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On estimating the intensity of oil-pollution in the North-Sea

R. Helmers

Department of Operations Reasearch, Statistics, and System Theory

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CWI
P.O. Box 94079
1090 GB Amsterdam
The Netherlands

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R.Helmers

CWI

P.O.Box 94079, 1090 GB Amsterdam, The Netherlands

Abstract

In a project, commissioned by the North Sea Directorate, Ministry of Transport and Public Works (RWS), the problem is to estimate the intensity of oil pollution in the Dutch part of the North Sea. A planar inhomogeneous Poisson point process with an intensity function, which is parametrized by a finite dimensional parameter, was used as a spatial model for the locations of the centres of the oilspots. The parametrization enables us to incorporate the available a priori knowledge about oil pollution, such as the location of sources of oil pollution (i.e. shipping areas or off-shore locations) and the intensity of shipping in various regions. The distribution of the volumes (marks) of oil spots is completely unknown, but we can use the volumes of the observed oil spots to estimate it (non parametric approach). Several estimators for the total amount of oil pollution are proposed and applied to the real data sets (marked planar point patterns), which were supplied by RWS. A relatively simple form of spatial bootstrapping was used in order to estimate the accuracy of the estimated total amount of oil pollution in the North Sea.

AMS Subject Classification (1991): 62M30, 62G09, 60G55, 62P99.
Keywords and Phrases: planar inhomogeneous Poisson process, marked planar point patterns, intensity function, maximum likelihood estimation, Voronoi tessellations, spatial bootstrapping, real data application, oil pollution.

0 Introduction

This report describes a project, commissioned by the North Sea Directorate, Ministry of Transport and Public Works (RWS).
Our aim is to develop a reliable statistical method for estimating the spatial variation and the total amount of oil pollution in the Dutch part W of the North Sea. We will employ a planar inhomogeneous Poisson point process as our spatial model for the locations of (the centres of) oil spots. The intensity function $\lambda(., \theta)$ of the Poisson

point process will be parametrized by a linear function of concomitant planar variables, which involves an unknown vector-valued parameter θ. The parametrization enables us to incorporate the available a priori knowledge about oil pollution, such as the location of sources of oil pollution (i.e. shipping areas or off-shore locations) and the intensity of shipping in various regions. However, (almost) nothing seems to be known about the distribution of the volumes of oil spots, but we can of course use the volumes of the observed oil spots to estimate it (non parametric approach).

In section 1 we describe our probabilistic model in detail. The estimation of the parameter θ of our model and of the total amount of oil pollution TA (a complicated function of θ and of the expected sizes of oil spots at various locations) is discussed in section 2. We also introduce in section 2 a spatial bootstrap resampling method for estimating the accuracy of our estimates of TA. In section 3 we present our results, both for 1993 and 1994. Some concluding remarks and directions for future research are given in section 4.

1 The model

Let the planar point pattern $\{s_i, i \in D\}$, where the index set D is a finite subset of the integer lattice $\mathbb{Z}^2$, denote the locations of observed oil spots in the Dutch part $W \subset \mathbb{R}^2$ (the plane) of the North Sea. (For the location of an oil spot we take the 'centre' of the oil spot).
We shall suppose that the planar point pattern $\{s_i,\ i \in D\}$ is a realization of an inhomogeneous Poisson point process, observed in W, with intensity function $\lambda(s,\theta)$. I.e., we assume that (N(A) denotes the number of points in a set A) (i) $N(A_1), N(A_2), \ldots, N(A_n)$ are *independent* random variables, for all disjoint subsets $A_1, \ldots, A_n$ of $\mathbb{R}^2$. (ii) For any set A, $N(A)$ has a Poisson distribution, with parameter (mean value) $\int_A \lambda(s,\theta)\,ds$: I.e.

$$P(N(A) = k) = \frac{(\int_A \lambda(s,\theta)\,ds)^k}{k!} exp(-\int_A \lambda(s,\theta)\,ds), \tag{1.1}$$

for $k = 0, 1, 2, \ldots$. So, the probability model we employ here is a planar inhomogeneous Poisson process, with an intensity function $\lambda(.,\theta)$ which is parametrized by a finite number (seven) of parameters $\theta = (\theta_1, \ldots, \theta_7)$, according to formula (1.3). In this manner we incorporate the available apriori knowledge about oil pollution in our stochastic model. Note that, because the random fluctuations are described by a Poisson point process, no interaction between disjoint parts of W is assumed to be present. Define the threshold value

$$C = 15\ (km) \tag{1.2}$$

This is according to RWS the maximal possible distance between an oil spot and a source of oil pollution (i.e. off-shore or shipping); it is determined by the maximal lifetime and speed of displacement of an oilspot. (see [5] for more details).

We shall assume that, for each $s \in W$, the intensity of oil pollution $\lambda(s, \theta)$ is given by (¡a, b¿ denotes the inner product between two vectors a and b):

$$\lambda(s, \theta) =< \theta, x(s) >= \sum_{l=1}^{7} \theta_l x_l(s) \tag{1.3}$$

where $\theta = (\theta_1, \ldots, \theta_7)$, with $\theta_l = \eta_l \beta_l$, $l = 1, 2, \ldots, 7$ and $x(s) = (x_1(s), \ldots, x_7(s))$, with $x_1(s) \equiv 1$ for all $s \in W$, while

$$x_2(s) = 1 - \frac{\min_{s_0}\{\| s - s_0 \|, s_0 \in \mathcal{O}\}}{C} \tag{1.4}$$

whenever $\min_{s_0}\{\| s - s_0 \|, s_0 \in \mathcal{O}\} \leq C$ (otherwise we take $x_2(s) = 0$), where $\mathcal{O}$ denotes the set of locations in W, which corresponds to off-shore activities. Similarly, the functions $x_l(s)$, $l = 3, \ldots, 7$ are given by

$$x_l(s) = 1 - \frac{\min_{s_0}\{\| s - s_0 \|, s_0 \in S_l\}}{C} \tag{1.5}$$

whenever $\min_{s_0}\{\| s - s_0 \|, s_0 \in S_l\} \leq C$ (otherwise we take $x_l(s) = 0$), with $S_l \subset \mathbb{R}^2$, where S_l denotes the region (in $\mathbb{R}^2$), with shipping intensity β_l. In addition β_1 corresponds to the general 'background' intensity of shipping in the whole region W, while β_2 is set (arbitrary) equal to 1; the parameter $\theta_2 = \eta_2$ simply describes the influence of off-shore activities. Clearly $0 \leq x_l(s) \leq 1$, $l = 1, \ldots, 7$. In other words: we assume that the intensity of oil-pollution $\lambda(s, \theta)$ at a given point is a linear combination of concomitant planar variables; i.e. (cf (1.3))

$$\lambda(s, \theta) = \sum_{l=1}^{7} \theta_l x_l(s) = \sum_{l=1}^{7} \eta_l \beta_l x_l(s);$$

$\eta = (\eta_1, \ldots, \eta_7)$ is an unknown parameter vector, while $\beta = (\beta_1, \ldots, \beta_7)$ are assumed to be known. In fact the following value for $\beta = (\beta_1, \ldots, \beta_7)$ was employed in the statistical analysis. (The RWS-terminology for the various shipping/offshore areas $S_1, \ldots, S_7$ can be found in the Appendix. Except for β_2, the dimension of the β_l's is ship /km^2).

