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Bounds for the Interval Availability Distribution

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Bounds are presented for the interval availability distribution of a two-state single component system which alternates between "up"- and "down"-state. Numerical computation is considered for the case in which both up and down times are distributed according to mixed Erlang distributions. Some numerical examples are presented and the results are compared with those obtained from the uniformization technique.

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1. Introduction

The long run fraction of time that a system will be operational does not provide enough information about the availability of the system in general (cf. van der Duyn Schouten & Wartenhorst [1]). For instance in production, to meet sales contracts, one may require the production-unit to be operational for at least 90 percent of time over the next two years. Not meeting this requirement may result in claims by clients, loss of goodwill and worse sales-perspectives in future. For the producer it is important to know the risk of not meeting the sales contracts. Similar reasoning holds for a computer vendor who sells computer systems with a 'guaranteed' level of availability in the 95-100 percent range (cf. Goyal & Tantawi [2]). To evaluate the probability of not meeting the availability requirement over a finite time interval one needs the *interval availability distribution,* which is hard to find in general.

Even for simple models there are no closed-form solutions. If the behaviour of a system is modeled as a continuous-time Markov chain with discrete finite state space, several techniques are available to numerically evaluate transient measures of the model. An overview of numerical methods and an extensive list of useful references on the subject can be found in Reibman et al. [6]. To obtain the interval availability distribution three methods prevail: discrete approximation of a system of partial differential equations, transform inversion, and uniformization (sometimes called randomization; cf. de Souza e Silva & Gail [8] and [9]).

However, if we consider non-Markovian models or if the state space grows too large, aggregation and approximation techniques become necessary to analyze the system (we restrict ourselves to analytic modeling, as measurement and simulation may be (computationally) infeasible in many situations). Promising results in this area have been obtained by van der Heijden [4] and van der Heijden & Schornagel [5]. They approximate a (probably complex) stochastic system by a two-state single component system which alternates between "up"- and "down"-state. From the information about the original system they extract approximating stationary distributions of sojourn times in up- and down-state. Assuming sojourn times in up- and down-state to be independent, they finally compute the interval availability distribution using a result of Takács [10]. However, numerically calculating the interval availability distribution in this way involves evaluation of an infinite summation consisting of multiple convolutions.

In this note we will show how to obtain bounds on the interval availability distribution which may be used to truncate the infinite summation properly. Evaluation of a large number of multiple

convolutions requires the use of computationally tractable distribution functions. Van der Heijden [4] uses the exponential distribution, whereas van der Heijden & Schornagel [5] use the gamma distribution. In section 3 we will present numerical computation of the bounds for mixed Erlang distributions. Some numerical examples will be presented and the results will be compared with those obtained from the uniformization technique.

2. Interval availability

2.1. Preliminaries

Consider a system consisting of one single component. This system can be in either one of two states: "up" or "down".
Define:

- $\alpha(t) :=$ total sojourn time in up-state during $[0,t]$, $t \geqslant 0$,
- $\beta(t) :=$ total sojourn time in down-state during $[0,t]$, $t \geqslant 0$,
- $\Omega(t,x) := P(\beta(t) \leqslant x)$, $0 \leqslant x \leqslant t$.

Clearly:

$$\alpha(t) + \beta(t) = t, \tag{2.1.1}$$

and the cumulative operational time distribution is given by:

$$P(\alpha(t) < x) = 1 - \Omega(t,t-x), \quad 0 \leqslant x \leqslant t.$$

Let $IA(t)$ denote the interval availability over the interval $[0,t]$, i.e.

$$IA(t) := \frac{\alpha(t)}{t}.$$

Obviously the interval availability distribution is given by:

$$P(IA(t) < z) = 1 - \Omega(t,(1-z)t), \quad 0 \leqslant z \leqslant 1. \tag{2.1.2}$$

Sojourn times in up- and down-state constitute two sequences of i.i.d. random variables which are assumed to be mutually independent. Let:

- $G(x) := P($ arbitrary up time $\leqslant x)$, $x \geqslant 0$,
- $G^{(n)}(x) :=$ n-fold convolution of $G(.)$, $G^{(0)}(x) \equiv 1$, $x \geqslant 0$,
- $H(x) := P($ arbitrary down time $\leqslant x)$, $x \geqslant 0$,
- $H^{(n)}(x) :=$ n-fold convolution of $H(.)$, $H^{(0)}(x) \equiv 1$, $x \geqslant 0$.

