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E.D. de Goede

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A Numerical Model of the Northwest European Continental Shelf on the CRAY Y-MP2E

E.D. de Goede
CWI
P.O. Box 4079, 1009 AB Amsterdam, The Netherlands

A numerical model for the shallow water equations in polar co-ordinates is applied to the northwest European Continental Shelf. Both a two-dimensional and a three-dimensional model of the Continental Shelf are investigated. A simulation is carried out for the period of 9 to 12 February 1989.
The numerical method used in this paper has been described extensively in [9]. This time splitting method consists of two stages and is unconditionally stable. Moreover, it can fully exploit vector and parallel facilities of supercomputers [10]. Three-dimensional shallow water models require a great computational effort and it is therefore necessary to use such fast computers.
In [15] the same Continental Shelf problem has been examined. The numerical results are in good agreement with each other. It appears that the amplitude and phase errors are small.
The numerical experiments are carried out on the one-processor CRAY Y-MP2E of ICIM (Informatics Centre for Civil Engineering and Environment). This supercomputer has recently been installed in the Netherlands for the simulation of large scale models of rivers and seas.

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1. INTRODUCTION

In the past, various two-dimensional numerical models have been developed for the northwest European Continental Shelf (e.g., in [3, 6, 12, 19]). Their main goal is the accurate prediction of the water levels. Presently, a two-dimensional model of the European Continental Shelf is being used for storm surge predictions along the Dutch coast. This model is operational at KNMI (Royal Dutch Meteorological Institute). Using wind and atmospheric pressure data from a numerical model of the atmosphere, the water elevations in the North Sea and especially along the Dutch coast are computed four times a day. Since these models are two-dimensional, they cannot be used to examine the vertical structure of the tidal currents.

In recent years, there has been a shift towards three-dimensional models to obtain more detailed information about the tidal currents. Three-dimensional models for the Continental Shelf can be found in e.g., [1,4,16]. With such three-dimensional models the interaction between wind and tides can be computed more accurately.

In this paper the fully nonlinear three-dimensional shallow water equations in polar co-ordinates are used. These hydrodynamical equations are solved by the numerical method described in [9]. This two-stage time splitting method is unconditionally stable. For the

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discretization in space, finite differences are used in both the horizontal and the vertical direction.

In [10] it was reported that this time splitting method can be implemented efficiently on a CRAY Y-MP4/464. In that paper a rectangular basin was investigated. Here, we examine the accuracy and computational efficiency of our method on an irregular domain. We use the geometry of the northwest European Continental Shelf with mesh sizes of about 16 km (the so-called CSM16 model). A simulation is carried out for the period of 9 to 12 February 1989. The input data (geometry, boundary conditions, depth values and Chezy coefficients) were supplied by the Tidal Waters Division of Rijkswaterstaat (Dutch Water Control and Public Works Department). The meteorological input (i.e., wind and atmospheric pressure) is neglected. Both a two-dimensional and a three-dimensional model are examined. For the two-dimensional case the same experiment has been carried out in [15].

The numerical experiments are carried out on the CRAY Y-MP2E installed at ICIM (Informatics Centre for Civil Engineering and Environment). Since May 1991 this supercomputer is used in the Netherlands for the simulation of estuarine, river and sea models. This CRAY Y-MP2E has one (vector-)processor and a clock cycle time of 6 ns. On this supercomputer the performance of our highly vectorizable numerical method is about 140 Mflops (millions of floating point operations per second), which is very satisfying. The four day simulation of the three-dimensional model requires about 157 seconds. In [15] a similar experiment on a PC with a 80386 chip and a mathematical co-processor requires a computation time of about 12 hours.

2. MATHEMATICAL MODEL

For the northwest European Continental Shelf model the three-dimensional shallow water equations are written in the form [4]

$$\frac{\partial u}{\partial t} = -\frac{u}{R\cos\varphi}\frac{\partial u}{\partial \chi} - \frac{v}{R}\frac{\partial u}{\partial \varphi} - \omega\frac{\partial u}{\partial \sigma} + 2\omega_e \sin\varphi\, v + \frac{uv\tan\varphi}{R} - \frac{g}{R\cos\varphi}\frac{\partial \zeta}{\partial \chi} + \frac{1}{h^2}\frac{\partial}{\partial \sigma}\left(\mu\frac{\partial u}{\partial \sigma}\right) \tag{2.1}$$

$$\frac{\partial v}{\partial t} = -\frac{u}{R\cos\varphi}\frac{\partial v}{\partial \chi} - \frac{v}{R}\frac{\partial v}{\partial \varphi} - \omega\frac{\partial v}{\partial \sigma} - 2\omega_e \sin\varphi\, u - \frac{u^2\tan\varphi}{R} - \frac{g}{R}\frac{\partial \zeta}{\partial \varphi} + \frac{1}{h^2}\frac{\partial}{\partial \sigma}\left(\mu\frac{\partial v}{\partial \sigma}\right) \tag{2.2}$$

$$\omega = \frac{1}{h}\left\{ \frac{\partial \zeta}{\partial t}(1-\sigma) + \frac{1}{R\cos\varphi}\frac{\partial}{\partial \chi}\left(h\int_\sigma^1 u\,d\sigma\right) + \frac{1}{R\cos\varphi}\frac{\partial}{\partial \varphi}\left(h\int_\sigma^1 v\,d\sigma\cos\varphi\right) \right\} \tag{2.3}$$

$$\frac{\partial \zeta}{\partial t} = -\frac{1}{R\cos\varphi}\frac{\partial}{\partial \chi}\left(h\int_0^1 u\,d\sigma\right) - \frac{1}{R\cos\varphi}\frac{\partial}{\partial \varphi}\left(h\int_0^1 v\,d\sigma\cos\varphi\right), \tag{2.4}$$

where the following notation is used:

C	Chezy coefficient
d	undisturbed depth of water

g	acceleration due to gravity
h	total depth ($= d + \zeta$)
R	radius of the Earth
t	time
u,v	velocity components in the χ- and φ-direction, respectively
w	vertical velocity component in the χ-φ-z co-ordinate system
μ	eddy viscosity coefficient in the σ-direction
χ,φ	east longitude and north latitude, respectively
ρ	water density
σ	vertical spatial co-ordinate (for a definition see (2.5))
ω	vertical velocity component in the χ-φ-σ co-ordinate system
ω_e	angular speed of the Earth's rotation
ζ	elevation above undisturbed depth.