j	1	2	3	4	5	6	7
β_j	0.002	1.000	0.006	0.010	0.014	0.018	0.029

Table 1: Values of β

The (model based) expected number of oil spots in W (as observed by the surveillance aircraft; see Figure 6 for the frequencies at which various locations in W were observed during 1993) is given by

$$A = \int_W \lambda(s, \theta) ds \tag{1.6}$$

Our main object of interest, the expected 'total amount of oil-pollution' in W (in a fixed observation period Q; i.e. each location in W is observed with exactly the same frequency) can be defined as:

$$TA = \int_W \lambda(s,\theta)\,\mu(s) r(s)\,ds \tag{1.7}$$

where $\mu(.)$ denotes the expected size of an oil spot observed at location s. Note that we also have to take into account the fact that different locations are generally observed with varying frequencies. To do this we argue as follows: since $lambda(s,\theta)\ ds$ can be interpreted as the expected number of oil spots in a very small region ds (as observed by the surveillance aircraft), $lambda(s,\theta)/$ Frequency$(s)\ ds$ is simply the expected number of oil spots in the region ds, when ds is observed only once, so that $lambda(s,\theta)r(s)ds$ (with $r(s) =$ as in (1.8)) is equal to the expected number of oil spots in a very small region ds, when the area ds is observed Q times. So the factor

$$r(s) = \text{Frequency(s)}^{-1}|W|^{-1}\int_W \text{Frequency(s)}ds = Q/\text{Frequency(s)} \tag{1.8}$$

is employed in (1.7) (see Figure 7), where the quotient $Q = \int_W \text{Frequency(s)}ds/|W|$ (see Table 5 for our estimation for 1993 and 1994) denotes the mean frequency of observation (in days); Frequency(s) of course denotes the number of times location s is observed (by a surveillance aircraft); $|W| = \int x_1(s)ds$ is nothing but the surface (in km^2) of W. By estimating TA (cf. (1.7)) we hopefully will get a realistic idea of the total amount of oil pollution present in the Dutch part of the North Sea during a fixed observation period Q (i.e. the same frequency at each location s).

2 Estimation of parameters

We will employ the classical method of maximum likelihood to obtain estimates for the unknown parameters $\eta_1,\ldots,\eta_7$ of our model. (An alternative nonparametric procedure for estimating the intensity function is proposed and studied in [2].)

From Theorem 12 of [4] it can be deduced that the maximum likelihood estimators (MLE) $\hat{\eta} = (\hat{\eta}_1,\ldots,\hat{\eta}_7)$ of $\eta = (\eta_1,\ldots,\eta_7)$ are given by the solution of the following set of non-linear equations:

$$\sum_{s_i, i\in D} \frac{d_{ji}}{\sum_{l=1}^{7}\eta_l\beta_l d_{li}} = \int_W x_j(s)\,ds \tag{2.1}$$

for $j = 1,\ldots,7$. Here $d_{ji} = x_j(s_i)$ for $j \geq 2$, while of course $d_{1i} = 1$ for all $i \in D$. We note in passing that to obtain the MLE the loglikelihood function (cf Rathbun & Cressie (1994))

$$\log L(\theta) = \sum_{s_i, i\in D} \log\lambda(s_i,\theta) - \int_W \lambda(s,\theta)\,ds \tag{2.2}$$

has to be maximized as a function of θ (see also Figure 3). Setting $\frac{d \log L(\theta)}{d\theta_j}$ equal to zero (for $j = 1, \ldots, 7$) easily leads us to (2.1). Because the set of equations (2.1) cannot be solved analytically, we have to employ an iteration method. In fact (see section 3) we applied the well-known Newton-Raphson method, with initial values determined by a 'random search'. (The loglikelihood function $\log L(\theta)$ is plotted in Figure 3 (for 1993) for the simpler model with only two parameters η_1, η_2; we set $\eta_l = 0$ for $l \geq 3$; i.e. (1.3) is replaced by the simpler parametrization $\lambda(s, \theta) = \sum_{l=1}^{2} \theta_l x_l(s)$, with $\theta_l = \beta_l \eta_l$ for $l = 1, 2$.)

The parameter A can be simply estimated by

$$\hat{A} = \int_W \lambda(s, \hat{\theta}) ds \tag{2.3}$$

where $\lambda(s, \hat{\theta})$ is given by (1.3), with $\theta = (\theta_1, \ldots, \theta_7)$ replaced by its estimate $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_7)$, where $\hat{\theta}_j = \beta_j \hat{\eta}_j, \ 1 \leq j \leq 7$.
Our main object of interest in this study, the total amount of oil pollution TA, must of course also be estimated from the data: An obvious estimate $\widehat{TA}_1$ of TA is given by

$$\widehat{TA}_1 = \sum_{i=1}^{n} \int_{W_i} \lambda(s, \hat{\theta}) r(s) ds . w(s_i) \tag{2.4}$$

where $w(s_i)$ denotes the observed volume of the oilspot at locations s_i, while the areas (polygons) W_i are defined by

$$W_i = \{s : ||s - s_i|| < ||s - s_j||, \ j \neq i, \ j = 1, \ldots, n\} \tag{2.5}$$

Note that $\{W_i, i = 1, \ldots, n\}$ is nothing but the Voronoi tessellation (see the Figures 8 and 9) given by $\{s_i, i = 1, \ldots, n\}$, the realization of the inhomogeneous planar Poisson point process at hand.
In (2.4) at each location s the factor $\lambda(s, \theta)\mu(s)$ appearing in the integrand of (1.7) is estimated by $\lambda(s, \hat{\theta}) w(s_i)$ for $s \in W_i$, the 'cell' of the Voronoi tessellation corresponding to s_i. (cf. Figure 8 and 9). The idea behind this is that hopefully $\mu(s)$ does not change very much locally, so that $w(s_i)$ also estimates $\mu(s)$, for $s \neq s_i$ $(i = 1, \ldots, n)$ in a neighbourhood (Voronoi 'cell') of s_i.

In contrast, if we suppose a priori that $\mu(s)$ is constant (independent of the location) on the whole region W, then

$$\widehat{TA}_2 = \int_W \lambda(s, \hat{\theta}) r(s) ds \ . \ \bar{w}_n \tag{2.6}$$

is an obvious estimate of TA; here $\bar{w}_n = n^{-1} \sum_{i=1}^{n} w_i$ denotes the sample mean of the observed volumes of oil spots $w_i = w(s_i), \ i = 1, \ldots, n$. We note in passing that $\tilde{A}$ can obviously be estimated by $\widehat{TA}_2 / \bar{w}_n$. Which of the two estimators $\widehat{TA}_1$ and $\widehat{TA}_2$ is to be preferred? If the observed volumes $w_1, \ldots, w_n$ form a random sample of (random) size from a fixed (unknown) distribution G then $\widehat{TA}_2$ appears to be reasonable. On the

other hand, if $\mu(s)$ the mean of the distribution of $w(s)$ does depends on its location (parameter) s, then $\widehat{TA}_1$ may be a better choice. Unfortunately however, at present, (almost) nothing seems to be known about the distribution of the volumes of oil spots (see, however, [7] for an alternative approach). For this very reason, it was decided to employ both $\widehat{TA}_1$ and $\widehat{TA}_2$ in the statistical analysis.
To estimate the accuracy of our estimates we employed a bootstrap resampling method. This is a modern computer-intensive alternative to the traditional way of setting confidence intervals. The idea is to simulate many realizations (B=600) of an inhomogeneous planar Poisson process in a window W, with (estimated) intensity $\lambda(s,\hat{\theta})$ (cf. [1], p653): Let $\hat{\lambda}_{max} = \max_{s \in W} \lambda(s,\hat{\theta})$ and simulate first a homogeneous Poisson process on W with intensity $\hat{\lambda}_{max}$. Next, we keep each event s of the homogeneous process with probability $\lambda(s,\hat{\theta})/\hat{\lambda}_{max}$ and reject otherwise (with probability $1 - \lambda(s,\hat{\theta})/\hat{\lambda}_{max}$). This simulation procedure leads to a single realization of a planar Poisson point process with (estimated) intensity $\lambda(.,\hat{\theta})$; the number of points in this simulated planar point pattern (denoted by n^*) is a realization of a Poisson random variable with parameter $\hat{A} = \int_W \lambda(s,\hat{\theta})ds$ (cf. (2.3)). For each Monte-Carlo realization - a planar point pattern $s^* = \{s_i^*, i \in D\}$ - we solved (2.1), with the d_{ji}'s replaced by $d_{ji}^* = x_j(s_i^*)$ and B bootstrap versions $\hat{\theta}^*$ of $\hat{\theta}$ were obtained. For each realization s^*, w^* is obtained by naively resampling the observed spot volumes w_i, $(1 \leq i \leq n)$: a bootstrap resample w^* of size n^* is drawn with replacement from $\{w_1, \ldots, w_n\}$. With the aid of the resampled values $\hat{\theta}^*$ of $\hat{\theta}$ and the resampled volumes w^* we computed $\widehat{TA}_1^*$ and $\widehat{TA}_2^*$ accordingly, by replacing $\hat{\theta}$ by $\hat{\theta}^*$ and w by w^* in (2.4) and (2.6).

