Centrum voor Wiskunde en Informatica

REPORT*RAPPORT*

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Report BS-R9416 May 1994

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SMC is sponsored by the Netherlands Organization for Scientific Research (NWO). CWI is a member of ERCIM, the European Research Consortium for Informatics and Mathematics.

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Top-Down Image Analysis by Cost Minimization in Hierarchical Graph Structures

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Abstract

This paper presents a top-down search method for boundary refinement based on cost minimization. Many image segmentation methods produce a series of successively coarser image subdivisions, which can be represented in a hierarchy of graphs. Sometimes it is necessary to refine boundaries in the coarsest segmentation by considering the successively finer segmentations presented in the hierarchy. We consider the problem of the detection of fiber boundaries in microscope images of muscle tissue. Based on a model for muscle fibers, a cost function suitable for application in hierarchies of graphs is derived. This cost function depends on both the grey value gradient along the boundary and on boundary shape. It is shown that minimization of this cost function reliably detects fiber boundaries.

AMS Subject Classification (1991): 68U10, 90C39
Keywords & Phrases: dynamic programming, curve detection, hierarchy of graphs
Note: This research was supported by the Foundation for Computer Science in the Netherlands (SION) with financial support from the Netherlands Organisation for Scientific Research (NWO). This research was part of a project in which the TNO Institute for Human Factors, CWI and the University of Amsterdam cooperate.

1. Introduction

Many image segmentation techniques [11, 14, 13, 1] produce a series of coarser and coarser image representations by initially considering individual pixels as regions, and subsequently merging clusters of adjacent regions into larger regions. This yields a sequence of image subdivisions, which can be represented conveniently through a hierarchy of region adjacency graphs [12].

Such a description reflects the structure of the image, but sometimes the final, top level segmentation must be refined and some effort must be made to extract relevant information. Each level in the hierarchy represents the subdivision of the image plane in a number of regions. Often a significant object in the image will coincide with such a region, but sometimes this is not the case. It is possible that an object is represented by several regions in one level, while it is only a part of a region in the next level. It is also possible, that the location of region boundaries in higher levels of the hierarchy is not accurate and some refinement is required.

Report BS-R9416
ISSN 0924-0659
CWI
P.O. Box 94079, 1090 GB Amsterdam, The Netherlands

Moreover, not all regions have the same significance. Some search method is required to detect the most significant structures in the hierarchy.

In this paper, a top-down boundary localization method in a hierarchy of region adjacency graphs is discussed. Such a method uses a priori knowledge of image content and must therefore be adapted to a particular problem. Our specific application is the detection of fibers in a microscope image of muscle tissue (figure 1.1). The individual fibers appear in the image as more or less round regions with a uniform grey value, which are bounded by a curve on which the image gradient is large.

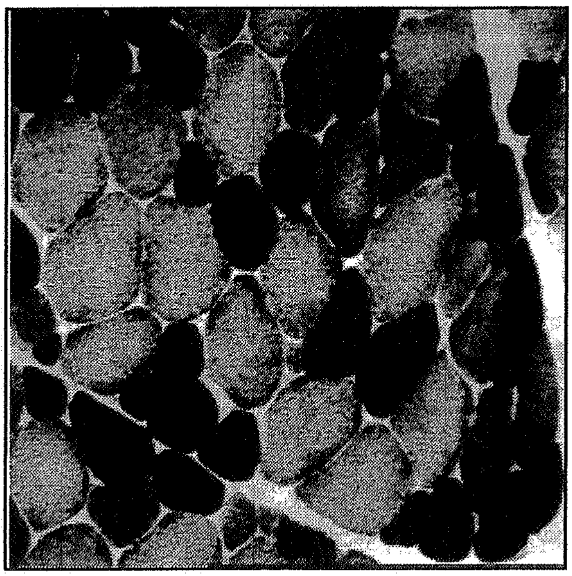


Figure 1.1: An image of muscle tissue, on which our top-down methods will be tested.

The top-down method presented in this paper is based on cost minimization by dynamic programming. Dynamic programming (e.g. [2]) is a well-known technique for many optimization problems and has been used for a long time in image processing for edge detection, e.g. by Montanari [10] and Martelli [8, 9]. Recent contributions have been made by Gerbrands [3] and Orange [15].

