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On ϕ–Amicable Pairs (with Appendix)

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Abstract

Let $\phi(n)$ denote Euler's totient function, i.e., the number of positive integers $< n$ and prime to n. We study pairs of positive integers (a_0, a_1) with $a_0 \le a_1$ such that $\phi(a_0) = \phi(a_1) = (a_0 + a_1)/k$ for some integer $k \ge 1$. We call these numbers ***ϕ–amicable pairs with multiplier*** k, analogously to Carmichael's multiply amicable pairs for the σ–function (which sums all the divisors of n).

We have computed all the ϕ–amicable pairs with larger member $\le 10^9$ and found 812 pairs for which the greatest common divisor is squarefree. With any such pair infinitely many other ϕ–amicable pairs can be associated. We present a table of the 58 so-called primitive ϕ–amicable pairs for which the larger member does not exceed 10^6. Next, ϕ–amicable pairs with a given prime structure are studied. It is proved that a relatively prime ϕ–amicable pair has at least twelve distinct prime factors and that, with the exception of the pair $(2^2, 2 \cdot 3)$, if one member of a ϕ–amicable pair has two distinct prime factors, then the other has at least four distinct prime factors. Finally, analogies with construction methods for the classical amicable numbers are shown; application of these methods yields another 79 primitive ϕ–amicable pairs with larger member $> 10^9$, the largest pair consisting of two 46-digit numbers.

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1. Introduction

Let $\phi(n)$ be Euler's totient function. The pair (a_0, a_1) with $1 < a_0 \le a_1$ is called *ϕ–amicable with multiplier k* if

$$\phi(a_0) = \phi(a_1) = \frac{a_0 + a_1}{k} \text{ for some integer } k \ge 1. \tag{1}$$

Since $\phi(n) < n$, we cannot have $k = 1$. To see that in fact $k > 2$, notice that if $k = 2$, then

$$\frac{a_0 + a_1}{2} > \frac{\phi(a_0) + \phi(a_1)}{2} = \phi(a_0).$$

If $a_0 = a_1 = a$, we have the equation $\phi(a) = 2a/k = a/l$ provided that k is even. This is known [6] to have the (only) solutions $a = 2^\alpha$ for $l = 2$ and $a = 2^\alpha 3^\beta$ for $l = 3$. If k is

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odd, $k = pk'$ say, where p is an odd prime, then $p \mid a$, $a = p^\gamma b$ with $\gcd(p, b) = 1$, and the equation easily reduces to the form $\phi(b) = b/l$ (where $l = k'(p-1)/2$). We assume from now that $a_0 < a_1$.

An analogous definition for the σ–function was given by Carmichael [4, p. 399], who called two positive integers a_0 and a_1 a *multiply amicable pair* if $\sigma(a_0) = \sigma(a_1) = l(a_0 + a_1)$ for some positive integer l. For $l = 1$, we obtain the "classical" amicable pairs, like $(220, 284) = (2^2 5 \cdot 11, 2^2 71)$, which was known already to the ancient Greeks. Mason [7] gives various multiply amicable pairs for $l = 2$ and $l = 3$.

This paper is organized as follows. In Section 2 the results are presented of an exhaustive computation of all the ϕ–amicable pairs with larger member $\le 10^9$. From the pairs found one readily sees that if (a_0, a_1) is a ϕ–amicable pair such that $p^n \mid a_0$ and $p^n \mid a_1$ for some prime p and positive integer n, then p^n may be replaced by p^{n+m} for any positive integer m, yielding infinitely many other ϕ–amicable pairs. The smallest such example is $(2^2, 2 \cdot 3)$ inducing the pairs $(2^{n+1}, 2^n 3)$, $n = 1, 2, \ldots$. So-called *primitive* ϕ–amicable pairs are introduced next, i.e., pairs (a_0, a_1) which cannot be generated from a smaller ϕ–amicable pair with some substitution like the above one. There are 499 primitive ϕ–amicable pairs $\le 10^9$.

In Section 3 ten basic properties of primitive ϕ–amicable pairs are given, most of them being used in the sequel. In Section 4 we derive several theoretical results which were partly suggested by our numerical results. We prove, among other things, that there are finitely many primitive ϕ–amicable pairs with a given number of different prime factors, that the number of different prime factors in the members of a relatively prime ϕ–amicable pair is at least twelve, and that if one member of a ϕ–amicable pair has two distinct prime factors, the other has at least four. In Section 5, finally, analogies with construction methods for ordinary amicable pairs are derived. Application yields another 79 primitive ϕ–amicable pairs in addition to those found with our exhaustive search.

The notation $a \parallel b$ denotes that a is a unitary divisor of b, that is, $\gcd(a, b/a) = 1$. We shall regularly use the following well-known properties of Euler's ϕ–function: $\phi(n) < n$ for $n > 1$; if $d < n$ and $d \mid n$, then $\phi(d) < \phi(n)$; ϕ is multiplicative; if p is a prime and m a positive integer, then $\phi(p^m) = p^{m-1}(p-1)$; $\phi(n)/n = \prod_{p \mid n}(p-1)/p$.

2. Exhaustive computation of ϕ-amicable pairs

We have computed a complete list of ϕ–amicable pairs with larger member $\le 10^9$ as follows. Suppose a_1 is given, and we wish to test if it is the larger member of a ϕ–amicable pair (a_0, a_1). We test if there is an integer k such that, for $a_0 = k\phi(a_1) - a_1$, $\phi(a_0) = \phi(a_1)$. If $a_0 \le \phi(a_1)$, $\phi(a_0) < \phi(a_1)$. Therefore, we must have $\phi(a_1) < a_0 < a_1$, so that

$$1 + \frac{a_1}{\phi(a_1)} < k < \frac{2a_1}{\phi(a_1)}. \tag{2}$$

For all a_1 with $2 \le a_1 \le 10^9$ we computed $\phi(a_1)$ and for k in the range (2): $a_0 = k\phi(a_1) - a_1$ and $\phi(a_0)$. If $\phi(a_0) = \phi(a_1)$, (a_0, a_1) is a ϕ–amicable pair with multiplier k.

Inspection of these pairs suggested the following two propositions which are easily proved with the help of the defining equations (1).

Proposition 1. *If (a_0, a_1) is ϕ–amicable with multiplier k, then for any prime p for which $p \mid \gcd(a_0, a_1)$, also (pa_0, pa_1) is ϕ–amicable, with the same multiplier. Conversely, if $p^2 \mid \gcd(a_0, a_1)$, then also $(a_0/p, a_1/p)$ is a ϕ–amicable pair.*

Proposition 2. *Let (a_0, a_1) be ϕ–amicable with multiplier k and let $b > 1$ be such that $\gcd(b, a_0) = \gcd(b, a_1) = 1$. If $kb/\phi(b) =: \overline{k}$ is an integer, then (ba_0, ba_1) is ϕ–amicable with multiplier $\overline{k} > k$. Conversely, if $c > 1$ is such that $c \parallel a_0$ and $c \parallel a_1$ and if $k\phi(c)/c =: \underline{k}$ is an integer, then $(a_0/c, a_1/c)$ is also ϕ–amicable, with multiplier $\underline{k} < k$.*

This in turn suggests the following

Definition. *A ϕ–amicable pair (a_0, a_1) is called primitive if there is no common divisor $g > 1$ of a_0 and a_1 such that $(a_0/g, a_1/g)$ is also ϕ–amicable.*

The second part of Proposition 1 shows that the greatest common divisor of a primitive ϕ–amicable pair must be square-free.

With the first part of Proposition 2 we may generate new (non-primitive) ϕ–amicable pairs from known primitive pairs pairs by finding numbers b satisfying the required conditions. It is no loss of generality to take b square-free (because of Proposition 1). If $b \bmod \phi(b) = 0$ then the only squarefree solutions 6 are $b = 2$ and $b = 6$, with $\overline{k} = 2k$, and $\overline{k} = 3k$, respectively. If $b \bmod \phi(b) \neq 0$ then the denominator of the reduced fraction of $b/\phi(b)$ should divide k.

To assist with the application of the first part of Proposition 2, in the first four columns of Table 1 we present, for $3 \leq k \leq 12$, the squarefree values of b for which $kb/\phi(b) = \overline{k}$ is integral, with $\overline{k} \leq 12$. These values of b are easily found by solving the equation $kb = \overline{k}\phi(b)$ for given values of k and $\overline{k}$. For the second part of Proposition 2, the last four columns of Table 1 give the squarefree values of c for which $k\phi(c)/c = \underline{k}$ is integral, with $\underline{k} \leq 12$. This is just a reverse reordering of the left half of Table 1.

k	b	$b/\phi(b)$	$\overline{k} = kb/\phi(b)$	k	c	$\phi(c)/c$	$\underline{k} = k\phi(c)/c$
3	2	2	6	5	5	4/5	4
3	14	7/3	7	6	2	1/2	3
3	6	3	9	6	3	2/3	4
4	5	5/4	5	7	14	3/7	3
4	3	3/2	6	7	21	4/7	4
4	21	7/4	7	7	7	6/7	6
4	2	2	8	8	2	1/2	4
4	10	5/2	10	9	6	1/3	3
4	110	11/4	11	9	3	2/3	6
4	6	3	12	10	10	2/5	4
5	2	2	10	10	2	1/2	5
5	22	11/5	11	10	5	4/5	8
6	7	7/6	7	11	110	4/11	4
6	3	3/2	9	11	22	5/11	5
6	2	2	12	11	55	8/11	8
8	5	5/4	10	11	11	10/11	10
8	55	11/8	11	12	6	1/3	4
8	3	3/2	12	12	2	1/2	6
10	11	11/10	11	12	3	2/3	8

TABLE 1. The pairs (k, b), $3 \leq k \leq 12$, for which $kb/\phi(b) = \overline{k}$ is integral, with $\overline{k} \leq 12$, and the converse, for use in Proposition 2.

In Tables 2–3 the 58 primitive ϕ–amicable pairs (a_0, a_1) with $a_1 \leq 10^6$ are listed, for increasing values of a_1. The last column gives the values of b for which Proposition 2 yields other, non-primitive, pairs (with squarefree greatest common divisor), and the corresponding multiplier $\overline{k}$.

