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Report on a window-on-science trip

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Report on a Window-On-Science Trip

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Abstract

An account is given of four working visits in the USA. The theme of the visits was computational fluid dynamics.

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1 Introduction

During the Third European Conference on Multigrid Methods, Bonn, October 1-4, 1990, I met Mr. Owen Coté, chief Geophysics and Space at the European Office of Aerospace Research and Development of the Department of Defense of the United States of America. Mr. Coté told that he could assist Europeans in the organization of so-called Window-On-Science (WOS) trips to the USA. A WOS trip basically consists of presenting some seminars at US Air Force Bases and of submitting a written report on the discussions had. In return, the US Air Force pays an honorarium intended to help defray travel expenses. Mr. Coté assisted me in organizing a WOS trip made from September 2 to September 19, 1991. The present report gives an account of this trip. The trip consisted of:

- giving a seminar on September 3 at the Phillips Laboratory, Hanscom Air Force Base, Bedford, Massachusetts,
- giving a seminar on September 5 at the Wright Laboratory, Wright-Patterson Air Force Base, Dayton, Ohio,
- presenting a paper at the Fourth International Symposium on Computational Fluid Dynamics, Davis, California, September 9-12, and
- participating in the ICASE/NASA-LaRC Workshop on Algorithmic Trends for Computational Fluid Dynamics in the 90's, Hampton, Virginia, September 15-17.

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2 Visit to the Phillips Laboratory, Hanscom Air Force Base, Bedford, Massachusetts, September 3

2.1 Laboratory and point of contact

The Phillips Laboratory is the US research laboratory on all kinds of geophysical forecasting for the US Air Force. The total number of employees working at the laboratory is about 550. The laboratory is divided into six scientific divisions: (i) the Atmospheric Science Division, (ii) the Advanced Weather Prediction Division, (iii) the Space Physics Division (up to heights of 10^5 ft), (iv) the Ionospheric Division, (v) the Optical Infrared Division and (vi) the Geological Division.

Point of contact for this visit is Mr. Samuel Yee, research scientist in the Atmospheric Prediction Branch of the Atmospheric Science Division. The common research interest of Yee and myself is the use of multigrid methods in computational fluid dynamics (CFD). Yee's interest in multigrid methods is inspired by the increasing importance of weather prediction on mesoscales ($\sim$ 500 km $\times$ 500 km). Mesoscale computations are very computation-intensive.

2.2 Seminar

Depending on the interests of some researchers in the Atmospheric Science Division, a presentation is given on the following two topics: (i) efficient multi-dimensional (multi-D) upwinding for the steady Euler equations and (ii) a local, solution-adaptive multigrid technique for the steady Euler equations. The abstract of the presentation given on both topics follows below. Details and results on the first topic can be found in [29, 31, 32], details and some preliminary results on the second topic are given in [20, 33, 39, 40, 41].

'Efficient multi-D upwinding for the steady Euler equations':
Some multi-D upwind discretizations for the steady Euler equations are presented, with the emphasis on both a good accuracy and a good efficiency. The discretizations consist of a 1-D Riemann solver with locally rotated left and right cell face states, the rotation angle depending on the local flow solution. On the basis of a model equation, first a study is made of the accuracy and stability properties of some possible multi-D upwind schemes. One multi-D scheme is derived for which smoothing analysis of point Gauss-Seidel relaxation shows that despite its rather low numerical diffusion, it still enables a good acceleration by multigrid. Another multi-D scheme is derived which has no numerical diffusion in crosswind direction, and of which convergence analysis shows that its corresponding discretized equations can be solved efficiently by means of defect correction iteration with in the inner multigrid iteration the aforementioned multi-D scheme. Further, it is shown that for Euler flows, an appropriate local rotation angle can be found by maximizing a Riemann invariant along the middle subpath of the wave path in state space. For the steady, 2-D Euler equations, numerical results are presented for some supersonic test cases with an oblique contact discontinuity and for some supersonic test cases with an oblique shock wave. The numerical results presented are in good agreement with the theoretical predictions. Comparisons are made with results obtained by standard 1-D upwind schemes. The multi-D results obtained compare very well, both with respect to accuracy and efficiency.

'A local, solution-adaptive multigrid technique for the steady Euler equations':
A solution-adaptive multigrid technique is presented, which makes use of locally refined, nested grids which may cover only parts of the entire computational domain. The locally refined grid regions deliver defects to the underlying coarse grids. In return, the coarse grids deliver solution corrections and boundary conditions to the overlying fine grid regions. It is shown that multigrid convergence acceleration by this adaptive technique works equally well as by the original, non-adaptive technique. Concerning computational efficiency, of course a great gain is obtained by refining only locally (instead of globally). In the process of locally refining the grid, we may strive for e.g. an equi-distribution of the estimated local truncation error at some user-defined level. Also for estimating the local trunca-

tion error, practical use can be made of the nested grid sequence by making an extrapolation of the relative truncation error between the finest and the one-but-finest grid. In equi-distributing the local truncation error over the domain, we may also coarsen regions of low-error estimates.

The solution-adaptive multigrid technique has already shown to be well-suited for an accurate and efficient resolution of detailed flow features in the case of steady subsonic Euler flow along a kinked wall [41]. Applying a standard finite volume discretization, at the kink the global discretization error appears to be zeroth-order in mesh size. As a remedy, in the solution-adaptive multigrid setting, we applied an appropriate sequence of continuously curved walls which converge to the kinked wall in the limit of zero mesh width. A good accuracy and still good efficiency is obtained in this way.

2.3 Discussions

At the Phillips Laboratory, besides with Mr. Samuel Yee, I also have discussions with three of his colleagues: Dr. Jean King, Mr. Malcolm MacLeod and Dr. Ken Yang.

Mr. Samuel Yee:
At the Fifth Copper Mountain Conference on Multigrid Methods, Copper Mountain, Colorado, March 31 - April 5, 1991, Mr. Yee had a presentation entitled: 'A dual-resolution semi-implicit method for numerical weather prediction models'. The dual-resolution approach is based on the fact that within geophysical fluid dynamics systems, Rossby waves propagate much slower than gravity waves and also change much slower (both in space and in time). Therefore, it seems useful to discretize the equations on a dual grid, both spatially and temporally. Yee has proposed now to discretize on a coarse grid: the Rossby terms and the nonlinear convection and divergence terms, and to discretize on a fine grid: the horizontal pressure gradient and the linear divergence and diffusion terms. Further, concerning the time integration he has proposed to apply an explicit method for the slow Rossby waves and an implicit method for the fast gravity waves. Preliminary dual-resolution results have been obtained for a shallow water model. Herewith, comparisons have been made with a single-grid approach. So far, all grids applied by Yee are uniform. Further, he does not apply two-grid convergence acceleration by transferring defects and corrections between the two grids. A property of the dual-resolution approach that we discuss in some detail is to what extent the overall solution accuracy is affected by the coarseness of the coarse grid.