In these techniques, some initial guess for the approximate location of the curve must be supplied. The boundary of a region in a high level of the hierarchy is used as an initial guess, while the boundary parts in an intermediate level of the hierarchy are used as possible parts of the exact boundary.

The organization of the rest of this paper is as follows. A series of coarser and coarser image segmentations can be represented by a hierarchy of region adjacency graphs. Object boundaries correspond to closed paths in the dual of the region adjacency graph. This is explained in section 2.

Section 3 presents an optimization approach for the detection of fiber boundaries. Based on our model of boundaries—more or less round curves through points with a large gradient in the image—a cost function is constructed, such that minimal cost paths correspond to fiber boundaries. Dynamic programming is used to detect optimal paths.

In section 4, the optimization approach is used in a hierarchical structure in order to achieve a stepwise refinement of object boundaries. Some results are shown and the effect of parameter choices is illustrated.

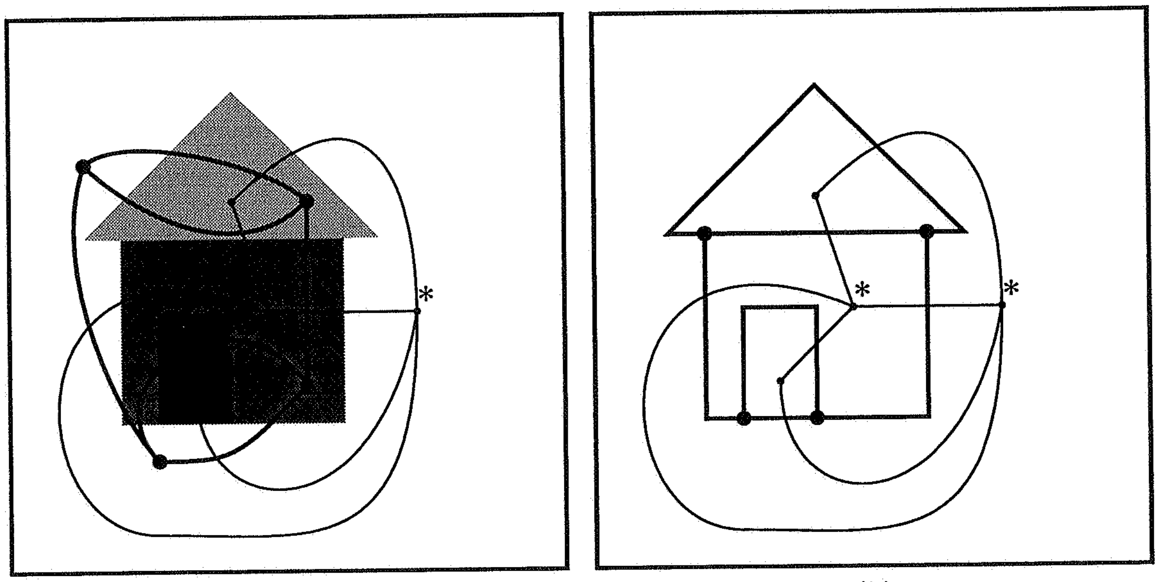
Section 5 presents the conclusions of this paper.

2. Hierarchies of Region Adjacency Graphs and their Duals.

A partition of the image plane into connected regions can be represented by a region adjacency graph. In this representation, each region acts as a vertex in a graph. Two vertices are connected by an edge if the corresponfing regions are adjacent.

The edges in a region adjacency graph correspond with boundaries between regions in the image. By selecting a proper set of edges in the region adjacency graph, the outline of an object in the image can be constructed. This section presents an optimization approach for the computation of edge sets which correspond to fiber boundaries.

Edges in the region adjacency graph correspond to boundary parts in the image plane. In order to select series of consecutive boundary parts, a representation is required in which it can be seen, which edges of the region adjacency graph represent consecutive curves in the image plane. In the region adjacency graph, this ordering is not represented explicitly.



(a) (b)

Figure 2.1: Figure (a) shows a region adjacency graph (in thin lines) and its dual (in bold lines). Note the correspondence between faces of one graph and vertices of the other one. Note also the correspondence between mutually intersecting edges of both graphs. In figure (b), the edges of the dual graph have been redrawn in the shape of the curves they represent in the image plane. There is a double edge between the vertices marked by a $*$ because the boundary between the corresponding regions consists of two connected components.