The total number of primitive ϕ–amicable pairs with larger member $\leq 10^9$ is 499. They occur with multipliers 3, 4, 5, and 7, and frequencies 109, 158, 144, and 88, respectively. We have applied the first part of Proposition 2 to these pairs, and found 475 non-primitive pairs with multipliers 5, 6, 7, 8, 9, 10, and 11, and frequencies 34, 109, 20, 158, 109, 34, and 11, respectively. The smallest pair with multiplier 10 comes from pair number 37 in

#	a_0	a_1	k	b, $\overline{k}$
1	$4 = 2^2$	$6 = 2 \cdot 3$	5	
2	$78 = 2 \cdot 3 \cdot 13$	$90 = 2 \cdot 3^2 5$	7	
3	$465 = 3 \cdot 5 \cdot 31$	$495 = 3^2 5 \cdot 11$	4	2, 8
4	$438 = 2 \cdot 3 \cdot 73$	$570 = 2 \cdot 3 \cdot 5 \cdot 19$	7	
5	$609 = 3 \cdot 7 \cdot 29$	$735 = 3 \cdot 5 \cdot 7^2$	4	2, 8
6	$158 = 2 \cdot 3 \cdot 193$	$1530 = 2 \cdot 3^2 5 \cdot 17$	7	
7	$530 = 2 \cdot 5 \cdot 11 \cdot 23$	$3630 = 2 \cdot 3 \cdot 5 \cdot 11^2$	7	
8	$3685 = 5 \cdot 11 \cdot 67$	$4235 = 5 \cdot 7 \cdot 11^2$	3	2, 6; 6, 9
9	$3934 = 2 \cdot 7 \cdot 281$	$4466 = 2 \cdot 7 \cdot 11 \cdot 29$	5	
10	$5475 = 3 \cdot 5^2 73$	$6045 = 3 \cdot 5 \cdot 13 \cdot 31$	4	2, 8
11	$5978 = 2 \cdot 7^2 61$	$6622 = 2 \cdot 7 \cdot 11 \cdot 43$	5	
12	$7525 = 5^2 7 \cdot 43$	$7595 = 5 \cdot 7^2 31$	3	2, 6; 6, 9
13	$11925 = 3^2 5^2 53$	$13035 = 3 \cdot 5 \cdot 11 \cdot 79$	4	2, 8
14	$12207 = 3 \cdot 13 \cdot 313$	$17745 = 3 \cdot 5 \cdot 7 \cdot 13^2$	4	2, 8
15	$21035 = 5 \cdot 7 \cdot 601$	$22165 = 5 \cdot 11 \cdot 13 \cdot 31$	3	2, 6; 6, 9
16	$19815 = 3 \cdot 5 \cdot 1321$	$22425 = 3 \cdot 5^2 13 \cdot 23$	4	2, 8
17	$22085 = 5 \cdot 7 \cdot 631$	$23275 = 5^2 7^2 19$	3	2, 6; 6, 9
18	$21189 = 3 \cdot 7 \cdot 1009$	$27195 = 3 \cdot 5 \cdot 7^2 37$	4	2, 8
19	$40334 = 2 \cdot 7 \cdot 43 \cdot 67$	$42826 = 2 \cdot 7^2 19 \cdot 23$	5	
20	$59045 = 5 \cdot 7^2 241$	$61915 = 5 \cdot 7 \cdot 29 \cdot 61$	3	2, 6; 6, 9
21	$66795 = 3 \cdot 5 \cdot 61 \cdot 73$	$71445 = 3 \cdot 5 \cdot 11 \cdot 433$	4	2, 8
22	$60726 = 2 \cdot 3 \cdot 29 \cdot 349$	$75690 = 2 \cdot 3^2 5 \cdot 29^2$	7	
23	$65945 = 5 \cdot 11^2 109$	$76615 = 5 \cdot 7 \cdot 11 \cdot 199$	3	2, 6; 6, 9
24	$70422 = 2 \cdot 3 \cdot 11^2 97$	$77418 = 2 \cdot 3^2 11 \cdot 17 \cdot 23$	7	
25	$73486 = 2 \cdot 7 \cdot 29 \cdot 181$	$77714 = 2 \cdot 7^2 13 \cdot 61$	5	
26	$70334 = 2 \cdot 11 \cdot 23 \cdot 139$	$81466 = 2 \cdot 7 \cdot 11 \cdot 23^2$	5	
27	$89745 = 3 \cdot 5 \cdot 31 \cdot 193$	$94575 = 3 \cdot 5^2 13 \cdot 97$	4	2, 8
28	$94666 = 2 \cdot 11 \cdot 13 \cdot 331$	$103334 = 2 \cdot 7 \cdot 11^2 61$	5	
29	$87591 = 3 \cdot 7 \cdot 43 \cdot 97$	$105945 = 3 \cdot 5 \cdot 7 \cdot 1009$	4	2, 8
30	$109298 = 2 \cdot 7 \cdot 37 \cdot 211$	$117502 = 2 \cdot 7^2 11 \cdot 109$	5	

TABLE 2. The first 30 primitive ϕ–amicable pairs of the 58 with larger member $\leq 10^6$.

Table 3, by multiplication of both members by $b = 10$. The smallest pair with multiplier 11 comes from pair number 136:

$$13290459 = 3 \cdot 7 \cdot 13 \cdot 89 \cdot 547,\ 14385189 = 3 \cdot 7 \cdot 13 \cdot 23 \cdot 29 \cdot 79,\ k = 4,$$

by multiplication of both members by $b = 110$. Furthermore, with the factors $b = 5, 2, 10$, this pair gives three other pairs with multipliers 5, 8, and 10, respectively. The complete list of 499 primitive ϕ–amicable pairs with larger member $\leq 10^9$ is given in the appendix to this paper. For each primitive pair (a_0, a_1) the pairs $(b, \overline{k})$ are given for which (ba_0, ba_1) is a ϕ–amicable pair with multiplier $\overline{k}$, according to Proposition 2.

It is easy to show that pairs with multipliers 3 and 4, and with squarefree greatest common divisor, must be primitive. All the pairs with multiplier 6, 8, 9, and 10 which we found are non-primitive, but we do not know whether there exist primitive pairs with such a multiplier. Notice that we found both primitive and non-primitive pairs with multipliers 5 and 7. The smallest non-primitive pairs with multipliers 5 and 7 are

$$1438815 = 3 \cdot 5 \cdot 7 \cdot 71 \cdot 193,\ 1786785 = 3 \cdot 5 \cdot 7^2 11 \cdot 13 \cdot 17,\ k = 5,$$

#	a_0	a_1	k	$b, \overline{k}$
31	$119434 = 2 \cdot 7 \cdot 19 \cdot 449$	$122486 = 2 \cdot 7 \cdot 13 \cdot 673$	5	
32	$164486 = 2 \cdot 7 \cdot 31 \cdot 379$	$175714 = 2 \cdot 7^2 11 \cdot 163$	5	
33	$211002 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 139$	$214038 = 2 \cdot 3^2 11 \cdot 23 \cdot 47$	7	
34	$239775 = 3 \cdot 5^2 23 \cdot 139$	$245985 = 3 \cdot 5 \cdot 23^2 31$	4	2, 8
35	$326325 = 3 \cdot 5^2 19 \cdot 229$	$330315 = 3 \cdot 5 \cdot 19^2 61$	4	2, 8
36	$267486 = 2 \cdot 3 \cdot 109 \cdot 409$	$349410 = 2 \cdot 3 \cdot 5 \cdot 19 \cdot 613$	7	
37	$287763 = 3 \cdot 7 \cdot 71 \cdot 193$	$357357 = 3 \cdot 7^2 11 \cdot 13 \cdot 17$	4	5, 5; 2, 8; 10, 10
38	$350405 = 5 \cdot 11 \cdot 23 \cdot 277$	$378235 = 5 \cdot 11 \cdot 13 \cdot 23^2$	3	2, 6; 14, 7; 6, 9
39	$367114 = 2 \cdot 11^2 37 \cdot 41$	$424886 = 2 \cdot 7 \cdot 11 \cdot 31 \cdot 89$	5	
40	$335814 = 2 \cdot 3 \cdot 97 \cdot 577$	$438330 = 2 \cdot 3 \cdot 5 \cdot 19 \cdot 769$	7	
41	$363486 = 2 \cdot 3 \cdot 29 \cdot 2089$	$455010 = 2 \cdot 3 \cdot 5 \cdot 29 \cdot 523$	7	
42	$441035 = 5 \cdot 7 \cdot 12601$	$466165 = 5 \cdot 7 \cdot 19 \cdot 701$	3	2, 6; 6, 9
43	$444414 = 2 \cdot 3 \cdot 17 \cdot 4357$	$531330 = 2 \cdot 3 \cdot 5 \cdot 89 \cdot 199$	7	
44	$545566 = 2 \cdot 7^2 19 \cdot 293$	$558194 = 2 \cdot 7 \cdot 13 \cdot 3067$	5	
45	$494054 = 2 \cdot 11 \cdot 17 \cdot 1321$	$561946 = 2 \cdot 7 \cdot 11 \cdot 41 \cdot 89$	5	
46	$344810 = 2 \cdot 5 \cdot 29^2 41$	$564630 = 2 \cdot 3 \cdot 5 \cdot 11 \cdot 29 \cdot 59$	7	
47	$442686 = 2 \cdot 3 \cdot 89 \cdot 829$	$577410 = 2 \cdot 3 \cdot 5 \cdot 19 \cdot 1013$	7	
48	$546675 = 3 \cdot 5^2 37 \cdot 197$	$582285 = 3 \cdot 5 \cdot 11 \cdot 3529$	4	2, 8
49	$573545 = 5 \cdot 7^2 2341$	$605815 = 5 \cdot 7 \cdot 19 \cdot 911$	3	2, 6; 6, 9
50	$614185 = 5 \cdot 11 \cdot 13 \cdot 859$	$621335 = 5 \cdot 11^2 13 \cdot 79$	3	2, 6; 14, 7; 6, 9
51	$489363 = 3 \cdot 7^2 3329$	$628845 = 3 \cdot 5 \cdot 7 \cdot 53 \cdot 113$	4	2, 8
52	$624358 = 2 \cdot 7^2 23 \cdot 277$	$650762 = 2 \cdot 7 \cdot 23 \cdot 43 \cdot 47$	5	
53	$722502 = 2 \cdot 3^2 11 \cdot 41 \cdot 89$	$755898 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 881$	7	
54	$756925 = 5^2 13 \cdot 17 \cdot 137$	$809795 = 5 \cdot 7 \cdot 17 \cdot 1361$	3	2, 6; 6, 9
55	$793914 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 523$	$813846 = 2 \cdot 3 \cdot 11^2 19 \cdot 59$	7	
56	$806386 = 2 \cdot 7 \cdot 239 \cdot 241$	$907214 = 2 \cdot 7 \cdot 11 \cdot 43 \cdot 137$	5	
57	$886006 = 2 \cdot 11 \cdot 17 \cdot 23 \cdot 103$	$909194 = 2 \cdot 11^2 13 \cdot 17^2$	5	
58	$898835 = 5 \cdot 7 \cdot 61 \cdot 421$	$915565 = 5 \cdot 7^2 37 \cdot 101$	3	2, 6; 6, 9

TABLE 3. The final 28 primitive ϕ–amicable pairs of the 58 with larger member $\leq 10^6$.

and

$$4905670 = 2 \cdot 5 \cdot 7 \cdot 11 \cdot 23 \cdot 277, \ 5295290 = 2 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 23^2, \ k = 7,$$

respectively; their "mother pairs" are numbers 37 and 38 in Table 3, respectively.

From each ϕ–amicable pair it is possible to generate infinitely many others with the rule of Proposition 1. For example, from the primitive pair $(2 \cdot 5 \cdot 11 \cdot 23, 2 \cdot 3 \cdot 5 \cdot 11^2)$ with multiplier $k = 7$ (number 7 in Table 2) we generate the non-primitive pairs:

$$(2^{i_1+1}5^{i_2+1}11^{i_3+1}23, 2^{i_1+1}3 \cdot 5^{i_2+1}11^{i_3+2}), \ i_1, i_2, i_3 \geq 0, \ i_1 + i_2 + i_3 > 0, \ k = 7.$$

3. Basic properties of primitive ϕ–amicable pairs

In this section we list ten basic properties of primitive ϕ–amicable pairs.

Let (a_0, a_1) be a primitive ϕ-amicable pair with $a_0 < a_1$ and multiplier k, and let p denote a prime.

B1 We cannot have $a_0 = 2$ or $a_0 = 3$. *Proof:* If $a_0 = 2$ then $1 = \phi(a_0) = \phi(a_1)$, so that $a_1 = 2$. But $a_1 > a_0$. If $a_0 = 3$ then $2 = \phi(a_0) = \phi(a_1)$, so that $a_1 = 3$, 4, or 6. None of these allows $a_0 < a_1$, or $a_0 + a_1$ to be even.

B2 For $i = 0$ or 1, if $p^2 \parallel a_i$ then $p \parallel a_{1-i}$. *Proof:* Write $a_i = p^2 a$. Then $kp(p-1)\phi(a) = p^2 a + a_{1-i}$, so $p \mid a_{1-i}$. If $p^2 \mid a_{1-i}$, then (a_0, a_1) is not primitive, by Proposition 1.

