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Routing control of a motorway network

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Abstract

The problem of setting route directives in a motorway network is formulated as a dynamic game problem. After restricting attention to a Nash equilibrium, a search procedure for such an equilibrium is proposed.

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1 Introduction

The purpose of this report is to describe the research of the CWI team of project DYNA for the routing control problem. This report is a slightly revised version of the CWI contribution to the final report of this project.

The aim of the DRIVE II project DYNA is to develop, implement, and test a real-time prediction algorithm for a motorway network. The 1994 extension of project DYNA defines as aim also control of a motorway network. The research of project DYNA is motivated by the use of telematics to deal with the steady increase of traffic demand and with the limited capacity of the existing motorway network.

It was the task of the DYNA team of the Centre for Mathematics and Computer Science (CWI) in Amsterdam, The Netherlands, to perform research on routing control of a motorway network.

The CWI team consists of the team leader J.H. van Schuppen and the senior researcher Dr. H.J.C. Huijberts. The latter is with the Eindhoven University of Technology and was affiliated with CWI for the project in a part time appointment of 40%.

The CWI team of DYNA has worked closely with the TUD team of DYNA (P.H.L. Bovy, N.J. van der Zijpp) in the modeling of the behavior of road users in response to control measures. The relation of the CWI team with the team of the University of Lancaster (P.C. Young, A. Chotai, C.J. Taylor) is less direct, the latter team concentrates attention on control for a small part of a network and considers other control measures. Meetings with the TUD team took place on June 28 and July 27, 1994 and with the University of Lancaster team in the period July 13-15 at Lancaster and in the period September 7-8 at CWI. In addition several meetings took place between all DYNA teams involved in the control extension of the project.

A summary of the achievements of the CWI team follows. An overview has been made of the available and foreseen control measures for routing control. Modeling for routing has resulted in a mathematical model called the *control system*. A proposal has been formulated for the structure of a routing control law. An explicit control law has been defined. The time available for the investigation was about four months throughout the year 1994. No simulation of the control system was undertaken.

Further research on routing control is required. The algorithm must be tested on the road. At the theoretical level the existence and the search procedure for the Nash equilibrium need attention.

The report is written both for an audience in traffic and transportation, and for an audience in control. Therefore both reader groups will find information that is well known to them but not to the other reader group. The report contains an extensive discussion on control measures for routing control because this is essential for the control problem considered.

An outline by sections follows. Section 2 contains the problem formulation. Section 3 presents the modeling for control. Section 4 deals with the routing control problem. The last section contains concluding remarks. Appendix A contains a summary of the model of DYNA. Appendix B contains a description of a state space model for routing control. Appendix C describes particular control studies. Appendix D contains a list of abbreviations and Appendix E the figures.

2 Problem formulation

2.1 Problem of routing control

In this investigation attention is limited to routing control of a motorway network. *Routing control* is defined as a control effort that seeks to influence the route choice of users of a motorway network. Routing control is a form of traffic control and a tool of traffic management. (The terms of *traffic control* and *traffic management* have been defined by ERTICO [20].)

Problem 2.1 *Synthesize and analyze algorithms for routing control in a motorway network. The controller must use the DYNA predictor. The main control objective is to minimize travel time. The control measures include providing information and directions to drivers, and road control measures. Attention is restricted to control measures that provide route directives.*

The routing control problem may be considered for the following motorway networks in The Netherlands:

1. A motorway network in the region of Rotterdam and The Hague. This network is the subject of the pilot project of DYNA, see [22].

2. The motorway network around the city of Rotterdam. This network is a subnet of the previously mentioned network.

3. The motorway network around the city of Amsterdam. Rijkswaterstaat is performing experiments with this network, see [52].

The main *control measures* taken into consideration for routing control are:

1. *VDS*. Display of variable direction signs at motorway intersections.
2. *DRIP*. Providing information to drivers by display of message signs just before motorway intersections.
3. *Radio broadcasts*. Providing information to drivers by regular radio broadcasts.
4. *RDS*. Providing information to drivers by the Radio Data System.

In Subsection 3.2 the reader will find a more extensive discussion of control measures.

The main control objective of routing control is to minimize the travel time of network users. In Subsection 4.1 the reader will find a detailed discussion on control objectives.

Because of the aim and of the tools of project DYNA attention is restricted to the problem of routing control by route directives. DRIP's can also be used as variable direction signs (VDS's).

The motivation of Problem 2.1 is the efficient use of a motorway network especially in conditions in and near congestion. A solution to this problem will provide information on what the benefit is in the case of the use of VDS's. There already is experience with providing actual state information to drivers, see Subsection 2.2. Route guidance systems, see Subsection 3.2, will also benefit from the solution to the problem. Additional discussion on the problem may be found in the Sections 3 and 4.

One may distinguish routing control and route guidance. Routing control is exercised by the road operator using the public control measures mentioned above. It will direct a driver only through part of a network. Route guidance may be defined as a way to influence the route choice of a selected set of drivers and is delivered to an individual driver by a private agency. It will guide a driver to a final destination. Route guidance seems particularly well suited for routing in an urban network. Examples of route guidance systems are EURO-SCOUT and services provided via the communication network SOCRATES, both discussed below. Routing control is still in the design phase.

2.2 Current state of knowledge

Experiments with routing control

An experiment with routing control was performed near Amsterdam, The Netherlands, see [52]. On November 14, 1991 a variable message sign was put in use called the *RIA-sign* for Route choice Information Amsterdam. The sign is located on motorway A8 between Zaandam and the Amsterdam ringway A10 in the direction of Amsterdam. The sign provides information about traffic conditions on the ringway, in particular, on the length of traffic jams at the Coentunnel and at the Zeeburgertunnel and on possible obstructions in these tunnels. On the basis of this information car drivers can make an en-route choice by either turning left to the A10 Noord or right to the A10 West.

A study has been made of the effectiveness of the RIA-sign [52] by interviewing a panel of drivers. The overall conclusion is that the sign is valued positively by drivers and has a positive effect on traffic flow. After the introduction of the sign the panel members were less frequently caught up in queues than before. The average travel time of the panel decreased by eight percent. The duration and severity of congestion decreased from period to period. Five to seven percent of all trips were influenced by the sign. Remarkably, in case no queue was present at the Coentunnel in the morning peak its use increased by nine percent.

Recently Rijkswaterstaat has started to experiment with providing information to drivers at several other locations of the motorway network near Amsterdam.

Overview of literature on the subject

For the routing control problem as considered here the classical approach of route selection given arrival intensities by optimization techniques is not suitable. Arguments for this statement are that in this approach no use is made of the actual state of the network and neither of the time-varying arrival intensities at entry points. Attention in this report is restricted to feedback control laws that use actual state information. Several synthesis approaches have been suggested, see [16, 17, 42, 48] and the references provided in these papers. For an introduction to routing control see [2, 66].

M. Papageorgiou [48] has proposed to use optimal control theory to synthesize a control law that meets the control objectives. See that paper for an example and for a simulation of the controlled system. The conclusion of that paper is that the routing advice is oscillating between the alternate routes considered. Other disadvantages are the high computational cost of the optimal control law and that the structure of the control law is hard to determine. Therefore this approach has its limitations. Papageorgiou in [48] has also proposed linearization of the nonlinear control law and application of a time-invariant LQG-control law. This approach has serious disadvantages because near the state of traffic congestion the control system is highly nonlinear. Papageorgiou has also proposed a state feedback law consisting of a proportional and integral term. The parameters of this control law must be determined by trial and error. In the paper [48] a simulation is shown of the control system with this control law. Difficulties with this approach are the computation of the control law parameters and the suitability of a linear control law for a nonlinear control system. Therefore this synthesis approach has also its limitations. Nevertheless, the authors value the pioneering work of Papageorgiou as presented in [48]. For recent work by M. Papageorgiou and co-workers see [39, 30, 40]. The authors became aware of the latter publications only when this report was essentially completed.

For related publications on routing control see [32, 21, 45, 61].

At the time the final report of this project is to be submitted to the Commission of the European Communities, the First World Congress on Applications of Transport Telematics and Intelligent Vehicle-Highway Systems has been held, November 30 - December 3, 1994. Several presentation and proceedings papers discuss the routing control problem. The proceedings papers do not always provide sufficient information on the details of the routing control law used. The paper [33] on the RHAPIT field trial discusses routing control using the communication system SOCRATES. As part of the DRIVE Project QUO VADIS several routing control laws have been proposed by M. Papageorgiou and are in the process of being evaluated by simulation or by road testing, see [43, 49]. In the DRIVE Project EUROCOR attention is also given to routing control, see [41]. The papers of the DRIVE Project DYNA at the proceedings are [6, 27, 12].

The analogy of routing in a motorway network with routing in a communication network may be exploited. For references on this approach see [24, 64].

Character of this project

The novelty of this investigation into routing control is in the extended problem formulation, in the discussion about routing control measures, in the use of the Nash equilibrium concept for dynamic user optimization, and in the proposed control law.

The program DYNA offers the perspective of future implementation of routing control. This program produces estimates of the traffic intensity per origin destination (OD) pair and predictions of the arrival intensities per OD pair. By the time project DYNA is completed the DYNA predictor will become available. Note that in the paper of M. Papageorgiou [48] control is based on the availability of traffic intensities per destination and per section and that an estimation procedure for these intensities is not provided in that paper.

3 Modeling for control of traffic flow in a motorway network

In this section a model for routing control is formulated. The model is to be used for control synthesis and for online computations with DYNA. In addition an extensive discussion is included on control measures.

Parts of the research described in this section have been carried out in cooperation with P.H.L. Bovy and N.J. van der Zijpp of the TUD team of project DYNA.

3.1 Model for traffic flow without control

The models for traffic flow in a motorway network with control to be derived below are based on similar models without control. The model of project DYNA and the state space model of appendix B are models for traffic flow without control. Both models are briefly summarized below. The state space model was developed prior to the start of project DYNA.

Terminology on a motorway network follows. Consider a motorway network on which a control and signalling system is installed. Such a system is assumed to have a detector station about every 500 m. of the motorway. At a detector station information is collected on the traffic flow. Detector stations are also assumed present at on-ramps, at off-ramps, and at motorway intersections. A model of a motorway network consists of a graph in which the nodes correspond to locations of detector stations. A *section* of the network is an ordered pair of nodes such that there is a motorway connection between these nodes that does not pass another node. An *origin-destination pair* (OD) is an ordered set of nodes where the first node corresponds to a location with an on-ramp and the second to a location with an off-ramp. A *route* of an OD pair is a path from the associated origin to the associated destination. It is specified by a chain of sections. In general an OD pair may have several routes. A *link* is a chain of sections between two adjacent intersections of a motorway network.

There follows a summary of the model of project DYNA. For details on this model the reader is referred to [22, 23, 34]. For a short description of the results of project DYNA see [4, 5].

Within the project DYNA two models have been developed for a motorway network: A behavioral model with a prediction horizon of up to two hours and a statistical model with a prediction horizon of up to 45 minutes. In the sequel only the behavioral model is considered. It is the task of the DYNA team from the University of Lancaster to perform research on control of motorway traffic with the statistical model.

The forecasting algorithm of DYNA has two major components: (1) dynamic prediction algorithm for origin-destination (OD) demand; and (2) a dynamic traffic assignment model (see appendix A for references). The data of the algorithms are historic data for the network and real-time traffic data. The outputs of the forecasting system are traffic density, average speed, and other variables for each link of the network. The intended user of the forecasting system is the road operator.

A summary follows of a state space model for traffic flow in a motorway network. The detailed state space model is described in appendix B. At the end of this subsection the model of DYNA and the state space model are compared.

For motorway traffic flow a macroscopic model of H.J. Payne will be used that has been modified by S.A. Smulders. The state variables of this model are traffic density and average speed per section. Because the purpose of this model is routing, the traffic density per section must be distinguished per OD pair and per route. Denote the traffic density of OD pair (i, j), of route k, and of section (l, m) by $x((i, j), k, (l, m), .) : T \rightarrow R_+$ in veh/km.lane. The density in section (l, m) may then be calculated by

$$x((l, m), t) = \sum x((i, j), k, (l, m), t),$$

where the sum is over all OD pairs and all routes that use the section (l, m). The state vector x contains the traffic densities defined above and the average speed in all sections.

The state transition equations are deduced from Payne's model and from the structure of the network. The resulting state space model is described by the dynamic system

$$x(t+1) = f(x(t), \lambda(t)), \; x(t_0) = x_0. \tag{1}$$

In this model the state vector x contains the traffic densities and the average speeds, while the vector λ specifies the entry flows at origins.