For this purpose, the *dual* [4] of a region adjacency graph must be used.

Definition 2.1 *Let G be a planar graph with vertices V and edges E. Its dual graph is denoted by $\overline{G}$ and its vertex and edge sets by $\overline{V}$ and $\overline{E}$, respectively. Each vertex in $\overline{G}$ corresponds with a face (a region surrounded by a closed path and with no interior edges) of G. Two vertices of $\overline{G}$ are connected by an edge if the corresponding faces in G share an edge.*

The faces of the dual graph correspond to vertices in the original graph. There is a one-to-one relation between the edges of G and $\overline{G}$.

Dual graphs are defined for graphs which are embedded in the plane; for regions adjacency graphs, such an embedding exists obviously. This is illustrated in figure 2.1. In the present context, it is necessary to take double edges into account. Double edges occur when the boundary between two regions consists of more than one connected component. This is the case, for example, for the two vertices marked by a $*$ in figure 2.1: the boundaries between the corresponding regions consists of two connected components.

It is useful here to consider the image grid in terms of pixels, cracks and points. In this view, pixels correspond to open squares of the form $(x, x+1) \times (y, y+1)$ with $x, y \in \mathbb{Z}$, in the Euclidean plane. Pairs of directly adjacent pixels (4-neighbors) meet at *cracks*, which are line segments of the form $(x, x+1) \times \{y\}$ or $\{x\} \times (y, y+1)$. Four pixels meet in points (x, y). This description corresponds with the topological notion of a *cell complex*, as noted by Kovalevsky [6].

Edges in the dual region adjacency graph correspond to boundaries between regions in the image plane. These boundaries are formed by series of cracks, connected at points. The vertices in the dual region adjacency graph correspond to points in the image plane where boundaries between regions meet.

Let $\overline{v}_0, \overline{v}_1, \ldots, \overline{v}_n$ be a path in the dual graph and let $\overline{e}_i$ be the edge between $\overline{v}_{i-1}$ and $\overline{v}_i$. Then $\overline{e}_1, \overline{e}_2, \ldots, \overline{e}_n$ corresponds with a series of contiguous curves in the image plane.

From a hierarchy of region adjacency graphs, a hierarchy of dual region adjacency graphs can be constructed. This has been described in detail by Kropatsch *et al.* [7] for the situation where the hierarchy is built by graph decimation, but their description is also valid for more general hierarchies. The levels of the dual hierarchy are constructed by constructing the dual graph for each level independently. Each edge in a higher level of the dual hierarchy corresponds to a series of contiguous edges in a lower level. In terms of curves in the image plane, this means that each edge in a higher level represents a curve which consists of a number of consecutive parts, which are each represented by an edge in a lower level.

Consider a path in some higher level of a hierarchy of dual region adjacency graphs and the curve it represents in the image plane. This curve can also be represented by a path in some lower level of the dual hierarchy. This path can be constructed by considering all the edges occurring in the higher level, and computing the corresponding edges in the lower level graph. Concatenating all of these edges yields the desired path in the lower level.

3. Construction of the Cost Function

Closed paths in a dual region adjacency graph correspond with closed curves in the image plane. In this section, a cost minimization procedure will be used in order to detect closed curves which correspond to fiber boundaries. This will be done by means of a hierarchical method. This section describes the procedure used in each level; the next sections describe the hierarchical procedure and presents some results.

In this section, three issues must be addressed: a suitable cost function must be constructed, a set of allowed paths must be defined and an optimization algorithm must be chosen. The optimization procedure will be carried out by dynamic programming. Using dynamic programming, we will minimize a cost function of the form

$$C(\overline{v}_0, \overline{v}_1, \ldots, \overline{v}_n) = (1-\alpha) \sum_i P(\overline{v}_i, \overline{v}_{i+1}) + \alpha \sum_i Q(\overline{v}_{i-1}, \overline{v}_i, \overline{v}_{i+1}). \tag{3.1}$$

We consider closed paths, so $\overline{v}_{n+i}$ must be read as $\overline{v}_i$ for $i = 1, 2$. The terms $P(\overline{v}_i, \overline{v}_{i+1})$ represent the cost contributed by pairs of adjacent vertices, i.e. by edges. The terms $Q(\overline{v}_i, \overline{v}_{i+1}, \overline{v}_{i+2})$

represent the cost contributed by triples of consecutive vertices, i.e. by pairs of contiguous edges. In the rest of this section, P and Q will be chosen such that P depends on the extent to which the curve follows maximal gradient paths, while Q depends on the circularity of the curve. The weight factor α can be adapted to make one of these curve characteristics more dominant. As the costs P and Q will be positive, dynamic programming can be used for the computation of a minimal cost path.