3 The results

In Figures 1 and 2 the available real data sets ('marked planar point patterns') for 1993 and 1994 are presented. First we have to compute the integrals $(j = 1, \ldots, 7)$ appearing on the right hand side of (2.1). The results are given in Table 2. (Recall that the RWS terminology for the various shipping/offshore areas $S_1, \ldots, S_7$ is given in the Appendix.

Note that the first row of Table 2 yields that the surface of W equals $|W| = 57207km^2$. Next we proceed with the solution of the set of non-linear equations (2.1). To solve (2.1) for $j = 1, \ldots, 7$ we first obtained initial estimates $(\hat{\eta}_{10}, \ldots, \hat{\eta}_{70})$ by performing a 'random search' (10^5 random points) in the 7-dimensional cube $[0,2]^7$. With the aid of these initial estimates a Newton-Raphson iteration was performed, which converges in about 10 steps to the values in Table 3 and Table 4. In Figure 4 (for 1993) and Figure 5 (for 1994) the estimated intensity function $\lambda(.,\hat{\theta})$ of our planar inhomogeneous Poisson process is plotted; here $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_7)$ with $\hat{\theta}_j = \beta_j \hat{\eta}_j$, $1 \leq j \leq 7$ (cf. Table 4). (Note that - at first sight perhaps rather unexpectedly - $\hat{\theta}_7$ (for 1993) is negative; the explanation for this fact appears to be that there are no oil spots in the vicinity of S_7 (cf Figure 1). The point estimates $\widehat{TA}_1$ and $\widehat{TA}_2$ (the values of $\bar{w}_n$,

j	$\int_W x_j(s)\,ds$
1	57207
2	6270
3	902
4	4522
5	2627
6	3465
7	877

Table 2: Numerical estimates for $\int_W x_j(s)\,ds; C = 15$

the mean of the observed volumes of oil spots (which plays a role in the definition of $\widehat{TA}_2$), are 2.00 for 1993 and 1.08 for 1994) for the total amount of oil pollution (in the observation period Q) are given in Table 6, whereas the accuracy of these estimates can be inferred from Figure 10 (1993) and Figure 11 (1994) (see, however, also the second remark in section 4). Because the iteration required for the computation of our estimates, sometimes fails to converge, the numbers of bootstrapped values $\widehat{TA}_1^*$ and $\widehat{TA}_2^*$ are in fact somewhat less than B=600.

Solutions of (2.1): $\hat{\eta}$		
	1993	1994
$\hat{\eta}_1$	0.51660	0.53859
$\hat{\eta}_2$	0.00196	0.00041
$\hat{\eta}_3$	2.58059	0.53768
$\hat{\eta}_4$	0.48388	0.23274
$\hat{\eta}_5$	0.32078	0.24229
$\hat{\eta}_6$	0.31182	0.25710
$\hat{\eta}_7$	-0.17727	0.05707

Table 3: Estimation of $\eta = (\eta_1, \ldots, \eta_7)$

$\hat{\theta},\ (\hat{\theta}_i = \hat{\eta}_i * \beta_i)$		
	1993	1994
$\hat{\theta}_1$	0.00103	0.00108
$\hat{\theta}_2$	0.00196	0.00041
$\hat{\theta}_3$	0.01548	0.00323
$\hat{\theta}_4$	0.00483	0.00233
$\hat{\theta}_5$	0.00449	0.00339
$\hat{\theta}_6$	0.00561	0.00463
$\hat{\theta}_7$	-0.00514	0.00166
$\hat{A}$	133.79	104.21

Table 4: Estimation of $\theta = (\theta_1, \ldots, \theta_7)$ and A

	1993	1994
Q	61.0	47.3

Table 5: Mean Frequency of observation Q in 1993 and 1994

Estimation of TA	1993	1994
$\widehat{TA}_1$	1893	350
$\widehat{TA}_2$	585	269

Table 6: Total Amount ($\widehat{TA}$) estimation in period Q

4 Concluding remarks

- the data set of 1993 $\{w(s_i), i = 1, \ldots, n\}, n = 134$, of the sizes of observed oil spots contains a few outliers. Notably a very big oil spot (of size 50.3 m^3; the observed total amount equals $\sum_{i=1}^{n} w_i = 267m^3$) (cf. Figure 1) occurs at a rather unlikely location, namely far away from the locations of various sources of oil-pollution (i.e. the regions $S_2, \ldots, S_7$). In a way, we can view this oil spot as a 'spatial outlier'. Also the (estimated) intensity in a neighbourhood of this oil spot is relatively low and accordingly no other oil spots were observed in the vicinity of the big oil spot. This means that our estimate $\widehat{TA}_1$ is largely determined by this single observation, because the 'cell' of the Voronoi tessellation corresponding to this oil spot is relatively large. As a consequence of this, estimation of TA by $\widehat{TA}_1$ gives a much bigger value then the estimate $\widehat{TA}_2$. A further investigation into the problem of modelling the 'big oil spots' appears to be worthwhile.

- The simple bootstrap resampling procedure employed in the statistical analysis does *not* take a possible dependence of the mean sizes $\mu(s)$ of the oil spots on the location s into account. As a result the histograms of bootstrapped $\widehat{TA}_2$-values will indeed enable us to assess correctly the accuracy of our estimate $\widehat{TA}_2$. The statistical variability in $\widehat{TA}_1$ however, is more difficult to estimate, because - by construction - this estimator is based on the assumption that $\mu(s)$ varies considerably over the whole region W. Indeed, it is clear from the histogram of the bootstrapped $\widehat{TA}_1$-estimates (Figure 10) for 1993, that our $\widehat{TA}_1$-estimate 1893 (for 1993) is apparently a rather extreme one. For 1994 the situation is simpler: the estimates $\widehat{TA}_1$ and $\widehat{TA}_2$ differ by a much smaller amount. A study of bootstrap resampling schemes, which will enable us to bootstrap statistics like $\widehat{TA}_1$ coorectly, is desired here.

- *Estimating the total amount of oil pollution in a given year*
  The estimates $\widehat{TA}_1$ and $\widehat{TA}_2$ can be viewed as estimates of the total amount of oil pollution in an observation period of Q days. So, to obtain a very rough estimate of the total amount of oil pollution TA_{year} *in a given year* (1993; 1994) we propose the estimates

$$\widehat{TA}_{i,\text{year}} = \frac{365}{Q.\nu} \widehat{TA}_i, \;\; i = 1, 2$$

  with Q as before and where ν is equal to the mean life time of an oil spot. According to RWS (see [5]) we may take $\nu = \frac{1}{3}$ day. The 'blow-up factor' $\frac{365}{Q.\nu}$ results in 18 in 1993 and 23 in 1994.
  A more detailed analysis (involving the distribution of the life time of an oil spot (which depends on its size)) is certainly needed at this point.

- The observation period Q as well as the correction factor $r(.)$ (i.e. the sampling plan employed by RWS to obtain the data sets used in this study) both clearly affect the quality of our estimation procedures. The question how to choose the sampling plan in an efficient way is still an interesting open problem at present. The autor hopes to report on these matters elsewhere.

