
 & & & & & - & & & & & \\
 & & & & - & & - & & & & \\
 & & & + & & - & & - & & & \\
 & & + & & - & & - & & - & & \\
 & + & & + & & - & & - & & - & \\
+ & & + & & - & & - & & - & & -
\end{array}
$$

An even number of the A are B

$$
\begin{array}{ccccccccccc}
 & & & & & + & & & & & \\
 & & & & + & & - & & & & \\
 & & & + & & - & & + & & & \\
 & & + & & - & & + & & - & & \\
 & + & & - & & + & & - & & + & \\
+ & & - & & + & & - & & + & & -
\end{array}
$$

- if $Q \rightsquigarrow E$ then [not Q] $\rightsquigarrow -E$.
- if $Q_1 \rightsquigarrow E_1$ and $Q_2 \rightsquigarrow E_2$ then
  [Q_1 and Q_2] $\rightsquigarrow E_1 \wedge E_2$ and [Q_1 or Q_2] $\rightsquigarrow E_1 \vee E_2$

For instance, according to these instructions, the arithmetical expression that translates *at least two but not all* is: $n \geq 2 \wedge -m = 0$.

8 Relational Properties

As quantifiers are relations, we can study their relational properties and the way in which these properties are reflected in the tree patterns. For example, a quantifier Q is reflexive if and only if:

$$\forall X QXX.$$

The quantifiers *all* and *some* are reflexive, the quantifiers *no* and *not all* are not. One can now study questions about tree patterns such as the following. If Q is reflexive, what will its tree pattern be like? Can it be shown that every quantifier with this tree pattern is reflexive? If some quantifier Q has a tree pattern with an outer north east diagonal consisting of $-$-signs, which relational property of Q does this reflect?

A relational property with linguistic interest is *symmetry.* A quantifier Q is *symmetric* if and only if:

$$\forall X \forall Y QXY \iff QYX.$$

It is left to the reader to establish the corresponding tree pattern. The linguistic interest of this class lies in the fact that the symmetric quantifiers are precisely the class of quantifiers which can occur at the Q position in 'there'-existential sentences (sentences of the form *There are* Q).

9 Monotonicity

Another example of a relational property of quantifiers with linguistic interest (to be illustrated below) is *upward right-monotonicity* in the second argument place:

MON$\uparrow$ If QAB and $B \subseteq B'$, then QAB'.

Examples: *all, some, at least five.* The tree pattern corresponding to **MON$\uparrow$** turns out to be the following:

- If a node has a $+$, then all nodes to the right on the same row have $+$-s.

A quantifier relation is downward right monotone in the second argument if the following holds:

MON$\downarrow$ If QAB and $B' \subseteq B$ then QAB'.

Examples: *not all, no.* Corresponding tree pattern property:

- If a node has a $+$, then all nodes to the left on the same row have $+$-s.

Inspection of the tree pattern makes clear that *an even number of* is neither **MON$\uparrow$** nor **MON$\downarrow$**.

Barwise and Cooper (1981) observed a certain correlation between the monotonicity properties of conjoined noun phrases on the one hand and the use of *and* versus *but* on the other: noun phrases that are monotone in the same direction are conjoined with *and*, monotonicity in opposite directions triggers conjunction by means of *but*. Examples: *all men and some women, many English but no Dutch.* Of course, other factors are at work as well.

A second linguistic application is the account of so-called negative and positive polarity phenomena in natural language.

(9) *Few people lifted a finger to help the wounded soldiers.*

(10) **Many people lifted a finger to help the wounded soldiers.*

(11) *No sailor refused any of the gifts.*

(12) **Every sailor refused any of the gifts.*

To lift a finger and *any* are negative polarity items: they must be in a 'negative context'. The noun phrases allowing negative polarity items in their scopes turn out to be, roughly at least, the $\mathbf{MON}{\downarrow}$ noun phrases.

Negative polarity items have positive counterparts: expressions allowed within the scope of a $\mathbf{MON}{\uparrow}$ noun phrase but awkward in the scope of a $\mathbf{MON}{\downarrow}$ noun phrase:

(13) *Some people could hardly believe it.*

(14) **Nobody could hardly believe it.*

A further linguistic question suggested here is the following. Given some list of positive polarity items in English, is it possible to arrive at more finegrained classifications by subdividing this list into items that are allowed within the scope of a noun phrase which is neither $\mathbf{MON}{\uparrow}$ nor $\mathbf{MON}{\downarrow}$ and items that are not allowed in such contexts?