A comparison can now be made of the model of DYNA and of the state space model. In the DYNA model the dynamics of the traffic flow is specified by the assignment matrix. The report [28] contains a description of this matrix. In the state space model the dynamics of the traffic flow follows the model proposed by Payne. The latter approach is more detailed for control synthesis. In the DYNA model the link flows are determined as the solution of an equation. In the state space model a recursion is specified for the state. In the state space model traffic density is distinguished per OD pair, per route, and per section. In the DYNA model there is a fractional representation per origin, per path, and per link. These representations are thus closely related and the complexity is comparable. The difference in the models is the consideration of links versus sections, and therefore the state space model has a higher complexity. Both models assume the availability of predictions of traffic demand for OD pairs at on-ramps.

Example 3.1 *Ring Amsterdam.* A model for traffic flow without control will be formulated for the motorway network near Amsterdam as an illustration of modeling. See Figure 2 for the motorway network.

On the motorway network near Amsterdam a Motorway Control and Signalling System (MCSS) has been installed. Identify the location of the detector stations of this network with the nodes of a graph. The entry and exit points of the motorway network are identified with origins and destination nodes of the graph. The modeling then proceeds as indicated in appendix B.

The model for the ring near Amsterdam has the following numerical characteristics. The ring around Amsterdam is about 36 km in length. With section lengths of about 500 m. this amounts for both directions to 144 sections. The number of entry and exit points is about 19. The average number of realistic routes per OD pair is about 1.2. The dimension of the state space is therefore approximately $(19 \times 19 - 19) \times 1.2 \times 144 + 144 \approx 59,242$.

3.2 Control measures

Attention in this report is restricted to those control measures that have a direct impact on the route selected by users of a motorway network. Control measures may be distinguished into information control measures and road control measures. An example of an information control measure is the transmission of a radio message with information on congestion in a network. An example of a road control measure is the lifting of a lane closure.

Below control measures for routing control are discussed. The information on the measures is structured by the mode, which information is provided, when it is provided, and where it can be received (which, when, where).

1. *Teletext.* In The Netherlands information is available via television by a program called *Teletekst.* Via regular tv channels digital messages are transmitted to TV viewers jointly with the video and audio signals. The viewer can by a selection mechanism display on his or her TV screen the pages with traffic information. The information provided concerns road works and congestion. It is available 24 hours a day, 7 days a week. The information may be consulted by travellers before the start of their trip. There is no report that it is used by travellers when en-route.

2. *Radio.* Transmission of messages on regular radio channels. The messages contain almost exclusively information on exceptional circumstances, such as congestion and unforeseen lane closures. A traffic jam may be reported when it exceeds 2 km in length in off-peak hours or 4 km in peak hours. The traffic information will in general cover a wide geographic area, usually a country or part of a country. Messages of the type mentioned are often broadcasted at fixed times, more frequently in peak hour traffic than outside peak hours. No increase in the frequency of radio messages is foreseen. These messages can be received by travellers before their trip or en-route by a car radio.

3. *RDS-TMC.* The Radio Data System (RDS) has recently come on the market as part of a car radio. Via a Travel Message Channel (TMC) information is communicated to the car radio and stored there. The user of a car equipped with RDS may by the push of a button start RDS to sound an audio message in his or her language with traffic information. The content of the information will include mainly the length of traffic jams and road works on the route ahead. The information provided is based on the current state of traffic at the moment the broadcast is made. No predictions of the traffic situation are provided. This information is available 24 hours a day, 7 days a week. It is updated regularly. In the future RDS messages can in principle also communicate estimates of travel times and route directives. For information on RDS see [25].

4. *DRIP.* A Dynamisch Route Informatie Paneel (DRIP; Dynamic route information board) is an example of a variable message sign (VMS). An example of a DRIP is the RIA sign discussed in Section 2. That sign displays the length of the traffic jams in the western and the northern link of the ring around Amsterdam. In 1994 three more DRIP's have been installed on the motorway network near Amsterdam.

   The information content of the RIA sign near Amsterdam is mainly the length of traffic jams on one of the four links of the ring or near a tunnel, or obstructions because of road works. A traffic jam at the RIA sign is displayed if it exceeds 1 km in length. The sign mentions the length as 2, 3, or 4 km where the last value is used also if the length is 5 km or larger. In case of incidents the RIA sign can be used to display any variable message sign. A DRIP can in principle display any message of limited length. It is active 24 hours a day, 7 days a week.

A DRIP can in principle also be used to display route directives although it has not been used as such. The limitations of a DRIP will allow display of route directives for about one or two destinations.

5. *VDS*. Variable Direction Signs (VDS) are currently not installed in The Netherlands. In principle DRIP's can be used as VDS's. The authors do not have information on the use of VDS's outside The Netherlands.

6. *Route guidance systems*. SOCRATES and EUROSCOUT are two examples of route guidance systems. The EUROSCOUT system uses a large number of beacons throughout a network. When a car with EUROSCOUT equipment enters the action area of a beacon it transmits at request the time and the location of its last contact with another beacon and its destination. The beacon provides the car with instructions and directives to travel to the next beacon. The route to the next beacon is computed at the control center based on actual traffic information. EUROSCOUT is currently still in the design and test phase. EUROSCOUT offers the opportunity to direct traffic equipped with this system to particular routes. An advantage of this system is that cars must specify their destination, so this information can be used in routing control.

   System Of Cellular Radio for Traffic Efficiency and Safety (SOCRATES) is another route guidance system. A car equipped with this system transmits via the Global System for Mobile telecommunication (GSM), the new digital telephone network, its position to a Traffic Control Centre (TCC). At the TCC average travel times are computed. The information transmitted to cars contains the average recent travel times between a large number of locations. The car user will be offered only the travel times for the possible routes to his destination.

   A car user with the system SOCRATES decides himself on the choice of a route. This choice is based on the travel time estimates. Biasing these estimates to influence route choice is not a realistic option in the opinion of the authors. Instead of travel time estimates based on recent past data one may use a travel time prediction for which the program of DYNA can be used. Because of these arguments SOCRATES is not an obvious control measure for routing control. For information on SOCRATES see [15, 14, 25].

   The implementation of route guidance systems is uncertain at the time this report is written. Not mentioned here are route guidance systems that do not use actual traffic data.

7. *Road control measures*. An example of a road control measure is the temporary blocking of one or more lanes because of road works or because of an accident. These measures will not be considered further in this report.

The information provided by the different control measures must be consistent.

The control measures mentioned may also be distinguished into whether they influence the pre-trip or the en-route choices. The measures discussed above may then be classified as:

- Control measures for pre-trip choices: Teletext, radio messages, RDS.

- Control measures for en-route choices: VDS, DRIP, Radio messages, RDS, and route guidance systems.

Not considered in this report are the control measures of ramp metering and variable speed limits.

Routing information

Routing information provided by control measures may be distinguished into the following forms:

1. *Actual information.* Information reflecting the state of traffic in the motorway network at the time the information is sent to the communication channel or immediately prior to display. Most current control measures mentioned operate in this mode.

2. *Predictions.* For example, prediction of travel times for one or more routes. Control measures that provide such information do not yet exist, although a DRIP and the route guidance system SOCRATES can in principle provide this information.

3. *Route directives.* A network user is provided a route directive.

A note on terminology follows. A *route directive* is information that points travellers to a particular route. A route directive may be provided by a variable direction sign, a DRIP, a radio message, or RDS. Attention here is focused on route directives that are determined by information from a motorway control and signalling system. Ordinary fixed direction signs are not considered. A route directive is considered as an advice, travellers may decide to follow or to ignore the directive. Thus a route directive should not be considered as mandatory. A route guidance system is a system that through a sequence of messages guides the traveller from his origin to his destination. Such a system may include a sequence of route directives.

How is the routing information mentioned above used by car drivers? In all cases the driver has to receive and to interpret the information and to determine whether or not it is relevant to his trip. In the case of a route directive a driver must decide whether or not to follow the route directive. In the case of information on the actual state of traffic in the network he must estimate his future travel time or cost based on this information and on his destination. In general this is difficult for a driver because the information provided is necessarily vague, very far from complete, and because the dynamics of a motorway network are complex. In the case of travel time predictions a driver must make a route choice by comparing only the travel times of the possible routes. Because travel time predictions can be provided only for one or two destinations, a driver must interpret this information for his trip.

Which type of routing information is to be preferred for routing control? Travel time predictions are considered with reservations. Car drivers may compare the travel time prediction with the experienced travel time. Because the predictions have a large uncertainty there will always be a difference between these travel times. Therefore drivers may lose confidence in the predictions. This will then affect the reliability of routing control. Potential liability suits of the road operator is another reason for the reservations. A route directive does not allow a driver to compare travel times. The confidence of drivers in the route directive system is influenced only by its overall performance. Information on the actual state of traffic is factual but the interpretation for car drivers is in general difficult. The experience with the RIA sign is positive, see Subsection 2.2, but this may be due to the small size of the network. In conclusion, the routing information of actual state information and of travel directions are preferred over travel time predictions. The advantages of travel directives over actual state information are as of yet not clear.

Route directives are the most extreme of the information control measures mentioned. The public may consider route directives as not acceptable, but this behavior may change. The tendency in traffic control is to go to more and more automated motorways. The increasing congestion may bring road users to accept route directives. For a study of this point see [21].

In the investigation attention will be limited to route information in the form of route directives. Note that route directives as defined here require online feedback control from measurements provided by a MCSS. Routing information in the form of actual state information is provided either always, as in case a DRIP is installed, or at regular times, in case of radio messages. Implementation of this form of routing information does not need online control. For this type of information only planning is necessary such as where to place a DRIP and what information to provide. Route directives require online control. These directives will be based on travel predictions such as provided by the program of project DYNA.

What should be the information content of a route directive? It is not practical to offer to drivers the sight of a board with route directives for 50 or more destinations. The current practice of route directives must be adopted in which, in a hierarchical manner, a driver is directed successively to the region, to the city, to the neighborhood, and to the street of his destination. For example, for the motorway network near Amsterdam considered before, traffic on the A8 destined for Utrecht or beyond may at a VDS on the A8 see a route directive for the city of Utrecht or for the A2. While travelling through a complex motorway network he may be offered route directives at several locations. For each VDS one will have to determine how many route directives to display.

How to convey a route directive to a network user? Preferably a route directive should be addressed to travellers for a particular OD pair. The route directive may vary with OD pairs. Route guidance systems offer the possibility to communicate a route directive only to travellers for the particular OD pair considered. A VDS, radio, and RDS will in general communicate a route directive to many travellers, also to those for whom it is not directly relevant.

3.3 Modeling of control system

In this subsection a control system will be formulated. This control system will be used in the next section for synthesis of a control algorithm for route directives.

A reference on route finding in transportation networks is [8]. A discussion on modeling of information on network users in the approach of network analysis is presented in [7].

Modeling of pre-trip choices

The control measures that have an effect on the pre-trip choices of travellers have been determined in the previous subsection. These measures are: Teletext, radio , and RDS (RDS messages may be received in the car before departure). The route information received by these control measures will have an effect on

1. The decision whether or not to make the trip.
2. The departure time.

3. The entrance (origin) and exit (destination) of the motorway network to be used.
4. The route to be followed.

The reaction of persons that consider a trip may then be one of the following options:

1. To start travelling at the intended time along the planned route.
2. To start travelling at the intended time along an alternate route.
3. To start travelling at the intended time but to enter and/or exit from the motorway network at a new destination and hence by another route.
4. To delay the departure time, or, if this is still possible, to advance the departure time.
5. To cancel the trip.

Of all persons travelling from a certain entry to a certain exit point of a motorway network only a fraction will receive the information via the control measures mentioned. The reaction of persons that receive the information will be distributed over the possibilities mentioned above. The percentages occurring in these choices are dependent on the OD pair and on the time of departure. These percentages may change over the years as the fraction of persons having teletext and/or RDS in their car radio increases, and as people learn to cope with the information. Only interviews of car users can provide estimates of these percentages. It is recommended to hold regularly such panel interviews to determine such estimates.

Modeling of en-route choices

The control measures that have an effect on the en-route choices are: VDS, DRIP, radio, RDS, and route guidance systems. The route information received by these control measures will have an effect on:

1. The exit of the motorway network to be used for the trip.
2. The route to be followed.

The possible reactions of car drivers to messages are:

1. To proceed to the intended destination (exit) by the route followed so far.
2. To proceed to the intended destination by an alternate route.
3. To change the destination and hence select a new route (The new route may partly overlap with the original route).
4. To return to the origin because this option is preferred over continuation of the trip, say in case of a road block.
5. To make a temporary stop along the motorway.

From the literature [8] it is known that the reactions of car drivers to en-route messages is mixed. Some car drivers will not see or will not notice the message, some who do receive it do not know how to interpret the message, etc. Experience with routing information and variable direction signs is needed in combination with interviews of road users for modeling of drivers reactions.

The control system

The effect of control measures on the traffic flow in a motorway network is modelled for the state space representation. The modeling of this effect in the model of DYNA is completely analogous.