Appendix

The various shipping/offshore areas $S_1, \ldots, S_7$ are in RWS-terminology referred as:

1. Background shipping
2. Offshore mining area
3. IJ-Geul
4. Noord Hinder TSS
5. Terschelling/German Bight TSS + Wandelaar/Wielingen
6. TSS Off Texel & Off Vlieland + Maas West Inner & Outer TSS/Euro-Geul
7. Maas North TSS/Goeree

TSS = Traffic Separation Scheme

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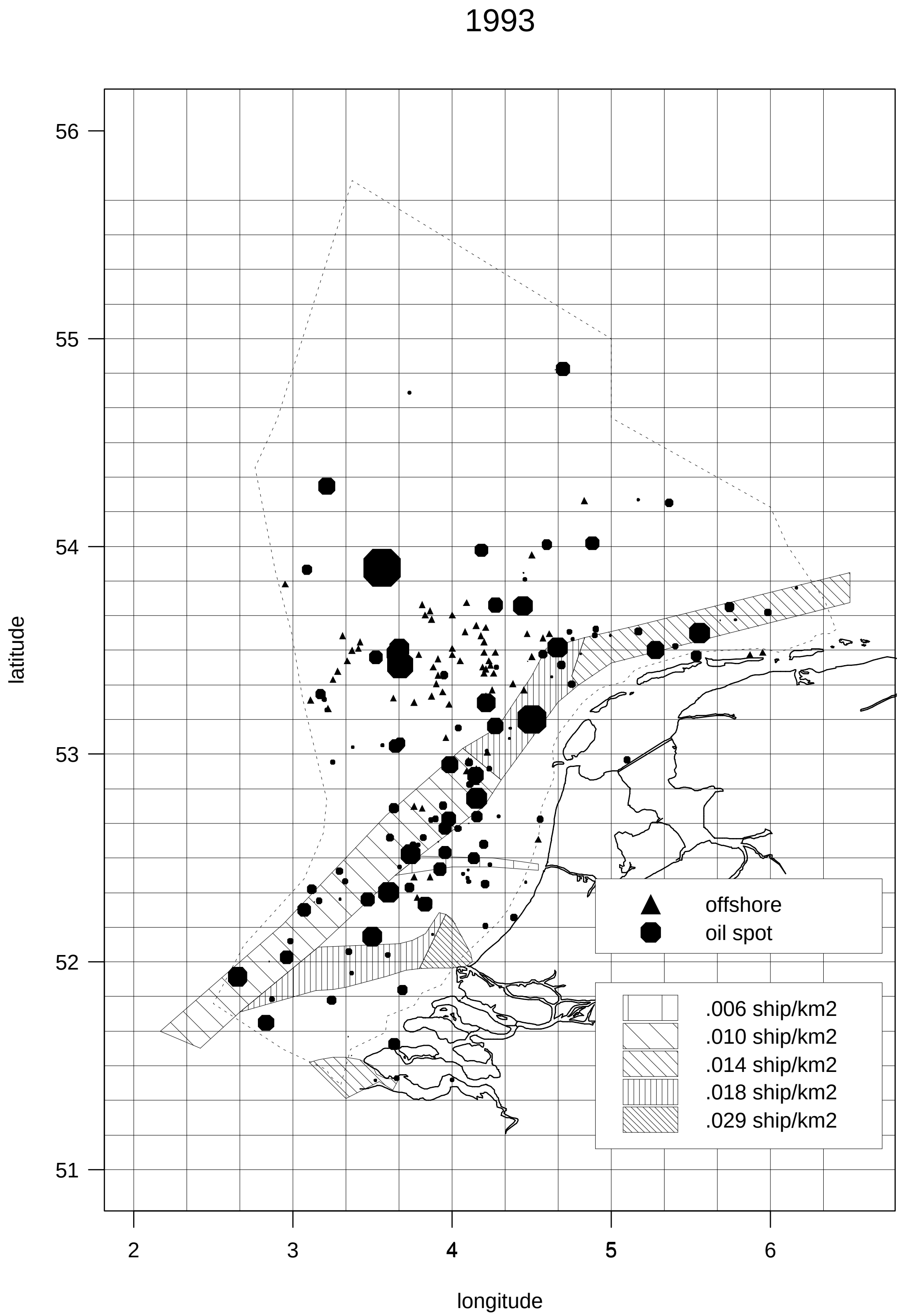

Figure 1: 134 Oilspots (the plotted size of an oilspot is proportional to its volume)

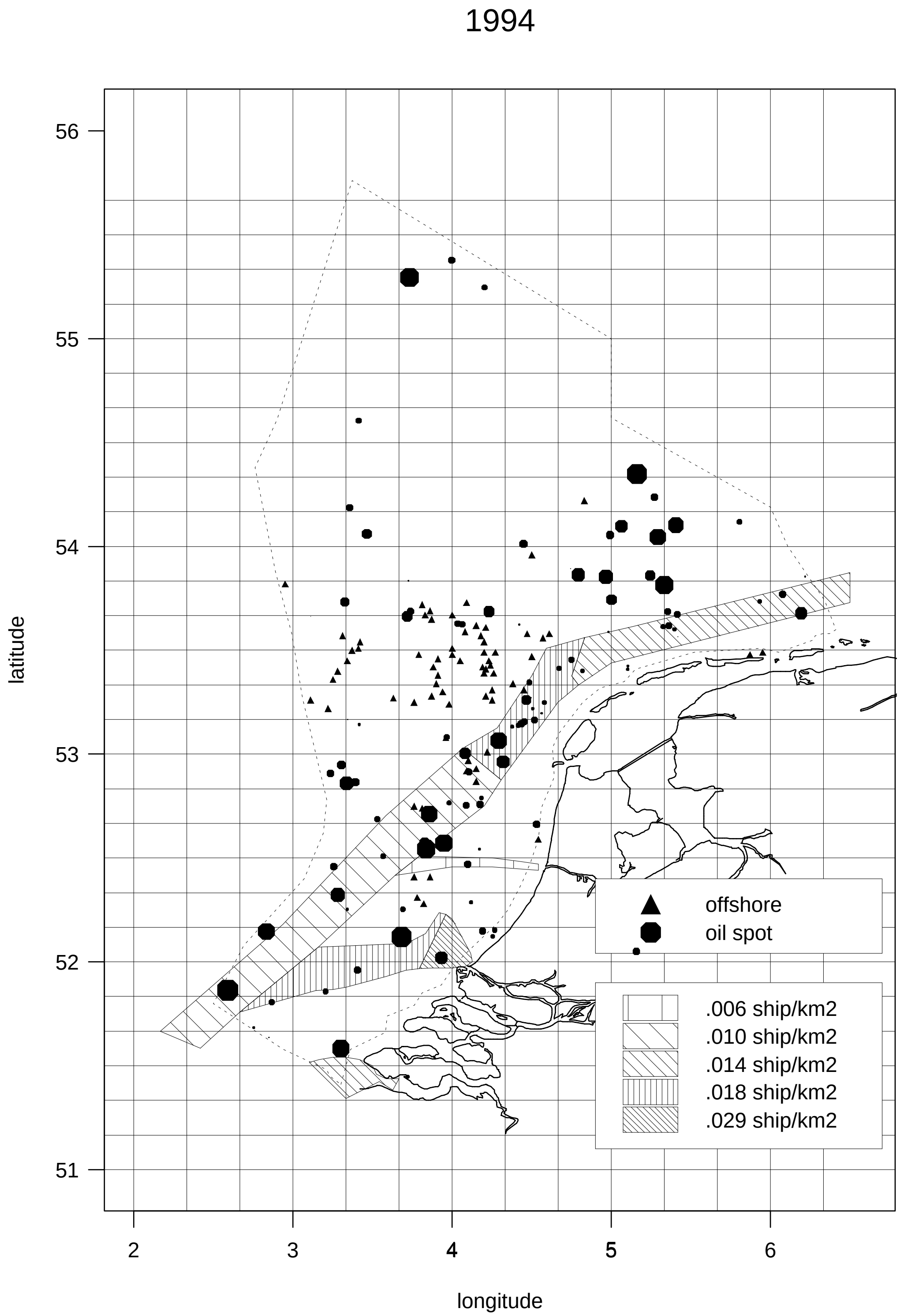

Figure 2: 104 Oilspots (the plotted size of an oilspot is proportional to its volume)