Fiber boundaries are characterized by two properties:

1. Fiber boundaries separate regions of different brightness.
2. Fiber boundaries are more or less round.

The cost function will be chosen such that this type of curves can be detected.

The terms $P(\overline{v}_i, \overline{v}_{i+1})$ will be used to express the extent to which the curve separates regions of different brightness. Each pair $(\overline{v}_i, \overline{v}_{i+1})$ corresponds with a curve in the image plane. Let $c_1, c_2, \ldots, c_k$ be the cracks that constitute this curve, where k is the length of the curve. As we are interested in the actual localization of curves in the image plane, and not in the number of edges by which they are represented, the cost function $\sum_i P(\overline{v}_i, \overline{v}_{i+1})$ should not depend on the number of parts from which a curve is constitutes, but only on layout of the curve on the image plane. Therefore, P must have the form

$$P(\overline{v}_i, \overline{v}_{i+1}) = \sum_{j=1}^{k} p(c_j), \tag{3.2}$$

where $p(c_j)$ is a measure for the contrast across a single crack. A common choice in literature for the contrast function $p(c_j)$ is $M - |\nabla f(c_j)|$, where M is some large constant and $\nabla f(c_j)$ is a measure of the image gradient at the crack; usually, it is simply the difference of the grey values of the pixels on both sides of the crack. This choice has an undesirable property: because all $p(c_j)$ must be positive, M must be larger than the largest gradient value in the image. This can be a large number, implying that most of the time, $p(c_j)$ will be quite large. Therefore, short curves will be favored, which may lead to artifacts. In our application, parts of fibers might be cut off, as a shorter path through the interior of a fiber may have a lower cost than the actual boundary, which is longer.

A more suitable choice is therefore

$$p(c_j) = \frac{1}{\epsilon + |\nabla f(c_j)|}, \tag{3.3}$$

where ϵ is some small number (in our implementation 0.1) which avoids division by zero, and $\nabla f(c)$ is an estimator for the image gradient. In our case the difference of the grey values of the pixels on both sides of the crack. With this choice, the presence of large gradients in the image does not enforce an increase of the cost of all the cracks. Moreover, this choice has a clear interpretation: a path with a given length and grey value contrast has the same cost as a path which is twice as long, but has also twice the contrast.

The second criterion for fiber boundaries is that they are more or less round. The roundness measure proposed here uses an estimation $c = (x_c, y_c)$ of the center of the fiber. Consider a curve segment with end points $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$. If this curve part is part of a round curve around c, the vector $p_2 - p_1$ is perpendicular to the vector between c and the center $(1/2)(p_1 + p_2)$ of the line segment $p_1 p_2$. The deviation θ of the angle between the vectors $p_2 - p_1$ and $(1/2)(p_1 + p_2) - c$ from the optimal value of $\pi/2$ is a measure for the non-roundness of the curve.

This observation is used for the construction of $Q(\overline{v}_i, \overline{v}_{i+1}, \overline{v}_{i+2})$. The three vertices define two adjacent curve segments. The curve represented by the concatenation of the two corresponding edges is considered, and the direction of the line segment between its end points is compared with the optimal direction given the estimated center c. Let θ denote the deviation from the optimal angle and let l denote the length of the line segment between both end points. Then Q is defined by

$$Q(\overline{v}_i, \overline{v}_{i+1}, \overline{v}_{i+2}) = l\theta^2. \tag{3.4}$$

The square is introduced to reduce the cost of small deviations, while increasing the cost of large deviations. This allows more deviations from exact circularity than a linear cost function. The length l is used again because the cost must be proportional to the actual length of the curve in the image plane and not to the number of edges by which it is represented.