Since the Continental Shelf covers a wide area (see Figure 1), the equations have been transformed from Cartesian into polar co-ordinates. The equations (2.1)-(2.3) are the momentum equations and (2.4) denotes the continuity equation. In our model the accelerations in the vertical direction have been neglected, because they are very small, particularly when compared with the acceleration due to gravity. This is known as the so-called shallow water approximation.

In the vertical, the domain is bounded by the bottom topography and the time-dependent water elevation. To ensure that the three-dimensional domain is constant in the vertical direction, system (2.1)-(2.4) has been transformed into the constant interval [0,1] by the sigma transformation [17]

$$\sigma = \frac{\zeta - z}{d + \zeta}, \quad \text{where } -d \le z \le \zeta \text{ and } 1 \ge \sigma \ge 0. \tag{2.5}$$

The relation between the untransformed (physical) vertical velocity w and the transformed velocity ω is given by [4,20]

$$w = -\omega h + \frac{\partial \zeta}{\partial t} - \sigma \frac{\partial h}{\partial t} + \frac{u}{R\cos\varphi}\left(\frac{\partial \zeta}{\partial \chi} - \sigma \frac{\partial h}{\partial \chi}\right) + \frac{v}{R}\left(\frac{\partial \zeta}{\partial \varphi} - \sigma \frac{\partial h}{\partial \varphi}\right),$$

which leads to [4]

$$w = -\frac{1}{R\cos\varphi}\frac{\partial}{\partial \chi}\left(h\int_\sigma^1 u\,d\sigma\right) - \frac{1}{R}\frac{\partial}{\partial \varphi}\left(h\int_\sigma^1 v\,d\sigma\right) + \frac{h\tan\varphi}{R}\int_\sigma^1 v\,d\sigma$$
$$+ \frac{u}{R\cos\varphi}\left(\frac{\partial \zeta}{\partial \chi} - \sigma \frac{\partial h}{\partial \chi}\right) + \frac{v}{R}\left(\frac{\partial \zeta}{\partial \varphi} - \sigma \frac{\partial h}{\partial \varphi}\right).$$

The boundary conditions at the sea surface ($\sigma = 0$) are given by

$$\omega(\chi,\varphi,0,t) = 0, \quad \left(\mu\frac{\partial u}{\partial \sigma}\right)_{\sigma=0} = -\frac{h}{\rho}W_f\cos(\phi) \quad \text{and} \quad \left(\mu\frac{\partial v}{\partial \sigma}\right)_{\sigma=0} = -\frac{h}{\rho}W_f\sin(\phi),$$

where W_f denotes the wind stress and ϕ the angle between the wind direction and the positive χ-axis. Similarly, at the bottom ($\sigma = 1$) we have

$$\omega(\chi,\varphi,1,t) = 0\,,\ \left(\mu\frac{\partial u}{\partial\sigma}\right)_{\sigma=1} = -h\frac{g\,u_d}{C^2}\sqrt{u_d^2 + v_d^2}\ \text{and}\ \left(\mu\frac{\partial v}{\partial\sigma}\right)_{\sigma=1} = -h\frac{g\,v_d}{C^2}\sqrt{u_d^2 + v_d^2}\,,$$

where u_d and v_d represent components of the velocity at some depth near the bottom.

3. NUMERICAL DISCRETIZATION

To discretize system (2.1)-(2.4), we first apply a finite difference space discretization on a spatial grid that is staggered in both the horizontal and the vertical direction. Figure 2 shows the structure of the horizontal grid. The computational domain is covered by an nx·ny·ns rectangular grid. On this grid the spatial derivatives are replaced by second-order finite differences, which results into a semi-discretized system of dimension nx·ny·(3ns+1). Owing to the sigma transformation (2.5), we have a constant number of grid layers in the vertical direction.

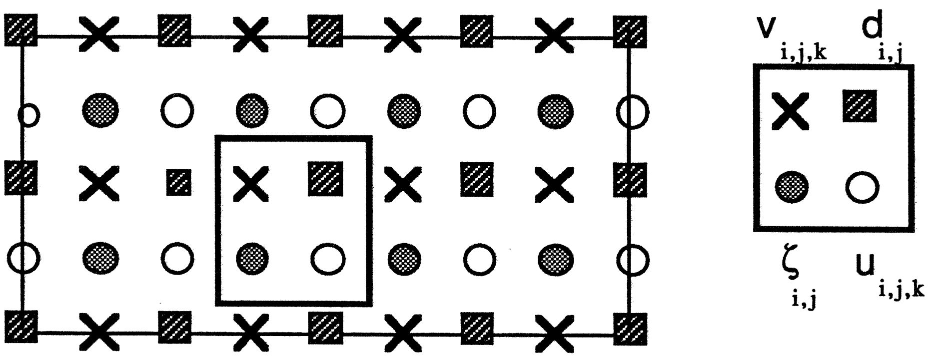

Figure 2. The staggered grid in the horizontal direction.