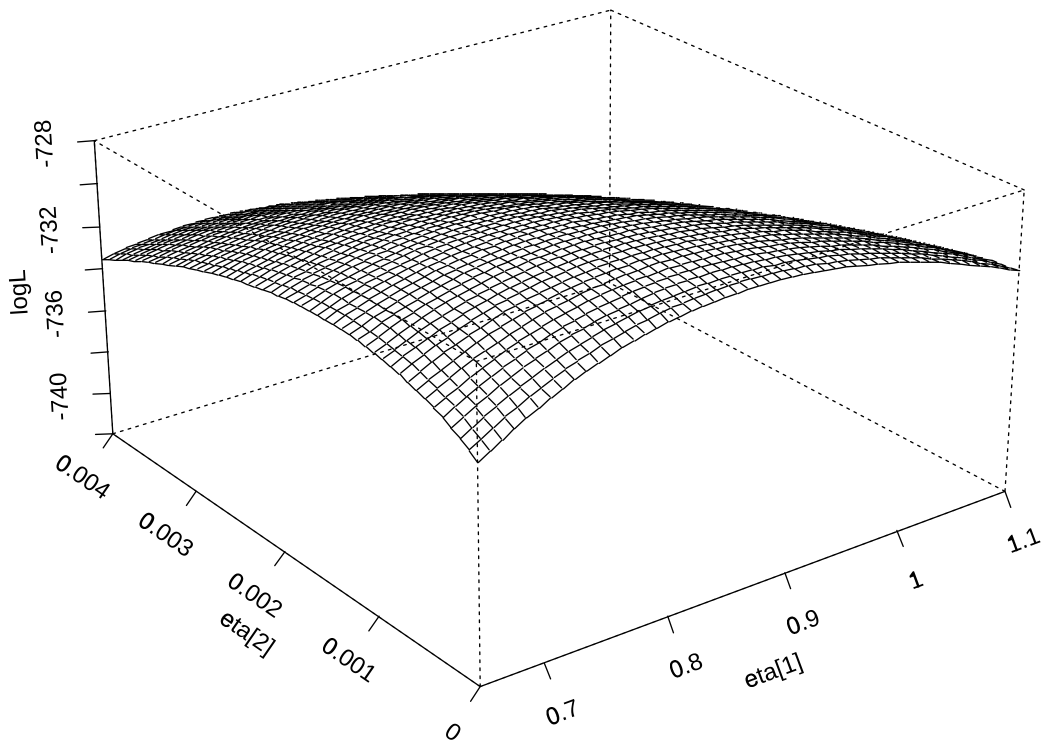

Figure 3: 1993: $\log L(\theta)$ as function of (η_1, η_2); $\eta_1 = \theta_1/\beta_1,\ \eta_2 = \theta_2/\beta_2$