It is from a practical point convenient that route directives are not adjusted continuously but only at regularly spaced times and held fixed in between these times. One may think of updating the route directives every 5 minutes.

The effect of the control measures for the en-route choices is modelled in Subsection B.6 of appendix B. Below follows a summary of that subsection. Let T denote the time axis.

The effect of route directives on the traffic flow is modelled by splitting fractions. At each intersection of a motorway where the possible routes of an OD pair branch the splitting fraction describes which part of the flow follows each route. Let

$$u((i,j),k,m,.) : T \to [0,1],$$

represent the route directives for OD pair (i,j), for route k, at intersection m. Let the vector u contain all routing fractions. The *control system* for the state space model is then described by the recursion

$$x(t+1) = f_1(x(t), u(t), \lambda(t)), \quad x(t_0) = x_0. \tag{2}$$

The effect of the control measures for the pre-trip choices is modelled as follows. Consider the reference system of Subsection **??**. This reference system is extended to a control system

$$x_2(t+1) = f_2(x_2(t), u(t)), \quad x_2(t_0) = x_{20}. \tag{3}$$

A specification of this system follows. If a route directive points all traffic to the route with the shortest distance then no changes are made to this system with respect to the uncontrolled system. If the route directive for a particular OD pair points all traffic to one alternate route then the arrival intensities have to be adjusted as specified in Subsection 3.3. This adjustment is not detailed here. It will involve compliance rates as in the case of en-route choices. For intermediate values of the route directives the adjustments are correspondingly.

The compliance rates mentioned above model several effects: the message of the control measure may not reach a car driver, even if it reaches him he may not be able to interpret the information, and even if he is able to interpret the information he may for particular reasons prefer another route. The compliance rate will vary with the OD pair, the route, the location where the information is provided, and the time of day. As argued before, compliance rates have to be estimated by opinion surveys and by analyzing traffic data. The compliance rates may slowly change over time when network users become more familiar with route directives and learn to cope with the information.

Geographical aggregation

It is not practical to display at a motorway intersection route directives for a large number of destinations. Therefore route directives will be set in a geographically hierarchical manner as remarked before. In a motorway network one should therefore distinguish a small number of regions for which route directives may be issued. Every region may in the network be associated with a destination to be called an origin or destination *node*. A pair of such nodes in which the first item is an origin network node and the second a destination network node is called an *Origin-Destination Network Node pair* (ODNN). The set of ODNN pairs so obtained is small compared to the number of OD pairs of the network. The reader may want to look at Appendix C where this approach is detailed for the Amsterdam ring network. Denote by ODNN the set of OD pairs that represent origin-destination network node pairs.

4 Routing control

In this section the routing control problem is formulated and a solution for the problem is presented.

4.1 Problem formulation

In Subsection 2.1 the control problem was formulated to synthesize and evaluate control algorithms for route directives in a motorway network.

The main control measure for route directives is the VDS, but also DRIP, teletext, radio, and RDS may be used. Route guidance is another control measure, but because no route guidance system is in operation at the time this report is written, it will not be considered here. The motivation for this problem is to assess the effect of route directives on the traffic flow in a motorway network. The control algorithm can be implemented online.

Routing control will be used when it is advantageous to do so as measured by the control objectives. It can be used in peak hour traffic and in traffic situations with incidents. A study has been made on when routing control is most effective (private communication with Dr. S.A. Smulders of Rijkswaterstaat). A conclusion of this study is that in peak hour traffic routing control may not be so useful because congestion occurs always at the same locations and at the same periods. Network users have already optimized their route choice for this situation. Another conclusion of this study is that in the case of unforeseen incidents routing control may be very effective. A reason for this is that an unforeseen incident may require an unusual alternate route that an ordinary network user would not consider.

The authors agree with the conclusion that routing control can be extremely effective for network users in case of incidents. However, they are also of the opinion that routing control can be effective in peak hour traffic. In such a situation the traffic conditions can be widely varying, congestion does not occur at the same location every day and is also dependent on the weather and social activities. The evaluation of the variable message sign RIA, see [52], indicates that the route choice of drivers during congestion can be unexpected for researchers in traffic control.

The routing control algorithms must be credible and reliable. The users of the routing information are human beings many of whom will be driving through the network regularly.

Control objectives

The routing control problem may be formulated as having one decisionmaker or many decisionmakers. In the formulation of one decisionmaker, the road operator who is responsible to the government for the network, selects the control objectives and decides on routing. In the formulation of multidecisionmaking a decisionmaker corresponds to the flow of an OD pair. Although in principle the road users make the route choice individually, the choice will be similar for all users travelling on the same OD pair asuming that they are provided with the same routing information and assuming rational behaviour. Because of this distinction one must work with the decision criteria of *user optimality* and of *system optimality*. These concepts were introduced in [65] with the names of user optimality and network optimality. System optimality refers to one decisionmaker, the road operator, and user optimality refers to multi decisionmakers, one decisionmaker for each OD pair. In game and team theory the concepts of user optimality and of system optimality are discussed in more detail.

For the criterion of user optimality one distinguishes between single class user optimality and multi class user optimality depending on whether or not one distinguishes groups of network users. An example of a pair of network users is passenger cars and freight cars. With a single class user optimum every decisionmaker has the same control objective, while with a multi class user optimum the decisionmakers have in general different control objectives.

In traffic and transportation theory one calls a decision criterion *dynamic* if it refers to a sequence of decisions taken over a specified time period. Hence one speaks of a dynamic user optimality with a single class etc. The adjective dynamic is not used in control theory.

In this report attention will be focused on dynamic user optimality with a single class. This criterion represents a form of market economics which the authors regard as a useful concept for control of a motorway network.

Control objectives of the flow for a particular OD pair are:

1. Travel time
2. Travel distance
3. Travel costs
4. Road surface (quality)
5. Safety
6. Congestion and stress
7. Ease of driving
8. Environmental factors
9. Pleasant scenery

Reliability of the control system and the information provided will influence the way the control objectives are attained. Yet, reliability itself is not a control objective of the users.

In this report attention is limited to a motorway network. In practice any driver will also consider the network of non-motorways. This may be taken into consideration in a future investigation.

Control objectives of the operator of a motorway network are:

1. Total travel time of all network users
2. Network utilization
3. Safety
4. Road maintenance
5. Environmental factors

6. Congestion, occurence and frequency
7. Fairness to all network users
8. Stability of the traffic flow through the network
9. Robustness of the closed-loop control system for modeling uncertainty

For the investigation on which this paper reports a combination of travel time and travel cost (tolls) will be taken as the cost function. Many criteria are highly correlated with travel time.

Let G denote the class of control laws and for OD pair (i,j), $G(i,j)$ the corresponding class of control laws. Elements of G and $G(i,j)$ are denoted by g and $g(i,j)$ respectively. Let

$$J((i,j),g)$$

represent the cost, say the total travel time, over a specified time horizon for the traffic flow of OD pair (i,j) when control law g is used. The cost is to be minimized.

The criterion of dynamic user optimality with a single class will be specified further. The cost function per OD pair, the user, is specified above. The decision variable is the control law. The authors have decided to narrow down the dynamic user optimality concept to the Nash equilibrium concept.

Definition 4.1 *For the routing problem formulated above a control law $g^* \in G$ is said to be a* Nash equilibrium *if for all OD pairs $(i,j) \in OD$*

$$J((i,j),g^*) \leq J((i,j),g), \tag{4}$$

where

$$g(k,l) = \begin{cases} g^*(k,l), & \forall (k,l) \in OD, \quad (k,l) \neq (i,j), \\ g_1(k,l), & \text{if } (k,l) = (i,j), \end{cases} \tag{5}$$

and $g_1(i,j) \in G(i,j)$ is arbitrary.

Thus a control law is a Nash equilibrium if, for each OD pair, the cost increases when for only that OD pair another control law than the optimal one is used. The restriction to a Nash equilibrium seems quite reasonable considering the facts that in any reasonable size network there are in general many OD flows and that drivers on different OD flows do not directly communicate.

The concept of a Nash equilibrium formulated in a paper that was published in 1951 by J. Nash is related to the concept of user equilibrium proposed by J.G. Wardrop in a paper published in 1952. The concept of a Nash equilibrium as formulated above applies to the traffic flows for ODNN pairs. The concept of user equilibrium has been defined for one OD pair with two or more routes, see [65, pp. 344-345]. In the latter case the individual drivers are the decisionmakers. The usage of a Nash equilibrium to routing control in a motorway network seems therefore novel although analogous to the concept of user equilibrium.

An equilibrium property that is used in the literature is that the travel times along all possible routes are equal, see [48]. That travel times along the possible routes are equal may in some cases be the result of the use of a control law. However, there will also be cases in which it is optimal to direct all traffic to the route with the shortest travel time while the travel time along any other route is strictly larger.

Open questions are whether a Nash equilibrium exists, and, if so, whether it is unique. In general a Nash equilibrium, if it exists, is not unique.

Control problem

Problem 4.2 *Consider the routing control problem for route directives with the Nash equilibrium concept formulated above. Determine a control law $g^* \in G$ that is a Nash equilibrium.*

The routing control problem with the dynamic user optimality concept is now seen to be a dynamic game problem. The different decisionmakers, flows per OD pair, have partly conflicting control objectives. Indeed, it is advantageous for one OD flow if the flow for another pair follows a route that lowers the traffic density on the route of the first OD pair. A game problem is called dynamic if it refers to a sequence of decisions taken over time.

Control synthesis by feedback control accounts for:

1. The actual state of traffic in the network.
2. Future arrival intensities at origins (on-ramps).
3. The use of VDS's and route guidance.
4. In case of optimal control, traffic control actions to be implemented in the near future.
5. Reduction of network capacity such as lane closures in case of accidents, if such events have been detected.

As stated in [32] many of the existing or considered routing control and route guidance systems do not account for these points.

In the literature there is much confusion on the formulation of the routing control problem. The authors believe that the formulation as stated above is the proper one.

4.2 Control synthesis

In this subsection the procedure is described to synthesize a control law for Problem 4.2. A proposal for a routing control law is stated in the next subsection.

Structure control law

In this investigation control synthesis will be based on a version of the separation principle. According to this principle a control law is synthesized by combining a predictor or filter with a control law based on state feedback. The control law, see Figure 1, consists of the DYNA predictor and the DYNA controller. The DYNA predictor has been developed in the project DYNA. It provides estimates of intensities and average speeds in links of the motorway network and predictions of the intensities at entry points of the network. It also can provide predictions of intensities and average speeds in links of the network. The DYNA controller uses the predictions of the state variables and produces the input, the variable direction settings. This input is then applied to the real motorway network and to the DYNA predictor.

The expression principle refers to a procedure to be followed. There is no result in the literature that states that for a dynamic game problem of a deterministic nonlinear control system the separation property holds. The use of the principle is therefore a control synthesis approximation.

According to the separation principle, control synthesis of a control law may be separated into the synthesis of a predictor and of a control law based on state predictions. The predictor has already been developed within the DYNA project. What remains to be done is to synthesize the DYNA controller, a control law based on state predictions.

Control synthesis as sketched above also involves the internal model principle formulated by W.M. Wonham. The motorway network is excited by the arrival intensities at entry points or origins. These intensties vary in time, rather strongly so during peak hour traffic. The control law should therefore contain a model for these intensities that may then be used for predictions and control. The DYNA predictor contains a model for the intensities at entry points, based on historic data, and produces predictions of these intensities.

Simplified problem

According to the previous subsubsection Problem 4.2 is now reduced to a new problem. Consider the control system consisting of a model of the motorway network and of the reference system

$$x(t+1) = f(x(t), u(t)), \quad x(t_0) = x_0. \tag{6}$$

This control system is deterministic and nonlinear. Even for simple networks the dimension of its state space runs in the thousands. The state x of this system may be assumed available for control. A control law for this system is a map $g : X \to U$ and the set of control laws is again denoted by G. The resulting input is then

$$u(t) = g(x(t)). \tag{7}$$

Let for $g \in G$, $J(g)$ be the cost function for the time horizon considered. The problem is then to determine a control law $g^* \in G$ that is a Nash equilibriuma. Special cases of this problem are presented in appendix C.

Control synthesis procedures

Control theory provides synthesis procedures for control laws such that the closed-loop system meets the specified control objectives or optimizes the cost function. The closed-loop system consists of a control system in combination with a control law, say (6) and (7).

The theory for dynamic game problems is underdeveloped. For a book on this theory see [3]. No solution is known for the dynamic game problem formulated above. Therefore a control synthesis procedure must be proposed and it must be argued or proven that the resulting control law meets the control objectives in a satisfactory way.

The following control synthesis procedures are used in control theory for nonlinear systems:

1. Engineering synthesis.
2. Linearization of the control system in combination with control synthesis for linear systems.
3. Optimal control.