The boundary shape is evaluated using two edges instead of one. This is done in order to avoid undesirable effects which can occur if the curves represented by edges become short at the lower levels of the hierarchy. This can be especially disadvantageous in the lowest level, where edges correspond with individual cracks and only horizontal and vertical directions are possible.

We still have to define the set of allowed paths over which cost minimization must be performed. As we will work in a top-down procedure, there will be an initial coarse guess, which was computed at some higher level. This guess is some closed curve in a dual region adjacency graph. Let $X \subset \overline{E}$ be the set of edges in this path. Then a set $Y \subset \overline{E}$ of edges is constructed which lie in a strip around the coarse path. An edge $\overline{e} \in \overline{E}$ is in Y if and only there is a face in the dual region adjacency graph which is bounded by both e and some edge in X. Thus, the allowed paths all lie in a narrow strip which follows the initial guess.

The set of allowed paths is the set of all paths containing only edges in Y. One restriction must be made: in order to avoid artifacts, paths which do not go around the estimated center c must be excluded. This is done by allowing certain pairs of adjacent vertices to occur only in a particular order in the path, effectively making the edges in the dual graph directed.

4. Results of the Hierarchical Search Method

The cost minimization approach described in the previous sections can be used for top-down boundary refinement in a hierarchical structure. In the examples presented here, we use a hierarchy of region adjacency graphs constructed by the method presented in Nacken [13]. Figure 4.1 shows some levels in this hierarchy.

As the initial guess for the curve boundary, the boundary of the receptive field of some vertex in the hierarchy of region adjacency graphs is used. An estimation for the center of the corresponding fiber must also be given. At present, these initial guesses are generated by a human operator, but they can also be generated automatically.

The circumference of the selected region corresponds with a closed path in the top level of the dual region adjacency graph. Top-down search is started at some intermediate level in the hierarchy, typically the third one. The curve corresponding to the initial guess is represented as a closed path in this level. Then the allowed paths are defined by selecting a set of allowed edges, and the minimal cost path is computed, yielding the first refinement of the initial guess. This path is then represented at the next lowest level, where it is again used as an initial guess, which is refined by cost minimization. This procedure is repeated, until the final curve in the base level of the hierarchy is reached.

Some results are presented in figure 4.2. The pictures show the initial guess, generated at the top level, the allowed edges in the third level of the hierarchy, the minimal cost path

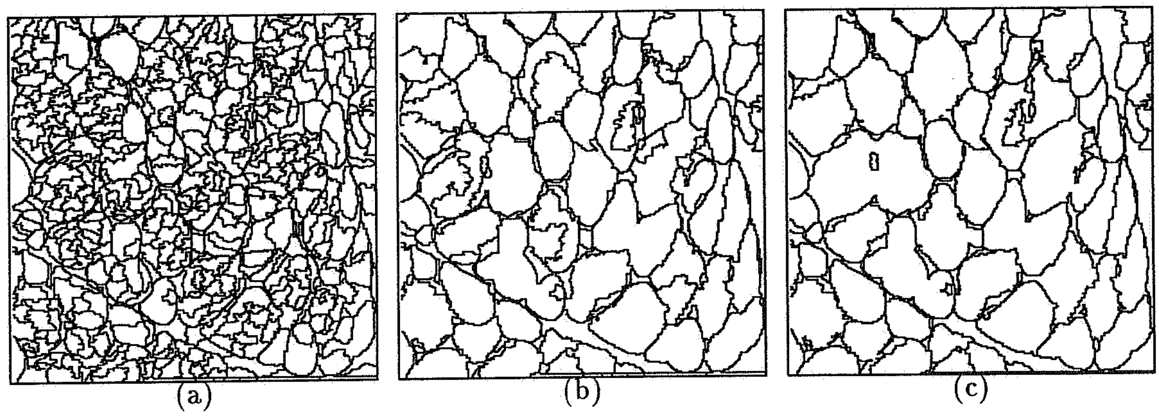


Figure 4.1: The segmentations corresponding to several levels of the hierarchy. The segmentation 6 levels below the top (a), the segmentation 4 levels below the top (b) and the top level segmentation (c) are shown. Note that the base level segmentation corresponds with the pixel grid.

detected at the third level of the hierarchy and the final path, detected at the base level. Note that the initial guess shows some large deviations from the true fiber boundary. In terms of the hierarchy, these deviations are small, because they correspond to just a few 'wrong' parent-child links. Therefore, the path refinement procedure has no difficulties in finding the correct boundary. The method described here detects almost all fibers correctly. In practice, errors occur only when the structure of the hierarchy is deformed so much that the correct fiber boundaries cannot be detected by repeatedly applying relatively small changes in successive levels.