Assume $G(.)$ and $H(.)$ to be absolutely continuous. We extend the definition of $\Omega(t,x)$:

- $\Omega(t,x,y) := P(\beta(t) \leqslant x \mid y)$, $\quad 0 \leqslant x \leqslant t; y \in \{U,D\}$,

where $y = U$ (D) denotes that the system starts at time 0 with an up (down) period. Takács [10] proved the following lemma. The proof that we present here was briefly mentioned already in remark 1 of Takács [10]. It is presented here, because it provides insight and it facilitates the understanding of what follows.

LEMMA 2.1.

Assume that the system starts at time 0 with an up period. The random variable $\beta(t)$ has the following distribution:

$$\Omega(t,x,U) := \sum_{n=0}^{\infty} H^{(n)}(x) \left[G^{(n)}(t-x) - G^{(n+1)}(t-x) \right], \quad 0 \leqslant x \leqslant t. \tag{2.1.3}$$

PROOF:

Choose $0 \leqslant x \leqslant t$ and let:

$$\tau_x := \text{first instant for which } \alpha(\tau_x) = t - x.$$

Clearly the system is up at time τ_x. From (2.1.1) we have

$$\begin{aligned} P(\beta(t) \leqslant x) &= P(\alpha(t) \geqslant t - x) \\ &= P(\alpha(t) \geqslant \alpha(\tau_x)) \\ &= P(\tau_x \leqslant t) \\ &= P(\alpha(\tau_x) + \beta(\tau_x) \leqslant t) \\ &= P(\beta(\tau_x) \leqslant x). \end{aligned}$$

Let the system start at time 0 with an up period. Note that the system is up at time τ_x and that $G(.)$ and $H(.)$ are independent. Then

$$\begin{aligned} \Omega(t,x,U) &= \sum_{n=0}^{\infty} P(\beta(\tau_x) \leqslant x \mid n \text{ complete down intervals before } \tau_x) \\ &\qquad * P(n \text{ complete down intervals before } \tau_x) \\ &= \sum_{n=0}^{\infty} H^{(n)}(x) P(\text{total length of } n \text{ complete up intervals} < t - x \\ &\qquad \leqslant \text{total length of } n+1 \text{ complete up intervals}) \\ &= \sum_{n=0}^{\infty} H^{(n)}(x) \left[G^{(n)}(t-x) - G^{(n+1)}(t-x) \right]. \end{aligned}$$

□

2.2. Bounds

Computation of the interval availability distribution (2.1.2) using lemma 2.1 involves evaluation of an infinite summation. In theorem 2.1 we provide bounds on $\Omega(t,x,U)$ which may be used to truncate the infinite summation properly.

THEOREM 2.1.

Assume that the system starts with an up period at time 0. Let:

$$L(N) := \sum_{n=0}^{N} H^{(n)}(x) \left[G^{(n)}(t-x) - G^{(n+1)}(t-x) \right],$$

$$U(N) := L(N) + G^{(N+1)}(t-x) H^{(N+1)}(x).$$

Then for $0 \leqslant x \leqslant t$, $N \geqslant 0$:

$$L(N) \leqslant \Omega(t,x,U) \leqslant U(N). \tag{2.2.1}$$

PROOF:

From lemma 2.1 we know:

$$\Omega(t,x,U) = \sum_{n=0}^{\infty} H^{(n)}(x) \left[G^{(n)}(t-x) - G^{(n+1)}(t-x) \right], \quad 0 \leqslant x \leqslant t.$$

From (the proof of) lemma 2.1 we can easily see that all terms in this summation are non-negative

(probabilities). So, if the summation is truncated after N steps ($N \geqslant 0$) the intermediate result gives us a lower bound for $\Omega(t,x,U)$.
Next, to obtain an upper bound for $\Omega(t,x,U)$, from (2.1.1) we note that:

$$P(\beta(t) \leqslant x) = P(\alpha(t) \geqslant t-x)$$
$$= 1 - P(\alpha(t) < t-x). \tag{2.2.2}$$