We now briefly describe the time integration method for the semi-discrete system. For a detailed description we refer to [9]. The first stage of the two-stage time splitting method requires the successive solution of two non-symmetric, linear systems of dimension nx·ny·ns (for the u- and v-component, respectively). This system is solved by a Jacobi-type method, which offers the facility of both an explicit and an implicit treatment of the advective terms. At the second stage a nonlinear system has to be solved. A linearization process is introduced and the resulting linear systems are solved by a preconditioned conjugate gradient method.

In [8] it has been proved that this method is unconditionally stable for a model in which the advective terms and the Coriolis term have been omitted. For a model including these terms, our method appears to have the same good stability properties [9].

The time integration method is first-order accurate in time. It is possible to obtain second-order accuracy by adjusting the discretization of the Coriolis term, the mixed advective terms and the bottom friction term. However, such a second-order treatment would decrease the computational efficiency dramatically. For the sake of efficiency we therefore decided to only use a first-order discretization. It should be noted that the diffusion terms and the terms describing the propagation of the surface waves are second-order accurate in time.

Almost all spatial derivatives are discretized in a symmetric and therefore non-dissipative way. However, following the approach of Stelling [18], at the first stage the mixed advective terms $v\partial u/(R\partial\varphi)$ and $u\partial v/(R\cos\varphi\ \partial\chi)$ are approximated by an upwind discretization. The resulting dissipation is of fourth-order magnitude and does on the one hand not lead to an undesirable damping of the solution and is on the other hand just enough to suppress spurious oscillations.

4. IMPLEMENTATION ON VECTOR COMPUTERS

It is well-known that long vectors are crucial for an efficient use of vector computers. The time splitting method discussed in the previous section has been constructed in such a way that in the horizontal direction the computations are independent of each other [9]. For example, at the first stage the Jacobi-type method requires the solution of $nx \cdot ny$ independent tridiagonal systems, all of dimension ns. Thus, a long vector length of $nx \cdot ny$ is obtained by solving the tridiagonal systems at the same time. In [10] the computational efficiency of this method has been demonstrated on a CRAY Y-MP4/464 for a *rectangular* domain.

We now discuss the implementation on vector computers for *irregular* domains. On such domains, we may perform the computations on a surrounding rectangular domain, which contains both sea and land regions. At the end of each time step, the values in the land regions should be neglected. Then, direct addressing can be used which again leads to vector operations over the whole domain. On the other hand, one may strip out the sea regions and only solve the shallow water equations in these regions (i.e., indirect addressing). Although this leads to shorter vectors, no additional operations are required in the land regions.

We have chosen the direct addressing approach, because on many computers the performance for direct addressing is significantly higher. Obviously, as the land to sea ratio increases, then the indirect addressing technique will become more attractive. On such domains we propose a domain decomposition approach in the horizontal to obtain a better ratio of sea to land regions.

5. APPLICATION

In this section we examine both a two-dimensional and a three-dimensional model of the northwest European Continental Shelf. This model covers the same computational grid as the CSM16 model (average mesh size of about 16 km) of Rijkswaterstaat. The input data (geometry, boundary conditions, depth values and Chezy coefficients) were supplied by the Tidal Waters Division of Rijkswaterstaat. The boundaries of the model are parallel to the geographical co-ordinates 48° N, 62°20' N, 12° W and 13° E. Along the north, west and south (open) boundaries, water elevations are prescribed (see Figure 1). A simulation is carried out for the period of 9 to 12 February 1989.

For our model in polar co-ordinates (see (2.1)-(2.4)), we choose $\Delta\chi=1/4°$ and $\Delta\varphi=1/6°$, where $\Delta\chi$ and $\Delta\varphi$ denote the grid sizes in χ- and φ-direction, respectively. This leads to a mesh size of 18553 m in the north-south direction and to mesh sizes ranging from 12922 m to 18621 m in the east-west direction. The other parameters are $g=9.81$ m/s^2, $W_f=0$ kg/ms^2 (thus, no wind), $\rho=1025$ kg/m^3, $\omega_e=7.27e\text{-}5$ s^{-1} and $R=6378000$ m. The water is initially at rest and the motion on the Continental Shelf is generated by the water elevations prescribed along the open boundaries. These (weakly reflective) boundary conditions allow disturbances from the interior of the model to propagate outwards.

The computations are performed on a grid with nx=100 and ny=87. In the vertical direction we use various values for ns, ranging from 1 (thus, a two-dimensional experiment) to 25. In the two-dimensional case, the number of active grid cells, which represent sea regions, is about 5500.

In the three-dimensional case, it is necessary to specify how the vertical diffusion coefficient μ varies with χ, φ, σ and t. The horizontal variation of μ over the North Sea has been investigated in [13]. Using these results, an appropriate parametrization of the vertical eddy viscosity is

$$\mu = c\left\{ \left(h\int_0^1 u\,d\sigma \right)^2 + \left(h\int_0^1 v\,d\sigma \right)^2 \right\},$$

where c is some parameter. In our experiments we choose c=0.4 s, which is twice the value used in [4].

The numerical experiments are carried out on the one-processor CRAY Y-MP2E installed at ICIM. Since May 1991 this supercomputer is operational for the simulation of large scale water models.

We also compare the numerical solution with a reference solution computed on the same grid with a very small time step of 30 s. Thus, the difference between the reference solution and numerical solution computed with a larger time step represents the error due to the time integration.

The numerical results are examined in the following stations: Wick, Aberdeen, Cromer, Innerdowsing, Dover, Le Havre, Dieppe, Boulogne, Calais, Oostende, Zeebrugge, Hoek van Holland, IJmuiden, Den Helder and Borkum. These stations have been used in [15] too.