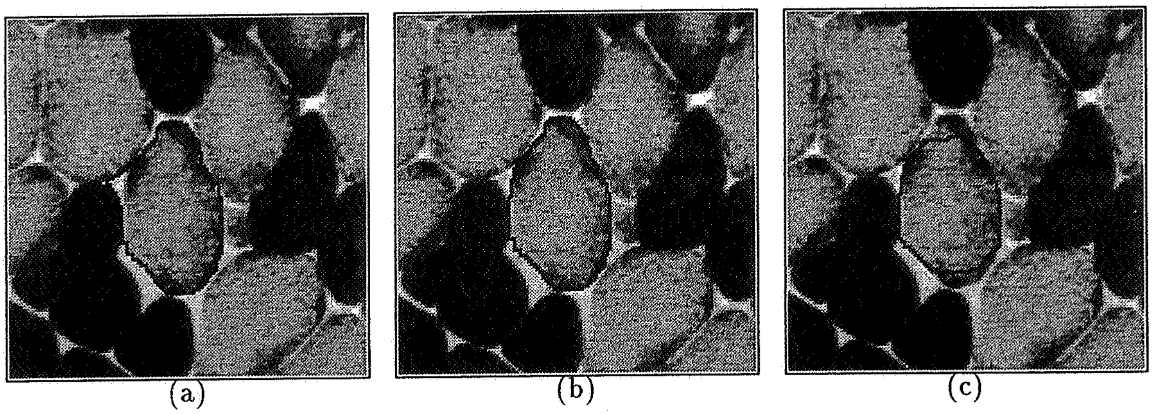
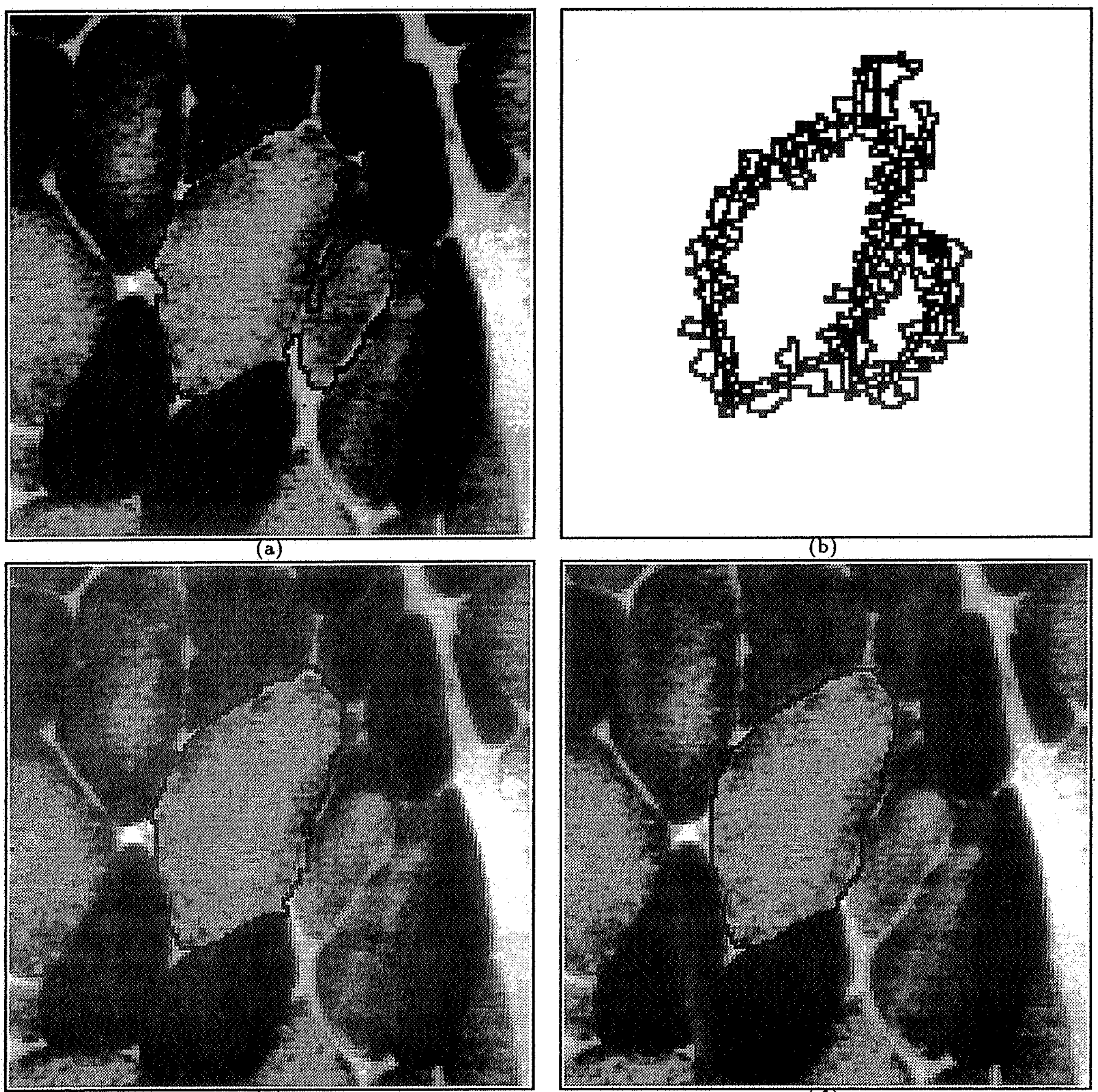