Besides in mesoscale flow computations, Yee also has an interest in global flow computations, particularly in avoiding severe stability restrictions to the time step when approaching the poles of the earth. For this purpose he applies a special gridding in which the mesh sizes remain finite when going to the poles.

Dr. Jean King:
Dr. King, senior research scientist in the Atmospheric Science Division, works on techniques for solving the inverse problem of computing an atmospheric temperature profile from measured data (e.g. infrared data gathered by a satellite). A standard technique for solving this inverse problem is matrix inversion with physical constraints imposed on the temperature profile. According to King, in practice, the unsatisfactory property of this technique is that these physical constraints often have to be such that the whole inversion process boils down to looking first at the answer in the back of the book and next computing the answer. As an alternative, King has developed an inversion technique which needs no constraints, a so-called physical (or differential) inversion technique. (Quite recently, King found that this technique has also been proposed by the British astrophysicist Eddington in the context of a problem from astrophysics.) The coefficients in King's inversion matrix are related to the Riemann-ζ function.

Mr. Malcolm MacLeod:
Mr. MacLeod, research scientist in the Space Physics Division, studies force-free electromagnetic fields ($(J \times B) \times B = 0$ or $J \times B = \alpha B$ with α constant). The practical purpose of this study is to predict

the electromagnetic impact on satellites of sun-ejected clouds of plasma. So far, MacLeod's study considers continuous equations only. Numerics may be invoked in the near future.

Dr. Ken Yang:
Dr. Yang, research scientist in the Atmospheric Science Division, appears to be interested in the extension of steady techniques (such as presented in the seminar) to his unsteady problems.

2.4 Conclusions

With Yee, the appointment is made that relevant research reports will be exchanged. Concerning reports on work already done, I give Yee a package on some multigrid-CFD work done at CWI. Further, a policy document [1], some annual reports [2] and a publications catalogue will be sent.

3 Visit to the Wright Laboratory, Wright-Patterson Air Force Base, Dayton, Ohio, September 5-6

3.1 Laboratory and point of contact

At the Wright Laboratory, research is done on aerospace technology for the US Air Force. The total number of employees working at the laboratory is about 2750. The laboratory is divided into four scientific divisions: (i) the Flight Dynamics Division, (ii) the Propulsion Division, (iii) the Materials Division and (iv) the Avionics Division.

Point of contact for my visit to the Wright Laboratory is Dr. Joseph Shang, head of the Computational Aerodynamics Group in the Flight Dynamics Division. Our common interest is the computation of the external aerodynamics of aircraft. Shang's group consists of 15 people, of whom 10 are working on a permanent basis and 5 on contract basis. Some research topics in Shang's group are: unsteady flows (in relation to aeroelasticity), hypersonics (with special interests for flux splitting schemes, TVD schemes, real gas flows and ionization) and parallel processing.

The Computational Aerodynamics Group is not the only group active in the field of CFD at the Wright Laboratory. Other CFD groups are the Aerodynamical Methods Group (which, compared to the Computational Aerodynamics Group, does rather applied work), the Thermal Protection Group (which mainly considers hypersonic applications) and two groups at the Propulsion Laboratory (which mainly study high free-stream turbulence and the effects of unsteadyness on heat transfer, with the aim to improve heat transfer computations).

3.2 Seminar

Depending on the interests of Dr. Shang, a presentation is given on the following topics: (i) multigrid and defect correction for the steady Navier-Stokes equations, (ii) damped, direction-dependent multigrid for hypersonic flow computations and (iii) efficient multi-D upwinding for the steady Euler equations. The abstract of the presentation given on the first two topics follows hereafter. Details on both topics can be found in [26, 27] and [30], respectively. The abstract on the last topic is given in section 2.2.

'Multigrid and defect correction for the steady Navier-Stokes equations':
An accurate and efficient numerical method for the steady Navier-Stokes equations is presented. The method is based on an existing method for the Euler equations. The Euler method is quickly evaluated by showing results obtained for a wind tunnel flow and some airfoil flows. The flows vary from fully subsonic to almost fully supersonic. The Euler flow results obtained are satisfactory and - hence - allow the extension to Navier-Stokes.

The Navier-Stokes method uses a finite volume discretization. For the evaluation of the convective fluxes an approximate Riemann solver is applied, and for the diffusive fluxes use is made of a second-order accurate central discretization. The solution method requires the flux functions to be continuously differentiable. Well-known approximate Riemann solvers satisfying this requirement are Van Leer's and Osher's. Analytical and numerical results are presented which show that for reasons of accuracy Osher's scheme is to be preferred above Van Leer's. The convective discretization is first-order accurate by taking the left and right Riemann state equal to that in the corresponding adjacent volume. Higher-order accuracy is obtained by applying higher-order piecewise polynomial state interpolation. A limiter is presented which allows monotone, third-order accurate flux evaluations.

Nonlinear multigrid is applied for an efficient solution of the system of discretized Navier-Stokes equations. Herewith, collective symmetric point Gauss-Seidel relaxation is used as the smoother. The solution method is very efficient for the first-order discretized Navier-Stokes equations. Difficulties arising when applying the solution method to higher-order discretized equations are circumvented by introducing defect correction as an outer iteration.

Results are given for a subsonic and supersonic flat plate flow, the latter with oblique shock wave - boundary layer interaction. The results are satisfactory; good accuracy and efficiency can be obtained for a wide range of practically interesting Mach and Reynolds numbers.

'Damped, direction-dependent multigrid for hypersonic flow computations':
A nonlinear multigrid technique with improved robustness is presented for the solution of the steady Euler equations. The system of nonlinear equations is discretized by an upwind finite volume method. Collective symmetric point Gauss-Seidel relaxation is applied as the standard smoothing technique. In case of failure of the point relaxation, a switch is made to a local evolution technique. The robustness improvements to the nonlinear multigrid method are a local damping of the restricted defect, a global upwind prolongation of the correction and a global upwind restriction of the defect. The defect damping operator is derived from a two-grid convergence analysis. The upwind prolongation operator is made such that it is consistent with the upwind finite volume discretization. It makes efficient use of the P-variant of Osher's approximate Riemann solver. The upwind restriction operator is an approximate adjoint of the upwind prolongation operator. Satisfactory convergence results are shown for the computation of a hypersonic launch and reentry flow around a blunt forebody with canopy. For the test cases considered, it appears that the improved multigrid method performs significantly better than a standard nonlinear multigrid method. For all test cases considered it appears that the most significant improvement comes from the upwind prolongation, rather than from the upwind restriction and the defect damping.