Below these procedures are discussed. General references on control synthesis for nonlinear systems are [19, 29, 37, 44, 53].

Engineering synthesis In this procedure attempts are made to construct control laws by engineering design. The literature contains many papers in which this approach is followed. Because the approach is rather problem dependent no further comments will be made.

Linearization Two well known synthesis procedures for nonlinear control systems are linearization and feedback linearization.

The synthesis procedure of linearization of a control system is a local approach. At a particular state, say x_s, the nonlinear control system

$$x(t+1) = f_1(x(t), u(t), \lambda(t)), \; x(t_0) = x_0,$$

is linearized with the result being represented by

$$\bar{x}(t+1) = A(t)\bar{x}(t) + B_1(t)u(t) + B_2(t)\lambda(t), \; \bar{x}(t_0) = \bar{x}_0. \tag{8}$$

Control theory for linear systems can then be used to synthesize a linear control law of the form

$$u(t) = F_1(t)\bar{x}(t) + F_2(t)z(t), \tag{9}$$

where z contains also the future traffic demand λ. The role of the system representing the traffic demand will not be discussed here. The linear control law should be such that the closed-loop system moves the state back to a good equilibrium point. Control objectives in the synthesis of the linear control law should include stability and cost minimization.

The usefulness of the linearization approach for routing control is not clear. The actual computations required for the linearization need to be investigated. The resulting linearized system may be time-varying. In addition it is of a high dimension. A linear control law may be constructed in several ways. The region of the state space in which the linear control law may be used while safeguarding stability is yet to be determined. A linear control law may have a limited region of applicability. These points remain to be investigated.

Linearization for routing control has been proposed by M. Papageorgiou [48].

Feedback linearization is a synthesis procedure according to which a nonlinear control system is transformed by a feedback law and a state space transformation to a linear control system. At the time this report is written it is not clear whether this procedure is useful for routing control. The nonlinearities in the control system are only approximately known and therefore the procedure may yield a control system that is only approximately linear.

Optimal control In the synthesis procedure of optimal control one wants to determine a control law for a given control system that optimizes a cost criterion. Optimal control theory provides necessary and sufficient conditions for the existence of an optimal control law. In general it is difficult to obtain an analytic expression for the control law. In practice one tries to obtain a control law by a numerical procedure. This approach requires a discretization of the state space and the input space, and use of the dynamic programming algorithm. The dimensionality of the state space model for a realistic motorway network leads to the conclusion that for such a network this procedure is computationally extremely time consuming. Possibly a considerable simplification of the control system may limit the computational complexity to a manageable form. The usefulness of this approach is as of yet uncertain.

For references on the optimal control approach to routing control see [48].

4.3 Routing control law

In this subsection a control law is formulated for routing control with route directives. Recall that a control law is defined to consist of a predictor and a controller. Below attention is focused on a controller that uses state predictions.

A control law for the simplified control system of Subsection 4.2 is a function of the state of that control system. The state contains that information that together with the input determines the future evolution of the system. Recall that in both the state space control system and the model of DYNA the state consists of:

1. The traffic densities per OD pair, per route, per section, and the average speed.
2. The state of the reference system that generates the time-varying arrival intensities at origins of the network.

Recall from Subsection 3.3 that route directives are assumed to be updated at regularly spaced times, say every 5 minutes, and held fixed in between these times.

The Nash equilibrium

As stated in Subsection 4.1, open questions for Problem 4.2 are whether a Nash equilibrium exists and, if so, whether it is unique?

The question of existence of a Nash equilibrium is difficult. In Appendix C a proof is presented of the existence of a Nash equilibrium for a simple network and a single decision routing problem. The existence of a Nash equilibrium has not been established in general.

If a Nash equilibrium exists, how can it be determined? For a dynamic game problem one can derive a set of dynamic programming equations that characterize a Nash equilibrium, see [3, Section 6.6.2 Th. 6, Section 6.5.2 Th. A-6]. Considering the size of the state space of the system, for the Amsterdam network of the order of 60.10^3, this approach is not feasible in practice. Therefore a realistic approximation procedure for the Nash equilibrium has to be formulated.

Principle for control law synthesis

The definition of a Nash equilibrium suggests a search procedure to determine such an equilibrium. It consists of an optimization per ODNN pair and a search procedure over ODNN pairs.

Principle 4.3 Routing control law synthesis.

1. *Select and order a set of origin-destination network nodes pairs.*
2. *Perform per ODNN pair selected in step 1 an online input design based on a finite-horizon look ahead procedure using travel time predictions produced by the DYNA predictor.*
3. *Repeat the steps 1 and 2 until for no ODNN pair a further improvement can be made.*
4. *Fix the route directives for the next discretization interval.*
5. *Repeat the first four steps of this procedure in the subsequent intervals.*

Comments and details of the steps of the principle follow in the remainder of this subsection.

Consider a particular ODNN pair. Suppose that the control law for the other ODNN pairs are fixed. This way one obtains for this ODNN pair an optimal control problem. Because of the complexity of the model a finite-horizon look ahead input design is used to solve this problem. In this input design procedure the DYNA predictor can be used to predict travel times along all possible routes. Then that route is selected that has the lowest travel cost and the route directives are set accordingly. Step 2 of the principle is herewith explained.

If the route directives are such that they satisfy the conditions of a Nash equilibrium for a control law then the steps 1 and 2 of the principle do not lead to further improvement for any ODNN pair. If the route directives do not satisfy the conditions of a Nash equilibrium for a control law then one may consider alternate sets of route directives. Step 1 of the principle suggests a search or optimization procedure over all ODNN pairs. The objective of this search is to determine a Nash equilibrium if one exists. The specifics of this search procedure are detailed below.

The principle formulated above leads to a control law that can be computed with the DYNA predictor and is therefore practically realisable. No claim of optimality for this control law is made. A control law based on this principle may not be a Nash equilibrium. A further theoretical investigation must establish whether or not the synthesis procedure will yield a Nash equilibrium.

Finite-horizon look ahead design and travel time prediction

The search procedure for a Nash equilibrium requires the solution of a control problem for each selected ODNN pair. Consider a control law g_0 and $(i,j) \in ODNN$. Keep the control law g_0 fixed for all other pairs in ODNN. The optimal control problem is then to determine the control law $g^*(i,j) \in G(i,j)$ that solves

$$\inf_{g(i,j) \in G(i,j)} J((i,j),(g(i,j),g_0(i,j))).$$

Because the dynamics of the control system for the motorway network is nonlinear and of high dimension it is as of yet not clear how to solve this optimal control problem. For the moment it is suggested to use a finite-horizon look ahead input design. This procedure is well known in optimal control theory, and a rather special variant of it is used in model based predictive control.

Because the cost function is based on travel time, an important problem is to predict the travel time that is to be experienced by a traveller along a route. If the cost function is the sum of travel time and travel cost then also travel time has to be estimated.

How to estimate the travel time along a route? What is needed is the travel time experienced by a traveller who at a certain time departs from a particular node of the network. It is therefore clear that the travel time depends on the route directives, the input, and hence on the control law used, not only for the ODNN flow along which the driver is travelling but also for other ODNN flows.

Definition 4.4 Instanteneous travel time prediction *Predict the travel time for a particular ODNN pair along a route based on the state of the network at the time the estimate is made by the formula*

$$\hat{\tau}(x(t)) = \sum_i \frac{L_i}{v_i(t)}, \qquad (10)$$

where the sum is over all sections of the route, L_i is the length of section i, and $v_i(t)$ is the average speed of all cars in section i at time $t \in T$.

The term instanteneous travel time prediction was suggested by M. Ben-Akiva. This prediction is an approximation of the to be experienced travel time. It neglects the future evolution of the traffic flow in the network. Thus, if a car is held up in congestion for some time then the travel time experienced may deviate considerably from the prediction.

Definition 4.5 Travel time predictor 2. *Predict the travel time for an origin-destination network node pair along a route by forward simulation of the traffic network using the DYNA predictor starting from the current state of traffic in the network while keeping the route directives for the other origin-destination network node pairs fixed in the horizon considered.*

It should be clear that the simulation of travel time predictor 2 can be performed only if the route directives of the other origin-destination network node pairs are available. Hence the assumption made in the definition. Distinguish between the availability for the other ODNN pairs of (1) their control law and (2) their input in the form of route directives. Because the control law for an ODNN pair is not available as an analytic function, the simulation can make use only of the route directives for such a pair.

How realistic are these travel time predictions? The accuracy of Predictor 2 depends of course on the accuracy of the DYNA predictor. The fact that for the other ODNN pairs only route directives are used has an effect at a later stage of the computation of the control law. Suppose that after ODNN pair (i, j) the pair (k, l) is considered. For the latter pair travel time predictions are computed and the route with the lowest travel cost is selected. Suppose that for the latter pair a new route is selected. As a consequence the future states of the network change. Hence the travel time prediction of the first ODNN pair is no longer consistent with the state of the network over the horizon and consequently the route directives may no longer be optimal in the sense of pointing to the route with the lowest travel cost.

How do the two travel time predictors compare? The instantenous travel time predictor is easy to compute yet not so realistic. It neglects the future evolution of the traffic flow in the network. Travel time predictor 2 is more realistic in that it accounts for the future traffic flow of the network and the predicted arrival intensities at the entry points. Travel time predictor 2 is realistic only if combined with an extensive search procedure over ODNN pairs. Simulation and testing will have to establish how realistic the travel time predictors are. In this investigation travel time predictor 2 is preferred over the instanteneous travel time predictor.

The travel time estimate can in principle be written as a function of the state of the network and of the state of the reference system at the time the estimate is computed, say $\hat{\tau}(x(t))$ where $x(t)$ is the state of the network at the time the prediction is made. This is true because the future travel time is determined by the current state of the network and by the predictions of arrival intensities at entry points.

An interesting discussion on the role of these travel time predictions is presented in [32]. For travel time estimates see also [54].

Travel time predictors and predictors of congestion have been investigated in the DRIVE Project GERDIEN, see [62].

Search procedure over ODNN pairs

According to Principle 4.3 for routing control a search must be made over ODNN pairs. It must therefore be specified which ODNN pairs are investigated in which order.

The search procedure defined below requires introduction of a few terms. A motorway network is distinguished into links, stretches of motorway between intersections. The traffic state of a section is said to be in *congestion* or *congested* if the average speed in the section averaged over a time period is below a specified value. For example, if the average speed is below 60 km/h over a three minute period. A link is said to be *congested* if any of its sections is congested.

Within the control law computation the network may be simulated over a finite horizon, say 30 minutes. From this simulation one may deduce which links are predicted to experience congestion in that horizon.

Procedure 4.6 Search over ODNN pairs. *Assume that from a previous step there is available a set of links in which congestion is predicted to occur in a specified finite-horizon.*

1. *Determine the set of all links that currently are congested or that are predicted to experience congestion in the finite horizon considered.*
2. *Determine the set of ODNN pairs for which the shortest distance route passes through a link determined in step 1. These ODNN pairs may be ordered according to decreasing size of traffic flow intensity.*
3. *For each ODNN pair selected perform a finite-horizon look ahead input design as described above.*
4. *Iterate the steps 1 through 3 till there is no further change in the route directives. In practice one will stop after a few iterations.*

Comments on the procedure follow. According to Principle 4.3 an input optimization is performed over all possible routes for each ODNN pair. Such an optimization is necessary only if the shortest distance route passes through a congested link or a link with predicted congestion. This argument motivates the steps 1 and 2 of the above procedure. These steps lead to a considerable reduction in the subsequent search.

As argued before, if the input in the form of route directives satisfy the conditions of a Nash equilibrium then no further improvement can be made by changes in the input. If the conditions of a Nash equilibrium are not satisfied then a route with a lower travel cost for one or more ODNN pairs can be obtained. Procedure 4.6 is thus a search procedure for a Nash equilibrium.

It is necessary to iterate the steps 1 through 3 of the principle. This may be seen from the discussion on the travel time prediction. The selection of different route directives by a search procedure may lead to congestion in links that before did not experience congestion.

Will Procedure 4.6 lead to a Nash equilibrium? It has not been established that a Nash equilibrium exists. Even if a Nash equilibrium exists it is not clear whether the procedure converges to a Nash equilibrium. The convergence speed towards a more acceptable equilibrium may be accelerated by ordering the ODNN pairs over which the search is made. Hence the suggestion to order the ODNN pairs selected in decreasing order of traffic intensity. In practice one will stop the search after one or a few iterations. See [46] for a critical comment on this procedure.

The questions on the search procedure formulated above are both of practical and of theoretical interest. In practice the travel time predictions that are used in the finite-horizon look ahead input design have a large uncertainty. With such predictions a Nash equilibrium may not exist and one would have to accept the route directives produced by a short search procedure.

Proposal control law

Based on the discussion presented so far we propose the following control law.