B3 For $i = 0$ or 1, if $2^2 \parallel a_i$ then $(a_0, a_1) = (2^2, 2 \cdot 3)$. *Proof:* Using **B2**, write $a_i = 2^2 a$ and $a_{1-i} = 2b$, where a and b are odd. Then $2\phi(a) = \phi(b) = (4a + 2b)/k$, so $k\phi(a) = 2a + b$. The right-hand side is odd and the left-hand side is even unless $a = 1$, and then $b = 3$.

B4 a_0 is even if and only if a_1 is even. *Proof:* Let $i = 0$ or 1. Using **B3**, we may suppose that $2 \parallel a_i$. Write $a_i = 2a$, where a is odd, $a > 1$. Then $k\phi(a) = 2a + a_{1-i}$. The left-hand side is even, so a_{1-i} must be even.

B5 a_1 is not prime. *Proof:* This is true for any a_0, a_1 satisfying $1 < a_0 < a_1$ and $\phi(a_0) = \phi(a_1)$, for then $\phi(a_1) < a_0$. If $a_1 = p$ then $\phi(a_1) = p - 1 < a_0 < p = a_1$, which is impossible.

B6 For $i = 0$ or 1, a_i cannot be a perfect square, or twice a perfect square, except if $(a_0, a_1) = (2^2, 2 \cdot 3)$. *Proof:* Suppose $a_i = n^2$ is a perfect square. By **B3**, if n is even then $(a_0, a_1) = (2^2, 2 \cdot 3)$. Suppose n is odd. By **B2**, $a_{1-i} = na$ with $\gcd(n, a) = 1$. Then

$$\phi(n^2) = n^2 \prod_{p \mid n} \left(1 - \frac{1}{p}\right),$$

$$\phi(na) = \phi(n)\phi(a) = n\phi(a) \prod_{p \mid n} \left(1 - \frac{1}{p}\right),$$

and these are equal, so $\phi(a) = n$, an odd number. Since $a_i > 1$, this is a contradiction. Suppose next that $a_i = 2n^2$, where n is odd since (a_0, a_1) is primitive. The proof proceeds in the same way, since $\phi(2n^2) = \phi(n^2)$ and $\phi(2na) = \phi(na)$.

B7 We cannot have $a_0 \mid a_1$. *Proof:* If $a_0 \mid a_1$ then $\phi(a_0) \le \phi(a_1)$, with equality only when $a_0 = a_1$. But $a_0 < a_1$, so $\phi(a_0) < \phi(a_1)$, a contradiction.

B8 For at least one prime p, $p \parallel a_0 a_1$.

B9 If a_0 and a_1 are squarefree, then $3 \mid a_0$ if and only if $3 \mid a_1$. *Proof:* Suppose $3 \parallel a_0$ and write $a_0 = 3a$. We have

$$2\phi(a) = \phi(a_1) = \frac{3a + a_1}{k}$$

for some k. Suppose $3 \nmid a_1$. Then $3 \nmid \phi(a_1)$ so $p \bmod 3 = 2$ for all p with $p \mid a_1$, and $\phi(a_1) \bmod 3 = 1$. Also, $3 \nmid \phi(a)$ so $q \bmod 3 = 2$ for all primes q with $q \mid a$, and $\phi(a) \bmod 3 = 1$. But then, mod 3, $\phi(a_1) = 2\phi(a) = 2$, a contradiction. Hence $3 \mid a_1$. Similarly, $3 \mid a_0$ if $3 \mid a_1$.

B10 If (a_0, a_1) has multiplier $k = 3$, then the smallest prime divisor of $a_0 a_1$ is at least 5. If (a_0, a_1) has multiplier $k = 4$, then $a_0 a_1$ is odd.

4. ϕ–AMICABLE PAIRS WITH A GIVEN PRIME STRUCTURE

Property **B5**, given in Section 3, shows that there exists no ϕ–amicable pair for which the larger member is a prime. Here, we shall study pairs with a given prime structure more generally. First we have the following general finiteness result:

Theorem 1. *There are only finitely many primitive ϕ–amicable pairs with a given number of different prime factors.*

Proof. This proof is inspired by analogous results of Borho [1] for ordinary and unitary amicable pairs. First we notice that if the total number t of different prime factors of a primitive ϕ–amicable pair (a_0, a_1) is prescribed, then there are only finitely many values of k, r, and s with $r + s = t$ for which

$$k = \frac{a_0}{\phi(a_0)} + \frac{a_1}{\phi(a_1)} = \prod_{i=1}^{r} \frac{p_i}{p_i - 1} + \prod_{j=1}^{s} \frac{q_j}{q_j - 1}$$

can hold. Therefore we are done if we can show that the equation

$$k = \frac{x_1}{x_1 - 1} \cdots \frac{x_r}{x_r - 1} + \frac{y_1}{y_1 - 1} \cdots \frac{y_s}{y_s - 1} \tag{4}$$

has finitely many solutions in integers ≥ 2, for given k, r, and s. Here, we should realize that the only solutions of (4) that can actually correspond to a primitive ϕ–amicable pair are those for which all the x_i's and y_j's are prime. In those cases in which $x_{i_1} = y_{j_1} = p$, say, with corresponding $a_0 = \prod_{i=1}^r x_i$ and $a_1 = \prod_{j=1}^s y_j$, also (a_0p, a_1), and (a_0, a_1p) are potential primitive ϕ–amicable pairs (leading to the same equation (4)). However, only at most one of these three can actually be a primitive ϕ–amicable pair (if one of them satisfies the first equality in (1), the others do not).

Suppose (4) has infinitely many solutions. Let $z = (z_1, \ldots, z_t)$ be a solution of (4) and denote by $z^{(1)}, z^{(2)}, \ldots$, an infinite sequence of different solutions. Then this contains an infinite subsequence in which the last component is non-decreasing, i.e., without loss of generality we may assume

$$z_t^{(1)} \leq z_t^{(2)} \leq \ldots .$$

This reasoning can be repeated for the components $t-1, t-2, \ldots, 1$, so that after t subsequence-transitions we finally have a (still) infinite subsequence of different solutions ordered in such a way that each component of one solution is not greater than the corresponding component of the next solution. However, since the right-hand side of (4) is monotonically decreasing in each of its variables, if $z^{(1)}$ is a solution, then $z^{(2)}$ cannot be a solution. This is a contradiction. □

The next theorem gives an upper bound for the smallest odd prime divisor of a ϕ–amicable pair, as a function of the multiplier and the maximum of the numbers of different prime factors in the two members.

Theorem 2. *Let (a_0, a_1) be a ϕ–amicable pair with multiplier k, where a_0 and a_1 have r and s different prime divisors, respectively. Let P be the smallest odd prime divisor of a_0a_1 and $m = \max\{r, s\}$. Then*

$$P \leq \frac{k + 4m - 8}{k - 4} \text{ with } k \geq 5, \text{ if the pair is even,} \tag{5}$$

and

$$P \leq \frac{k + 2m - 2}{k - 2} \text{ with } k \geq 3, \text{ if the pair is odd.} \tag{6}$$

Proof. We have

$$k = \prod_{p|a_0} \frac{p}{p-1} + \prod_{q|a_1} \frac{q}{q-1} \quad (p, q \text{ primes}).$$

If both a_0 and a_1 are even, **B10** implies that $k \geq 5$. Furthermore, since consecutive primes differ at least by 1, it follows that

$$k \leq 2 \prod_{i=1}^{r-1} \frac{P+i-1}{P+i-2} + 2 \prod_{i=1}^{s-1} \frac{P+i-1}{P+i-2} \leq 4 \prod_{i=1}^{m-1} \frac{P+i-1}{P+i-2}.$$

Cancellation in the last product yields

$$k \leq 4 \frac{P+m-2}{P-1},$$

so that

$$P \leq \frac{k+4m-8}{k-4} \text{ with } k \geq 5.$$

If both a_0 and a_1 are odd it similarly follows that

$$k \leq 2\prod_{i=1}^{m} \frac{P+i-1}{P+i-2} = 2\frac{P+m-1}{P-1},$$

so that

$$P \leq \frac{k+2m-2}{k-2} \text{ with } k \geq 3. \quad \square$$

Remark. Using the fact that consecutive odd primes differ at least by 2, we can show that $P \leq (2k^2+32m-64)/(k^2-16)$ if the pair is even (which is sharper than (5) for $k < 4(m-2)$) and $P \leq (2k^2+8m-8)/(k^2-4)$ if the pair is odd (which is sharper than (6) for $k < 2(m-1)$).

The next theorem deals with ϕ–amicable pairs (a_0, a_1) in which $\gcd(a_0, a_1) = 1$. A corollary concerns ϕ–amicable pairs of the form (p, a) in which p is a prime number with $p < a$.

Theorem 3. *If (a_0, a_1) is a relatively prime ϕ-amicable pair with multiplier k, then*

$$k-1 < \frac{a_0a_1}{\phi(a_0a_1)} < \frac{k^2}{4}, \tag{7}$$

and a_0a_1 has at least twelve different prime factors.

Proof. Recall that, for any real $x > 0$, $x + (1/x) \geq 2$, with equality only when $x = 1$. Since $\gcd(a_0, a_1) = 1$, we have, from (1),

$$\frac{\phi(a_0a_1)}{a_0a_1} = \frac{\phi(a_0)\phi(a_1)}{a_0a_1} = \frac{(a_0+a_1)^2}{k^2a_0a_1} = \frac{1}{k^2}\left(\frac{a_0}{a_1} + 2 + \frac{a_1}{a_0}\right) \geq \frac{4}{k^2}.$$

Since $a_0 < a_1$, we have the right-hand inequality in (7). For the left-hand inequality, we note simply that $(a_i/\phi(a_i)) - 1 > 0$ for $i = 1, 2$. Then

$$k = \frac{a_0}{\phi(a_0)} + \frac{a_1}{\phi(a_1)} < \frac{a_0a_1}{\phi(a_0)\phi(a_1)} + 1 = \frac{a_0a_1}{\phi(a_0a_1)} + 1.$$

To prove that a_0a_1 has at least twelve different prime factors, we shall set $A = a_0a_1$, $F(A) = A/\phi(A) = \prod_{p|A} p/(p-1)$, and ω equal to the number of different prime factors of A. Let $A = p_1p_2 \ldots p_\omega$, with $p_1 < p_2 < \cdots < p_\omega$. We require the following observations. (i) By **B2**, **B4** and **B9**, A is squarefree and not divisible by 2 or 3. (ii) Using (1), if p and q are primes with $p \mid A$ and $q \bmod p = 1$ then $q \nmid A$. (iii) Since $\phi(a_0) = \phi(a_1)$ and $\phi(a_0a_1) = \phi(a_0)\phi(a_1)$, $\phi(A)$ is a perfect square.

It is also easy to see that A must be divisible by an odd number of primes congruent to 2 mod 3, and by an odd number of primes congruent to 3 mod 4. However, we do not use this below.

Suppose $k \geq 4$. We have $p_1 \geq 5$, $p_2 \geq 7$, $\ldots$. If $\omega \leq 32$, then

$$F(A) \leq F(5 \cdot 7 \cdot 11 \cdot \ldots \cdot 139) < 2.994,$$

(there being 32 primes from 5 to 139, inclusive) and we have a contradiction of the left-hand inequality in (7). So $\omega \geq 33$ when $k \geq 4$.