3.3 Discussions

Within the Computational Aerodynamics Group, I have discussions with the following co-workers of Dr. Shang: Dr. Miguel Visbal, Dr. Phil Webster, Dr. Steve Scherr, Dr. Datta Gaitonde and Dr. Ray Gordnier. Further, I have discussions with some CFD people from the Propulsion Division.

Dr. Miguel Visbal:
Visbal's research is concerned with low-subsonic vortex flows over delta wings and ogives, with as governing equations the unsteady, 3-D Navier-Stokes equations [15, 49]. Flows considered earlier by Visbal are pitching airfoil flows [53] and bifurcating horse-shoe vortex flows generated by cylinder/flat-plate junctures [52]. Asking for Visbal's opinion about the alternative approach of applying a vortex method [36] (i.e. a Lagrangian instead of an Eulerian formulation), he answers that although a good property of vortex methods is their potential to accurately resolve vortices over arbitrarily long distances, a poor property still is their difficulty in generating vorticity along body surfaces. A method which is hybrid in the sense that it is Eulerian along body surfaces and Lagrangian away from those might be a good alternative. When sticking to an Eulerian formulation in the whole computational domain, an accurate and still efficient resolution of vortices might also be obtained by applying the

zero-crosswind diffusion scheme from [32]. The aspect of time integration in Visbal's work reminds me of the work done at CWI in the project group Numerical Solution of Evolution Problems [1, 2]. Further, the bifurcating flows and the vortex flows over delta wings remind me of the works of Bakker, Hoeijmakers and Powell [3, 22, 44].

Dr. Phil Webster:
Since recently, Webster considers vortex breakdown over delta wings in the low-subsonic speed regime. He has a publication on it [54]. Before that he considered thin layer and thick layer Navier-Stokes flows. Webster's work on vortex breakdown reminds me of the efforts put for some years into this now by Hänel and his co-workers (see e.g. [4, 17]). A scheme like e.g. the zero-crosswind diffusion scheme from [32] might also be of some interest to this work. (This impression particularly arises when Webster tells that the streamwise location of the point of vortex breakdown is very sensitive to the amount of artificial dissipation added to the central difference scheme that he applies.)

Dr. Steve Scherr:
Scherr's work concerns the parallel processing of CFD codes on distributed memory machines. For this purpose he uses an in-house Intel 8 machine. Future plans are to work on a 500 processors machine at Caltech (the Delta). For his parallelization work, Scherr considers two CFD codes: an explicit MacCormack type code and an implicit Beam-Warming type code. A difficulty of the latter implicit code in its implementation on a distributed memory computer is the required simultaneous access to each grid point. For the explicit MacCormack code no such difficulty exists of course. The first results obtained with the explicit code on the Intel 8 are promising. The speed-up as a function of the number of processors appears to be linear. Given this linear speed-up and given the result found that a single-processor run on the Intel machine is only 50 times slower than a single-processor run on the Cray XMP (where the Cray processor has run in vector mode and the Intel processor in scalar mode), it can be concluded that only 50 Intel processors may be sufficient to equal a single Cray XMP processor. (With the 500 processors Delta machine, the Cray may be definitely beaten.)

Concerning the difficulty in parallelizing the implicit Beam-Warming code, some relevant papers may already exist for this in the context of parallelizing general BLAS routines. At least for shared memory computers, even a text book exists for this [12]. In CWI's project group Numerical Software, so far work has been done on the vectorization of BLAS routines [1, 2]. Another approach might be to replace the global implicitness by a local implicitness. (For speed-up, besides of parallelization, proper use may then also be made of a nonlinear multigrid technique [28, 48].)

Dr. Datta Gaitonde:
Gaitonde's work appears to be close to CFD work done in CWI's project group Numerical Solution of Stationary Problems [1, 2]. For various flow problems, Gaitonde has studied the accuracy of: (i) the Steger-Warming scheme (with a modification of MacCormack and Kennler), (ii) the Van Leer scheme (in the original variant) and (iii) the Roe scheme. To obtain higher than first-order accuracy in space, Gaitonde applies piecewise polynomial state interpolation through the fully one-sided upwind scheme ($\kappa = -1$). For this he has investigated what is a proper choice of state variables to be interpolated. State vectors that he considered for this purpose are: (i) the conservative vector $(\rho, \rho u, \rho v, \rho e)^T$, (ii) the primitive vector $(\rho, u, v, p)^T$ and (iii) the semi-primitive/semi-conservative vector $(\rho, \rho u, \rho v, p)^T$. Concerning these validation experiments on upwind schemes, it might be interesting if also would be considered: (i) the Osher scheme, (ii) the modified Van Leer scheme as proposed by Hänel et al. [18] and (iii) the Lombard scheme [10]. Concerning the choice of variables to be interpolated, it would be interesting to also consider Riemann-invariant type variables, e.g. the state vector $(u, v, \sqrt{\gamma p/\rho}, p\rho^{-\gamma})^T$. Riemann-invariant type variables are, by their very nature, more smooth than other variables and hence more suited for higher-order polynomial interpolation. Yet, it might well be that the differences in solution accuracy for the various possible state vectors, can only be observed for very discriminating test cases. Of probably greater influence to the solution accuracy, is the value of κ. For the best accuracy, $\kappa = \frac{1}{3}$ is to be preferred above $\kappa = -1$. Further, for reasons

of both accuracy and efficiency $\kappa = 0$ may be preferable [9].

Gaitonde continues the presentation of his work by showing numerical results he obtained for: (i) a shock-shock interaction in front of a blunt body, (ii) a supersonic Navier-Stokes flow along a kinked wall and (iii) a single bow shock flow in front of a blunt body. A peculiarity in the bow shocks that Gaitonde obtained for the latter flow, is the occurrence of a bubble in the shock just upstream of the stagnation point. This bubble might be related to the sonic glitch as observed (and removed) in [6]. Concerning the flow along the kinked wall, the question arises whether there is a zeroth-order error, hidden near the kink, in the underlying Euler flow solution. In [41], a study is made of such a spurious, zeroth-order error behavior. Finally we still discuss the observed mutual difference in order of accuracy between computed surface heat transfer and computed surface pressure.

Dr. Ray Gordnier:
The last co-worker of Dr. Shang with whom I have a discussion, Dr. Gordnier, explains me his work on vortex flows over delta wings [15], for which he uses codes of Visbal.

Propulsion Division:
Within the Propulsion Division, I have discussions with some people working on CFD in the groups of Dr. R. Rivir and Dr. W.M. Roquemore. As opposed to the work done in Dr. Shang's group, the work done in these two groups is much more related to experiments.