Some questions that can be solved by looking at the tree pattern characterisations of the monotonicity properties are the following. What is the effect of negation on monotonicity? What monotonicity property does the conjunction of two $\mathbf{MON}{\uparrow}$ ($\mathbf{MON}{\downarrow}$) noun phrases have? What is the monotonicity behaviour of the disjunction of two $\mathbf{MON}{\uparrow}$ ($\mathbf{MON}{\downarrow}$) noun phrases? What is the monotonicity property of the conjunction (disjunction) of a $\mathbf{MON}{\uparrow}$ and a $\mathbf{MON}{\downarrow}$ noun phrase? The answers to these questions can be 'tested empirically' by substituting the resulting noun phrases in sentences containing negative or positive polarity items.

One can also study monotonicity in the first argument:

${\uparrow}\mathbf{MON}$ If QAB and $A \subseteq A'$ then $QA'B$.

${\downarrow}\mathbf{MON}$ If QAB and $A' \subseteq A$ then $QA'B$.

Examples of ${\uparrow}\mathbf{MON}$ determiners are *some* and *not all*. *All* and *no* are ${\downarrow}\mathbf{MON}$ determiners. It is left to the reader to establish the tree patterns corresponding to the ${\uparrow}\mathbf{MON}$ and ${\downarrow}\mathbf{MON}$ properties.

(15) *All sailors who refused any of the gifts were despised.*

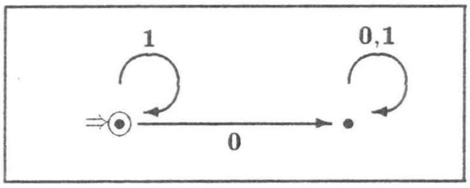
(16) **Some sailors who refused any of the gifts were despised.*

Examples (15) and (16) illustrate that monotonicity properties in the left argument can be used to explain polarity phenomena within the syntactic restriction of the determiners, i.e. within the noun phrases that have these determiners as their heads.

10 Quantifiers and Automata

Quantifiers correspond to automata that accept strings over a binary alphabet $\{0, 1\}$: a string s with m zeros and n ones in it is accepted if and only if position $\langle m, n\rangle$ in the numerical tree for the quantifier has a $+$. To give an example, the quantifier *all* corresponds to the regular language 1^* (the set of all strings consisting of just 1-s). Figure (6) give a finite state machine (see the article FINITE STATE MACHINES) for this quantifier.

Figure 6: Finite State Machine for Computing 'All'



The reader is encouraged to construct finite state machines for computing *at least two*, *at most five* and *between three and seven*. Would these finite state machines still work if one would wish to allow strings of infinite length?

The languages accepted by quantifier automata are closed under permutation: it is the *number* of zeros and ones in the string that counts, not the order in which they are presented. Call a finite state machine *permutation invariant* if it has the following property: if reading a string s will get the machine from state p to state q, then reading any permutation of s will also get the machine from state p to state q. Quantifier automata must be permutation invariant. A finite state machine is *acyclic* if the machine does never return to a given state once it has left that state (in other words: 1-cycles are allowed, but all other cycles are out). An example of a quantifier that can be computed by a cyclic finite state machine but not by an acyclic one is *an even number of*.

A quantifier is called *first order definable* if it is definable in terms of the Fregean quantifiers $\forall$ and $\exists$, the identity predicate, and the two predicates for the restriction and the body of the quantifier. The question of first order definability is relevant for the semantics of natural language, because the suitability of logical representation languages for given natural language fragments depends on it. The first order definable quantifiers are exactly those that can be computed by an acyclic permutation-invariant finite state machine (Van Benthem 1986). It follows from this that *an even number of* is not first order definable (a cyclic automaton is needed for its computation), nor are quantifiers like *half* and *most*, which cannot be computed on a finite state machine at all (a memory stack is needed to 'remember' the numbers of elements in $A - B$ and $A \cap B$). The reader is encouraged to design a push down stack automaton for computing *most*.

The automata-perspective can be exploited to give an account of *semi-quantifiers* involving ordinals:

(17) Every tenth passenger *will receive a free bottle of champagne on board.*

(18) The first ten passengers *will receive a free bottle of champagne on board.*

It is not difficult to design finite state machines for computing these semi-quantifiers. Note that semi-quantifiers do not observe **ISOM**. The reader is encouraged to formulate a weaker condition that they do satisfy.