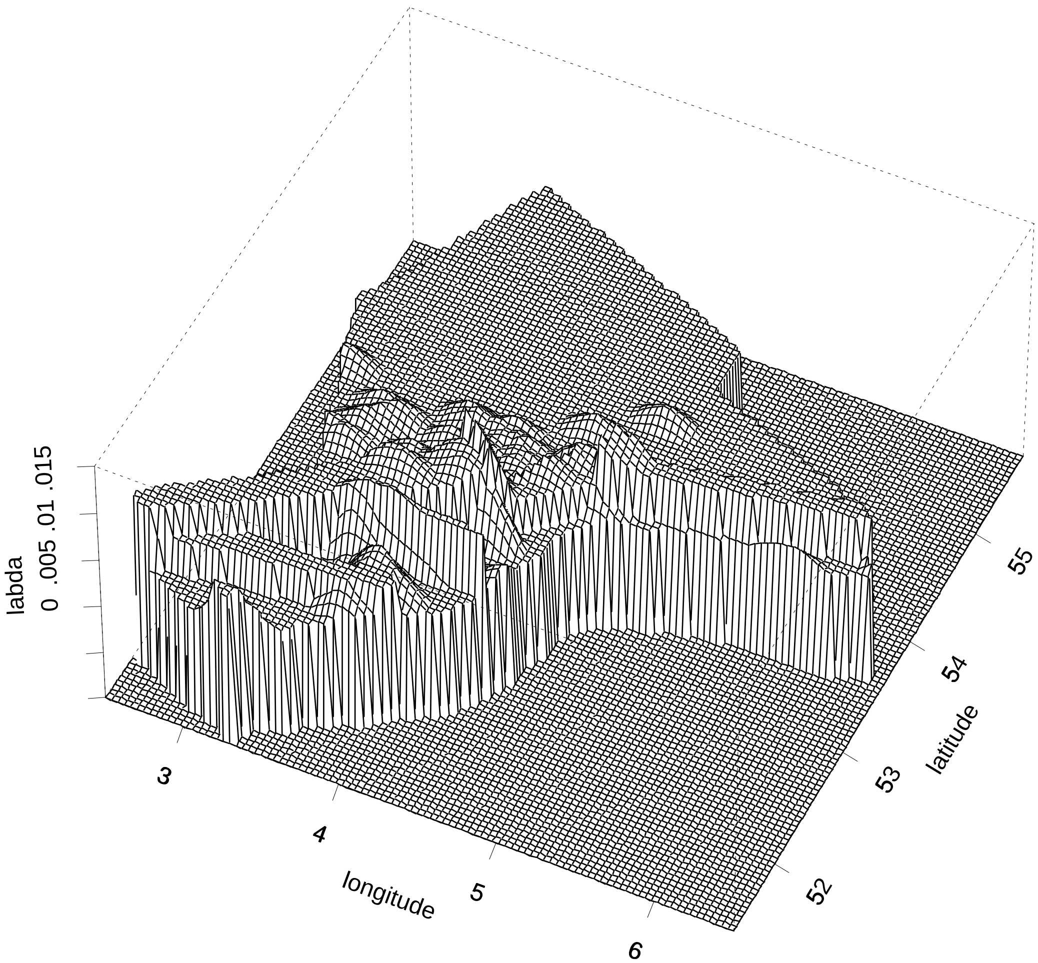

Figure 4: 1993: Estimated intensity function $\lambda(., \hat{\theta})$

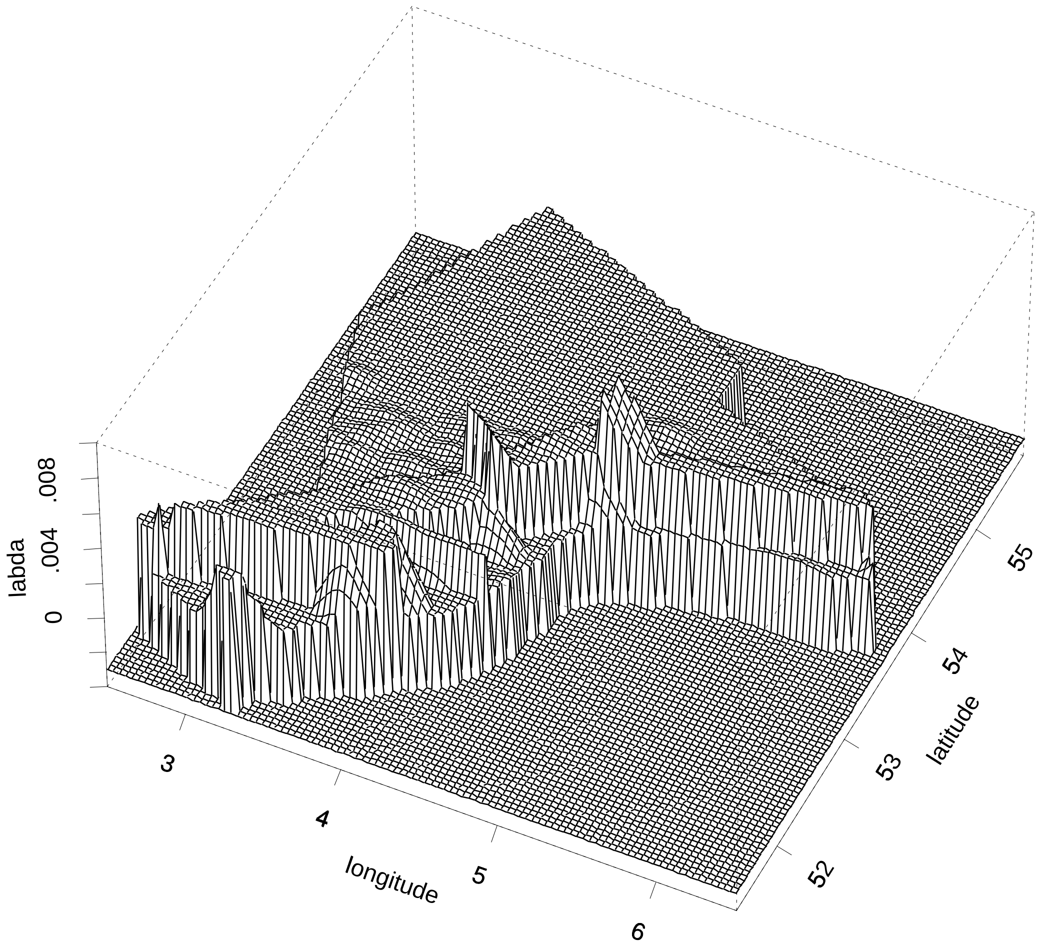

Figure 5: 1994: Estimated intensity function $\lambda(.,\hat{\theta})$

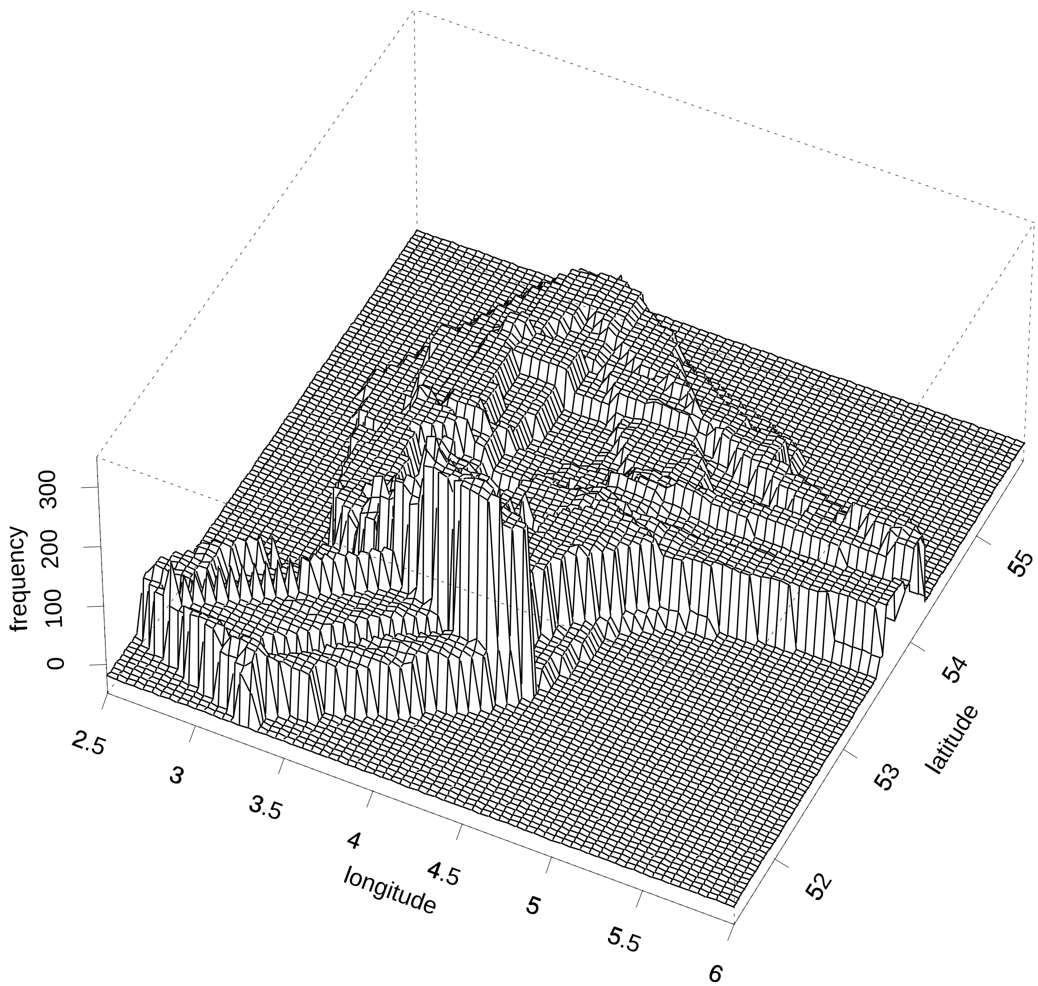

Figure 6: Frequency at which locations in Dutch part W of the North Sea are observed, during 1993