Consider given the set of origin-destination network node pairs. Recall that each origin or destination in ODNN represents a network sector and that for all pairs in ODNN there are two or more routes through the network. Suppose that the route directives are set every 5 minutes and held constant between updates.

Algorithm 4.7 Routing control with DYNA. *Assume available from a previous stage of the control law computation route directives over a finite-horizon, say 30 minutes, for all ODNN pairs and a set of links in which congestion is predicted to occur within this horizon.*

1. *Select and order a set of ODNN pairs as described in steps 1 and 2 of the Procedure 4.6.*
2. *For each $ODNN$ selected in step 1 perform a finite-horizon look ahead input design:*
   (a) *For all possible routes of this $ODNN$ pair set the route directive to the route, predict the travel time along that route by using travel time Predictor 2, including the DYNA predictor, and compute the travel cost of that route.*
   (b) *Select for the next discretization interval the route directive that minimizes the travel cost.*
   (c) *Select similarly route directives for the subsequent discretization intervals of the finite-horizon of this $ODNN$ pair.*
3. *Repeat the steps 1 and 2 till there is no further improvement in the travel cost for any ODNN pair.*
4. *Apply the route directives for the next discretization interval.*
5. *Repeat the first four steps of this algorithm in the subsequent intervals.*

The proposed algorithm is based on a modeling approach that, for reasons of exposition, is explained here again. A Nash equilibrium of a control is formulated in terms of an optimality property that is to hold for the control law of each ODNN pair. For routing control the control law $g : X \to U$ is not available in analytic form or as a formula because it is extremely complex. The use of the finite horizon look ahead input design leads to route directives for each of the discretization intervals of the horizon considered. The procedure proposed above therefore compares per ODNN pair not two control laws but two input settings over a finite horizon. The consequences of this modeling approximation are not yet clear. However, given the complexity of the routing problem, the limitations of the project, and the available tools, the proposed algorithm is a step towards a realistic solution.

Example 4.8 Consider the motorway ring of Amsterdam. See also Appendix C. The geographical aggregation of this ring is indicated in the appendix as consisting of the sectors A1, A2, A4, and A8. The set of $ODNN$ pairs with two or more routes is

$$\begin{aligned} ODNN &= \{(A1,A2),\ (A1,A4),\ (A1,A8),\ (A2,A1),\ (A2,A4),\ (A2,A8). \\ &\quad (A4,A1),\ (A4,A2),\ (A2,A8),\ (A8,A1),\ (A8,A2),\ (A8,A4)\}. \end{aligned}$$

The above algorithm then prescribes for $(A8, A2) \in ODNN$, for example, to consider the input that directs all traffic to route 1 along the western and southern ring, and the input that directs all traffic to route 2 along the northern and eastern ring. Next the travel times for both routes are determined, say $\hat{\tau}_1(x(t))$ and $\hat{\tau}_2(x(t))$. Then

$$u((A8, A2), t) = \begin{cases} 0, & \hat{\tau}_1(x(t)) \leq \hat{\tau}_2(x(t)), \\ 1, & \text{else.} \end{cases}$$

Similarly the optimal route directives are computed for the other ODNN pairs.

Comparison with other control laws

In other projects there is ongoing research into routing control. Because the details of these investigations are not yet published we briefly describe other control laws.

In the DRIVE Project LLAMD-MARGOT routing control has been described for a network near Munich in Germany. The control law uses a finite-horizon look ahead input design for route directives combined with an optimization over ODNN pairs. The optimization procedure is over all ODNN pairs. There is no publication available yet on the control law. This control law differs from that proposed in this report in that it does not refer to the Nash equilibrium, that the optimization procedure over all ODNN pairs is less detailed, and that the travel time prediction is not specified.

Other routing control laws are under development, see the DRIVE Projects EUROCOR, QUO VADIS, and the RHAPIT field trial. For information on these projects see the references quoted in Subsection 2.2.

Link level

The link with the control at a lower level in the hierarchy may be mentioned at this point. The simulation of the motorway network produces also trajectories for the densities and average speeds in all links of the network. These trajectories may be considered as set points for local controllers. Deviations of these set points can be controlled at the local level using the control measures of on-ramp metering and speed control. Local control at the link level in project DYNA was the task of the team of the University of Lancaster.

4.4 Analysis of a motorway network with route directives

How does a motorway network with route directives process the flow of traffic? How does travel time and travel cost compare between a network with and without route directives? Is providing route directives significantly better for the OD flows and the network than providing only actual state information on the network? These and other questions must be considered in an evaluation of routing control.

How to evaluate routing control? The most realistic test is to perform an experiment in a motorway network with route directives provided by variable message signs. Within the project DYNA there is no time to perform such an experiment also because the DYNA predictor will be completed towards the end of the project life time. One may test the performance of a controlled motorway network by simulation. For any realistic network such a simulation is a time consuming operation. Because of the short duration of the investigation no such simulation study has been made. There remains a theoretical analysis of the closed-loop system of a motorway network. This analysis will be outlined below.

For an analysis of a controlled motorway network two situations may be considered. The first closed-loop system consists of the control system of a motorway network and a control law operating on the state of the control system. This is the model of Subsection 4.2. The second closed-loop system consists of a model of the actual network, the DYNA predictor, and the DYNA controller, see Figure 1. The second system is more complex than the first one but more realistic.

A closed-loop system should be evaluated on all the control objectives stated in Subsection 4.1 and not only on the cost function.

Stability analysis

For any control law the closed-loop control system is a nonlinear system. If the OD demands are constant and if a constant feedback law is used then a time-invariant nonlinear system is obtained

$$x(t+1) = f_1(x(t), u(t), \lambda(t)) = f_1(x(t), g(x(t)), \lambda_e) = f_2(x(t)), \quad x(t_0) = x_0. \tag{11}$$

A stability analysis of this system is of interest to control synthesis. An equilibrium point is then any element $x_e \in X$ in the state space such that

$$x_e = f_2(x_e). \tag{12}$$

There will probably be two or more equilibrium points. The computation of these equilibrium points is an open question. Local stability and the regions of attraction of equilibrium points are other questions. A Lyapunov function for stability analysis may possibly be based on the flow. It seems that the presence of congestion in a motorway network is a form of a local equilibrium that is not stable.

References on stability of nonlinear systems are [19, 26, 67].

5 Concluding remarks

The CWI team of DRIVE II Project DYNA has been asked to perform research on the routing control problem for a motorway network. Only a theoretical investigation has been requested, no simulation had to be performed. The control law is required to use the DYNA predictor.

The contribution of the CWI team to project DYNA is in the extended problem formulation, in the discussion on routing control measures, in the use of the Nash equilibrium concept for dynamic user optimality, and in the proposed control law.

The proposed control law needs further investigation, testing by simulation, and testing on the road. It will take several more years of development work before it comes to a road test.

A future investigation into routing control may address the following points:

1. At the theoretical level there is the question of existence of a Nash equilibrium for the routing control problem. See Appendix C for a starting point of this problem.

2. The search procedure over selected ODNN pairs as proposed in Procedure 4.6. A theoretical question is whether the search procedure terminates in a finite number of steps if there exists a Nash equilibrium. A practical question is whether a few steps of the procedure produces an acceptable solution to the routing control problem and, if so, how these steps should be performed.

3. Can the finite-horizon look ahead input design using travel time predictions be improved upon?

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A The traffic model of project DYNA

Estimation and prediction of OD traffic demand See [28] for information on this topic. For OD estimation see [13].

Dynamic traffic assignment Traffic assignment is a method from operations research and transportation research by which one may compute link flows in a network. The method requires the specification of a demand and a supply model. The link flows are then defined as the solution of the equation that results if one equates the demand and the supply function. The solution is often referred to as a fixpoint of this equation. A general reference on equilibrium analysis is [1].

Traffic assignment models may be distinguished into within-day static models, with uniform demand and supply, and within-day dynamic models, with varying demand and/or supply. Dynamic traffic assignment is a method to determine link flows in a network using a within-day dynamic model. The description of DYNA's traffic assignment method presented below is taken from [9]. For general references on dynamic traffic assignment see [10, 11, 7, 5].

Link performance models Within DYNA travel times are needed on individual links for different traffic flow levels. For procedures to compute travel times see [35].

Path choice models Within the model of DYNA there is a module that generates a set of paths for each origin-destination pair. For a description of this module see [18]. Drivers at an entry point to the network select a path according to a discrete probability choice model. At branching points in the network drivers may reconsider their path choice.

B State space model for traffic flow in a motorway network

The text of this appendix is a modified version of part of the report [63]. Section B.6 is new.

In this section a state space model for traffic flow in a motorway network will be presented. This model was developed prior to the start of project DYNA. The objectives of the model are that it can be used for routing control of motorway traffic and that it is not too complex. Because a routing advice is in general issued only if there is congestion or if such a traffic condition is anticipated, the model should be realistic in describing the occurence and buildup of congestion. Because traffic intensities fluctuate during peak hours when routing is mainly needed, a model that incorporates traffic densities seems quite appropriate.

Example B.1 By way of example a simple urban motorway network will be treated consisting of a rectangle or ring around a city with four links to neighbouring cities, see figure 3. This network has the advantage that for every origin-destination pair there are at most two routes through the network.

Since the figure provides only limited information, a brief description of the network follows. The nodes 1,2,3,4 represent starting and termination nodes of the network on links to other cities. The nodes 9,10,11,12 represent entries and exits. The nodes 5,6,7,8 represent motorway intersections at which there are neither exits nor entries. Freeway traffic may flow from node 1 via intersection 5 to node 9, or to node 12. There is also the flow from node 9 via intersection 5 to node 1 or to node 12. The flow form node 1 to node 3 has two possible routes, one via the intersections 5, 6, and 7 and another via the intersections 5, 8, and 7.

B.1 A model of a motorway network

Consider a motorway network. It is assumed that a control and signalling system is installed on this network. Such a system has a detector station at about every 500 m. of the motorway. At the location of this detector station there are two detection loops in every lane. The information provided by these loops are passage times and passage speeds of vehicles. It will also be assumed that detector loops are installed on every entry, exit, and branch of a motorway intersection of the motorway network. This assumption may not be realistic but for this investigation it will be adopted. A section of the network is a stretch of motorway between two adjacent detector stations. It follows from these assumptions that every entry, exit, or intersection is located at the beginning or end of a section.

Consider a model of a motorway network consisting of an ordered graph. The nodes of the graph correspond to detector stations. Let $I \subset Z_+$ denote the index set of the nodes. At a node there may be an entry, exit, intersection, or only the joint of two sections. Let $I_O \subset I$ denote the index set of the nodes at which there are entries or origins and starting points of the network. Let $I_D \subset I$ denote the index set of the nodes at which there are exits or destinations or terminating points of the network. Let $I_I \subset I$ denote the index set of the nodes at which there are motorway intersections. For example 2.1 these sets are

$$I = \{1, 2, \ldots, 12\}, I_O = I_D = \{1, 2, 3, 4, 9, 10, 11, 12\}, I_I = \{5, 6, 7, 8\}.$$

A *section* of the network model is an ordered pair $(l, m) \in I \times I$ such that there is a path from node l to node m not passing another node. The set of network sections is denoted by $S \subset (I \times I)$. A section of the network model corresponds to a motorway section. For example 2.1 the set of sections consists of

$$S = \{(1,5),(5,1),(5,9),(9,5),\ldots\}.$$

An *origin-destination pair* (OD-pair) is an ordered tuple $(i,j) \in I_O \times I_D$ where $i \in I_O$ is a node with an entry or origin and $j \in I_D$ is a node with an exit or destination. Let $OD \subset (I_O \times I_D)$ be the set of OD-pairs. For example 2.1 this set is given by

$$OD = \{(1,2),(1,3),(1,4),(1,9),(1,10),(1,11),(1,12),\ldots\}.$$

A *route* of the OD-pair (i,j) is a path from node i to node j. A route is specified by a chain of sections. In general an OD-pair may admit several routes. For $(i,j) \in OD$ let $K(i,j) \subset Z_+$ be the index set of distinct possible routes and let

$$R(i,j) = \{r((i,j),k), k \in K(i,j)\}$$

denote the corresponding set of routes. A route is described as a chain of sections

$$r((i,j),k) = \{(i,i_2),(i_2,i_3),\ldots,(i_s,j)\}.$$

Consider example 2.1. For $(1,3) \in OD$ there are only two routes specified by $K(1,3) = \{1,2\}$,

$$r((1,3),1) = \{(1,5),(5,9),(9,6),(6,10),(10,7),(7,3)\}, \quad (13)$$

$$r((1,3),1) = \{(1,5),(5,12),(12,8),(8,11),(11,7),(7,3)\}. \quad (14)$$

B.2 A model of traffic flow in a motorway network

H.J. Payne's model for motorway traffic flow will be used. This model is a modification of one introduced by M.J. Lighthill and G.B. Whitham [36]. The latter model has only traffic density as a state variable. Payne extended this model with the state variable of average speed. There is a continuous-time version [50] and a discrete-time version [51] of this model. Changes to Payne's model were proposed by M. Papageorgiou [47], by S.A. Smulders [57] whose model was validated for a motorway in The Netherlands, and by P. Varaiya and co-workers [31]. Below yet another version of Payne's model is defined that is close to that of [57]. For related work see [55, 56, 58, 57, 59, 60].