3.4 Conclusions

The Computational Aerodynamics Group at the Wright Laboratory produces a large amount of papers. Given the fact that it does not concern here an academic research environment but a research environment with obligations towards groups which are much more applied than the Computational Aerodynamics Group itself, the level of these papers is impressive.

The appointment is made that relevant research reports will be exchanged. Concerning reports on work already done, I give Shang a package on some of our CFD work. Further, a policy document [1], a few annual reports [2] and a publications catalogue will be sent.

4 Visit to the Fourth International Symposium on Computational Fluid Dynamics, Davis, California, September 9-12

4.1 Symposium

The symposium appears to be huge; scheduled within $3\frac{1}{2}$ days are: about 200 oral presentations (organized in some six parallel sessions) and about 100 poster presentations. This report only covers a small subset of all presentations. A complete account of the papers presented at the symposium is given in the (about 1500 A4-pages thick) preliminary proceedings. Besides the presentations there are also four $1\frac{1}{2}$-hour panel discussions.

4.2 CWI presentation

B. Koren and H.T.M. van der Maarel: 'Analysis of transonic shock configurations by a solution-adaptive multigrid technique':
In our paper, [33], an efficient computational tool is presented for analyzing the intriguing configurations of transonic shock waves appearing with local supersonic zones along continuously curved, convex walls. Of special interest herewith are: (i) the flow in the region where the shock wave abuts the continuously curved wall (the shock foot flow) and (ii) the flow near the other end of the shock wave (the shock tip flow). The computational method applied uses local grid enrichment. A detailed description of the data structure used for this is given in [20]. The discretization and solution method

applied are basically the same as those developed earlier for a non-adaptive Euler algorithm. These earlier developed methods are: an upwind finite volume method for the discretization, a multigrid accelerated point relaxation method for the solution of first-order discretized equations and an iterative defect correction method for the solution of higher-order discretized equations. For details on these existing numerical methods we refer to [19, 25]. A new numerical ingredient in the present method is the discretization at the internal fine-coarse grid boundaries. For details on this we refer to [39]. Numerical results are shown for a di-atomic perfect gas around a NACA0012-airfoil. A remarkable result found is that for decreasing mesh width, at the downstream shock foot face an expansion occurs of which the gradient seems to remain finite, in contradiction to the known analytical results from [57]. A doubt that we already had about the correctness of the singular solution found in [57] is that the infinite shock curvature conflicts with the fact that standard 1-D shock relations were applied in deriving it. Possibly, very large shock curvatures require multi-D shock relations, i.e. shock relations with jumps in tangential direction as well. Secondly, although the analysis in [57] starts with the full Euler equations, in becoming local (i.e. in approaching the wall along the shock wave), to these starting equations it applies some simplifications which are based on questionable pre-assumptions about how the flow locally behaves there. In [33] we apply the full Euler equations throughout the complete computational domain. Further, in [39] the order of accuracy of the Euler equations discretized on an adaptive grid has been analyzed such that in the numerical results presented in [33] it is known to be non-zero everywhere. During the symposium, quite a lot of interest is shown by people familiar with the Zierep expansion. Concerning the (simultaneously computed) shock tip flows, no remarkable results are observed.

4.3 Other presentations

Ch. Hirsch, A. Rizzi, C. Lacor, P. Eliasson, I. Lindblad and J. Häuser: 'A multiblock/multigrid code to simulate complex 3-D Navier-Stokes flows on structured meshes':
The paper, [21], is presented by Rizzi. It describes a general software system for the simulation of 3-D Reynolds-averaged Navier-Stokes flows around complex geometries. The software system is developed for the European Space Agency. A data structure has been designed which allows the use of various numerical techniques, such as e.g. upwind or central differencing, and explicit or implicit time stepping. Further, to accelerate convergence to steady state, use can be made of residual averaging or multigridding. The numerical results shown look very good. Noteworthy is that multigrid is not applied in a hypersonic test case considered.

P.R. Garabedian: 'Comparison of numerical methods in transonic aerodynamics':
Garabedian gives an interesting presentation on: (i) non-uniqueness of steady Euler flow solutions and on (ii) a new method for estimating wave drag through transformation of the entropy inequality [14]. Concerning the non-uniqueness of steady Euler flow computations, Garabedian points out that this may be an explanation for buffeting and thus may be of real practical interest.

S.J. Osher: 'High order accurate non-oscillatory and fast methods for computing flows and steep gradients':
Osher reviews his work on ENO schemes and illustrates the application of it to various mathematical problems (not only flow problems). Although accurate multi-D results have been obtained now with ENO schemes, to my opinion, a weak spot of the ENO approach is that it still is a 1-D technique. (Extensions to multi-D are still made by dimensional splitting.) Hence, like all other grid-aligned 1-D upwind schemes it leads to the inconsistency that whereas much mathematical rigor is involved in the 1-D upwind scheme itself, hardly any mathematics is involved in the final extension to multi-D. Further, it also leads to the inconsistency that whereas upwind discretizations should be dependent on the solution solely, a grid-aligned 1-D upwind scheme applied in multi-D also depends on the grid.

Y. Tamura and K. Fujii: 'A multi-dimensional upwind scheme for the Euler equations on structured grids':
Tamura presents a multi-D upwind scheme on a structured grid [51]. The scheme is based on a grid-independent scheme for scalar convection equations and on a conventional 1-D approximate Riemann solver with MUSCL extension. Results for both scalar equations and the Euler equations are presented to show improved resolution of discontinuities. An interesting test case considered by Tamura (in his presentation, not in his paper) is a symmetric flow around a full NACA0012-airfoil. Comparing for this test case the results obtained by a standard, grid-aligned 1-D upwind scheme with the results obtained by the multi-D upwind scheme of Tamura et al., it clearly appears that the multi-D upwind scheme leads to a sharper resolution of discontinuities. However, it also appears that the multi-D upwind scheme gives no perfect symmetry, whereas the standard, grid-aligned 1-D upwind scheme does. The observed asymmetry indicates either an error in the multi-D upwind method or - most probably - a convergence problem. It seems a good idea that in validating multi-D upwind methods, at least one symmetrical test case is considered.

J.A. Sethian: 'Computing turbulent flows in complex geometries: vortex methods and massively parallel processors':
Sethian considers a nice collection of unsteady, 2-D test cases: a backward facing step flow, the Helmholtz flow and flows around an impulsively started airfoil, a pitching airfoil, an airfoil at high angle of attack and a circular cylinder. Of these test cases, in particular the two moving airfoil cases seem to be well-suited for illustrating the merits of a vortex method. (When a fixed frame of reference can be chosen, an Eulerian approach often seems to be preferable.) In visualizing his flows, Sethian makes use of particle injection by a mouse. An interesting possibility of a vortex method combined with a flexible visualization system is to make comparisons with classical flow visualizations made in experiments (e.g. those from [13, 45]).