If we assume that the system starts with a *down* period at time 0, we can prove in an obvious way similar to lemma 2.1:

$$P(\beta(t) \leqslant x \mid y = D) = \sum_{n=0}^{\infty} H^{(n+1)}(x)\left[G^{(n)}(t-x) - G^{(n+1)}(t-x)\right],\ 0 \leqslant x \leqslant t. \tag{2.2.3}$$

Because of symmetry and using the assumption of absolute continuity of the probability distribution functions $G(.)$ and $H(.)$:

$$P(\alpha(t) < x \mid y = U) = \sum_{n=0}^{\infty} G^{(n+1)}(x)\left[H^{(n)}(t-x) - H^{(n+1)}(t-x)\right],\ 0 \leqslant x \leqslant t. \tag{2.2.4}$$

So, assuming that the system starts with an *up* period we obtain from (2.2.2) and (2.2.4):

$$\Omega(t,x,U) = 1 - \sum_{n=0}^{\infty} G^{(n+1)}(t-x)\left[H^{(n)}(x) - H^{(n+1)}(x)\right],\ 0 \leqslant x \leqslant t. \tag{2.2.5}$$

Again all terms in this summation are non-negative, so truncating (2.2.5) after N steps ($N \geqslant 0$) gives us an upper bound for $\Omega(t,x,U)$. By definition:

$$G^{(0)}(x) = H^{(0)}(x) = 1, \quad x \geqslant 0.$$

So, the upper bound can easily be rewritten to:

$$U(N) = L(N) + G^{(N+1)}(t-x)\,H^{(N+1)}(x). \qquad \square$$

REMARK 2.1: Theorem 2.1 can also be proved by overestimation of the error obtained by truncating (2.1.3) (cf. de Souza e Silva & Gail [8]). Let $e(N)$ denote the error obtained with the truncation of the infinite summation (2.1.3) to N steps; $\Omega(t,x,U) = L(N) + e(N)$. Then we see that $e(N)$ satisfies:

$$\begin{aligned}
e(N) &:= \sum_{n=N+1}^{\infty} H^{(n)}(x)\left[G^{(n)}(t-x) - G^{(n+1)}(t-x)\right] \\
&= \sum_{n=N+1}^{\infty} G^{(n)}(t-x)H^{(n)}(x) - \sum_{n=N+1}^{\infty} G^{(n+1)}(t-x)H^{(n)}(x) \\
&\leqslant \sum_{n=N+1}^{\infty} G^{(n)}(t-x)H^{(n)}(x) - \sum_{n=N+1}^{\infty} G^{(n+1)}(t-x)H^{(n+1)}(x) \\
&= \sum_{n=N+1}^{\infty} G^{(n)}(t-x)H^{(n)}(x) - \sum_{n=N+2}^{\infty} G^{(n)}(t-x)H^{(n)}(x) \\
&= G^{(N+1)}(t-x)H^{(N+1)}(x).
\end{aligned}$$

The same result can be obtained in an alternative way of overestimating $e(N)$:

$$\begin{aligned}
e(N) &:= \sum_{n=N+1}^{\infty} H^{(n)}(x)\left[G^{(n)}(t-x) - G^{(n+1)}(t-x)\right] \\
&\leqslant \sum_{n=N+1}^{\infty} H^{(N+1)}(x)\left[G^{(n)}(t-x) - G^{(n+1)}(t-x)\right]
\end{aligned}$$

$$= \quad G^{(N+1)}(t-x)H^{(N+1)}(x).$$

REMARK 2.2: If we consider the computation of $U(n)$, $n \geqslant 0$, we note that $G^{(n)}(t-x)$ is used already in the computation of $L(n)$, and $H^{(n+1)}(x)$ is needed to compute $L(n+1)$. So, if (2.1.3) is truncated after N steps, $N \geqslant 0$, only one additional convolution ($H^{(N+1)}(x)$) has to be evaluated to obtain an upper bound on $\Omega(t,x,U)$. In fact, one may truncate the iteration procedure if the gap between $L(N)$ and $U(N)$ (2.2.1) reaches some prespecified level. This might lead to computational savings, because of truncation of the iteration procedure at an earlier stage.