Figure 4.3: Optimal boundaries detected with $\alpha = 0.002$ (a), $\alpha = 0.01$ (b) and $\alpha = 0.05$ (c).

The parameter α in equation (3.1) was chosen to be 0.01. With this choice the P and Q terms have about the same order of magnitude, and both shape and gradient information are taken into account. The value of α for which both terms have the same order of magnitude is image dependent. The contrast cost function depends on the grey values present in the image, while the shape cost function does not. Therefore, the optimal value of α is related to the contrast or the grey value range of the image. In practice, it appears that one value of α suffices for all fibers in an image.

Figure 4.3 shows the effect of modification of α. In the middle image, $\alpha = 0.01$ and the



(a) (b) (c) (d)

Figure 4.2: Boundary refinement by cost minimization: the initial guess (a), the set of allowed edges in the intermediate level (b), the minimal cost path at the intermediate level (c) and the final path in the base level (d).

fiber boundary is detected correctly. In the left image, $\alpha = 0.002$ was used. The cost function is dominated by contrast terms and the boundary traces strong contrasts, yielding an undesirable bump on the boundary. In the right image, $\alpha = 0.05$ was used. The cost function is dominated by the shape terms and a more or less circular boundary is generated, although this one does not follow brightness edges.

5. Conclusions

If a hierarchical image description is constructed in a bottom-up fashion, some post-processing or top-down search methods are required for the extraction of image content from this description. In this paper, it has been shown how this can be done with a cost minimization approach.

Top-down search methods require some image model or a priori knowledge of image content and are therefore always used for a particular problem or image type. We have described a method for the detection of more or less round objects with smooth boundaries and step edges at their boundaries.

The method searches for object boundaries which are optimal with respect to a cost function, which is constructed based on a priori knowledge of image content. This method requires the extension of well known curve detection methods, used on pixels grids, to region adjacency graphs. It consists of a number of optimization steps, performed in top-down order on the levels of the hierarchy. The objects are detected with satisfactory accuracy and the method is able to correct for errors in the tree structure of the hierarchy, which originate from the bottom-up procedure. There is one free parameter. It has been argued that this parameter must be chosen in such a way that the two terms in the cost function have the same order of magnitude. It has been demonstrated that changing this parameter has the expected effect on the result.

Model-based image processing techniques, such as those based on active contours or snakes [5] and parametrically deformable models [16] have become popular recently, and their combination with hierarchical methods might become a fruitful field of research.

Acknowledgements: The author thanks C. Orange, F. Groen, A. Toet and H. Heijmans for their useful suggestions and A. Montanvert for supplying the muscle image.

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