Now suppose that $k = 3$. We show first that $\omega \geq 11$. If $5 \nmid A$, then $p_1 \geq 7$, $p_2 \geq 11$, $\dots$. If $\omega \leq 14$, then

$$F(A) \leq F(7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 37 \cdot 41 \cdot 43 \cdot 47 \cdot 53 \cdot 59) < 1.994,$$

similarly contradicting (7). Thus, $\omega \geq 15$ in this case. If $5 \mid A$ and $7 \nmid A$, then $p_1 = 5$, $p_2 \geq 13$, $\dots$, bearing in mind that $q \nmid A$ when $q \bmod 5 = 1$. Then, if $\omega \leq 18$,

$$F(A) \leq F(5 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 37 \cdot 43 \cdot 47 \cdot 53 \cdot 59 \cdot 67 \cdot 73 \cdot 79 \cdot 83 \cdot 89 \cdot 97 \cdot 103) < 1.996,$$

so $\omega \geq 19$. We now continue in a similar fashion, but the reporting will be abbreviated. *Including* the preceding two cases, the full proof that $\omega \geq 11$ is as follows.

$$\begin{aligned}
&5 \nmid A: \\
&\quad F(7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 37 \cdot 41 \cdot 43 \cdot 47 \cdot 53 \cdot 59) < 1.994 \Rightarrow \omega \geq 15, \\
&5 \mid A,\ 7 \nmid A: \\
&\quad F(5 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 37 \cdot 43 \cdot 47 \cdot 53 \cdot 59 \cdot 67 \cdot 73 \cdot 79 \cdot 83 \cdot 89 \cdot 97 \cdot 103) \\
&\qquad < 1.996 \Rightarrow \omega \geq 19, \\
&5 \cdot 7 \mid A,\ 13 \nmid A: \\
&\quad F(5 \cdot 7 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 47 \cdot 53 \cdot 59 \cdot 67 \cdot 73 \cdot 79 \cdot 83 \cdot 89) < 1.99 \Rightarrow \omega \geq 15, \\
&5 \cdot 7 \cdot 13 \mid A,\ 17 \nmid A: \\
&\quad F(5 \cdot 7 \cdot 13 \cdot 19 \cdot 23 \cdot 37 \cdot 47 \cdot 59 \cdot 67 \cdot 73 \cdot 83 \cdot 89 \cdot 97) < 1.99 \Rightarrow \omega \geq 14, \\
&5 \cdot 7 \cdot 13 \cdot 17 \mid A,\ 19 \nmid A: \\
&\quad F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 23 \cdot 37 \cdot 47 \cdot 59 \cdot 67 \cdot 73 \cdot 83 \cdot 89 \cdot 97) < 1.996 \Rightarrow \omega \geq 14, \\
&5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \mid A,\ 23 \nmid A: \\
&\quad F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 37 \cdot 47 \cdot 59 \cdot 67 \cdot 73 \cdot 83 \cdot 89) < 1.995 \Rightarrow \omega \geq 13, \\
&5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \mid A,\ 37 \nmid A: \\
(8)\quad &\quad F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 59 \cdot 67 \cdot 73 \cdot 83 \cdot 89) < 1.99 \Rightarrow \omega \geq 12, \\
&5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \mid A,\ 59 \nmid A: \\
&\quad F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 67 \cdot 73 \cdot 83) < 1.99 \Rightarrow \omega \geq 11, \\
&5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \mid A,\ 67 \nmid A: \\
&\quad F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 73 \cdot 83) < 1.99 \Rightarrow \omega \geq 11, \\
&5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \mid A,\ 73 \nmid A: \\
&\quad F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 83) < 1.99 \Rightarrow \omega \geq 11, \\
&5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 73 \mid A: \\
&\quad F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 73) < 1.994 \Rightarrow \omega \geq 11.
\end{aligned}$$

To show that in fact $\omega \geq 12$, we now assume that $\omega = 11$ and will show this to be untenable. The above calculations show that we must have $5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \mid A$. We cannot have $p_8 \geq 73$, for in that case $p_1 = 5$, $\dots$, $p_7 = 37$, $p_8 \geq 73$, $p_9 \geq 83$, $p_{10} \geq 89$, $p_{11} \geq 97$, and

$$F(A) \leq F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 73 \cdot 83 \cdot 89 \cdot 97) < 1.997, \tag{9}$$

contradicting the left-hand inequality in (7). Therefore, $p_8 = 59$ or 67.

(i) Suppose $p_8 = 67$. Then $p_9 = 73$, since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 67 \cdot 83 \cdot 89 \cdot 97) < 1.9992,$$

and $p_{10} = 83$ or 89, since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 67 \cdot 73 \cdot 97 \cdot 107) < 1.999.$$

(i) (a) Suppose $p_{10} = 89$. Then $p_{11} \in \{97, 107, 109\}$, since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 67 \cdot 73 \cdot 89 \cdot 163) < 1.995.$$

In all three cases, since $23 \cdot 67 \cdot 89 \mid A$, we have $11^3 \parallel \phi(A)$.

(i) (b) Suppose $p_{10} = 83$. Then $p_{11} \in \{89, 97, 107, 109\}$, since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 67 \cdot 73 \cdot 83 \cdot 163) < 1.996.$$

In these cases, since $83 \mid A$, we have $41 \parallel \phi(A)$.

(ii) Suppose $p_8 = 59$. Then $p_9 \in \{67, 73, 83\}$, since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 89 \cdot 97 \cdot 107) < 1.998.$$

In all of the following cases, except one to be noted, since $59 \mid A$ we have $29 \parallel \phi(A)$.

(ii) (a) If $p_9 = 83$, then $p_{10} = 89$ since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 83 \cdot 97 \cdot 107) < 1.9995,$$

and $p_{11} \in \{97, 107, 109\}$ since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 83 \cdot 89 \cdot 163) < 1.995.$$

(ii) (b) If $p_9 = 73$, then $p_{10} \in \{83, 89, 97, 107\}$, since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 73 \cdot 109 \cdot 163) < 1.994.$$

The inequalities

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 73 \cdot 107 \cdot 163) < 1.995,$$
$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 73 \cdot 97 \cdot 163) < 1.997,$$
$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 73 \cdot 89 \cdot 163) < 1.999,$$
$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 73 \cdot 83 \cdot 163) < 1.9998,$$

then show, respectively, that if $p_{10} = 107$, then $p_{11} = 109$; if $p_{10} = 97$, then $p_{11} \in \{107, 109\}$; if $p_{10} = 89$, then $p_{11} \in \{97, 107, 109\}$; if $p_{10} = 83$, then $p_{11} \in \{89, 97, 107, 109\}$.

(ii) (c) If, finally, $p_9 = 67$, then $p_{10} \in \{73, 83, 89, 97, 107\}$, since

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 109 \cdot 163) < 1.997.$$

The inequalities

$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 107 \cdot 163) < 1.997,$$
$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 97 \cdot 163) < 1.999,$$
$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 89 \cdot 173) < 1.99993,$$
$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 83 \cdot 227) < 1.999,$$
$$F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 37 \cdot 59 \cdot 67 \cdot 73 \cdot 317) < 1.9997,$$

then show, respectively, that if $p_{10} = 107$, then $p_{11} = 109$; if $p_{10} = 97$, then $p_{11} \in \{107, 109\}$; if $p_{10} = 89$, then $p_{11} \in \{97, 107, 109, 163, 167\}$; if $p_{10} = 83$, then $p_{11} \le 199$; if $p_{10} = 73$, then $p_{11} \le 283$. In this final subcase, if $p_{11} = 233$ then $29^2 \parallel \phi(A)$; however, here $3^9 \parallel \phi(A)$.

We have shown in all cases that $\phi(A)$ is not a perfect square, so $\omega \ne 11$. Hence, $\omega \ge 12$. □

Corollary. *If p is prime, $p < a$ and (p, a) is a ϕ-amicable pair, then a has at least twelve different prime factors.*

Proof. It is clear that $p \nmid a$ and that p exceeds all prime factors of a. We write $A = pa$, and use the calculations in the proof of Theorem 3. Thus, we must show in this special case that $\omega \geq 13$. This is known to be true when $k \geq 4$. Assume that $k = 3$ and $\omega = 12$. Then, from the work immediately preceding (8), $5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \mid a$ and certainly

$$p = 1 + \phi(a) \geq 1 + 4 \cdot 6 \cdot 12 \cdot 16 \cdot 18 \cdot 22 \cdot 2^5 = 58392577,$$

from which

$$\frac{p}{p-1} < 1.00000002.$$

We must have $37 \mid a$, since otherwise, by (8),

$$F(A) \leq F(5 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 59 \cdot 67 \cdot 73 \cdot 83 \cdot 89 \cdot p) < 1.99 \cdot 1.00000002 < 2,$$

contradicting the left-hand inequality in (7). All the calculations from (9) onwards may then be used in the same way, since, in the worst case, $1.99993 \cdot 1.00000002 < 2$. Since $p - 1 = \phi(a) = (p + a)/3$, we have $2\phi(a) = a + 1$. For every possible set $\{p_1, \ldots, p_{11}\}$ of prime factors of a, we checked whether the equation $2\phi(a) = a + 1$ was satisfied, and it never was. □

The odd numbers > 1 we know satisfying the equation $a + 1 = 2\phi(a)$ are $a = 3$, $3 \cdot 5$, $3{\cdot}5{\cdot}17$, $3{\cdot}5{\cdot}17{\cdot}257$, $3{\cdot}5{\cdot}17{\cdot}257{\cdot}65537$, $3{\cdot}5{\cdot}17{\cdot}353{\cdot}929$, and $3{\cdot}5{\cdot}17{\cdot}353{\cdot}929{\cdot}83623937$ (cf. [5, Problem B37]), but for all of them $3 \mid a$, so that these cannot be a member of a ϕ–amicable pair (p, a) with multiplier 3. The first five numbers are the products of the first j Fermat primes, for $j = 1, 2, 3, 4, 5$, respectively, and it is easy to prove that such a product satisfies our equation. No further such products will, since the next Fermat number is $2^{32} + 1$, which is composite (with smallest prime divisor 641). In Section 5 we will see that solutions of the equation $a + 1 = 2\phi(a)$ sometimes can help to generate new ϕ–amicable pairs.

The following propositions show how from a number a satisfying $a + 1 = 2\phi(a)$ other numbers with that property can be found.

Proposition 3. *If $a + 1 = 2\phi(a)$ and if $q = a + 2$ is a prime number, then $aq + 1 = 2\phi(aq)$.*

Proof. We have

$$2\phi(aq) = 2\phi(a)\phi(q) = (a + 1)(q - 1) = aq + (q - a - 1) = aq + 1. \quad \square$$

Proposition 4. *Let $a + 1 = 2\phi(a)$ and write $a^2 + a + 1 = D_1 D_2$ with $0 < D_1 < D_2$. If both $q = a + 1 + D_1$ and $r = a + 1 + D_2$ are prime numbers, then $aqr + 1 = 2\phi(aqr)$.*

Proof. We have

$$2\phi(aqr) = 2\phi(a)(q - 1)(r - 1) = (a + 1)(q - 1)(r - 1),$$

so we wish to prove that

$$aqr + 1 = (a + 1)(q - 1)(r - 1).$$

Some rearranging of the terms shows that this is equivalent to

$$[q - (a + 1)]\ [r - (a + 1)] = a^2 + a + 1,$$

which is true, since $[q - (a + 1)]\ [r - (a + 1)] = D_1 D_2 = a^2 + a + 1$. □

The following theorem is concerned with a ϕ–amicable pair in which one of the members has precisely two distinct prime factors.