S.M. Belotserkovsky and L.I. Turchak: 'Method of discrete vortices and applications':
Turchak gives the presentation. The flows that he considers are incompressible potential flows with vorticity shed from the body. A difficulty with these flows is the prediction of the point of separation. Turchak remarks that vortex methods are of particular importance to aeroacoustics.

D. Hänel, M. Breuer, J. Klöker and M. Meinke: 'Computation of unsteady vortical flows':
Hänel presents paper [17]. As opposed to the two previous speakers, Hänel does not consider a Lagrangian, but an Eulerian approach. Though not in his paper, Hänel also shows results of vortex breakdown over a delta wing. Hänel mentions that due to the very small length scales which he still considers, his computing times are enormous.

H. Deconinck, P.L. Roe and R. Struijs: 'A multidimensional generalization of Roe's flux-difference splitter for the Euler equations':
Deconinck, who presents paper [8], clearly explains the extension of Roe's upwind scheme for the 1-D Euler equations to the 2-D Euler equations. In particular he clearly shows that a space discretization which uses triangles, gives computational efficiency because the resulting linear solution distributions on the triangles lead to constant solution gradients which can be taken out of the finite volume integrals.

F.T. Johnson, M.B. Bieterman, J.E. Bussoletti, C.L. Hilmes, R.G. Melvin, D.P. Young: 'Application experiences related to solution adaptive grids':
The speaker from Boeing presents experiences related to Boeing's solution adaptive grid code TRANAIR [24]. The code solves the full potential equation employing a finite element discretization in conjunction with a locally refined rectangular grid. It is capable of analyzing flows around arbitrary 3-D geometries and it is routinely based for that purpose in many application areas. The code contains an algorithm which can adaptively refine or de-refine each grid cell as the solution proceeds. The algo-

rithm is based on an oct-tree data structure. (Semi-refinement is not applied.) To take into account viscous effects, the potential flow method may be coupled with a boundary layer code. The Boeing people do not apply multigrid acceleration, neither do they apply estimates of the local truncation error for their refinement criterion. (For refinement, physical criteria are applied.) In relation to the solution-adaptive multigrid work that we do at CWI (see section 2.2), it seems useful to exchange reports.

H. Guillard, J.M. Malé and R. Peyret: 'Numerical simulation of compressible mixing layers using adaptive spectral methods':
Peyret gives a clear presentation on the application of spectral methods to non-smooth problems [16]. A general comment made by Peyret is that the further development of spectral methods slows down.

4.4 Panel discussions

Parallel architectures versus mainframe supercomputers:
chairman: J.-J. Chattot (University of California, Davis).
panel members: M. Leca (ONERA),
M. Merriam (NASA Ames Research Center).

The chairman and the two panel members give their view on the parallelization of CFD codes. Chattot mentions that a close cooperation has been set up on the parallelization of CFD codes, between the University of California at Davis and the NASA Ames Research Center. Chattot emphasizes that discretizations with compact stencils are important for parallel processing, this because of their low message passing across internal boundaries. After Leca has explained the status of parallel processing in CFD at ONERA, Merriam speaks about the status at NASA Ames. At NASA Ames much effort is put into the implementation of CFD codes on massively parallel, distributed memory computers. (A large project has been started into this direction: the Touchstone project.) Merriam points out that the upper limit for speed-up which is dictated by the amount of intrinsic sequentialness in an algorithm (Amdahl's law), is always very far away in CFD applications. (According to Merriam, for CFD applications the number of parallel processors that is wanted, will always be very large.) In response to a question from the audience whether interactive visualization will inhibit a general applicability of massively parallel, distributed memory machines, Merriam answers that he does not see problems here, because - as he says - for visualization one generally does not need more data than one can show on a workstation screen.

CFD on complex grids:
chairman: J.L. Steger (University of California, Davis).
panel members: J. Benek (Arnold Air Force Station),
K. Nakahashi (University of Osaka),
N. Weatherill (University College of Wales).

Weatherill states that the more complex geometries are, the more preferable are unstructured grids above structured multi-block grids. Benek points out that even more difficult than the grid generation is the mathematical definition of geometries from e.g. CAD output or wind tunnel models. In response to a question from the audience why not to get rid of all grid generation problems by applying grid-free formulations, Benek answers that, to his opinion, there is a conservation of difficulties. In response to this question, Steger still remarks that the alternative approach of applying Cartesian grid methods with sliced-off cells (see e.g. [38, 42, 56]) isn't a remedy either, this because Cartesian grid methods do not work well for Navier-Stokes. (To my opinion, the latter problem is certainly something that still can be improved; Cartesian grid methods are still relatively underdeveloped.)

Centered versus upwind schemes:

chairman: M.M. Hafez (University of California, Davis).
panel members: J. South (NASA Langley Research Center),
R.W. MacCormack (Stanford University),
H. Deconinck (Von Karman Institute for Fluid Dynamics),
S.T. Zalesak (NASA Goddard Space Flight Center).

South states that multigrid methods work better in case of central differencing than in case of upwind differencing. He tries to elucidate this statement by saying that it is in particular the presence of tuning parameters in central difference methods which makes multigridding and central differencing such a good combination. Further, the statement seems to be supported by the widely spread use of Jameson's central-difference-based multigrid codes. A comment that can be made on the statement is that various publications exist now in which multigrid (both linear and nonlinear) is successfully combined with dimensionally split 1-D upwind schemes (see e.g. [34, 43] for linear multigrid and [11, 19] for nonlinear multigrid). Even some early results are available now in which multigrid is successfully combined with multi-D upwind schemes [5, 29]. What is essential for a successful application of multigrid is good smoothing. First-order accurate upwind schemes (both 1-D and multi-D) lead (in a natural way) to better smoothing than central schemes.

Another statement made by South is that solution-adaptive grid techniques will make genuinely multi-D upwind schemes obsolete before becoming mature. A comment that can be made on this is that future CFD codes may very well be codes in which different numerical techniques are combined for a single numerical purpose, for instance for obtaining a good accuracy. Multi-D upwinding plus solution-adaptive gridding is an example of such a combination. (In fact, the work reported in [20, 33, 39, 40] was carried in the framework of a project which aims at combining just these two techniques.)

MacCormack states that upwind schemes are naturally first-order accurate. (The scalar linear convection equation already shows that this is not true in general.) A different statement of MacCormack is that upwind differencing can be seen as central differencing with artificial diffusion, but that in practice central discretizations do not get added such sophisticated artificial diffusion as naturally occurring in upwind discretizations.