To represent the results we define

ERROR : absolute amplitude error for the water elevation ζ averaged over the time and over the stations

COMP : computation time on the CRAY Y-MP2E.

In Table 5.1 we list the amplitude errors for various values of the time step τ.

Table 5.1. Amplitude errors and computation times.

τ	**ns=1** (2D)		**ns=10** (3D)	
(s)	ERROR (m)	COMP (s)	ERROR (m)	COMP (s)
300	0.005	57.3	0.01	282.3
600	0.01	44.6	0.02	157.2
900	0.015	41.9	0.03	116.7
1200	0.02	42.8	0.04	102.2

Even for a large time step of 1200 s the amplitude errors are small. In the three-dimensional experiment the errors are about twice as large. The numerical results show that the phase errors for our time integration method are also small.

In the case of τ=1200 s, the three-dimensional experiment is only three times more expensive than the two-dimensional one, whereas this factor is about six for τ=300 s. This is due to the fact that in the two-dimensional experiment a major part of the computation time is involved in the computation of the water elevation, which is particularly the case when large time steps are used. The computational complexity of the water elevation is the same for both two-dimensional and three-dimensional models [8]. In three-dimensional experiments the computation time for the advective terms is proportional to the number of vertical grid layers. Therefore, our numerical method is more efficient for three-dimensional models than for two-dimensional ones.

In Figures 3.1 to 3.15 we show the water elevations for the aforementioned stations during the fourth day of the simulation. Both two-dimensional (i.e., ns=1) and three-dimensional results (with ns=10) are given. For the two-dimensional case, the same experiment has been carried out in [15]. The numerical results are in good agreement. In [15] the three-dimensional mathematical model contains an additional equation for the turbulent energy. Owing to this different parametrization of the vertical eddy viscosity, the numerical results can not be compared for the three-dimensional case.

Some oscillations have been observed at the station Wick (see Figure 3.1). These oscillations are due to the choice of the vertical diffusion coefficient. In the case of a constant value for the vertical diffusion we have not observed such oscillations. If we integrate over a longer period than four days, then these oscillations disappear.

Figures 3.1 to 3.15 show small differences between the two-dimensional and the three-dimensional results. In the vertical direction we have observed only a small variation of the tidal currents. At the bottom the velocities are approximately ten percent smaller than at the water surface. For such test problems one may equally well apply two-dimensional models instead of three-dimensional ones in order to estimate the water elevations. However, in order to take into account more detailed physics (e.g., near the sea bottom) three-dimensional models are essential [5,16].

In the experiments we have used both the scalar and the vector optimization of the CRAY Y-MP2E. For our integration method the computation time reduces by about a factor of 5 due to the scalar optimization, and by an additional factor of 9 due to the vectorization. This again shows that our method can be implemented efficiently on vector computers. The gain factor is more or less independent of the number of layers in the vertical direction. The performance of our time splitting method is approximately 140 Mflops. Owing to the direct addressing approach (see Section 4), which introduces operations in land regions, the Mflops rate is somewhat misleading. Therefore, we will consider computation times in the next paragraphs.

The WAQUA system, which is a widely applied package in the Netherlands for the simulation of (two-dimensional) shallow water flows, has been implemented on vector–parallel computers too (see e.g., [19]). On the CRAY Y-MP2E a similar (two-dimensional) Continental Shelf experiment as in this paper (with a time step of 600 s) requires about 46 s and yields a performance of about 40 Mflops. Since our method requires about 3 to 5 times more operations [21] and has a higher performance on the CRAY Y-MP2E of about 3.5, this is in agreement with each other. It should be noted that our numerical method has been developed for three-dimensional models and is more efficient for three-dimensional than for two-dimensional models [8]. The four day simulation of our three-dimensional model with a time step of 600 s requires about 157 seconds. In [15] the three-dimensional Continental Shelf experiment on a PC with a 80386 chip and a mathematical co-processor requires a computation time of about 12 hours.

For our three-dimensional Continental Shelf experiment we could not find any computation times on vector-parallel computers in the literature. To illustrate the computational efficiency of our numerical method we therefore make a comparison with the conditionally stable method described in [7]. This method is representative for various numerical methods described in the literature (see e.g., [2,14]). We remark that the first stage of the (unconditionally stable) method used in this paper is very similar to the method in [7].

Table 5.2. Computation times.

	time splitting method (advection implicit)	time splitting method (advection explicit)	conditionally stable method (τ=50 s)
ns=5 (τ=600)	124.0	99.8	425.5
ns=10 (τ=600)	218.2	157.2	836.2
ns=10 (τ=1200)	131.2	101.7	
ns=25 (τ=600)	505.2	342.8	2056.0

We now compare the computation times for the conditionally and the unconditionally stable method. Since our unconditionally stable time splitting method offers the facility of both an explicit and implicit treatment of the advective terms, we list computation times for both cases. The numerical results for both cases agree very well. Thus, the advective terms do not play an important role in this experiment. In Table 5.2 the computation times are presented for various values of ns.

The computation times clearly show that the unconditionally stable method is much more efficient. In the case of ns=10, the ratio in computation time is about a factor of eight. For the conditionally stable method the advective terms require a great computational effort, which is due to the time step restriction.

In [2] a fractional step approach has been developed to allow larger time steps for the advective terms. However, for time steps larger than 50 s we already encounter stability problems for the conditionally stable method. Instabilities occur in the relatively deep inlet near Glasgow. Thus, in practice, the limited stability of the conditionally stable method prevents large time steps.

In conclusion, the unconditionally stable time splitting method used in this paper turns out to be a robust and efficient method for the three-dimensional shallow water equations (see also [10,11]).