In the first phase of this investigation a continuous-time model was proposed. The discrete-time model proposed below is close to that of A. Messmer and M. Papageorgiou [38] and that of Papageorgiou [48]. The differences are pointed out below.

B.3 The state variables of the model

Consider a motorway network. As argued above, a model of such a network consists of sections. The length of a section will usually be about 500 m. The model proposed below is realistic only with relatively short sections of a length that is at most 1000 m.

According to Payne's model, the state variables of motorway traffic flow are density and average speed of each section. Because the purpose of the model is routing, the traffic density per section must be separated out per origin-destination pair and per route. The model of this paper therefore differs from that of Papageorgiou [48]. In the latter model the traffic density is distinguished only on destination. In [38] it seems that fixed fractions of the density are used for each origin. Considering the dynamics of traffic flow in a motorway network, it seems necessary to consider a model in which the density per origin-destination pair and per route are distinguished. The route is important in this also. There is a report that the model of [48] leads to the phenomenon that in a particular situation motorway traffic flows back in the direction it came from. This may occur because only local directions are used. If a route is prescribed this phenomenon cannot occur. Later in the paper the model will be extended so that vehicles may change their route.

Denote the density of OD-pair $(i,j) \in OD$, of route number $k \in K(i,j)$, and of section $(l,m) \in S$ by $x((i,j),k,(l,m),.) : T \to R_+$ in vehicles per km per lane, veh/km.lane. Denote the density in section $(l,m) \in S$ by $x((l,m),.) : T \to R_+$

$$x((l,m),t) = \sum x((i,j),k,(l,m),t)$$

where the sum is over all $(i,j) \in OD$ and all routes $k \in K(i,j)$ that use section (l,m).

Denote the branching fraction for OD-pair (i,j), for route number $k \in K(i,j)$, for traffic flowing in section $(l,m) \in S$ to intersection $m \in I_I$ according to route k by

$$p((i,j),k,(l,m),t) = \frac{x((i,j),k,(l,m),t)}{x((l,m),t)} \tag{15}$$

This fraction represents the chance that a vehicle travelling in section (l,m) at $t \in T$ uses $(i,j) \in OD$ and route k.

Denote the average speed of all cars in section (l,m) by $v((l,m),.) : T \to R_+$.

The flow of section $(l,m) \in S$ is defined as $q((l,m),.) : T \to R_+$

$$q((l,m),t) = lane(l,m)x((l,m),t)v((l,m),t) \quad veh/h,$$

where $lane(l,m)$ is the number of lanes of the section.

Consider two adjacent sections, say $(l,m),(m,s) \in S$ such that at node $m \in I$ there is no entry, no exit, and no intersection. The transition flow at node m is defined by

$$\alpha q((l,m),t) + (1-\alpha)q((m,s),t) \quad veh/h. \tag{16}$$

Here $\alpha \in [0,1]$ is a weighting factor. The transition flow is thus a convex combination of the flows in both sections. For a continuous-time model it was estimated in [57] that the value $\alpha = 0.85$ is appropriate.

B.4 The state transitions

The transitions of the state variables, density and average speed per section, are described next. An example is presented in B.2. The transition of the density is based on a continuity equation

$$\begin{aligned} x((i,j),k,(l,m),t+1) = x((i,j),k,(l,m),t) + \frac{\Delta t}{Le(l,m)lane(l,m)} \times \\ \times \left[q_{in}((i,j),k,(l,m),t) - q_{out}((i,j),k,(l,m),t) I_{(x((i,j),k,(l,m),t)>0)} \right], \end{aligned} \tag{17}$$

where $(i,j) \in OD, k \in K(i,j), (l,m) \in S, \Delta t$ is the length of the time step in hours, $Le(l,m)$ is the length of the section in km, and q_{in} and q_{out} are respectively the flow into and the flow out of the section in the interval $(t, t+1)$. These flows depend on the section considered, they are specified below for several special cases.

The transition of the average speed of section $(l,m) \in S$ is assumed to be given by the formula

$$\begin{aligned} v((l,m),t+1) &= v((l,m),t) + \frac{\Delta t}{\tau}[v^e(x((l,m),t)) - v((l,m),t)] \\ &\quad + \frac{\Delta t}{L_e(l,m)} v((s,l),t)[v((s,l),t) - v((l,m),t)] \\ &\quad + \Delta t (Le(l,m) lane(l,m))^2 [\beta x((l,m),t) + (1-\beta) x((m,r),t)] \times \\ &\quad \times [x((m,r),t) - x((l,m),t)] \end{aligned} \tag{18}$$

where $\tau \in (0,\infty)$ is a time constant, $\beta \in (0,1)$, (s,l) is the adjacent upstream section, and (m,r) the adjacent downstream section. When either node l or node m is an intersection then the terms of (18) that refer to an upstream or downstream section beyond the intersection are set to zero. Here $v^e : R_+ \to R_+$ is the equilibrium relation between density and average speed. The actual form for this relation suggested by S.A. Smulders [57] will be used,

$$v^e(x) = \begin{cases} v_{free} - ax, & 0 \le x \le x_{crit}, \\ b[\frac{1}{x} - \frac{1}{x_{jam}}], & x_{crit} \le x \le x_{jam}, \\ 0, & x_{jam} \le x. \end{cases} \tag{19}$$

For a particular stretch of motorway the parameter values of this function are $x_{crit} = 27$ veh/km.lane, $x_{jam} = 110$ veh/km.lane, $a = 0.58 km^2/h$, $b = 3197 veh/h.lane$. The second term on the right-hand side of (18) is called the *relaxation term*. If the average speed differs from the equilibrium speed according to the actual density via the equilibrium relation, then the average speed is adjusted. The third term is called the *convection term*. If in the upstream section the average speed is higher than in the section under consideration then the average speed is adjusted accordingly. The last term is called the *anticipation term*. If drivers notice that density is gradually increasing they are expected to anticipate on this by reducing their speed. This term is the most controversial one, several variants have been proposed. Additional validation of this term is needed.

Transition for a section not adjacent to an intersection Consider a section $(l,m) \in S$ such that at node $m \in I$ there is no intersection. Let $(i,j) \in OD, k \in K(i,j)$, and let $(m,s) \in S$ be the section downstream of (l,m). Then the flow out of section (l,m) for these variables is

$$\begin{aligned} & q_{out}((i,j),k,(l,m),t) \\ &\quad = p((i,j),k,(l,m),t)\alpha \times \\ &\qquad \times \left[q((l,m),t) - \sum_{i_1 \in I_O, k_1 \in K(i_1,m)} p((i_1,m),k_1,(l,m),t) q((l,m),t) \right] \\ &\qquad + p((i,j),k,(l,m),t)(1-\alpha) \left[q((m,s),t) - \sum_{j_1 \in I_D, k_1 \in K(m,j_1)} \lambda((m,j_1),k_1,t) \right] \\ &\qquad + p((i,m),k,(l,m),t) q((l,m),t) I_{\{j=m\}}. \end{aligned} \tag{20}$$

If $j \neq m$ then $q((l,m),t)$ represents the flow of section (l,m),

$$\sum_{i_1 \in I_O, k_1 \in K(i_1,m)} p((i_1,m),k_1,(l,m),t) q((l,m),t)$$

represents the exit flow at node m, and

$$\sum_{j_1 \in I_D, k_1 \in K(m,j_1)} \lambda((m,j_1),k_1,t)$$

represents the flow at node m into section (m,s) from outside the network. Here $\alpha \in [0,1]$ is a weighting factor. The expression (20) is similar to that of (16). If $j = m$ then there is a flow out of the section for state $x((i,m),k,(l,m),t)$ as indicated in (20). The flow into section (m,s) is

$$\begin{aligned} & q_{in}((i,j),k,(m,s),t) \\ & \quad = p((i,j),k,(l,m),t)\alpha \times \\ & \qquad \times \left[q((l,m),t) - \sum_{i_1 \in I_O, k_1 \in K(i_1,m)} p((i_1,m),k_1,(l,m),t) q((l,m),t) \right] \\ & \qquad + p((i,j),k,(l,m),t)(1-\alpha) \left[q((m,s),t) - \sum_{j_1 \in I_D, k_1 \in K(m,j_1)} \lambda((m,j_1),k_1,t) \right] \\ & \qquad + \lambda((m,j),k,t) I_{\{i=m\}}. \end{aligned} \tag{21}$$

If there is no exit or no entry at node m then the corresponding terms in (21) are zero. It is assumed that for the OD-pair (i,j) there is an initial assignment of traffic to a route $r((i,j),k) \in R(i,j)$ with route number $k \in K(i,j)$. For example, route $r((i,j),k)$ may be selected such that the travel distance along route $r((i,j),k)$ is the smallest of all those of routes in $R(i,j)$. Below the model will be extended such that vehicles can change route.

Transition for a section adjacent to an intersection Consider section $(l,m) \in S$ and suppose that there is an intersection at node $m \in I$. The flow out of the section for $(i,j) \in OD, k \in K(i,j)$, and $(l,m) \in S$ is

$$q_{out}((i,j),k,(l,m),t) = p((i,j),k,(l,m),t) q((l,m),t).$$

Consider next a section $(m,s) \in S$ that starts at a motorway intersection $m \in I_I$. For $(i,j) \in OD, k \in K(i,j)$ the flow into variable $x((i,j),k,(m,s),t)$ is given by

$$q_{in}((i,j),k,(m,s),t) = p((i,j),k,(l,m),t) q((l,m),t)$$

where (l,m) is the section directly upstream from (m,s) according to route k.

Boundary conditions

The intensities of the entry flows are assumed to be specified by the user of the model and not to depend on the state of the network. Consider next a section from which traffic leaves the network. The flow out of such a section $(l,j) \in S$ for $(i,j) \in OD$, $k \in K(i,j)$ is assumed to be given by

$$q_{out}((i,j),k,(l,j),t) = p((i,j),k,(l,j),t) q((l,j),t).$$

B.5 The full state space model

Consider the state variables $x((i,j),k,(l,m),.) : T \to R_+$, $v((l,m),.) : T \to R_+$, for $(i,j) \in OD$, $k \in K(i,j)$, and $(l,m) \in r((i,j),k)$. Collect these variables in the state vector $x : T \to R_+^M$ for some $M \in Z_+$. Similarly combine the arrival rates $\lambda((i,j),k,.) : T \to R_+$ into a vector $\lambda : T \to R_+^S$. These rates are assumed to be specified by the user of the model. The dynamic system may then be written as

$$x(t+1) = f(x(t), \lambda(t)), x(t_0) = x_0. \tag{22}$$

The differences can now be pointed out between the discrete-time model of this paper and that of M. Papageorgiou and co-workers [38, 48]. In [48] a density per section is used. Then on page 477 the density is decomposed into a density per destination and a recursion is proposed for this density. However, it is not stated in the paper that this density per destination is used in the remainder of the paper or whether the heuristic approach of page 476 is used. In this heuristic approach one divides the density into fixed fractions. For the transition of these fractions a heuristic rule is proposed. In [38] the same model is proposed as in [48]. In the model of this paper state variables are considered consisting of densities per OD-pair, per route, and per section. As mentioned before, the dependence on the route prevents unrealistic rerouting. Experience with modeling will have to show the usefulness of the state space model and answer the question of minimality of the state space.