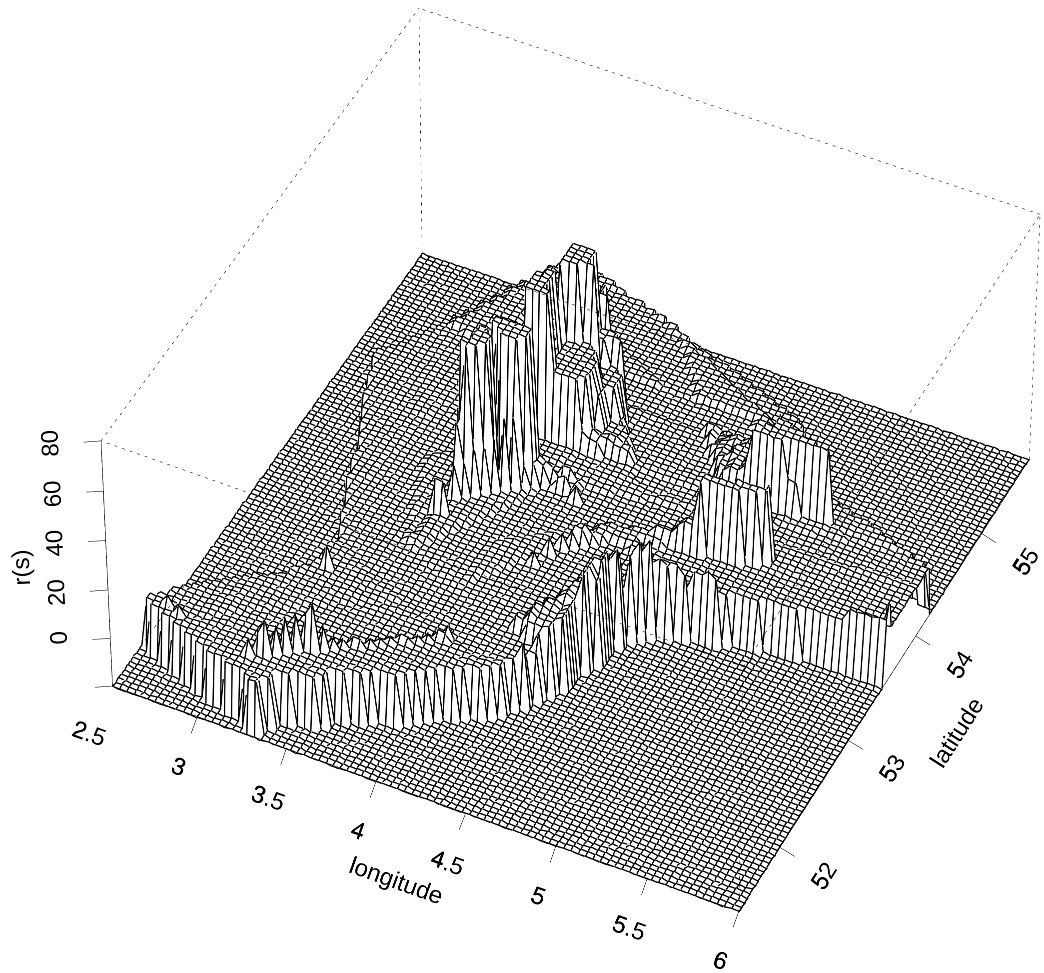

Figure 7: Correction factor r(.) on Dutch part W of the North Sea during 1993

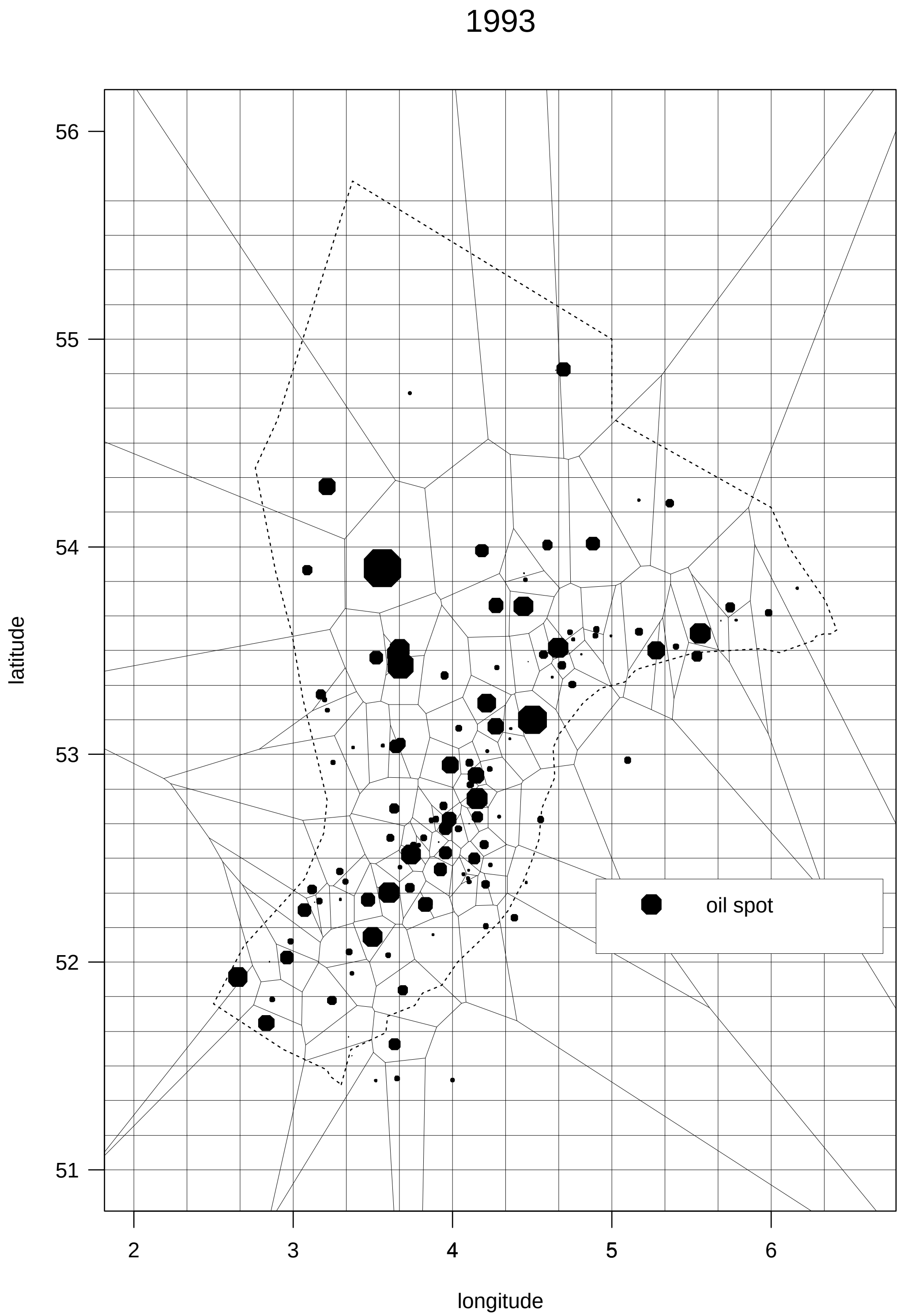

Figure 8: Voronoi tessellation of the oil spots

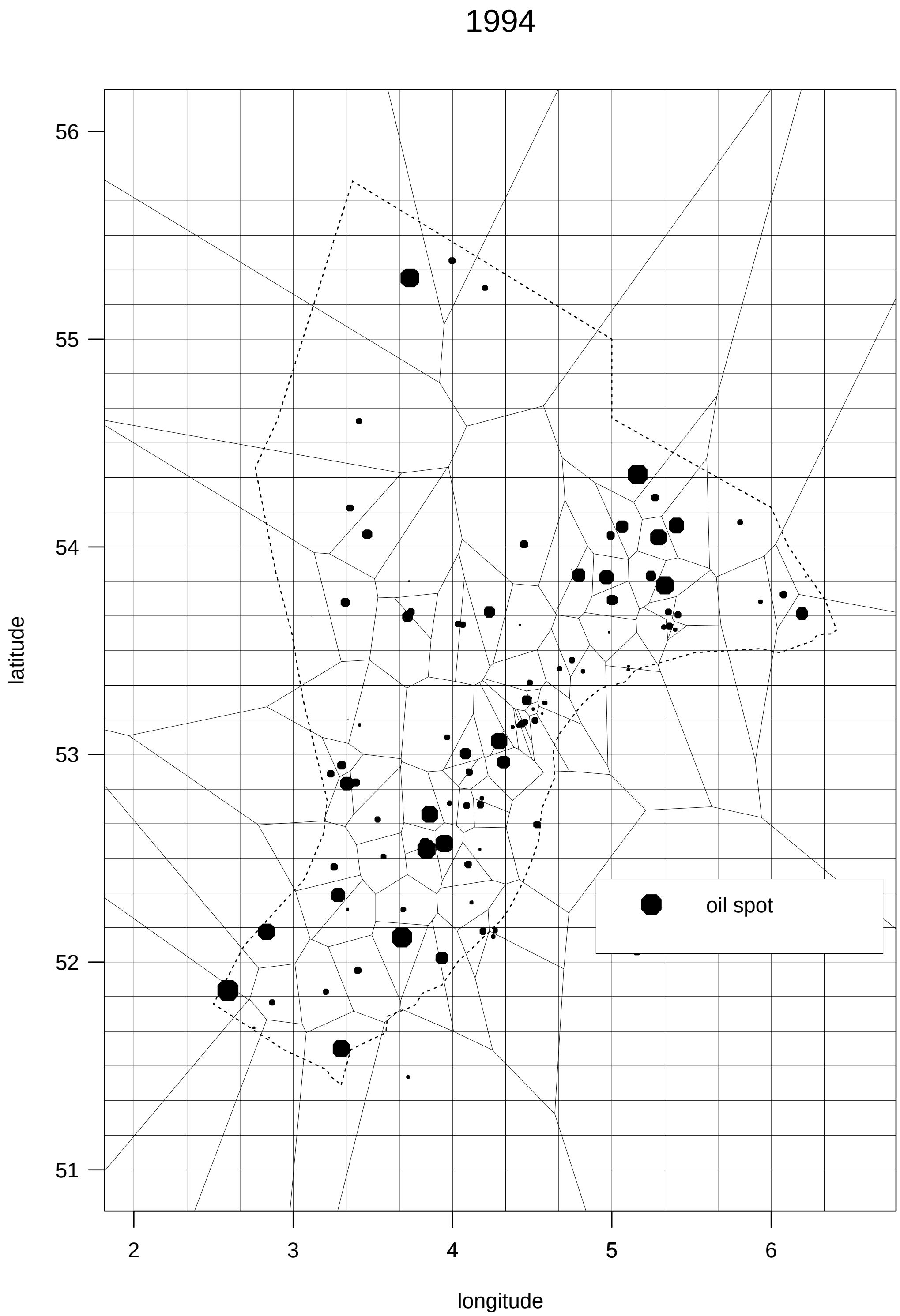