11 Quantifiers and Presupposition

It is sometimes profitable to distinguish between the content and the presupposition of a quantified phrase. The difference between *four men walked* and *the four men walked* can be expressed in terms of this distinction as follows. The first sentence is true if a set of four entities can be found that were walking men, false otherwise. The second is true if the domain of discourse contains exactly four entities which are men, and all those entities were walkers, false if the domain contains exactly four men and not all those entities were walkers, and undefined in case the domain does not contain exactly four men. Thus we see that quantifiers loaded with a presupposition introduce truth value gaps, or a 'third truth value': in case the presupposition does not hold the quantified expression is neither true nor false, but 'something else' (see the article PRESUPPOSITION).

Aristotle held that in the Square of Opposition the quantified expressions on the top row each imply the expressions immediately below them: *all men walked* should imply *some men walked*, and similarly for *no gallants sang* and *not all gallants sang*. This reflects the fact that the Aristotelian quantifiers are supposed to have existential import: QAB implies that there are As. The existential import of natural language quantifiers can be viewed as a presupposition associated with the use of those quantifiers. The quantifier QAB is true if there are A and A, B are in the Q relation; it is false if there are A and A, B are not in the Q relation, and it is undefined (or has a third value) if there are no A. Again, there is a value which is neither true nor false for cases where the presupposition is not fulfilled.

12 Quantifiers and Partiality

Studying quantifiers in a partial setting is necessary, among other things, to be able to deal with the semantics of perception reports.

(19) *I saw two bears prepare sandwiches.*

(20) *Two bears were preparing sandwiches.*

(21) *I saw no bears prepare sandwiches.*

(22) *No bears were preparing sandwiches.*

(23) *I saw nobody on the road.*

(24) *Nobody was on the road.*

From the truth of sentence (19) it follows that two bears were indeed preparing sandwiches, so (20) is a consequence of (19), but from the truth of (21) it does not follow that no bears were preparing sandwiches, so (22) is not a consequence of (21). The same holds for the relation between (23) and (24).

Obviously, some semantic difference between the quantifiers *two bears* and *no bears* (or *nobody*) must account for this difference in logical behaviour. One might think that the difference between (19) and (21) is simply a matter of scope: in the second sentence, but not in the first, the quantifier in the complement has scope over the sentence. The explanation will not do, however, as the infinitival subject position is not a scope sensitive position, witness the fact that neither (19) nor (21) exhibits a scope ambiguity. Rather, the fact that the quantifier in

(19) seems to have narrow scope and that the quantifier in (21) seems to have wide scope, while neither of the two sentences exhibits a scope ambiguity does itself call for a semantic explanation.

The starting point of the semantic investigation of perception reports is the the following question: which quantifier property licences the inference in (25)?

(25) *I saw QAB./ Therefore: QAB.*

A key feature of the semantics of perception reports is the fact that perception sentences like (26) do not imply variants where the perception complement is replaced by an equivalent complement (equivalent in classical logic, that is), as in (27).

(26) *I saw John help Mary.*

(27) *I saw John help Mary and help Bill or not help Bill.*

In a classical framework the complements in (26) and (27) are logically equivalent, so the semantic distinction between the two examples gets lost. To preserve it one must distinguish between (partial) models supported by what I saw, (partial) models refuted by what I saw, and (partial) models untouched by what I saw. This threefold distinction using partial models (or situations) accounts for the difference between (26) and (27), for it may be that a situation where John helps Mary is supported by what I saw, while on the other hand none of the situations supported by what I saw have Bill in it.

The switch to a partial perspective involves for every predicate P a distinction between things satisfying P, things not satisfying P, and things doing neither. Restricting attention to the case of two predicates A and B on a universe E, partial predicate A on E divides E in a region of things that do satify A (call this set A^+), a region of things that do not satisfy A (call this set A^-) and a region of things with unknown A status (call this set A^*). Similarly, the partial version of B carves up E in three regions B^+, B^- and B^*. Extending quantifier theory to cover this three valued case involves providing suitable extensions of the principles **EXT**, **CONS** and **ISOM**. Rather that spelling out these details, we sketch the application to perception reports.

Any proper handling of perception reports will have to involves something like the following principle of *Scenic Inclusion*:

SCENIC INCL If R is a relation between perceivers and situations they perceive, and s is a situation in which pRs' holds (with p a perceiver, R a relation of perception, and s' a scene perceived by p in s), then $s' \subseteq s$, i.e. everything which is true in s' is also true in s, and everything which is false in s is also false in s'.