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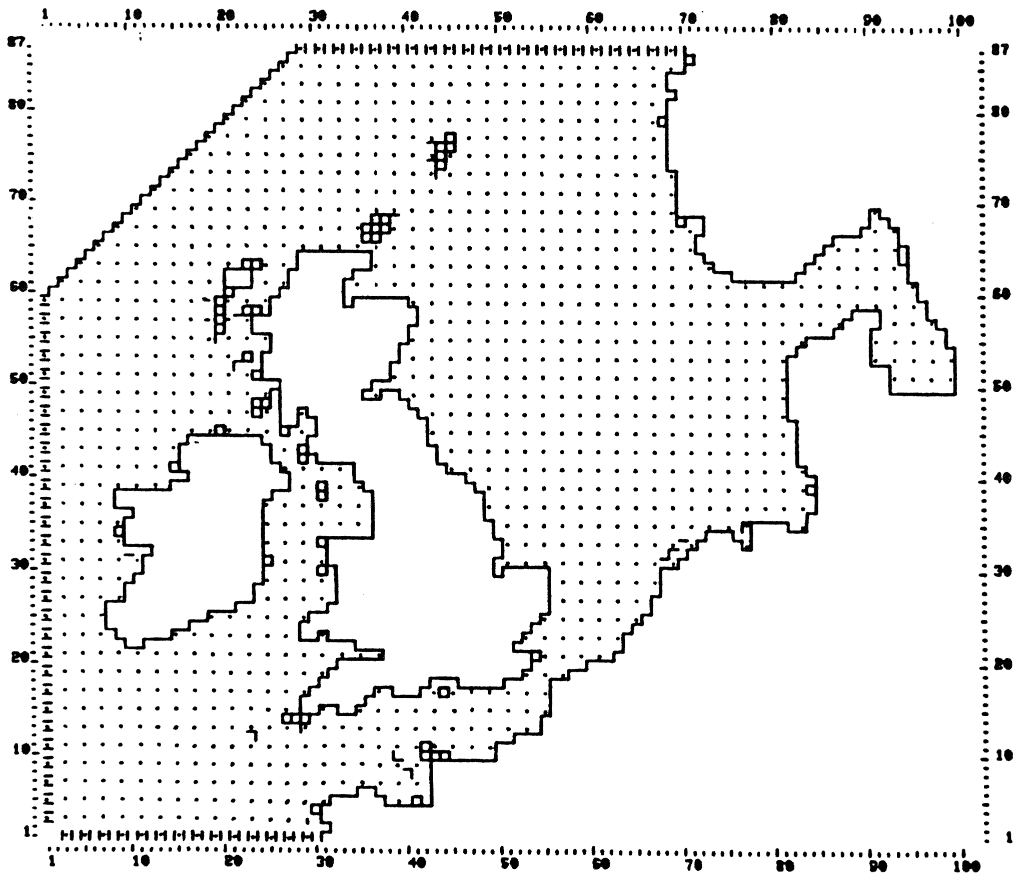

Figure 1. The computational grid of the Continental Shelf model.

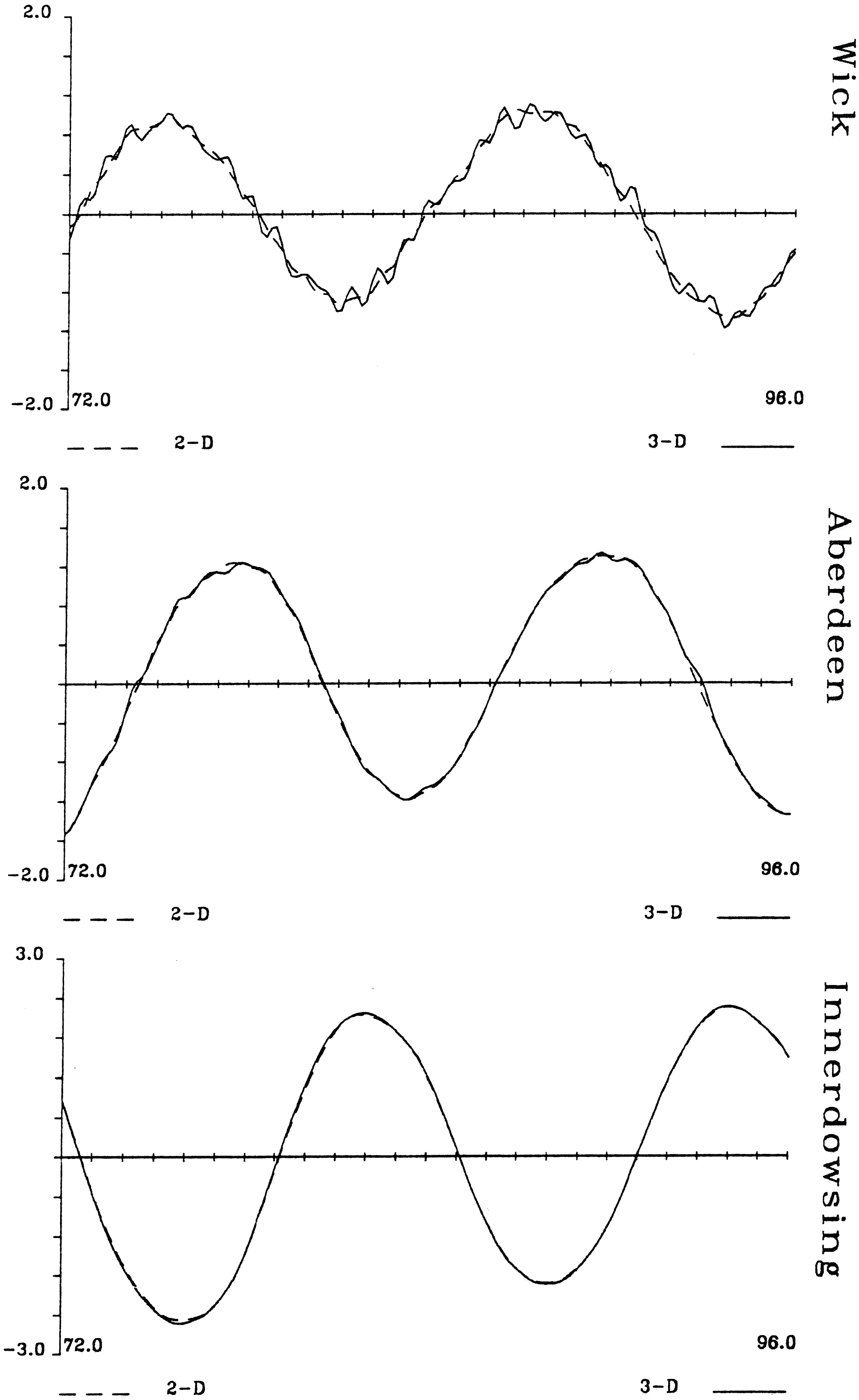
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Figure 3.1.–3.3.