Example B.2 *A state space model for a network with two routes.* Consider the motorway network of figure 4. The sets of node numbers are

$$I = \{1,2,3,4,5,6\}, I_O = \{1\}, I_D = \{6\}, I_I = \{2,5\}.$$

The set of OD-pairs is $OD = \{(1,6)\}$. The set of sections is

$$S = \{(1,2),(2,3),(3,5),(2,4),(4,5),(5,6)\}.$$

The set of routes for OD-pair (1,6) is

$$\begin{aligned}
r((1,6),1) &= \{(1,2),(2,3),(3,5),(5,6)\}, &(23)\\
r((1,6),2) &= \{(1,2),(2,4),(4,5),(5,6)\}, &(24)\\
R(1,6) &= \{r((1,6),1), r((1,6),2)\} = \{r((1,6),k), \quad k \in K(1,6)\}, &(25)\\
K(1,6) &= \{1,2\}. &(26)
\end{aligned}$$

The density state variables are

$$\begin{aligned}
&x((1,6),1,(1,2),t), x((1,6),1,(2,3),t), x((1,6),1,(3,5),t),\\
&x((1,6),1,(5,6),t), x((1,6),2,(1,2),t), x((1,6),2,(2,4),t),\\
&x((1,6),2,(4,5),t), x((1,6),2,(5,6),t).
\end{aligned}$$

The state transitions are specified by the recursions

$$\begin{aligned}
x((1,6),1,(1,2),t+1) &= x((1,6),1,(1,2),t) + \frac{\Delta t}{Le(1,2)lane(1,2)} \times\\
&\quad \times[\lambda((1,6),1,t) - p((1,6),1,(1,2),t)q((1,2),t)], &(27)\\
q((1,2),t) &= lane(1,2)x((1,2),t)v((1,2),t), &(28)\\
x((1,2),t) &= x((1,6),1,(1,2),t) + x((1,6),2,(1,2),t) &(29)\\
x((1,6),2,(1,2),t+1) &= x((1,6),2,(1,2),t) + \frac{\Delta t}{Le(1,2)lane(1,2)} \times\\
&\quad \times[\lambda((1,6),2,t) - p((1,6),2,(1,2),t)q((1,2),t)], &(30)\\
x((1,6),1,(2,3),t+1) &= x((1,6),1,(2,3),t) + \frac{\Delta t}{Le(2,3)lane(2,3)} \times\\
&\quad \times[p((1,6),1,(1,2),t)q((1,2),t)\\
&\quad -\alpha q((2,3),t) - (1-\alpha)q((3,5),t)], &(31)
\end{aligned}$$

$$q((2,3),t) = lane(2,3)x((1,6),1,(2,3),t)v((2,3),t), \quad (32)$$

$$x((1,6),1,(3,5),t+1) = x((1,6),1,(3,5),t) + \frac{\Delta t}{Le(3,5)lane(3,5)} \times \times[\alpha q((2,3),t) + (1-\alpha)q((3,5),t) - q((3,5),t)], \quad (33)$$

$$q((3,5),t) = lane(3,5)x((1,6),1,(3,5),t)v((3,5),t), \quad (34)$$

$$x((1,6),2,(2,4),t+1) = x((1,6),2,(2,4),t) + \frac{\Delta t}{Le(2,4)lane(2,4)} \times \times[p((1,6),2,(1,2),t)q((1,2),t) -\alpha q((2,4),t) - (1-\alpha)q((4,5),t)], \quad (35)$$

$$q((2,4),t) = lane(2,4)x((1,6),2,(2,4),t)v((2,4),t), \quad (36)$$

$$x((1,6),2,(4,5),t+1) = x((1,6),2,(4,5),t) + \frac{\Delta t}{Le(4,5)lane(4,5)} \times \times[\alpha q((2,4),t) + (1-\alpha)q((4,5),t) - q((4,5),t)], \quad (37)$$

$$q((4,5),t) = lane(4,5)x((1,6),2,(4,5),t)v((4,5),t), \quad (38)$$

$$x((1,6),1,(5,6),t+1) = x((1,6),1,(5,6),t) + \frac{\Delta t}{Le(5,6)lane(5,6)} \times \times[q((3,5),t) - p((1,6),1,(5,6),t)q((5,6),t)], \quad (39)$$

$$q((5,6),t) = lane(5,6)x((5,6),t)v((5,6),t), \quad (40)$$

$$x((5,6),t) = x((1,6),1,(5,6),t) + x((1,6),2,(5,6),t) \quad (41)$$

$$x((1,6),2,(5,6),t+1) = x((1,6),2,(5,6),t) + \frac{\Delta t}{Le(5,6)lane(5,6)} \times \times[q((4,5),t) - p((1,6),2,(5,6),t)q((5,6),t)]. \quad (42)$$

B.6 How variable direction signs affect the traffic flow in the network

Consider a motorway network on which a control and signalling system is installed. Suppose that it is possible to provide drivers with route directives. For example, at every motorway intersection it is possible to display variable directions signs for each destination. Or it may be possible to inform drivers by radio or by another communication channel that transmits messages from the control center to drivers. In the network of Example B.1 drivers on the link (1,5) for the OD-pair (1,3) may at Intersection 5 be advised to follow route $r((1,3),1)$ or route $r((1,3),2)$, see (13,14).

The state space model for motorway traffic flow in a network may be considered as a control system. To complete the specification of this system the effect of the input on the flows must be specified. This is first done for an example.

Consider Example B.2 with Figure 4. For the OD-pair $(1,6) \in OD$ there are two routes, $r((1,6),1)$ and $r((1,6),2)$. These routes have the sections (1,2) and (5,6) in common. There is a branching point of the routes at the intersection of Node 2. Therefore a route directive may be displayed at Intersection 2. The effect of this routing directive on the flow may be modelled in the state transitions by

$$x((1,6),1,(2,3),t+1) = x((1,6),1,(2,3),t) + \frac{\Delta t}{Le(2,3)lane(2,3)} \times \times[w((1,6),1,2,t)q((1,2),t) - \alpha q((2,3),t) - (1-\alpha)q((3,5),t)], \quad (43)$$

$$x((1,6),2,(2,4),t+1) = x((1,6),2,(2,4),t) + \frac{\Delta t}{Le(2,4)lane(2,4)} \times \times[w((1,6),2,2,t)q((1,2),t) - \alpha q((2,4),t) - (1-\alpha)q((4,5),t)]. \quad (44)$$

Here $u((1,6),1,2,.) : T \rightarrow [0,1]$ represents the route directive of the flow for OD pair $(1,6)$ that at Intersection 2 is directed to Route 1. If $u((1,6),1,2,t) = 1$ then all traffic is directed to Route 1, if $u(.) = 0$ then all traffic is directed to Route 2, while if $u(.) = 0.7$ then 70% of the time traffic is directed to Route 1 and 30% of the time to Route 2. Let $c((1,6),1,2,.) : U \rightarrow [0,1]$ be the *compliance rate* of traffic for OD pair $(1,6)$ that at Node 2 is directed to Route 1. Define the *splitting fraction* $w((1,6),1,2,.) : U \times T \rightarrow [0,1]$ as

$$w((1,6),1,2,u,t) = \begin{cases} c((1,6),1,2,1), & \text{if } u((1,6),1,2,t) = 1, \\ c((1,6),1,2,0), & \text{if } u((1,6),1,2,t) = 0, \\ c((1,6),1,2,0.7), & \text{if } u((1,6),1,2,t) = 0.7. \end{cases} \tag{45}$$

For example,

$$\begin{aligned} w((1,6),1,2,u,t) &= \begin{cases} 0.9, & \text{if } u(.) = 1, \\ 0.3, & \text{if } u(.) = 0, \\ 0.8, & \text{if } u(.) = 0.7, \end{cases} \\ w((1,6),2,2,u,t) &= \begin{cases} 0.1, & \text{if } u(.) = 1, \\ 0.7, & \text{if } u(.) = 0, \\ 0.2, & \text{if } u(.) = 0.7. \end{cases} \end{aligned}$$

Note that, if the route directive is $u(.) = 1$ then in this example 10% of the drivers take Route 2. Because the control system always needs an input, route directive $u(.) = 1$ also represents the case that no route directive is displayed, assuming that Route 1 is the shortest distance route.

In general the effect of routing control on the flow in a motorway network is described as follows. Consider the OD-pair (i,j). The set of routes for this OD-pair is $R(i,j) = \{r((i,j),k), k \in K(i,j)\}$. At each intersection where the traffic flow for (i,j) branches into two or more routes a route directive may be displayed. If the intersection at node m is such a branching point then the motorway operator must specify the routing directives $u((i,j),k,m,.) : T \rightarrow [0,1]$ for all $k \in K(i,j)$. Let $c((i,j),k,m,.) : U \rightarrow [0,1]$ be the associated compliance rates. Define the associated splitting fractions $w((i,j),k,m,.,.) : U \times T \rightarrow [0,1]$ as

$$w((i,j),k,m,u,t) = c((i,j),k,m,u), \quad \text{if } u((i,j),k,m,t) = u. \tag{46}$$

These fractions must satisfy the condition that for all $(i,j) \in OD$, $m \in I$, $t \in T$ fixed

$$\sum w((i,j),k,m,u,t) = 1,$$

where the sum is over all routes that branch at intersection m. In addition the route directives have to be consistent, all traffic destined for node j and beyond has to be directed to the same route. In the state transition equations the effect of routing control is modelled as

$$w((i,j),k,m,u,t)[\sum_{k_1} p((i,j),k_1,(l,m),t)q((l,m),t)]$$

where the sum is over all routes indexed by k_1 that use section (l,m) followed by section (m,s_1), in which node s_1 depends on route k_1.

Let $u : T \rightarrow U$ be the vector of all route directives for all $(i,j) \in OD$ and all relevant motorway intersections. The input u describes the set of all route directives, it will be called a *route scheme* or the input. The control system may then be written as

$$x(t+1) = f_1(x(t), u(t), \lambda(t)), \quad x(t_0) = x_0. \tag{47}$$

A *control law* is a map $g : X \to U$ that for each state x specifies the route directives in the network. A control law specifies the input to the control system via

$$u(t) = g(x(t)). \tag{48}$$

The control system (47) with the control law (48) then becomes a routing system given by

$$x(t+1) = f_1(x(t), g(x(t)), \lambda(t)) = f_2(x(t), \lambda(t)), \quad x(t_0) = x_0. \tag{49}$$

It is suggested that in route directives non-integer values of the input are useful. Consider first the case of integer route directives, in particular the case in which for an OD-pair there exist only two routes. At the branching point the input space is $U = \{0, 1\}$, and the input is $u((i,j), k, m, t) = 1$ or 0, corresponding to whether the routing directive points traffic to route 1 or to the alternate route. During periods of congestion when the future travel time along both routes is approximately equal, the input is likely to oscillate between both routes. In fact, the simulations mentioned in the paper [48] show precisely this phenomenon. Such oscillations seem detrimental to the stability of the traffic flow. In communication networks this phenomenon is less of a problem due to the faster dynamics.

With an input that may take non-integer values traffic is not likely to exhibit such oscillations. If the input space and the input at an intersection are respectively $U = [0, 1]$ and $u((i,j), k, m, t) = 0.7$, say, then road users are directed such that 70% of the time they are directed to route k and 30% of the time they are directed to the alternate route. This may be effected by time division: in each 10 minute interval traffic is directed to route 1 for 7 minutes and to route 2 for 3 minutes. In practice one may want to limit the number of input values, say to $U = \{0, 0.3, 0.5, 0.7, 1.0\}$.

C Control studies

The problem of routing control with route directives as formulated in Subsection 4.3 is in general too complex to be approached directly. Therefore routing control problems will be considered for several simple networks. It is expected that control laws for these simple networks will suggest ways to approach the control problem for arbitrary networks.

C.1 Routing control of traffic flow for one ODNN pair only

Consider the motorway network near Amsterdam as displayed in Figure 2. Consider in this network the flow (A8,A2) from the motorway A8 to the motorway A2. Route 1 for this OD pair, with the shortest distance, is via the western and southern ring, through the Coentunnel. Route 2, the only alternative in this network, is that via the northern and eastern ring of this network, through the Zeeburgertunnel.

In this control study the problem is posed to determine a control law for the flow on the OD pair (A8,A2) assuming that state measurements are available and that all other OD flows proceed along the route of shortest distance. Thus none of the flows for the other OD pairs is affected by the route directives for the flow of (A8,A2). This somewhat artificial problem has the advantage that it is a problem with only one decisionmaker and yet has the characteristics of a full network.

The routing control problem for this traffic flow may be considered as an optimal control problem for a nonlinear control system.

The control law of Subsection 4.3 will be formulated for this traffic flow. It is based on estimation of future travel times by the DYNA predictor as formulated in Subsection 4.3. Consider a time moment at which the input, the route directive for the flow of the OD pair (A8,A2), must be determined for the next time interval, say 5 minutes. The first input is to direct all traffic to Route 1. Estimate the travel time $\tau(1,x)$ for a car leaving the origin in the interval considered to reach the destination based on the current state of traffic. The second control input is to direct all traffic to Route 2. Determine again the travel time along this route, say $\tau(2,x)$. Then select the input for the current state of traffic and for the interval considered as those inputs or route directives that minimize the travel time. Then simulate this network with the selected control law for the interval. At the end of this interval the procedure is repeated. Thus again the travel time is estimated along both routes. In actual computations the travel times for subsequent intervals can be estimated by a smaller number of simulations.