REMARK 2.3: The gap between the lower and upper bound on $\Omega(t,x,U)$ (2.2.1) can be bounded in the following way:

$$\begin{aligned} U(N)-L(N) &= G^{(N+1)}(t-x)\, H^{(N+1)}(x) \\ &\leqslant [\, G(t-x)\, H(x)\,]^{N+1}\,. \end{aligned}$$

This provides us with a simple expression to compute a limit to the number of iteration steps (N) needed to obtain sufficiently close bounds for $\Omega(t,x,U)$. Note that this expression will be useless if both $G(t-x)$ and $H(x)$ are close to 1.

REMARK 2.4: Note that the bounds obtained from theorem 2.1 may still be useful in case of preliminary truncation of the iteration, e.g. due to excess of computation time or numerical instability during computation.

REMARK 2.5: Equivalent bounds on $\Omega(t,x,D)$ can be obtained easily. Truncating (2.2.3) returns a lower bound. Using the symmetry of the model one can obtain an upper bound on $\Omega(t,x,D)$ from (2.2.2) and (2.1.3).

3. Numerical aspects

Numerical calculation of bounds for the interval availability distribution using theorem 2.1, involves evaluation of n-fold convolutions, $n = 1, 2, \ldots, N+1$. This requires the use of computationally tractable probability distribution functions for the alternating up and down times. Van der Heijden [4] uses the exponential distribution (recursive schemes), whereas van der Heijden & Schornagel [5] use the gamma distribution (numerical integration). The case of Erlang distributions can be treated similar to the case of exponential distributions. In this section we will present numerical computation of the bounds derived in the previous section using mixed Erlang distributions for both up- and down times. Numerical results will be compared with those obtained with the uniformization technique (de Souza e Silva & Gail [8]).

Let $f(x)$ denote the probability density function of a nonnegative stochastic variable with probability distribution function $F(x)$. The corresponding Laplace transform is defined by:

$$F^*(s) := \int_0^\infty e^{-st} f(t)\, dt, \quad s \geqslant 0.$$

If we consider a mixture of two Erlang distributions with parameters $\{k,\lambda\}$ and $\{m,\mu\}$ respectively, $k,m = 1,2,\ldots$; $\lambda,\mu > 0$ (cf. Tijms [11], pp. 197), the distribution function and the Laplace transform are given by:

$$F(x) := p\left[1 - \sum_{i=0}^{k-1} e^{-\lambda x}\frac{(\lambda x)^i}{i!}\right] + (1-p)\left[1 - \sum_{i=0}^{m-1} e^{-\mu x}\frac{(\mu x)^i}{i!}\right], \quad 0 \leqslant p \leqslant 1,$$

$$F^*(s) := p\left[\frac{\lambda}{\lambda+s}\right]^k + (1-p)\left[\frac{\mu}{\mu+s}\right]^m, \quad 0 \leqslant p \leqslant 1.$$

Using Newton's Binomium we obtain the Laplace transform of the n-fold convolution $F^{(n)}(x)$:

$$\left[F^*(s)\right]^n := \sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i} \left[\frac{\lambda}{\lambda+s}\right]^{ki} \left[\frac{\mu}{\mu+s}\right]^{m(n-i)}. \tag{3.1}$$

We observe that in evaluating the n-fold convolution $F^{(n)}(x)$, convolutions of Erlang distributions are involved.

If $\lambda = \mu$, the n-fold convolution $F^{(n)}(x)$ is given by:

$$F^{(n)}(x) := \sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i} \left[1 - \sum_{j=0}^{ki+m(n-i)-1} e^{-\lambda x} \frac{(\lambda x)^j}{j!}\right]. \tag{3.2}$$

Evaluating (2.2.1) for the case in which $G^{(n)}(.)$ and $H^{(n)}(.)$, $n \geqslant 0$, are represented by (3.2), yields a numerically stable computational scheme. A numerical example is presented in table 3.1.