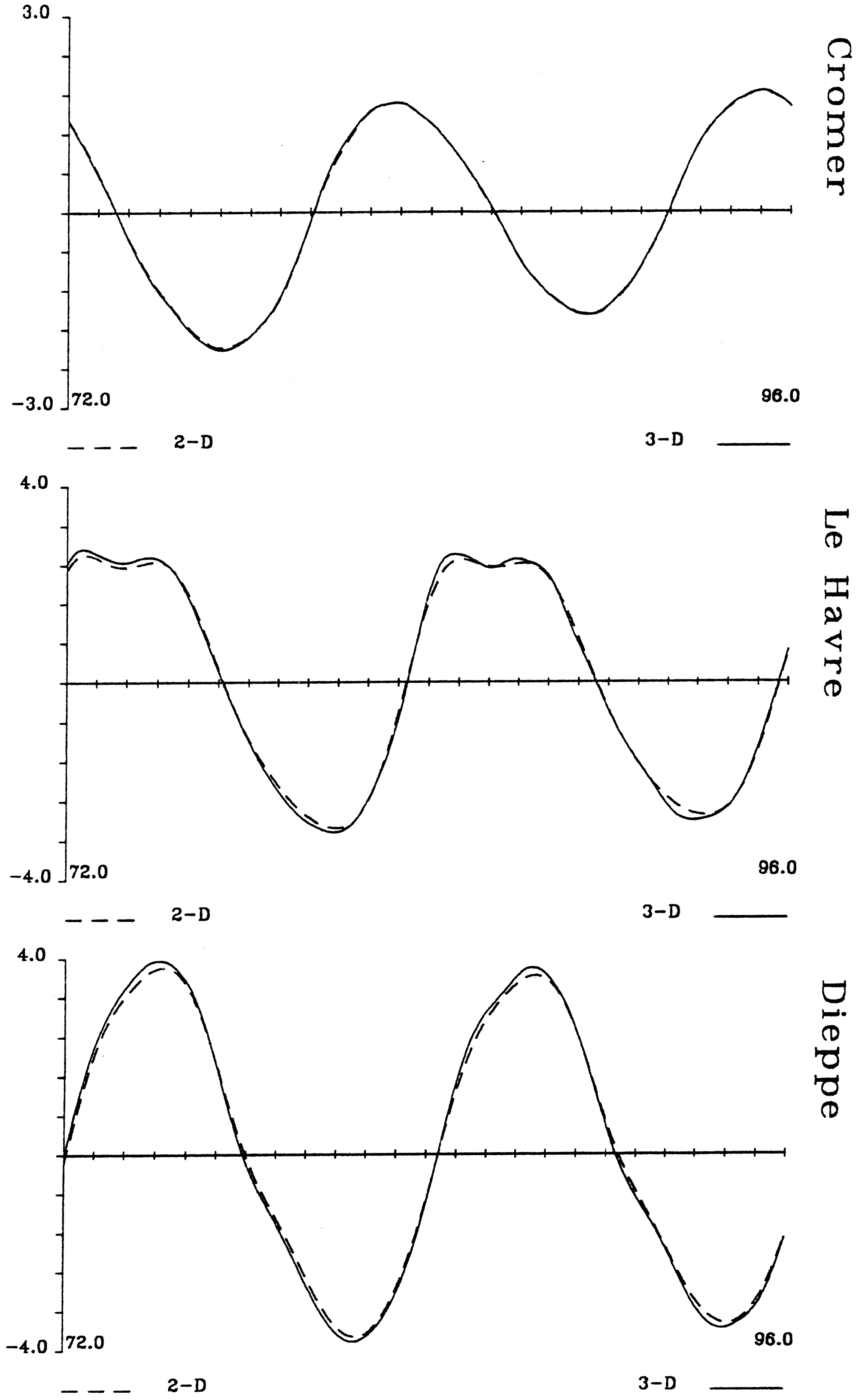
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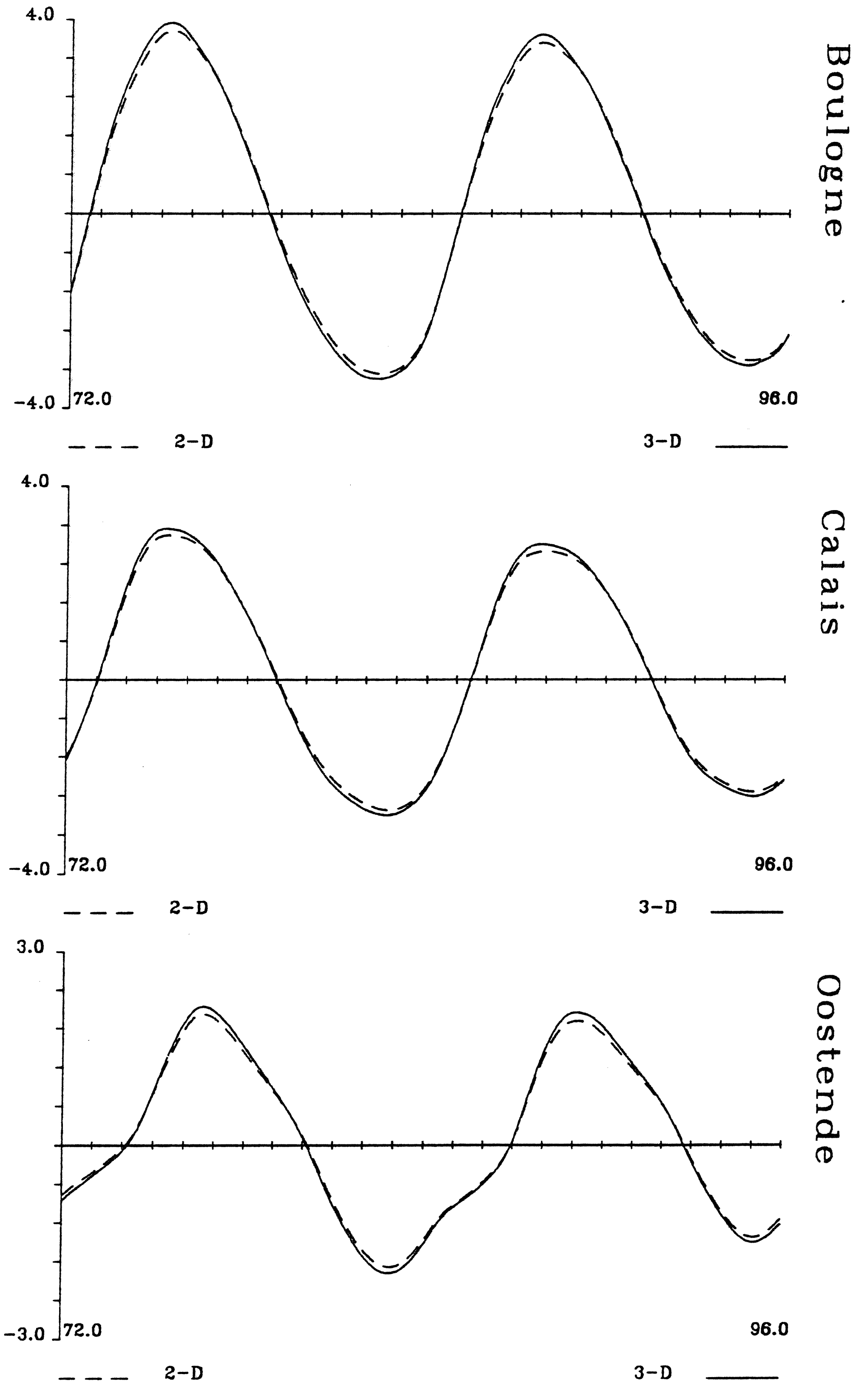
Figure 3.4.-3.6.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Calais
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Oostende
2-D
3-D