Deconinck gives a brief overview of multi-D upwind approaches with special emphasis on genuinely multi-D upwind methods. In response to Deconinck's survey presentation, Lerat remarks that phenomena such as acoustic waves have no directional preference (are isotropic) and therefore undermine the relevance of genuinely multi-D upwind methods. Deconinck answers that to his opinion it is a good striving to apply as much upwinding as physically admissible.

An opinion I have is that due to e.g. the explicit, centered work of Jameson et als. [23] and the implicit, centered work of Lerat et als. [37], centered methods can be considered as rather mature now, whereas upwind methods cannot.

5 Visit to the Workshop on Algorithmic Trends for Computational Fluid Dynamics in the 90's, Hampton, Virginia, September 15-17

5.1 Workshop

The workshop is organized by ICASE (M.Y. Hussaini) and the NASA Langley Research Center (A. Kumar and M.D. Salas). Citing the organizers, the purpose of the workshop is to bring together numerical analysts and computational fluid dynamicists, in order (i) to assess the state of the art in the areas of numerical analysis particularly relevant to CFD, (ii) to identify promising new developments in various areas of numerical analysis that will have impact on CFD and (iii) to establish future research directions for CFD. For this purpose, the workshop has five sessions with per session some four or five speakers with a known expertise in the corresponding field of research. The list of sessions and speakers is given below. Only invited presentations are given. Participation as listener is also on invitation. (The total number of participants is about 110.) The organization is intended to publish the proceedings in the ICASE/LaRC Series of Springer-Verlag.

Overviews:
chairman: R.W. MacCormack (Stanford University).
speakers: Ch. Hirsch (Vrije Universiteit Brussel),
O. Pironneau (INRIA Rocquencourt),
M.H. Schultz (Yale University),
J.L. Steger (University of California, Davis).

Acceleration techniques:
chairman: M.M. Hafez (University of California, Davis).
speakers: B. van Leer (University of Michigan),
R. Temam (Indiana University),
A. Jameson (Princeton University),
Y. Saad (University of Minnesota),
G.H. Golub (Stanford University).

Spectral and higher-order methods:
chairman: S.A. Orszag (Princeton University).
speakers: C. Canuto (Politecnico di Torino),
D.I. Gottlieb (Brown University),
S.J. Osher (University of California, Los Angeles),
S.T. Zalesak (NASA Goddard Space Flight Center).

Multi-resolution and subcell resolution schemes:
chairman: H.-O. Kreiss (University of California, Los Angeles).
speakers: R. Glowinski (University of Houston),
L.F. Greengard (Courant Institute of Mathematical Sciences),
E.V. Harabetian (University of Michigan),
A. Harten (Tel-Aviv University),
K.G. Powell (University of Michigan).

Inherently multi-dimensional schemes:
chairman: K.W. Morton (Oxford University).
speakers: P.L. Roe (University of Michigan),
H. Deconinck (Von Karman Institute for Fluid Dynamics),
R.A. Nicolaides (Carnegie-Mellon University).

Panel discussions:
chairman: M.D. Salas (NASA Langley Research Center).
panel members: R.W. MacCormack (Stanford University),
S.J. Osher (University of California, Los Angeles),
H.-O. Kreiss (University of California, Los Angeles),
M.M. Hafez (University of California, Davis),
K.W. Morton (Oxford University),
R. Peyret (Université de Nice).

5.2 Presentations

Ch. Hirsch:
Hirsch gives a general overview. He touches among others: Jameson's central discretization for the Euler equations (its 2nd-order dissipation for shock capturing and its 4th-order dissipation for damping

background noise) and the classical upwind approaches (the approaches which introduce physical information only at the level of discretization). Seen from a Taylor series expansion view-point, he discusses the position of flux-difference splitting schemes in between flux splitting schemes and central differencing schemes:

$$f_{i+\frac{1}{2}} = \frac{1}{2}(f_i + f_{i+1}) \tag{1}$$

for central schemes,

$$f_{i+\frac{1}{2}} = \frac{1}{2}(f_i + f_{i+1}) - \frac{1}{2} | A |_{i+\frac{1}{2}} (q_{i+1} - q_i) \tag{2}$$

for flux-difference splitting schemes and

$$f_{i+\frac{1}{2}} = f_i^+ + f_{i+1}^- = \frac{1}{2}(f_i + f_{i+1}) - \frac{1}{2} | A |_{i+\frac{1}{2}} (q_{i+1} - q_i) + O(h^2) \tag{3}$$

for flux-vector splitting schemes. He also points out basic differences between the flux-difference splitting schemes of Roe and Osher. (Whereas in Roe's scheme the numerical flux is approximated by a sum in the discrete space, expressed in terms of $(\sqrt{\rho}, \sqrt{\rho}u, \sqrt{\rho}v, \sqrt{\rho}H)^T$, in Osher's scheme it is an integration in the continuous state space, expressed in terms of $(u, v, \sqrt{\gamma p/\rho}, p\rho^{-\gamma})^T$.) Further, he discusses the TVD concept as introduced by Harten, and mentions the multigrid technique as presented by Radespiel at the 1991 AIAA CFD conference in Honolulu (a multigrid method which uses a central discretization on the finest grid and an upwind discretization on coarser grids). Hirsch concludes that to his opinion a major requirement for future CFD codes will be the capability to test for unsteadyness. Important future research directions he sees are: mesh adaptation (both structured and unstructured) and multi-D upwind methods.

O. Pironneau:
Pironneau presents a numerical method in which the compressible Navier-Stokes equations are interpreted as an extension of the incompressible Navier-Stokes equations. Further, he presents a novel discretization of the diffusive terms and a model equation which he calls a compressible Stokes equation.

M.H. Schultz:
Schultz mentions that the computer architecture considered in the Touchstone project (see section 4.4) is a hypercube.

B. van Leer:
In response to an early comment made during the workshop, that numerical techniques for the Euler equations are mature now, Van Leer opens his presentation by stating that to his opinion this is not yet the case. (He reveals that the night before, Euler has appeared to him in a dream, saying that he is still alive.)

Van Leer explains the characteristic time stepping technique for removing stiffness from the Euler equations. In relation to global and local time stepping, characteristic time stepping can be interpreted in the following way:

time stepping	Δt
global	same Δt in all cells
local	same CFL number in all cells
characteristic	same CFL number for all equations

Van Leer considers three separate stiff flow regimes: (i) a subsonic regime ($M < 0.5$), (ii) a transonic regime ($0.5 < M < 2$) and (iii) a supersonic regime ($M > 2$). By these choices of M it can be verified that in each of the three flow regimes, the lowest possible value of the Jacobian's condition number K is 3 (subsonic flow regime: $K = \frac{M+1}{M}$, transonic flow regime: $K = \frac{M+1}{|M-1|}$, supersonic flow regime: $K = \frac{M+1}{M-1}$). Van Leer remarks that the clustering of the Jacobian's eigenvalues (which is in fact what is done in characteristic time stepping) can be exploited in further optimizing the smoothing properties of multi-stage time stepping schemes. A problem that already starts to become clear now when applying characteristic time stepping in high-aspect ratio cells occurring at and near the bottom of boundary layers, is that though in such cells the separate wave speeds can still be equalized of course, the value of these equalized speeds may be quite small due to the cell's small height (Fig. 1). A remedy that remains to exist here is to take a column of such cells throughout the boundary layer and to treat these implicitly. In our point relaxation technique for the Euler equations we once considered a technique for removing stiffness at a *discrete* level: row balancing of 4×4-derivative matrices. Extension to Navier-Stokes of this latter approach is rather straightforward.