Theorem 4. *Except for the pair* $(2^2, 2\cdot 3)$, *if one member of a* ϕ*-amicable pair has exactly two distinct prime factors, then the other member has at least four distinct prime factors.*

Proof. The proof will be by contradiction. By **B1**, **B5**, **B6**, and the Corollary to Theorem 3, we may suppose, if $(a_0, a_1) \neq (2^2, 2\cdot 3)$, that (a_0, a_1) is a primitive ϕ-amicable pair with multiplier k, where a_0 has precisely two distinct prime factors and a_1 has two or three distinct prime factors. (The proof will be symmetric with regard to a_0 and a_1, and we do not require here that $a_0 < a_1$.) Let p, q, r, s be distinct odd primes.

By **B3**, **B4**, **B6**, and **B7**, if a_0 is even then $a_0 = 2p$ and either $a_1 = 2q$ or $a_1 = 2qr$. Since $\phi(a_0) = \phi(a_1) = (a_0 + a_1)/k$, in the former case we have $p = q$, a contradiction.

In the latter case, assume $q < r$. We have

$$p - 1 = (q-1)(r-1) = \frac{2p + 2qr}{k},$$

so that $p(k-2) = 2qr + k$, and

$$k(q-1)(r-1) = 2qr + 2\frac{2qr + k}{k-2}. \tag{10}$$

We write this as

$$(k-4)qr - (k-2)(q+r) = 4 - k,$$

multiply through by $k-4$ and add $(k-2)^2$ to both sides, to obtain

$$[(k-4)q - (k-2)][(k-4)r - (k-2)] = (k-2)^2 - (k-4)^2 = 4(k-3).$$

If $k = 3$, then $-q - 1 = 0$; if $k = 4$, then (10) gives us $(q-1)(r-1) = qr + 1$. These are impossible. If $k = 5$, then we must solve $(q-3)(r-3) = 8$: we obtain $q = 5$ and $r = 7$, and then $p = 1 + (q-1)(r-1) = 25$, not a prime. If $k \geq 6$, then, since $p = 1 + (q-1)(r-1) \geq 1 + 2\cdot 4 = 9$, we have $p \geq 11$ and

$$6 \leq k = \frac{2p}{p-1} + \frac{2qr}{(q-1)(r-1)} \leq \frac{2\cdot 11}{10} + \frac{2\cdot 3\cdot 5}{2\cdot 4} = 5.95.$$

Suppose now that a_0 is odd. We will treat here only the case that a_1 has three distinct prime factors, the other possibility (of two distinct prime factors) being much easier.

By **B4**, a_1 is also odd. Using **B2**, **B6**, **B7**, and Theorem 3, there are, without loss of generality, five possibilities for (a_0, a_1): (i) (p^2q, pq^2r), (ii) (p^2q, pqr), (iii) (p^2q, prs), (iv) (pq, p^2rs), (v) (pq, prs). We will consider these in turn.

In cases (i) and (ii), the equation $\phi(a_0) = \phi(a_1)$ quickly leads to contradictions.

In case (iii), we may assume $r < s$. We have

$$p(p-1)(q-1) = (p-1)(r-1)(s-1) = \frac{p^2q + prs}{k}.$$

Set

$$f(p,q,r,s) = \frac{pq}{(p-1)(q-1)} + \frac{prs}{(p-1)(r-1)(s-1)},$$

and suppose first that $k = 3$, so that, by **B10**, $\gcd(3, pqrs) = 1$. If $p = 5$, then $r \geq 7$, $s \geq 11$ and $q = 1 + (r-1)(s-1)/p \geq 13$. Then

$$3 = f(p,q,r,s) \leq f(5,13,7,11) < 2.96.$$

If $p = 7$, then $r \geq 5$, $s \geq 11$ and $q = 1 + (r-1)(s-1)/p$ implies $q \geq 11$. Then $3 = f(p,q,r,s) \leq f(7,11,5,11) < 2.89$. If $p \geq 11$, then $q \geq 5$, $r \geq 5$, $s \geq 7$ and $3 = f(p,q,r,s) \leq f(11,5,5,7) < 2.98$. Now suppose that $k \geq 4$. If $p = 3$, then $r \geq 5$, $s \geq 7$ and $q = 1 + (r-1)(s-1)/p$ implies $q \geq 11$. Then $4 \leq k = f(p,q,r,s) \leq f(3,11,5,7) < 3.9$. If $q = 3$, then $p \geq 5$, $r \geq 5$, $s \geq 7$ and $4 \leq k = f(p,q,r,s) \leq f(5,3,5,7) < 3.7$. Otherwise, $p \geq 5$, $q \geq 5$, $r \geq 3$, $s \geq 5$ and $4 \leq k = f(p,q,r,s) \leq f(5,5,3,5) < 3.91$.

In cases (iv) and (v), we again have, as in case (iii),

$$k = \frac{pq}{(p-1)(q-1)} + \frac{prs}{(p-1)(r-1)(s-1)} = f(p,q,r,s),$$

and, respectively, $q = 1 + p(r-1)(s-1)$, $q = 1 + (r-1)(s-1)$. The calculations for case (iii) must therefore hold true in these cases as well.

This completes the proof of Theorem 4. $\square$

5. Analogies with amicable pairs

We have determined some analogies with amicable pairs in the construction of ϕ–amicable pairs with a given prime structure. Assume, for example, that a_0 and a_1 have the form

$$a_0 = ar, \quad a_1 = apq,$$

where p, q, and r are distinct primes not dividing a. Examples are the pairs numbered 4 and 9 in Table 2. Substitution in (1) yields after some simple calculations:

$$r = (p-1)(q-1) + 1 \text{ and } (cp-d)(cq-d) = a(c+d), \tag{11}$$

where

$$c = k\phi(a) - 2a \text{ and } d = k\phi(a) - a. \tag{12}$$

It is convenient now to choose a and k such that c is a small positive number. For example, if we choose $a = 5 \cdot 7$ and $k = 3$, then $c = 2$ and $d = 37$. The second equation in (11) then reduces to:

$$(2p-37)(2q-37) = 1365 = 3 \cdot 5 \cdot 7 \cdot 13. \tag{13}$$

Writing the right-hand side as 1×1365 and equating with the two factors in the left-hand side yields $p = 19$ and $q = 701$, and, from the first equation in (11), $r = 12601$, p, q and r being primes. This gives the ϕ–amicable pair

$$441035 = 5 \cdot 7 \cdot 12601, \quad 466165 = 5 \cdot 7 \cdot 19 \cdot 701, \quad k = 3, \tag{14}$$

which is number 42 in Table 3. Other ways of writing the right-hand side of (13) as a product of two factors do not yield success. The two pairs of this form in Table 2 are obtained by choosing $a = 6$, $k = 7$ (number 4), and $a = 14$, $k = 5$ (number 9); both cases have $c = 2$.

For $c = 2, 4, 6, 8$, and 10, we have computed all the numbers a with $2 \leq a \leq 10^5$ for which $(c+2a)/\phi(a)$ is an integer (called k in (12)). We found 76 solutions, and for each of them, we checked whether there are primes p, q, and r satisfying (11). As a result we only found five primitive ϕ–amicable pairs of the form (ar, apq), and they all occur in our list of 499 primitive ϕ–amicable pairs below 10^9 (namely, as numbers 4, 9, 42, 109, and 148). In the case $c = 2$, $k = 4$, the first equation in (12) reduces to $1 = 2\phi(a) - a$, which occurs in the Corollary to Theorem 3. For the three largest solutions given below this Corollary (the other four have $a \le 10^5$), we also checked (12), and we found the 24-digit primitive ϕ–amicable pair with multiplier $k = 4$:

$$(15)\qquad \begin{aligned} 643433053433705010822135 &= 3 \cdot 5 \cdot 17 \cdot 353 \cdot 929 \cdot 7694364698739721, \\ 643433068822434241053705 &= 3 \cdot 5 \cdot 17 \cdot 353 \cdot 929 \cdot 64231061 \cdot 119791963. \end{aligned}$$

This construction method is analogous to similar methods known for (ordinary) amicable numbers. The simplest example is the so-called *rule of Thābit ibn Qurrah* [3] which constructs amicable pairs of the form (apq, ar) where a is a power of 2:

If the three numbers $p = 3 \cdot 2^{n-1} - 1$, $q = 3 \cdot 2^n - 1$, and $r = 9 \cdot 2^{2n-1} - 1$ are prime numbers, then $2^n pq$ and $2^n r$ form an amicable pair.

This rule yields amicable pairs for $n = 2, 4$, and 7, but for no other values of $n \le 20,000$ [2].

The ϕ–amicable pairs which we have constructed so far are squarefree (if we choose a to be squarefree). We also tried to find pairs of the form

$$a_1 = as^2 r, \quad a_2 = aspq,$$

where s is a prime not dividing a. Similarly to the above derivation, we found the two ϕ–amicable pairs

$$(16)\qquad 2609115 = 3 \cdot 5 \cdot 31^2 181, \quad 2747685 = 3 \cdot 5 \cdot 31 \cdot 19 \cdot 311, \quad k = 4,$$

$$(17)\qquad 11085135 = 3 \cdot 5 \cdot 31^2 769, \quad 11770545 = 3 \cdot 5 \cdot 31 \cdot 17 \cdot 1489, \quad k = 4.$$

From the pair (14), two more pairs with square-free greatest common divisor can be generated with the help of Proposition 2, namely for $b = 2$, and $b = 6$; from each of the pairs (16) and (17), one more such pair can be generated with Proposition 2, namely for $b = 2$.

With amicable pairs of the form (au, ap) where p is a prime and $\gcd(a, p) = 1$, it is often possible to associate many other amicable pairs of the form (auq, ars) with the following rule [9, Theorem 2]:

Let (au, ap) be a given amicable pair, where p is a prime with $\gcd(a, p) = 1$ and let $C = (p+1)(p+u)$. Write $C = D_1 D_2$ with $0 < D_1 < D_2$. If the three integers $r = p + D_1$, $s = p + D_2$, and $q = u + r + s$ are primes not dividing a, then (auq, ars) is also an amicable pair.

At present, we know 319 amicable pairs of the required form (au, ap), and for almost all of them the number C has extremely many divisors. Consequently, many thousands of new amicable pairs have been found with this rule, including several large pairs [8, Lemma 1].

Such a rule also exists for ϕ–amicable pairs. We have the following

Theorem 5. *Let (ap, au) be a given ϕ–amicable pair with multiplier k, where p is a prime with $\gcd(a, p) = 1$ (notice that a and u need not be coprime), and let $C = (p-1)(p+u)$. Write $C = D_1 D_2$ with $0 < D_1 < D_2$. If the three integers $r = p + D_1$, $s = p + D_2$, and $q = u + r + s$ are primes not dividing a, then (ars, auq) is also a ϕ–amicable pair with (the same) multiplier k.*

Proof. First we show that $\phi(auq) = \phi(ars)$. Since $\phi(au) = \phi(ap) = \phi(a)(p-1)$, and q does not divide u (because $q > u$), we have

$$\phi(auq) = \phi(au)\phi(q) = \phi(a)(p-1)(q-1)$$

and

$$\phi(ars) = \phi(a)(r-1)(s-1).$$

So we must show that $(p-1)(q-1) = (r-1)(s-1)$, or $(p-1)(u+r+s-1) = (r-1)(s-1)$. We rewrite this as:

$$(p-1)(p+u) = (r-p)(s-p), \tag{18}$$

which is true since $(r-p)(s-p) = D_1D_2 = (p-1)(p+u)$; this proves that $\phi(auq) = \phi(ars)$.