If $\lambda \neq \mu$, $F^{(n)}(x)$ is given by:

$$F^{(n)}(x) := \sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i} F_{(ki,\lambda)\otimes(m(n-i),\mu)}(x), \tag{3.3}$$

where $F_{(n_1,\mu_1)\otimes(n_2,\mu_2)}(x)$ denotes the distribution function of the convolution of an $Erlang(n_1,\mu_1)$ and an $Erlang(n_2,\mu_2)$ distributed variable; $n_1,n_2 = 1,2,\ldots$; $\mu_1,\mu_2 > 0$; $\mu_1 \neq \mu_2$. However, we haven't been able to obtain a numerically stable scheme to evaluate $F_{(n_1,\mu_1)\otimes(n_2,\mu_2)}(x)$. Using conditioning arguments we finally ended up with the following expression:

$$F_{(n_1,\mu_1)\otimes(n_2,\mu_2)}(x) := 1 - \sum_{i=0}^{n_1-1} e^{-\mu_1 x} \frac{(\mu_1 x)^i}{i!} - \left[\frac{\mu_1}{\mu_1-\mu_2}\right]^{n_1}$$
$$\times \sum_{l=n_1}^{n_1+n_2-1} \binom{l-1}{n_1-1} \left[\frac{\mu_2}{\mu_2-\mu_1}\right]^{l-n_1} \left[1 - \sum_{i=0}^{l-1} e^{-(\mu_1-\mu_2)x} \frac{((\mu_1-\mu_2)x)^i}{i!}\right] \sum_{i=0}^{n_1+n_2-l-1} e^{-\mu_2 x} \frac{(\mu_2 x)^i}{i!}.$$

Because of the presence of both positive and negative terms in the summation over l, this expression turns out to be numerically unstable, especially for the case μ_1 close to μ_2. Some numerical results are presented in table 3.2.

Harrison [3] obtained the probability density function of a finite convolution of exponential distributions by inversion of the Laplace transform. Using Harrison's results we obtain an alternative expression for $F_{(n_1,\mu_1)\otimes(n_2,\mu_2)}(x)$:

$$F_{(n_1,\mu_1)\otimes(n_2,\mu_2)}(x) := 1 - \mu_1^{n_1}\mu_2^{n_2}(D_1(x) + D_2(x)),$$

where $D_1(x)$ and $D_2(x)$ are defined by:

$$D_i(x) := \frac{(-1)^{n_j} e^{-\mu_i x}}{\mu_i(\mu_i-\mu_j)^{n_i+n_j-1}} \sum_{k=0}^{n_i-1} \binom{n_i+n_j-k-2}{n_j-1} \left[\frac{\mu_i-\mu_j}{\mu_i}\right]^k \sum_{l=0}^{k} \frac{(\mu_i x)^l}{l!}, \quad i,j = 1,2;\ i \neq j.$$

This expression also turns out not to be numerically stable. For example if $n_1 = n_2$ and if μ_1 is close to μ_2 $D_1(x)$ approaches $-D_2(x)$, which causes a loss of significant digits.

At this stage we note that the two-state single unit system (possibly obtained after a number of aggregation steps) with up and down times distributed according to mixtures of Erlang distributions can also be modeled as a continuous-time Markov chain (see for instance Ross [7]). This means that the interval availability distribution can also be calculated using one of the numerical methods described by Reibman et al. [6]. An application of the uniformization technique (also called randomization technique) is presented by de Souza e Silva & Gail [8]). This technique can be easily implemented and numerical errors are circumvented. A disadvantage of the uniformization technique is that the performance degrades for stiff models (a stiff model includes events that occur rapidly relative to the length of the solution interval (Reibman et al. [6])).

As with the uniformization technique the rate of convergence of $L(N)$ to $U(N)$ in (2.2.1) decreases if the stiffness of the model increases (e.g. because of an increase of the length of the observation

period). Numerical experiments, however, suggest that the (tailor-made) bounds presented in theorem 2.1 outperform the (generally applicable) uniformization technique with respect to both computation time and memory requirements. So, especially if $\lambda = \mu$ (no numerical instability) our bounds are to be preferred to the uniformization technique. If numerical instability occurs in the computation of convolutions during the iterations (e.g. because $\lambda \neq \mu$; λ close to μ, or large n_1 or n_2 in $F_{(n_1,\lambda)\otimes(n_2,\mu)}(x)$ in (3.3)), the uniformization technique may provide a good alternative.