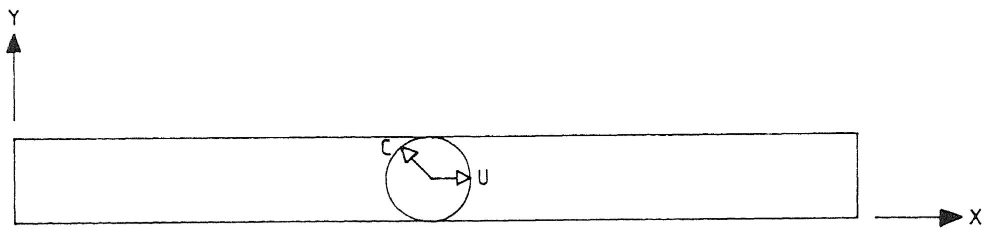

Fig. 1. Equalized speeds in a high-aspect ratio, boundary layer cell.

R. Temam:
Temam presents a new acceleration technique, called IU (Incremental Unknowns), which - as he says - is more flexible than multigrid techniques, in particular for nonlinear problems. Temam does not compare his method's performance with that of a multigrid method.

A. Jameson:
Jameson points out that the multi-stage time stepping schemes that he applies can be interpreted as Gauss-Seidel relaxation methods. Further he states that in multi-D applications, multigrid would need no smoother if a perfect multi-D upwind scheme would exist; i.e. an upwind scheme that perfectly transports the errors to and across the boundaries of the computational domain. Jameson shows Navier-Stokes flow results obtained by one of his central differencing methods. The results he shows are surface pressure distributions only and he compares those with measured wind tunnel data. As opposed to e.g. skin friction distributions, surface pressure distributions are not specific Navier-Stokes-type results. According to [35], Jameson-type convective flux formulas cannot do an accurate job in Navier-Stokes flow computations.

G.H. Golub:
Golub presents a special ordering of grid points for 2-D, topologically rectangular problems (a so-called torus ordering), which seems to allow a good vectorization of Gauss-Seidel relaxation.

S.J. Osher:
Like in Davis (see section 4.3), Osher starts to review ENO work. Some attendees mention the serious difficulties of ENO schemes. Van Leer states that ENO is a good method for 'bad weather' only, a very accurate scheme for discontinuities only. Away from discontinuities, ENO may appear to be an inaccurate scheme. Harten adds that switching from ENO to a more conventional scheme in smooth flow regions, is not only more accurate but also more efficient. Salas concludes on a transparancy that he himself has had ENO(UGH).

A novelty in Osher's presentation is his so-called fast solver via multiscale analysis. For this he considers the Kreiss equation $\frac{\partial u}{\partial t} = -\frac{\partial u}{\partial x}$, which he discretizes in wavelet space. Osher points out the multilevel aspect of wavelets in the time integration of the Kreiss equation. He remarks that in the resulting discrete system, a sparse initial matrix will remain sparse during time integration; fill-in will not occur. Osher says he has a report in preparation on this subject.

S.T. Zalesak:
Zalesak presents results for shock tube problems, obtained by ENO schemes of various orders of accuracy. By extending his stencils more and more, he goes to a higher and higher order of accuracy, $O(h^{16})$ e.g.. Instead of driving 1-D upwind schemes to such, for practice not very interesting limits, it seems more relevant these days to go to multi-D.

R. Glowinski:
According to Glowinski, in the family of existing discretization methods ordered according to increasing difficulty (finite difference, finite volume, finite element and spectral methods), wavelet methods should (as a new member) be placed at the end (so, after spectral methods). Since wavelets are more local than the approximating functions in spectral methods, this ordering is not obvious. (Intuitively one would expect that greater localness also leads to greater simplicity.) Glowinski mentions [50] as a good review article on wavelets. An impression that remains after Glowinski's presentation is that there is still much progress in grid methods. What is new today, may be obsolete next year already. There is probably less development in the field of grid-free methods.

A. Harten:
Analogous to Osher and Temam, Harten also presents an idea which shows similarities with the multigrid principle.

K.G. Powell:
Powell gives an impressive presentation on a solution-adaptive grid method for the 2-D Euler equations. In the first instance, the method he presents shows much agreement with the method under development at CWI [20, 33, 39, 40]. Powell also applies a tree-type data structure, but whereas we actually use nine pointers per cell, he only uses five pointers per cell. Each of his cells only has a pointer to its four children and to its parent, whereas our cells also have pointers to their direct neighbors. A benefit of a smaller number of pointers is of course a smaller amount of computer memory required. However, a probable disadvantage is a more cumbersome way to keep track of neighboring cells. So far, Powell does not exploit the existence of a nested family of grids for multigrid acceleration, neither does he apply it for estimating local truncation errors to be eventually used in a 'catch-all' grid refinement criterion. As opposed to Van der Maarel [39], who does apply a 'catch-all' refinement criterion based on estimated local truncation errors, Powell applies the velocity vector products $\nabla \cdot \mathbf{u}$ and $\nabla \times \mathbf{u}$. The first product is taken as a measure for compressibility and the Euclidian norm of the latter product as a measure for vorticity. Powell's choice of the treshold value for the refinement criterion is rather free. It may be determined by either computational resources available, by experiments, by a histogram with the number of cells flagged to be refined versus the treshold value for the refinement criterion [7], or by the standard deviation from a mean value for the refinement criterion considered. (The reasoning behind this last choice is that it guarantees that there is a bound for the number of refinements in that case; the number of refinements is known to decrease then with increasing grid level.) Similar to our method, Powell also does not consider internal boundaries (green boundaries [20]) with jumps in mesh size larger than a factor two. A difference along these internal boundaries may however exist with respect to the discretization of the equations. In [39] it has been found that one has to be careful in preserving the order of accuracy along these internal boundaries.

A very nice property of Powell's method is his application of a Cartesian grid approach such as in [38, 42, 56]. As a consequence, his basic grids (i.e. the grids on which he starts to introduce his solution-dependent grid refinements) may already have local, *geometry-dependent* refinements along bodies. (Here, the local body curvature is the refinement criterion.)