C.2 Routing control for two OD pairs

Consider as before the motorway network near Amsterdam as displayed in Figure 2. Consider the flows for the OD pairs (A1,A8) and (A2,A8). The shortest distance route from motorway A1 to motorway A8 is along the eastern and northern ring. The shortest distance route from motorway A2 to motorway A8 is along the southern and western ring. In peak hour traffic or at other congested times there may be a traffic jam on the western ring in the direction of the A8. In this case the flow for the OD pair (A2,A8) may consider the alternate route via the eastern and northern ring. Note that if traffic flow for OD pair (A2,A8) takes this alternate route then it will increase the traffic on the eastern and northern ring and in an unfortunate case may also cause a traffic jam there.

The routing control problem for this network is to determine a control law for route directives with the Nash equilibrium. This network has the property that two OD flows interact. It is suggested that the online optimization procedure be started with the OD pair (A2,A8). For this OD pair the route with the lowest cost is determined by estimating the future travel time if the route directive is either the southern and western route or the alternative. Subsequently the same online optimization procedure is performed for the OD pair (A1,A8).

C.3 Existence of a Nash equilibrium

Consider the motorway network near Amsterdam as before. Attention is limited to the ODNN pairs (A8,A2) and (A8,A4). For both ODNN pairs, denote the route along the western ring by r_1 and the route along the northern ring by r_2. Define $J_1(r,s)$ to be the travel time of (A8,A2) when for (A8,A2) route r is used and for (A8,A4) route s. Similarly, let $J_2(r,s)$ be the travel time of (A8,A4) when for (A8,A2) route r is used and for (A8,A4) route s.

The travel times have to satisfy certain conditions:

1. When less traffic is sent along a route then the travel time along this route decreases:

$$J_1(r_1,r_2) < J_1(r_1,r_1), \tag{50}$$
$$J_1(r_2,r_1) < J_1(r_2,r_2), \tag{51}$$
$$J_2(r_2,r_1) < J_2(r_1,r_1), \tag{52}$$
$$J_2(r_1,r_2) < J_2(r_2,r_2). \tag{53}$$

2. When of two routes one is identical to part of the other then a longer distance implies a longer travel time:

$$J_1(r_2,r_2) < J_2(r_2,r_2), \tag{54}$$
$$J_2(r_1,r_1) < J_1(r_1,r_1). \tag{55}$$

For the problem described above there are four possible Nash equilibria:

1. (r_1,r_1) or

$$J_1(r_1,r_1) \leq J_1(r_2,r_1), \tag{56}$$
$$J_2(r_1,r_1) \leq J_2(r_1,r_2); \tag{57}$$

2. (r_2,r_1) or

$$J_1(r_2,r_1) \leq J_1(r_1,r_1), \tag{58}$$
$$J_2(r_2,r_1) \leq J_2(r_2,r_2); \tag{59}$$

3. (r_2,r_2) or

$$J_1(r_2,r_2) \leq J_1(r_1,r_2), \tag{60}$$
$$J_2(r_2,r_2) \leq J_2(r_2,r_1); \tag{61}$$

4. (r_1,r_2) or

$$J_1(r_1,r_2) \leq J_1(r_1,r_2), \tag{62}$$
$$J_2(r_2,r_2) \leq J_2(r_2,r_1). \tag{63}$$

Proposition C.1 *For the problem defined above and with the conditions stated above, there always exists a Nash equilibrium.*

Proof Suppose that there exists no Nash equilibrium. Then

$$
\begin{aligned}
&((J_1(r_2,r_1) < J_1(r_1,r_1)) \vee (J_2(r_1,r_2) < J_2(r_1,r_1))) \\
&\wedge ((J_1(r_1,r_1) < J_1(r_2,r_1)) \vee (J_2(r_2,r_2) < J_2(r_2,r_1))) \\
&\wedge ((J_1(r_1,r_2) < J_1(r_2,r_2)) \vee (J_2(r_2,r_1) < J_2(r_2,r_2))) \\
&\wedge ((J_1(r_2,r_2) < J_1(r_1,r_2)) \vee (J_2(r_1,r_1) < J_2(r_1,r_2)))\,.
\end{aligned}
$$

Inspection yields that from this formula only two cases follow

1. Case 1

$$J_1(r_2,r_1) \quad < J_1(r_1,r_1), \tag{64}$$
$$J_2(r_2,r_2) \quad < J_2(r_2,r_1), \tag{65}$$
$$J_1(r_1,r_2) \quad < J_1(r_2,r_2), \tag{66}$$
$$J_2(r_1,r_1) \quad < J_2(r_1,r_2); \tag{67}$$

2. Case 2

$$J_2(r_1,r_2) \quad < J_2(r_1,r_1), \tag{68}$$
$$J_1(r_2,r_2) \quad < J_1(r_1,r_2), \tag{69}$$
$$J_2(r_2,r_1) \quad < J_2(r_2,r_2), \tag{70}$$
$$J_1(r_1,r_1) \quad < J_1(r_2,r_1). \tag{71}$$

Assume that case 1 holds. Then

$$J_2(r_2,r_2) <^{(65)} J_2(r_2,r_1) <^{(52)} J_2(r_1,r_1) <^{(67)} J_2(r_1,r_2) <^{(53)} J_2(r_2,r_2),$$

which yields a contradiction. Next, assume that case 2 holds. Then

$$J_1(r_1,r_1) <^{(71)} J_1(r_2,r_1) <^{(51)} J_1(r_2,r_2) <^{(69)} J_1(r_1,r_2) <^{(50)} J_1(r_1,r_1),$$

which again yiels a contradiction. Because neither case 1 nor case 2 can occur we conclude that there always exists a Nash equilibrium. □

D Abbreviations

ATIS	Advanced Travel Information Systems
ATT	Advanced Transport Telematics
CWI	Centrum voor Wiskunde en Informatica
	Centre for Mathematics and Computer Science
DYNA	DRIVE-II project V2036
GIS	Geographic Information System
GPS	Global Positioning System
GSM	Global System for Mobile telecommunication
MCSS	Motorway Control and Signalling System
RDS	Radio Data System
SOCRATES	System Of Cellular Radio for Traffic Efficiency and Safety
TCC	Traffic Control Centre
TMC	Travel Message Channel (formerly Traffic)
VDS	Variable Direction Sign
VMS	Variable Message Sign

E Figures

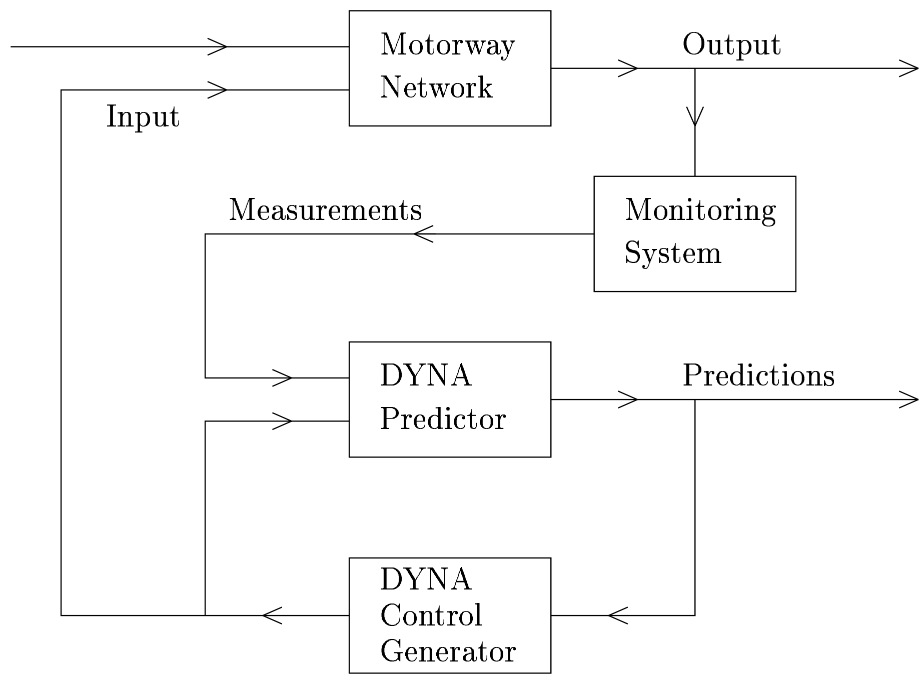

Figure 1: Closed-loop control system.

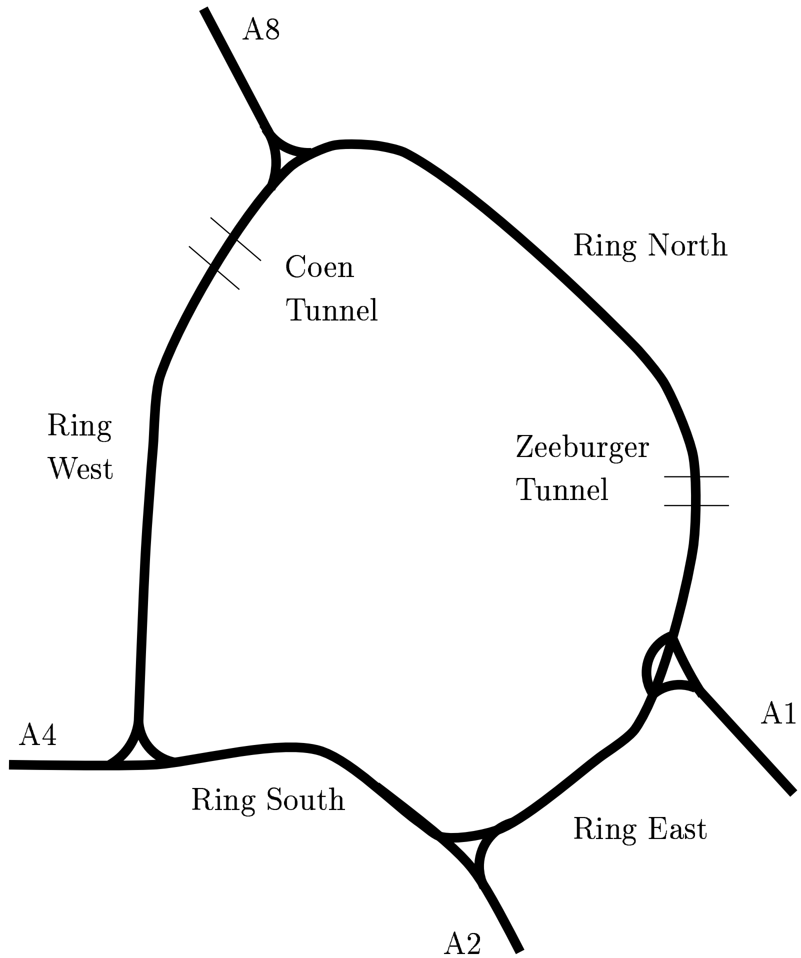

Figure 2: Schema of motorway network near Amsterdam.

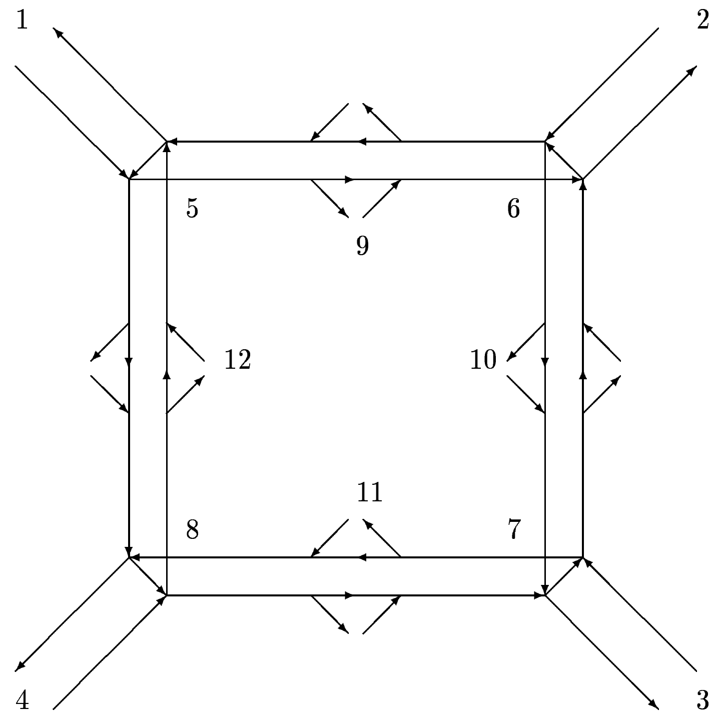

Figure 3: A motorway network in the form of a ring around an urban area.

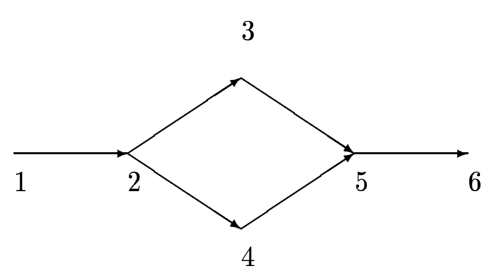

Figure 4: A motorway network consisting of two routes.

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