Secondly, we show that $\phi(auq) = (auq + ars)/k$. Since $\phi(au) = (au + ap)/k$, we have

$$\phi(auq) = \phi(au)(q-1) = (au + ap)(q-1)/k,$$

so that we must show that $(p+u)(q-1) = uq + rs$, or $pq = u + p + rs$. Substituting $q = u + r + s$ gives us to show that

$$p(u + r + s) = u + p + rs,$$

which is equivalent to (18). This proves that $\phi(auq) = (auq + ars)/k$, and completes the proof of Theorem 5. □

We have applied Theorem 5 to those primitive ϕ–amicable pairs with larger member $\leq 10^9$ which are of the required form, and to the large pair (15).

From the pairs with larger member $\leq 10^9$, 79 primitive ϕ–amicable pairs were generated. Table 4 gives the rank number of the "mother" pair of the form (ap, au) in column 1, the values of a, p, u, and k in columns 2–5, the number of primitive ϕ–amicable pairs generated from this mother pair in column 6, and the "successful" values of D_1 in column 7. To any listed value of D_1 corresponds a primitive ϕ–amicable pair of the form (ars, auq) with $r = p + D_1$, $s = p + (p-1)(p+u)/D_1$, and $q = u + r + s$, with the same multiplier as the mother pair. The three pairs generated by mother pair number 4 have larger member $\leq 10^6$, and occur as numbers 36, 40, and 47 (with $D_1 = 36$, 24, and 16, respectively) in Table 3. Furthermore, among the 79 pairs, there are six with larger member between 10^6 and 10^9 (also found with our exhaustive search). We list them in Table 5.

From pair (15) we found eight new large primitive ϕ–amicable pairs. The values of D_1 in Theorem 5 leading to these eight pairs are:

$$73914531840,\ 76666855680,\ 7394851553280,\ 123635643997056,$$
$$193847579836416,\ 865200857636352,\ 3982965255818208,\ 4194831919218688.$$

All pairs have multiplier $k = 4$. In the largest pair (with $D_1 = 73914531840$) both members have 46 decimal digits:

$$a_0 = 1030754714216455355643689856057807107041652655,$$
$$a_1 = 1030754738868600773437177012460443629727125585,$$

and $a_0 = ars$, $a_1 = auq$, with $a = 3 \cdot 5 \cdot 17 \cdot 353 \cdot 929$,

$$r = 7694438613271561, \quad s = 16019456990140998154 33,$$
$$u = 64231061 \cdot 119791963, \quad q = 1601961087817595849737.$$

Hence, in addition to the 499 primitive ϕ–amicable pairs with larger member $\leq 10^9$ we have found with the help of the method described in the beginning of this section and with Theorem 5, another 79 primitive ϕ–amicable pairs with larger member $> 10^9$.

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#	a	p	u	k	# D_1	the D_1's
4	6	73	95	7	3	16, 24, 36
8	55	67	77	3	1	16
9	14	281	319	5	2	32, 140
14	39	313	455	4	1	256
17	35	631	665	3	2	30, 486
26	506	139	161	5	1	138
38	1265	277	299	3	1	54
42	35	12601	13319	3	5	40, 810, 4032, 9450, 13440
50	715	859	869	3	2	78, 192
65	1518	829	851	7	5	24, 108, 168, 184, 630
89	273	14561	16159	4	6	32, 512, 1040, 1536, 3840, 7280
109	285	29641	30263	4	2	520, 9880
140	465	31249	33263	4	4	4, 378, 13392, 43008
141	598	21529	25991	5	1	4290
148	255	79561	80183	4	5	340, 1536, 11232, 17238, 97920
168	602	57793	63167	5	7	336, 774, 5418, 7840, 8428, 16254, 43008
203	285	199501	203699	4	10	240, 1536, 2128, 2880, 14250, 25536, 161280, 178752, 220500, 238336
225	6	10266913	13689215	7	2	653184, 1586304
267	6	18196993	24262655	7	4	86016, 180936, 5677056, 17842176
318	935	269281	283679	3	3	1350, 61440, 345600
332	6578	44851	45149	5	4	120, 720, 1170, 1656
365	506	642529	754271	5	3	270, 133860, 451632
409	5478	87649	96031	7	3	73920, 86592, 95284
488	2485	365509	375803	3	2	139392, 274912

TABLE 4. The 79 primitive ϕ–amicable pairs generated by applying Theorem 5 to the 38 primitive ϕ–amicable pairs $\leq 10^9$ which are of the form (ap, au) with p a prime, $\gcd(p, a) = 1$.

from #	a_0	a_1	k
8	$3017465 = 5 \cdot 11 \cdot 83 \cdot 661$	$3476935 = 5 \cdot 7 \cdot 11^2 821$	3
9	$8729014 = 2 \cdot 7 \cdot 421 \cdot 1481$	$9918986 = 2 \cdot 7 \cdot 11 \cdot 29 \cdot 2221$	5
9	$24236842 = 2 \cdot 7 \cdot 313 \cdot 5531$	$27523958 = 2 \cdot 7 \cdot 11 \cdot 29 \cdot 6163$	5
14	$27716559 = 3 \cdot 13 \cdot 569 \cdot 1249$	$40334385 = 3 \cdot 5 \cdot 7 \cdot 13^2 2273$	4
26	$61531118 = 2 \cdot 11 \cdot 23 \cdot 277 \cdot 439$	$71445682 = 2 \cdot 7 \cdot 11 \cdot 23^2 877$	5
17	$90348545 = 5 \cdot 7 \cdot 1117 \cdot 2311$	$95264575 = 5^2 7^2 19 \cdot 4093$	3

TABLE 5. Six primitive ϕ–amicable pairs with larger member $> 10^6$ and $\leq 10^9$, found with Theorem 5 from pairs 8, 9, 14, 17 and 26 in Table 2.

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ON ϕ–AMICABLE PAIRS

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APPENDIX

THE 499 PRIMITIVE ϕ–AMICABLE NUMBERS (a_0, a_1) WITH MULTIPLIER k AND LARGER MEMBER $\leq 10^9$

Column 5 gives the values of b and $\overline{k}$ such that (ba_0, ba_1) is a non-primitive ϕ–amicable pair with multiplier $\overline{k}$ (cf. Proposition 2).

#	a_0	a_1	k	b, $\overline{k}$
1	$4 = 2^2$	$6 = 2 \cdot 3$	5	
2	$78 = 2 \cdot 3 \cdot 13$	$90 = 2 \cdot 3^2 5$	7	
3	$465 = 3 \cdot 5 \cdot 31$	$495 = 3^2 5 \cdot 11$	4	2, 8
4	$438 = 2 \cdot 3 \cdot 73$	$570 = 2 \cdot 3 \cdot 5 \cdot 19$	7	
5	$609 = 3 \cdot 7 \cdot 29$	$735 = 3 \cdot 5 \cdot 7^2$	4	2, 8
6	$158 = 2 \cdot 3 \cdot 193$	$1530 = 2 \cdot 3^2 5 \cdot 17$	7	
7	$530 = 2 \cdot 5 \cdot 11 \cdot 23$	$3630 = 2 \cdot 3 \cdot 5 \cdot 11^2$	7	
8	$3685 = 5 \cdot 11 \cdot 67$	$4235 = 5 \cdot 7 \cdot 11^2$	3	2, 6; 6, 9
9	$3934 = 2 \cdot 7 \cdot 281$	$4466 = 2 \cdot 7 \cdot 11 \cdot 29$	5	
10	$5475 = 3 \cdot 5^2 73$	$6045 = 3 \cdot 5 \cdot 13 \cdot 31$	4	2, 8
11	$5978 = 2 \cdot 7^2 61$	$6622 = 2 \cdot 7 \cdot 11 \cdot 43$	5	
12	$7525 = 5^2 7 \cdot 43$	$7595 = 5 \cdot 7^2 31$	3	2, 6; 6, 9
13	$11925 = 3^2 5^2 53$	$13035 = 3 \cdot 5 \cdot 11 \cdot 79$	4	2, 8
14	$12207 = 3 \cdot 13 \cdot 313$	$17745 = 3 \cdot 5 \cdot 7 \cdot 13^2$	4	2, 8
15	$21035 = 5 \cdot 7 \cdot 601$	$22165 = 5 \cdot 11 \cdot 13 \cdot 31$	3	2, 6; 6, 9
16	$19815 = 3 \cdot 5 \cdot 1321$	$22425 = 3 \cdot 5^2 13 \cdot 23$	4	2, 8
17	$22085 = 5 \cdot 7 \cdot 631$	$23275 = 5^2 7^2 19$	3	2, 6; 6, 9
18	$21189 = 3 \cdot 7 \cdot 1009$	$27195 = 3 \cdot 5 \cdot 7^2 37$	4	2, 8
19	$40334 = 2 \cdot 7 \cdot 43 \cdot 67$	$42826 = 2 \cdot 7^2 19 \cdot 23$	5	
20	$59045 = 5 \cdot 7^2 241$	$61915 = 5 \cdot 7 \cdot 29 \cdot 61$	3	2, 6; 6, 9
21	$66795 = 3 \cdot 5 \cdot 61 \cdot 73$	$71445 = 3 \cdot 5 \cdot 11 \cdot 433$	4	2, 8
22	$60726 = 2 \cdot 3 \cdot 29 \cdot 349$	$75690 = 2 \cdot 3^2 5 \cdot 29^2$	7	
23	$65945 = 5 \cdot 11^2 109$	$76615 = 5 \cdot 7 \cdot 11 \cdot 199$	3	2, 6; 6, 9
24	$70422 = 2 \cdot 3 \cdot 11^2 97$	$77418 = 2 \cdot 3^2 11 \cdot 17 \cdot 23$	7	
25	$73486 = 2 \cdot 7 \cdot 29 \cdot 181$	$77714 = 2 \cdot 7^2 13 \cdot 61$	5	
26	$70334 = 2 \cdot 11 \cdot 23 \cdot 139$	$81466 = 2 \cdot 7 \cdot 11 \cdot 23^2$	5	
27	$89745 = 3 \cdot 5 \cdot 31 \cdot 193$	$94575 = 3 \cdot 5^2 13 \cdot 97$	4	2, 8
28	$94666 = 2 \cdot 11 \cdot 13 \cdot 331$	$103334 = 2 \cdot 7 \cdot 11^2 61$	5	
29	$87591 = 3 \cdot 7 \cdot 43 \cdot 97$	$105945 = 3 \cdot 5 \cdot 7 \cdot 1009$	4	2, 8
30	$109298 = 2 \cdot 7 \cdot 37 \cdot 211$	$117502 = 2 \cdot 7^2 11 \cdot 109$	5	
31	$119434 = 2 \cdot 7 \cdot 19 \cdot 449$	$122486 = 2 \cdot 7 \cdot 13 \cdot 673$	5	
32	$164486 = 2 \cdot 7 \cdot 31 \cdot 379$	$175714 = 2 \cdot 7^2 11 \cdot 163$	5	
33	$211002 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 139$	$214038 = 2 \cdot 3^2 11 \cdot 23 \cdot 47$	7	
34	$239775 = 3 \cdot 5^2 23 \cdot 139$	$245985 = 3 \cdot 5 \cdot 23^2 31$	4	2, 8
35	$326325 = 3 \cdot 5^2 19 \cdot 229$	$330315 = 3 \cdot 5 \cdot 19^2 61$	4	2, 8
36	$267486 = 2 \cdot 3 \cdot 109 \cdot 409$	$349410 = 2 \cdot 3 \cdot 5 \cdot 19 \cdot 613$	7	
37	$287763 = 3 \cdot 7 \cdot 71 \cdot 193$	$357357 = 3 \cdot 7^2 11 \cdot 13 \cdot 17$	4	5, 5; 2, 8; 10, 10
38	$350405 = 5 \cdot 11 \cdot 23 \cdot 277$	$378235 = 5 \cdot 11 \cdot 13 \cdot 23^2$	3	2, 6; 14, 7; 6, 9
39	$367114 = 2 \cdot 11^2 37 \cdot 41$	$424886 = 2 \cdot 7 \cdot 11 \cdot 31 \cdot 89$	5	
40	$335814 = 2 \cdot 3 \cdot 97 \cdot 577$	$438330 = 2 \cdot 3 \cdot 5 \cdot 19 \cdot 769$	7	
41	$363486 = 2 \cdot 3 \cdot 29 \cdot 2089$	$455010 = 2 \cdot 3 \cdot 5 \cdot 29 \cdot 523$	7	
42	$441035 = 5 \cdot 7 \cdot 12601$	$466165 = 5 \cdot 7 \cdot 19 \cdot 701$	3	2, 6; 6, 9
43	$444414 = 2 \cdot 3 \cdot 17 \cdot 4357$	$531330 = 2 \cdot 3 \cdot 5 \cdot 89 \cdot 199$	7	
44	$545566 = 2 \cdot 7^2 19 \cdot 293$	$558194 = 2 \cdot 7 \cdot 13 \cdot 3067$	5	
45	$494054 = 2 \cdot 11 \cdot 17 \cdot 1321$	$561946 = 2 \cdot 7 \cdot 11 \cdot 41 \cdot 89$	5	
46	$344810 = 2 \cdot 5 \cdot 29^2 41$	$564630 = 2 \cdot 3 \cdot 5 \cdot 11 \cdot 29 \cdot 59$	7	
47	$442686 = 2 \cdot 3 \cdot 89 \cdot 829$	$577410 = 2 \cdot 3 \cdot 5 \cdot 19 \cdot 1013$	7	
48	$546675 = 3 \cdot 5^2 37 \cdot 197$	$582285 = 3 \cdot 5 \cdot 11 \cdot 3529$	4	2, 8
49	$573545 = 5 \cdot 7^2 2341$	$605815 = 5 \cdot 7 \cdot 19 \cdot 911$	3	2, 6; 6, 9
50	$614185 = 5 \cdot 11 \cdot 13 \cdot 859$	$621335 = 5 \cdot 11^2 13 \cdot 79$	3	2, 6; 14, 7; 6, 9