Another nice property of Powell's method is its capability to compute unsteady flow problems. Making use of the wave speeds that he finds with an approximate Riemann solver (Roe's), he can also predict how the adapted grid for the next time step should look like. Powell shows some impressive unsteady flow results for the standard, supersonic, forward facing step flow. Impressive steady flow results which he shows are those for a standard, steady supersonic wedge-shoulder flow. A remarkable feature in the solutions found by Powell for the latter flow, is the occurrence of a Mach stem in the first shock wave reflecting at the upper surface. (So far, I have not seen this fine detail in the results of others.) Summarizing the computational results shown, we see that Powell has considered various non-smooth flow problems and has obtained very accurate results for these. It would be interesting now to see how his accurate discretization method still compares to other methods, for *smooth* flow problems.

Comparing the solution-adaptive grid methods under development at the University of Michigan and at CWI, it can be concluded that we do not fall foul of each other (see table below) and that it might be interesting to make a comparative study in the near future.

property	*University of Michigan*	*CWI*
flow model	2-D, steady and unsteady Euler	2-D, steady Navier-Stokes
1-D upwind scheme	Roe	Osher
solution method	explicit time stepping , single-grid	point relaxation, multigrid
grid	Cartesian, sliced-off at body	curvilinear, body-fitted
refinement criterion	velocity vector products	estimate local truncation error
number of pointers	five	nine

J.L. Steger:
Steger thinks that routine flow computations around complex configurations are going to be important. Steger mainly works on grid generation now. For this he prefers multi-domain approaches with overlapping grids. Types of subgrids that he considers are so-called collar grids (subgrids used for connecting neighboring subgrids), ray-like or hairy grids (subgrids generated by outward radiating surface grids) and background grids (regular, auxiliary grids used for connecting ray-like grids).

P.L. Roe:
In his presentation entitled 'Beyond the 1-D Riemann problem', Roe starts by stating that a major requirement for CFD codes in the 90's is that they should be robust and stand-alone. (Roe: 'Babysitting CFD codes should become history in the 90's.) Roe points out the difference in difficulties for upwind and central discretizations, when going from a nonlinear, 1-D, scalar convection equation to a nonlinear, multi-D system (see the two tables below, with the arrows indicating straightforward extensions and the question marks non-straightforward extensions).

		scalar	system
Central:	1-D	·······?·······	:
			↓
	multi-D		:

		scalar	system
Upwind:	1-D	······ → ······	:
			?
	multi-D		:

Roe continues by giving a review of good and bad properties related to the standard, dimensionally-split 1-D upwind methods. Some good properties:

- The first-order Godunov scheme is nonlinearly stable in any number of dimensions for a convex state space (which is what we have in case of a perfect gas).

- Shock waves are self-healing. Even when they are smeared, they still have a steepening mechanism. (Moretti: 'Shock waves are garbage collectors'.)

- Contact discontinuities aligned with the grid are treated exactly, which is favorable for the resolution of boundary layer flows.

Some bad properties:

- Multi-D physics is much richer than 1-D physics (omni-directional waves, vorticity, focusing).
- Shear waves oblique to the grid are misinterpreted. This is not healed. (This bad property deteriorates the resolution of vortical flows.)

Finally, Roe mainly discusses the work done by Rumsey et als. on rotated Riemann solvers [46]. Roe shows a poor result obtained by a standard, grid-aligned 1-D upwind scheme and the improved result obtained by a rotated upwind scheme. In response to this, MacCormack asks how the result would like for a central scheme. No central results seem to be available for this case.

R.A. Nicolaides:
Nicolaides presents a novel discretization method for the incompressible Navier-Stokes equations. As test case he considers the rather non-standard Hamel flow in a divergent channel [47].

5.3 Panel discussions

M.D. Salas:
Salas has observed the following:

- Whereas computational results obtained for complex, 3-D configurations are frequently shown now, numerical problems still seem to exist for the linear, scalar convection equation.
- Much attention still exists for initial value problems. Too little attention exists to his opinion for boundary value problems.
- On an average, capabilities of industrial products show a development in time as indicated in Fig. 2. Concerning the present-day capabilities of CFD codes, Salas has observed that the opinions differ about these. Some workshop attendees hold the view that CFD codes are still in their infancy (segment I in Fig. 2), whereas others think that they are almost mature now (segment II in Fig. 2). Salas thinks that the truth lies somewhere in between.

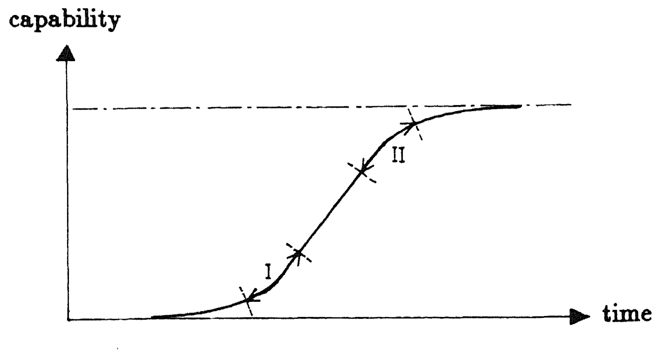

Fig. 2. Typical development curve of industrial products.

R.W. MacCormack:
MacCormack states the following:

- So far, turbulence specialists have not given useful tools to CFD. (All turbulence models actually being used in CFD, have been developed by people who are not turbulence specialists themselves.)
- An impact of parallel computing will be that it leads to a step back in algorithms.
- In industry, there is still little confidence in CFD.

To my opinion, in particular codes with tuning parameters might be conducive to the latter situation.

H.-O. Kreiss:
Kreiss is of the opinion that CFD codes are still in their infancy, simply because accurate and efficient computations of 3-D, turbulent flow problems are still far out of reach.

K.W. Morton:
Morton finally has the following opinions:

- Concerning genuinely, multi-D upwind methods - in agreement with Lerat (see section 4.4) - he thinks that these go too far by upwinding features, like sound, which should not be upwinded. Morton proposes to treat by upwind schemes only those flow features which have specific directions of propagation and to relax all those flow features which have not.
- More knowledge from finite element theory should be exploited.
- More attention should be paid to unsteady flow problems.

5.4 Conclusions

No common view appears to exist about what is the most promising new development in numerical analysis. Likewise, no common view exists about what is *the* future research direction for CFD. (For both, various opinions exist.)

Concerning the question what has contributed much to the progress of CFD in the 80's, it is a pleasure to see that an unanimous opinion among the attendees is that these are multigrid methods. (A multigrid text book is about to appear, which covers this last decade [55].)

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Contents