Figure 3.7.-3.9.

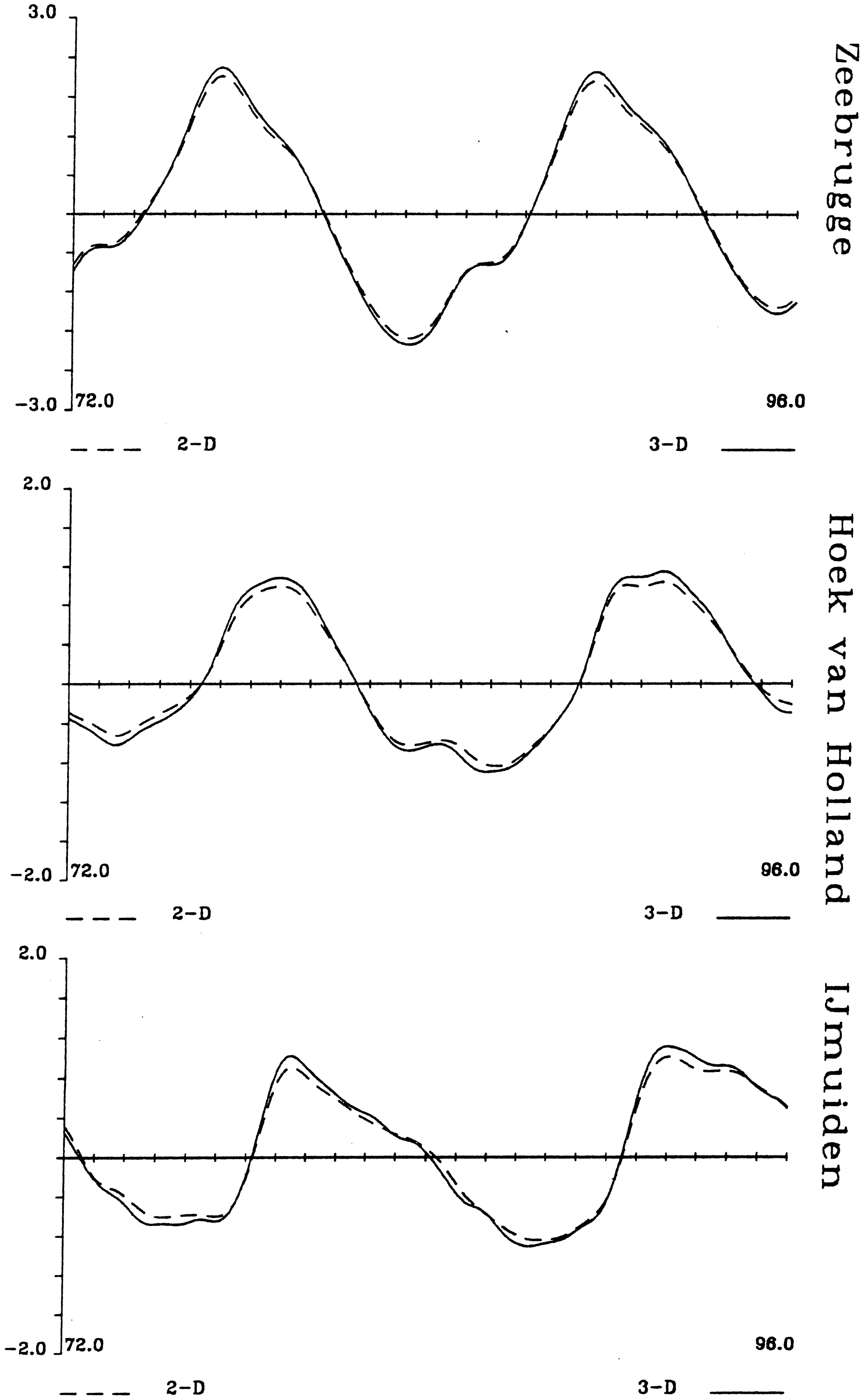