#	a_0	a_1	k	$b, \overline{k}$
51	$489363 = 3 \cdot 7^2 3329$	$628845 = 3 \cdot 5 \cdot 7 \cdot 53 \cdot 113$	4	2, 8
52	$624358 = 2 \cdot 7^2 23 \cdot 277$	$650762 = 2 \cdot 7 \cdot 23 \cdot 43 \cdot 47$	5	
53	$722502 = 2 \cdot 3^2 11 \cdot 41 \cdot 89$	$755898 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 881$	7	
54	$756925 = 5^2 13 \cdot 17 \cdot 137$	$809795 = 5 \cdot 7 \cdot 17 \cdot 1361$	3	2, 6; 6, 9
55	$793914 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 523$	$813846 = 2 \cdot 3 \cdot 11^2 19 \cdot 59$	7	
56	$806386 = 2 \cdot 7 \cdot 239 \cdot 241$	$907214 = 2 \cdot 7 \cdot 11 \cdot 43 \cdot 137$	5	
57	$886006 = 2 \cdot 11 \cdot 17 \cdot 23 \cdot 103$	$909194 = 2 \cdot 11^2 13 \cdot 17^2$	5	
58	$898835 = 5 \cdot 7 \cdot 61 \cdot 421$	$915565 = 5 \cdot 7^2 37 \cdot 101$	3	2, 6; 6, 9
59	$997934 = 2 \cdot 7^2 17 \cdot 599$	$1011346 = 2 \cdot 7 \cdot 29 \cdot 47 \cdot 53$	5	
60	$961245 = 3^2 5 \cdot 41 \cdot 521$	$1035555 = 3 \cdot 5 \cdot 17 \cdot 31 \cdot 131$	4	2, 8
61	$1086295 = 5 \cdot 7 \cdot 41 \cdot 757$	$1090985 = 5 \cdot 7^2 61 \cdot 73$	3	2, 6; 6, 9
62	$942081 = 3 \cdot 7 \cdot 113 \cdot 397$	$1186815 = 3 \cdot 5 \cdot 7 \cdot 89 \cdot 127$	4	2, 8
63	$1218014 = 2 \cdot 7 \cdot 19^2 241$	$1244386 = 2 \cdot 11 \cdot 13 \cdot 19 \cdot 229$	5	
64	$1142265 = 3 \cdot 5 \cdot 271 \cdot 281$	$1276935 = 3 \cdot 5 \cdot 11 \cdot 71 \cdot 109$	4	2, 8
65	$1258422 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 829$	$1291818 = 2 \cdot 3 \cdot 11 \cdot 23^2 37$	7	
66	$1219035 = 3 \cdot 5 \cdot 181 \cdot 449$	$1361445 = 3 \cdot 5 \cdot 17 \cdot 19 \cdot 281$	4	2, 8
67	$1288546 = 2 \cdot 7 \cdot 31 \cdot 2969$	$1382654 = 2 \cdot 7 \cdot 13 \cdot 71 \cdot 107$	5	
68	$1334571 = 3 \cdot 7 \cdot 103 \cdot 617$	$1681365 = 3 \cdot 5 \cdot 7 \cdot 67 \cdot 239$	4	2, 8
69	$1521845 = 5 \cdot 13^2 1801$	$1847755 = 5 \cdot 7 \cdot 13 \cdot 31 \cdot 131$	3	2, 6; 6, 9
70	$1645158 = 2 \cdot 3 \cdot 17 \cdot 127^2$	$1939290 = 2 \cdot 3 \cdot 5 \cdot 127 \cdot 509$	7	
71	$1966538 = 2 \cdot 7 \cdot 19 \cdot 7393$	$2025142 = 2 \cdot 7 \cdot 17 \cdot 67 \cdot 127$	5	
72	$2145385 = 5 \cdot 11 \cdot 19 \cdot 2053$	$2286935 = 5 \cdot 7 \cdot 19^2 181$	3	2, 6; 6, 9
73	$2106478 = 2 \cdot 11 \cdot 23^2 181$	$2447522 = 2 \cdot 7 \cdot 11 \cdot 23 \cdot 691$	5	
74	$2438765 = 5 \cdot 7 \cdot 59 \cdot 1181$	$2488915 = 5 \cdot 11 \cdot 13 \cdot 59^2$	3	2, 6; 6, 9
75	$2609115 = 3 \cdot 5 \cdot 31^2 181$	$2747685 = 3 \cdot 5 \cdot 19 \cdot 31 \cdot 311$	4	2, 8
76	$2487849 = 3 \cdot 7 \cdot 13^2 701$	$2753751 = 3 \cdot 7^2 11 \cdot 13 \cdot 131$	4	5, 5; 2, 8; 10, 10
77	$2462538 = 2 \cdot 3 \cdot 13 \cdot 131 \cdot 241$	$2779062 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 41 \cdot 79$	7	
78	$2891174 = 2 \cdot 11^2 13 \cdot 919$	$3167626 = 2 \cdot 7 \cdot 11 \cdot 67 \cdot 307$	5	
79	$3043075 = 5^2 7 \cdot 17389$	$3216605 = 5 \cdot 7^2 19 \cdot 691$	3	2, 6; 6, 9
80	$2664066 = 2 \cdot 3 \cdot 19 \cdot 23369$	$3224670 = 2 \cdot 3 \cdot 5 \cdot 47 \cdot 2287$	7	
81	$2972706 = 2 \cdot 3 \cdot 11 \cdot 73 \cdot 617$	$3236574 = 2 \cdot 3 \cdot 11 \cdot 19 \cdot 29 \cdot 89$	7	
82	$3265647 = 3 \cdot 7 \cdot 11 \cdot 67 \cdot 211$	$3387153 = 3 \cdot 7 \cdot 11^2 31 \cdot 43$	4	5, 5; 2, 8; 10, 10
83	$3105417 = 3 \cdot 7 \cdot 19 \cdot 43 \cdot 181$	$3426423 = 3 \cdot 7^2 11 \cdot 13 \cdot 163$	4	5, 5; 2, 8; 10, 10
84	$3017465 = 5 \cdot 11 \cdot 83 \cdot 661$	$3476935 = 5 \cdot 7 \cdot 11^2 821$	3	2, 6; 6, 9
85	$3496066 = 2 \cdot 7 \cdot 29 \cdot 79 \cdot 109$	$3580094 = 2 \cdot 7 \cdot 19 \cdot 43 \cdot 313$	5	
86	$3571665 = 3 \cdot 5 \cdot 31 \cdot 7681$	$3801135 = 3 \cdot 5 \cdot 13 \cdot 101 \cdot 193$	4	2, 8
87	$3795915 = 3 \cdot 5 \cdot 19^2 701$	$3864885 = 3 \cdot 5 \cdot 19 \cdot 71 \cdot 191$	4	2, 8
88	$3686514 = 2 \cdot 3 \cdot 13 \cdot 151 \cdot 313$	$4175886 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 31 \cdot 157$	7	
89	$3975153 = 3 \cdot 7 \cdot 13 \cdot 14561$	$4411407 = 3 \cdot 7 \cdot 11 \cdot 13^2 113$	4	5, 5; 2, 8; 10, 10
90	$3962295 = 3^2 5 \cdot 191 \cdot 461$	$4428105 = 3 \cdot 5 \cdot 11 \cdot 47 \cdot 571$	4	2, 8
91	$4167669 = 3 \cdot 11 \cdot 17^2 19 \cdot 23$	$4449291 = 3 \cdot 7 \cdot 11^2 17 \cdot 103$	4	5, 5; 2, 8; 10, 10
92	$3924654 = 2 \cdot 3 \cdot 17 \cdot 109 \cdot 353$	$4590930 = 2 \cdot 3 \cdot 5 \cdot 199 \cdot 769$	7	
93	$4646474 = 2 \cdot 7^2 17 \cdot 2789$	$4721206 = 2 \cdot 7 \cdot 17 \cdot 83 \cdot 239$	5	
94	$4933578 = 2 \cdot 3 \cdot 13 \cdot 19 \cdot 3329$	$5130294 = 2 \cdot 3 \cdot 13 \cdot 17 \cdot 53 \cdot 73$	7	
95	$4988895 = 3 \cdot 5 \cdot 37 \cdot 89 \cdot 101$	$5148705 = 3 \cdot 5 \cdot 17 \cdot 61 \cdot 331$	4	2, 8
96	$3964482 = 2 \cdot 3^2 257 \cdot 857$	$5239230 = 2 \cdot 3 \cdot 5 \cdot 17 \cdot 10273$	7	
97	$4822246 = 2 \cdot 11 \cdot 13^2 1297$	$5286554 = 2 \cdot 7 \cdot 13 \cdot 31 \cdot 937$	5	
98	$4675538 = 2 \cdot 7 \cdot 337 \cdot 991$	$5303662 = 2 \cdot 7^2 13 \cdot 23 \cdot 181$	5	
99	$5514275 = 5^2 13 \cdot 19^2 47$	$5812765 = 5 \cdot 7 \cdot 19 \cdot 8741$	3	2, 6; 6, 9
100	$5510834 = 2 \cdot 7^2 53 \cdot 1061$	$6064366 = 2 \cdot 7 \cdot 11 \cdot 53 \cdot 743$	5	