Figure 9: Voronoi tessellation of the oil spots

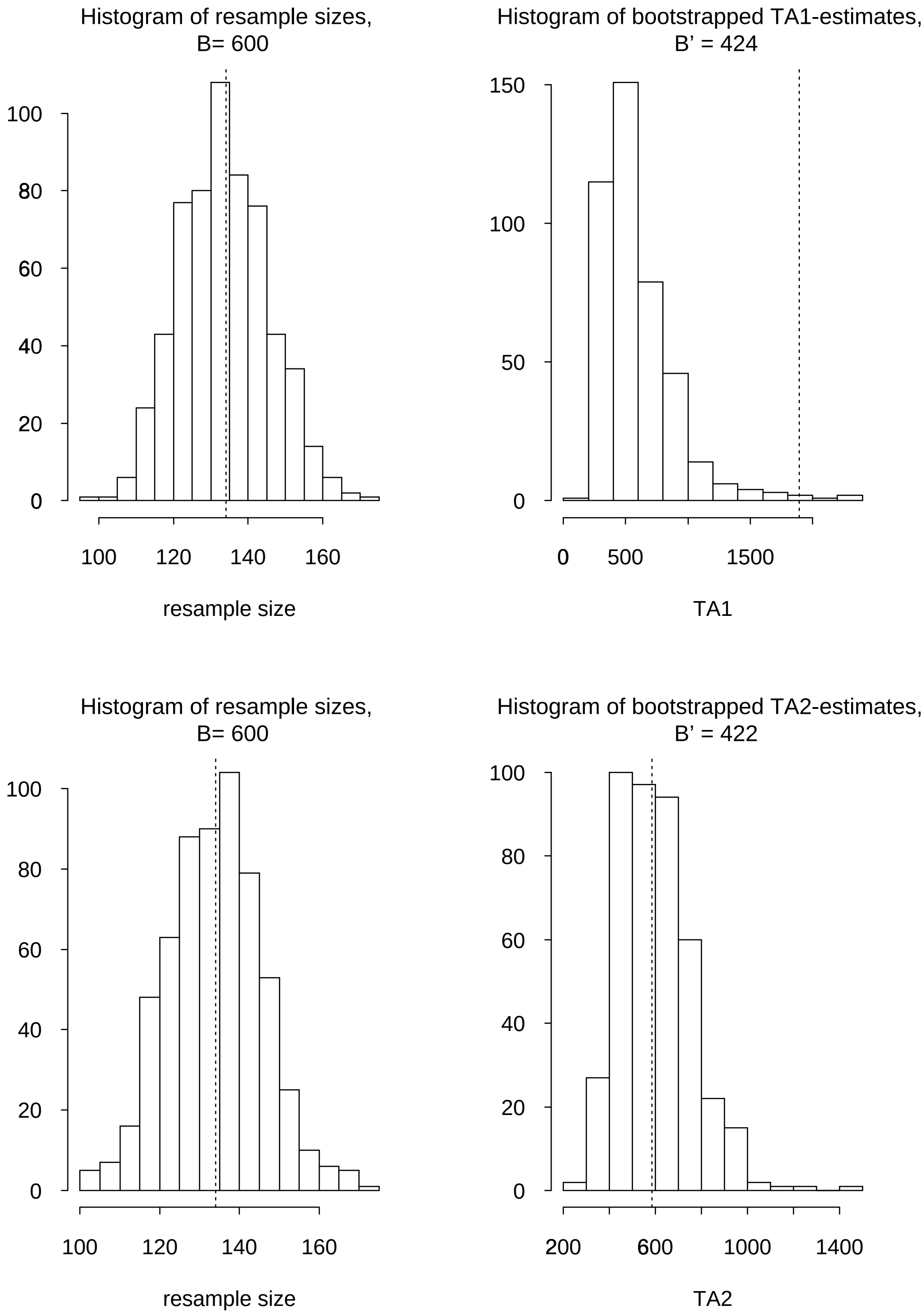

Figure 10: 1993; Bootstrapping with $\widehat{TA}_1$ and $\widehat{TA}_2$

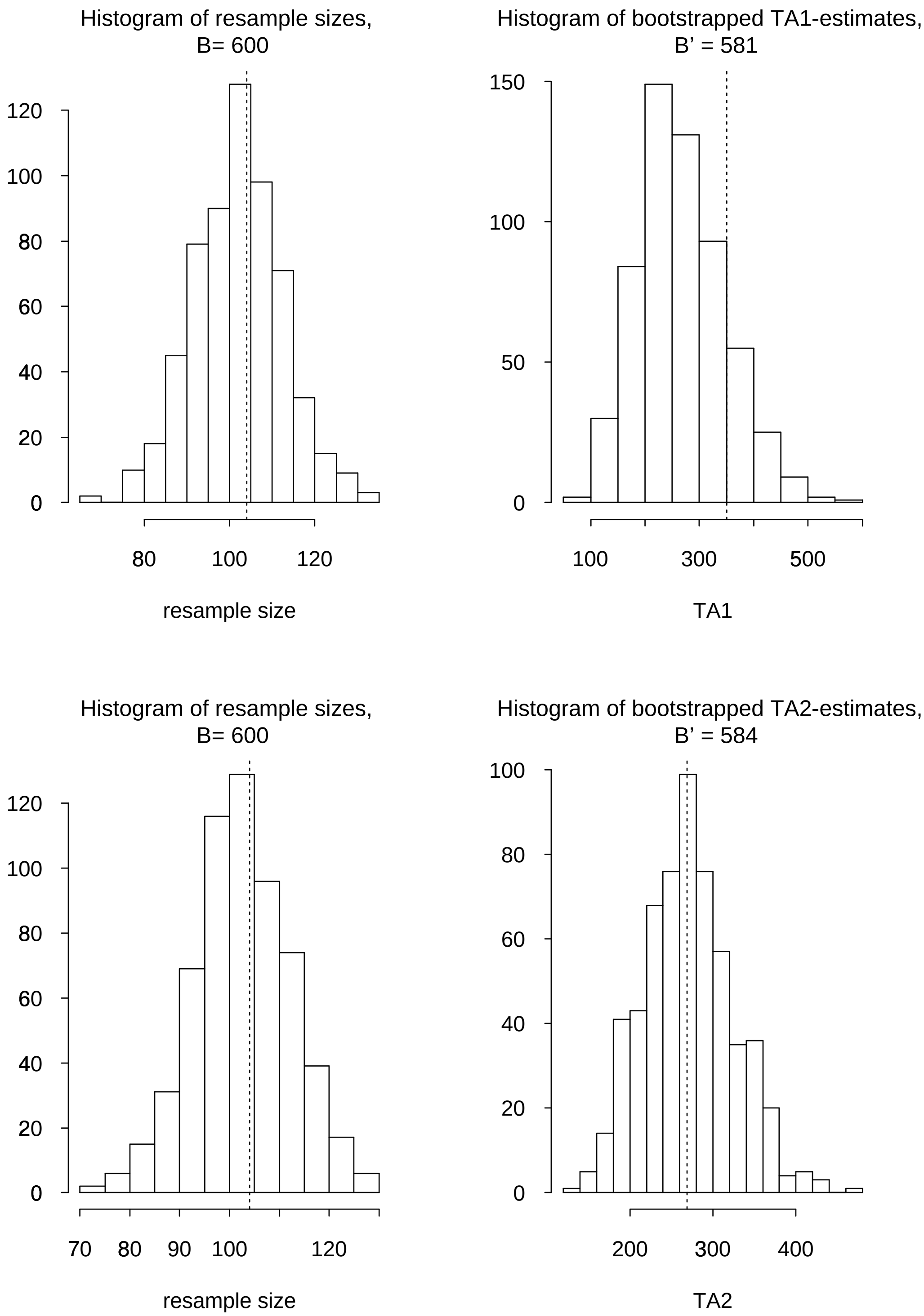

Figure 11: 1994; Bootstrapping with $\widehat{TA}_1$ and $\widehat{TA}_2$