For illustrational purposes, in table 3.1 and table 3.2 we present some numerical examples of the computation of $P(IA(t) < z \mid y = U)$ ($= 1 - \Omega(t, (1-z)t, U)$) for $z = avail - 0.025$, where *avail* denotes the long run availability of the system. We use both our bounds (2.2.1) based on Takács' results (T) and the uniformization technique (U). For method T the lower bounds (*Low*) and upper bounds (*Upp*) are computed from (2.2.1): $Low := 1 - U(N_T)$, $Upp := 1 - L(N_T)$; the number of iteration steps is given by N_T. The bounds for method U are computed from (4.6), (4.13) and (4.15) in de Souza e Silva & Gail [8]; N_U and C_U are indicators for the amount of computational effort involved with evaluating *Low* and *Upp* (for the definition of N_U and C_U we refer to de Souza e Silva & Gail [8]; note that N_T and N_U should not be compared because they have completely different meaning). *CPU* gives the computation time (seconds on an Encore Multimax 520) involved with each example. The final column tells us whether an error is obtained within a tolerance of 0.0001 (y) or the iteration procedure is truncated preliminarily due to numerical instability (n). With each table we give the expectation (E_x) and the coefficient of variation ($c_x^2 := \sigma_x^2 / [E_x]^2$) of the distributions of both up and down times. Also the long run availability ($E_{up} / (E_{up} + E_{down})$) of the corresponding two-state system is presented.

method	*Low*	*Upp*	N_T	N_U	C_U	*CPU*	*convergence*
T	0.182751	0.182794	7	*	*	0.04	y
U	0.182746	0.182810	*	155	30	3.51	y

$G(x) = 0.5\,Erl_{3,0.5}(x) + 0.5\,Erl_{6,0.5}(x)$ $\quad H(x) = 0.2\,Erl_{2,2.8}(x) + 0.8\,Erl_{3,2.8}(x)$
$E_{up} = 9.00$, $c_{up}^2 = 0.33$ $\quad E_{down} = 1.00$, $c_{down}^2 = 0.38$
availability (long run) = 0.9000

Table 3.1: bounds for $P(IA(40) < 0.875)$; $(\lambda = \mu)$

method	*Low*	*Upp*	N_T	N_U	C_U	*CPU*	*convergence*
T	0.168587	0.288682	4	*	*	0.08	n
U	0.192119	0.192205	*	157	31	7.22	y

$G(x) = 0.5\,Erl_{3,0.57}(x) + 0.5\,Erl_{6,0.47}(x)$ $\quad H(x) = 0.2\,Erl_{2,2.7}(x) + 0.8\,Erl_{3,2.83}(x)$
$E_{up} = 9.01$, $c_{up}^2 = 0.40$ $\quad E_{down} = 1.00$, $c_{down}^2 = 0.37$
availability (long run) = 0.9005

Table 3.2: bounds for $P(IA(40) < 0.875)$; $(\lambda \neq \mu)$

From these examples we see that T is much faster than U. The case $\lambda \neq \mu$ (table 3.2) does not give rise to numerical errors when U is applied. Note that the input for table 3.1 and table 3.2 is almost the same. Comparing the results for U in table 3.1 and 3.2 we note that the reduction in computation time for $\lambda = \mu$ in table 3.1 (3.51 s. instead of 7.22 s.) is caused by a more efficient way of modelling the system as a continuous time Markov chain (size of the state space is reduced from 14 to 9).

REMARK 3.1: In our examples we computed $P(IA(t) < z \mid y = U)$ for $z < avail$. In general for short observation periods this probability is close to 0. As the length of the observation period increases, the probability that the system enters the "down" state increases initially. In such

situations the interval availability distribution may be of use to compute the probability of not meeting some guaranteed level of availability. Since for long observation periods $IA(t)$ tends to the long run availability (*avail*), $P(IA(t) < z \mid y = U)$ would eventually become 0 for $z < avail$. Similar reasoning holds if $z \geqslant avail$. (cf. Goyal & Tantawi [2]).

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