Figure 3.10.–3.12.

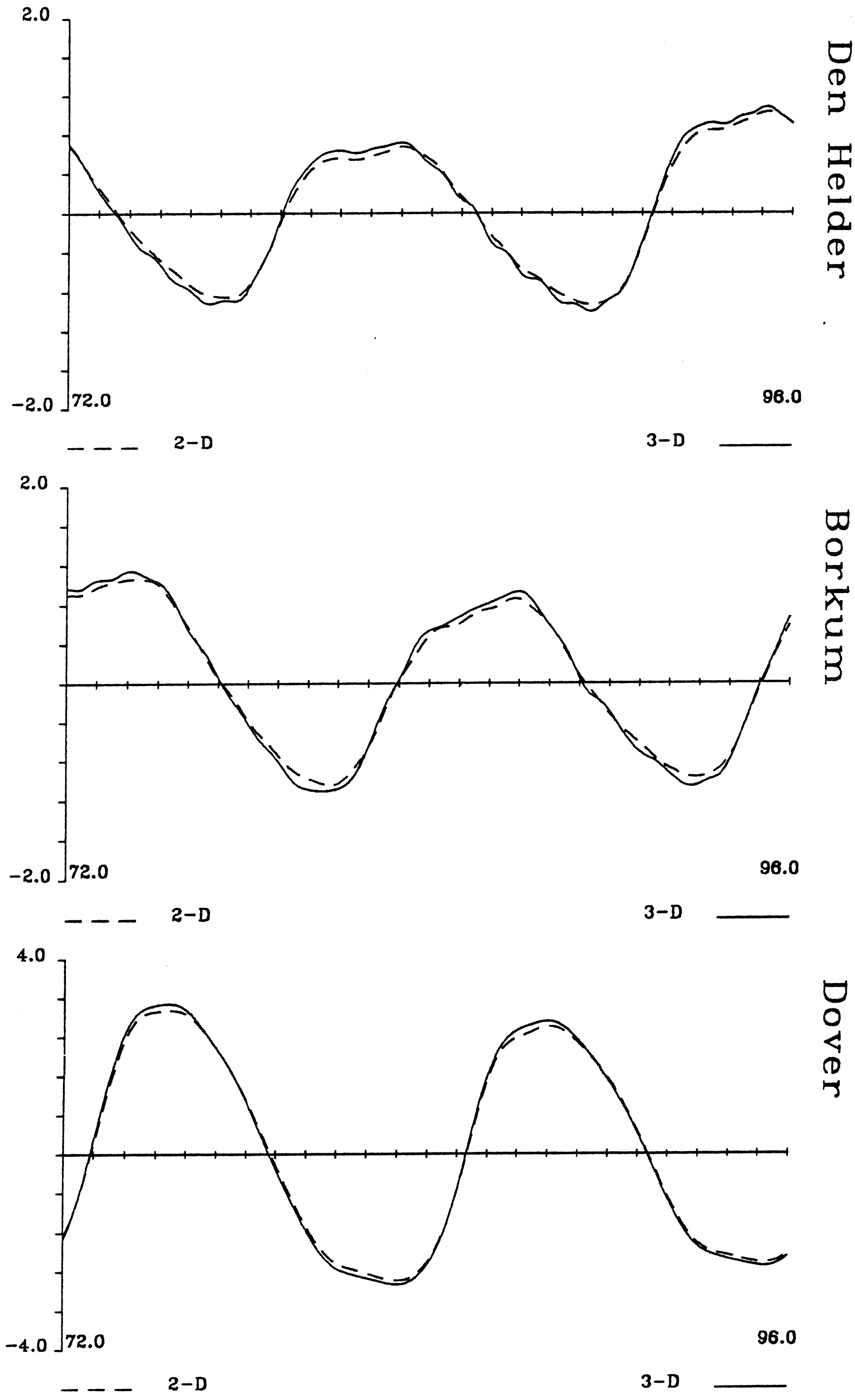

Figure 3.13.-3.15.