The principle says, in fact, that perception implies truth. It does account for the scope transparency of perception reports that was noted above. To see that errors in perception are irrelevant to the principle, note that a perception error is merely a case where the scene one believes to perceive is different from the scene one actually perceives. Cases of ironic reports such as *John saw ghosts on the cemetery again* will be ignore here, as pragmatic factors are involved in the fact that such reports do not imply truth.

By virtue of **SCENIC INCL**, the quantifiers licensing the inference in (25) are precisely the quantifiers which are persistent under situation inclusion: the quantifiers Q with the property that if QAB holds in a situation s and $s \subseteq s'$ then QAB holds in s' (because of the partial perspective one must add the dual relation for falsity, for good measure: if QAB is false in a situation s' and $s \subseteq s'$ then QAB is false in s). Under some suitable assumptions the $\subseteq$ persistent quantifiers turn out to be precisely the $\uparrow$**MON** quantifiers. This explains the logical implication relations between (19), (20), (21) and (22).

13 Implicit and Adverbial Quantification

For reasons of presentation, attention above was limited to noun phrase quantification. This limitation should not detract from the obvious fact that quantifiers show up explicitly or implicitly in a plethora of other natural language settings. Quantification is implicit in the semantics of tense (the present perfect tense in *John has lived in Cambridge* involves an implicit existential quantification over periods of time), in the semantics of modality (the semantics of *John knows how to swim* involves an implicit existential quantification over situations which are in accordance with John's abilities and in which John swims), and in lexical semantics in general (the semantics of *John is an ex-convict* involves an implicit existential quantification over a period of time in the past which John spent in jail after a conviction).

Explicit quantification can also be found outside noun phrases, in particular in adverbial modifiers. English has explicit adverbs of quantification for quantifying over locations (*everywhere*, *somewhere*, *nowhere*), over periods of time (*always*, *sometimes*, *never*, *often*), and over states of affairs (*necessarily*, *possibly*). Just like *all A*, *some A*, *no A* and *not all A*, the quartet *always*, *sometimes*, *never* and *not always* forms an Aristotelian square. In the same manner as with noun phrase quantifiers, these standard adverbial quantifiers have non standard cousins: *often*, *seldom*, *at least five times*, *more than once*, *exactly twice*, and so on.

Adverbial quantifiers behave very much like noun phrase quantifiers, the main difference being their different domain of quantification. An exact specification of the domain of quantification can be difficult. Contextual information may be needed to to determine whether an adverbial quantifier ranges over periods of time, events, or occasions, and to determine the 'granularity' of the domain of quantification: 'Are the periods of time measured in seconds, days, years, aeons?', etcetera.

(28) *Dinner is always served at six p.m. here.*

(29) *Everyone is expected to pay attention to the teacher.*

The fact that the temporal adverb in (28) ranges over days has to be inferred from the overall meaning of the sentence. But note that a similar overall constraint on the domain of quantification is present in (29) with respect to the quantified noun phrase: the overall meaning of the sentence has to be taken into account to establish that the quantifier ranges over everyone present in the situation except the teacher.

14 Quantifier Branching

Examples like the following have been accorded special status in the literature on natural language quantification.

(30) *Most men and most women like each other.*

(31) *Few men and few women like each other.*

(32) *Exactly four men and exactly three women like each other.*

The most plausible reading for example (30) is the one which is true just in case there are sets M and W with M consisting of a majority of the men and

W of a majority of the women, and the members of M and W like each other. Similarly, (31), in its most plausible reading, is true if and only if all possible sets M of men and W of women, with the members of M and W liking each other, are such that M contains only a small number of the men and W only a small number of the women (the sizes of these norm numbers depending on context). Finally, (32) is true if and only if the sets M of men and W of women such that M and W like each other (i.e., presumably, all members of M like all members of W, and vice versa) have 4 and 3 elements, respectively.

The common element in these analyses is that they involve sets picked independently of each other. This mutual independence suggests that for these examples neither putting the second noun phrase within the scope of the first one nor proceeding the other way around will produce the right result. What is needed, instead, is a so-called branching reading, where denotations for the quantifiers in the different branches are established indepently. Branching readings of quantifier pairs, or more generally quantifier n tuples, make sense only in cases where the quantifiers have similar monotonicity behaviour in the second argument: both **MON**$\uparrow$, as in (30), both **MON**$\downarrow$, as in (31), or both numerical (the numerical quantifier are conjunctions of **MON**$\uparrow$ and **MON**$\downarrow$ quantifiers).

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