#	a_0	a_1	k	$b, \overline{k}$
101	$6354145 = 5 \cdot 7 \cdot 71 \cdot 2557$	$6528095 = 5 \cdot 7 \cdot 37 \cdot 71^2$	3	2, 6; 6, 9
102	$6185178 = 2 \cdot 3^2 17^2 29 \cdot 41$	$6609702 = 2 \cdot 3 \cdot 11 \cdot 17 \cdot 43 \cdot 137$	7	
103	$6206325 = 3 \cdot 5^2 83 \cdot 997$	$6861195 = 3^2 5 \cdot 11 \cdot 83 \cdot 167$	4	2, 8
104	$6488742 = 2 \cdot 3 \cdot 13 \cdot 41 \cdot 2029$	$7139418 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 53 \cdot 157$	7	
105	$7365386 = 2 \cdot 7^2 17 \cdot 4421$	$7485814 = 2 \cdot 7 \cdot 17 \cdot 71 \cdot 443$	5	
106	$8041506 = 2 \cdot 3 \cdot 11 \cdot 37^2 89$	$8368734 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 37 \cdot 149$	7	
107	$7850626 = 2 \cdot 7 \cdot 31 \cdot 18089$	$8428574 = 2 \cdot 7 \cdot 11 \cdot 229 \cdot 239$	5	
108	$7996975 = 5^2 7 \cdot 45697$	$8453585 = 5 \cdot 7 \cdot 41 \cdot 43 \cdot 137$	3	2, 6; 6, 9
109	$8447685 = 3 \cdot 5 \cdot 19 \cdot 29641$	$8624955 = 3 \cdot 5 \cdot 19 \cdot 53 \cdot 571$	4	2, 8
110	$8514226 = 2 \cdot 7 \cdot 29 \cdot 67 \cdot 313$	$8783054 = 2 \cdot 7^2 19 \cdot 53 \cdot 89$	5	
111	$7009178 = 2 \cdot 11 \cdot 199 \cdot 1601$	$8830822 = 2 \cdot 7 \cdot 11^2 13 \cdot 401$	5	
112	$8570905 = 5 \cdot 7 \cdot 233 \cdot 1051$	$8968295 = 5 \cdot 7 \cdot 43 \cdot 59 \cdot 101$	3	2, 6; 6, 9
113	$7612046 = 2 \cdot 13 \cdot 19^2 811$	$9009154 = 2 \cdot 7 \cdot 11 \cdot 19 \cdot 3079$	5	
114	$8652658 = 2 \cdot 7 \cdot 31 \cdot 19937$	$9289742 = 2 \cdot 7 \cdot 11 \cdot 179 \cdot 337$	5	
115	$8433825 = 3 \cdot 5^2 139 \cdot 809$	$9406815 = 3 \cdot 5 \cdot 11 \cdot 47 \cdot 1213$	4	2, 8
116	$8614767 = 3 \cdot 7 \cdot 17 \cdot 59 \cdot 409$	$9559185 = 3 \cdot 5 \cdot 17 \cdot 19 \cdot 1973$	4	2, 8
117	$9135266 = 2 \cdot 7^2 31^2 97$	$9613534 = 2 \cdot 7 \cdot 17 \cdot 31 \cdot 1303$	5	
118	$8729014 = 2 \cdot 7 \cdot 421 \cdot 1481$	$9918986 = 2 \cdot 7 \cdot 11 \cdot 29 \cdot 2221$	5	
119	$9374865 = 3 \cdot 5 \cdot 31 \cdot 20161$	$9978735 = 3 \cdot 5 \cdot 13 \cdot 73 \cdot 701$	4	2, 8
120	$7715766 = 2 \cdot 3 \cdot 89 \cdot 14449$	$10084170 = 2 \cdot 3 \cdot 5 \cdot 29 \cdot 67 \cdot 173$	7	
121	$8887974 = 2 \cdot 3 \cdot 17 \cdot 79 \cdot 1103$	$10366170 = 2 \cdot 3 \cdot 5 \cdot 233 \cdot 1483$	7	
122	$9933686 = 2 \cdot 7 \cdot 37 \cdot 127 \cdot 151$	$10478314 = 2 \cdot 7 \cdot 11 \cdot 68041$	5	
123	$9007761 = 3 \cdot 7 \cdot 37 \cdot 11593$	$11023215 = 3 \cdot 5 \cdot 7 \cdot 277 \cdot 379$	4	2, 8
124	$10379486 = 2 \cdot 13 \cdot 17 \cdot 23 \cdot 1021$	$11162914 = 2 \cdot 7 \cdot 17^2 31 \cdot 89$	5	
125	$9541794 = 2 \cdot 3 \cdot 17 \cdot 139 \cdot 673$	$11231070 = 2 \cdot 3 \cdot 5 \cdot 113 \cdot 3313$	7	
126	$11268615 = 3 \cdot 5 \cdot 19^2 2081$	$11494905 = 3 \cdot 5 \cdot 19 \cdot 53 \cdot 761$	4	2, 8
127	$11085135 = 3 \cdot 5 \cdot 31^2 769$	$11770545 = 3 \cdot 5 \cdot 17 \cdot 31 \cdot 1489$	4	2, 8
128	$11750305 = 5 \cdot 7 \cdot 113 \cdot 2971$	$12199775 = 5^2 7^2 23 \cdot 433$	3	2, 6; 6, 9
129	$12251225 = 5^2 7^2 73 \cdot 137$	$12424615 = 5 \cdot 7 \cdot 29 \cdot 12241$	3	2, 6; 6, 9
130	$12651225 = 3 \cdot 5^2 37 \cdot 47 \cdot 97$	$12784935 = 3 \cdot 5 \cdot 17 \cdot 181 \cdot 277$	4	2, 8
131	$11051345 = 5 \cdot 31 \cdot 37 \cdot 41 \cdot 47$	$12795055 = 5 \cdot 7 \cdot 13 \cdot 61 \cdot 461$	3	2, 6; 6, 9
132	$12598135 = 5 \cdot 11 \cdot 23^2 433$	$13632905 = 5 \cdot 11 \cdot 13 \cdot 23 \cdot 829$	3	2, 6; 14, 7; 6, 9
133	$11902165 = 5 \cdot 11^2 103 \cdot 191$	$13679435 = 5 \cdot 7 \cdot 11 \cdot 35531$	3	2, 6; 6, 9
134	$12888678 = 2 \cdot 3 \cdot 11^2 41 \cdot 433$	$13722522 = 2 \cdot 3 \cdot 11 \cdot 19 \cdot 31 \cdot 353$	7	
135	$13785645 = 3 \cdot 5 \cdot 37 \cdot 59 \cdot 421$	$14277075 = 3 \cdot 5^2 19 \cdot 43 \cdot 233$	4	2, 8
136	$13290459 = 3 \cdot 7 \cdot 13 \cdot 89 \cdot 547$	$14385189 = 3 \cdot 7 \cdot 13 \cdot 23 \cdot 29 \cdot 79$	4	5, 5; 2, 8; 10, 10; 110, 11
137	$14698794 = 2 \cdot 3 \cdot 11 \cdot 23^2 421$	$15054006 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 47 \cdot 211$	7	
138	$13545825 = 3 \cdot 5^2 179 \cdot 1009$	$15162015 = 3 \cdot 5 \cdot 11 \cdot 43 \cdot 2137$	4	2, 8
139	$14059515 = 3 \cdot 5 \cdot 41 \cdot 22861$	$15201285 = 3 \cdot 5 \cdot 11 \cdot 181 \cdot 509$	4	2, 8
140	$14530785 = 3 \cdot 5 \cdot 31 \cdot 31249$	$15467295 = 3 \cdot 5 \cdot 29 \cdot 31^2 37$	4	2, 8
141	$12874342 = 2 \cdot 13 \cdot 23 \cdot 21529$	$15542618 = 2 \cdot 7 \cdot 13 \cdot 23 \cdot 47 \cdot 79$	5	
142	$14328202 = 2 \cdot 7 \cdot 43 \cdot 23801$	$15659798 = 2 \cdot 7 \cdot 11 \cdot 61 \cdot 1667$	5	
143	$11211177 = 3 \cdot 17^2 67 \cdot 193$	$16363095 = 3 \cdot 5 \cdot 7 \cdot 17 \cdot 89 \cdot 103$	4	2, 8
144	$14861094 = 2 \cdot 3 \cdot 17 \cdot 53 \cdot 2749$	$17147610 = 2 \cdot 3^2 5 \cdot 190529$	7	
145	$17240755 = 5 \cdot 7 \cdot 281 \cdot 1753$	$18079565 = 5 \cdot 7 \cdot 41 \cdot 43 \cdot 293$	3	2, 6; 6, 9
146	$16944315 = 3 \cdot 5 \cdot 79^2 181$	$18548805 = 3 \cdot 5 \cdot 11 \cdot 79 \cdot 1423$	4	2, 8
147	$19111295 = 5 \cdot 7 \cdot 421 \cdot 1297$	$20079745 = 5 \cdot 7 \cdot 29 \cdot 73 \cdot 271$	3	2, 6; 6, 9
148	$20288055 = 3 \cdot 5 \cdot 17 \cdot 79561$	$20446665 = 3 \cdot 5 \cdot 17 \cdot 181 \cdot 443$	4	2, 8
149	$18683655 = 3 \cdot 5 \cdot 97 \cdot 12841$	$20760825 = 3 \cdot 5^2 17 \cdot 19 \cdot 857$	4	2, 8
150	$19866602 = 2 \cdot 7 \cdot 43 \cdot 61 \cdot 541$	$20957398 = 2 \cdot 7^2 11 \cdot 19441$	5	

#	a_0	a_1	k	$b, \overline{k}$
151	$21870075 = 3 \cdot 5^2 17^2 1009$	$21998085 = 3 \cdot 5 \cdot 17 \cdot 281 \cdot 307$	4	2, 8
152	$17019474 = 2 \cdot 3 \cdot 67 \cdot 42337$	$22098990 = 2 \cdot 3 \cdot 5 \cdot 37 \cdot 43 \cdot 463$	7	
153	$19789305 = 3 \cdot 5 \cdot 151 \cdot 8737$	$22143495 = 3 \cdot 5 \cdot 11 \cdot 43 \cdot 3121$	4	2, 8
154	$20803846 = 2 \cdot 7 \cdot 29 \cdot 51241$	$22237754 = 2 \cdot 7 \cdot 11 \cdot 197 \cdot 733$	5	
155	$21942705 = 3 \cdot 5 \cdot 29 \cdot 73 \cdot 691$	$22570575 = 3 \cdot 5^2 19 \cdot 47 \cdot 337$	4	2, 8
156	$21829766 = 2 \cdot 7 \cdot 31 \cdot 179 \cdot 281$	$23026234 = 2 \cdot 7 \cdot 11 \cdot 149521$	5	
157	$24404886 = 2 \cdot 3^2 11 \cdot 23^2 233$	$24899754 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 47 \cdot 349$	7	
158	$24236842 = 2 \cdot 7 \cdot 313 \cdot 5531$	$27523958 = 2 \cdot 7 \cdot 11 \cdot 29 \cdot 6163$	5	
159	$26515695 = 3 \cdot 5 \cdot 31 \cdot 127 \cdot 449$	$27674385 = 3 \cdot 5 \cdot 17 \cdot 41 \cdot 2647$	4	2, 8
160	$22327557 = 3 \cdot 7 \cdot 97^2 113$	$27733755 = 3 \cdot 5 \cdot 7^2 97 \cdot 389$	4	2, 8
161	$23218382 = 2 \cdot 11 \cdot 41 \cdot 25741$	$28261618 = 2 \cdot 7 \cdot 11 \cdot 23 \cdot 79 \cdot 101$	5	
162	$27761162 = 2 \cdot 11 \cdot 13 \cdot 113 \cdot 859$	$29896438 = 2 \cdot 11^2 13^2 17 \cdot 43$	5	
163	$30268365 = 3 \cdot 5 \cdot 71 \cdot 97 \cdot 293$	$32523315 = 3 \cdot 5 \cdot 11 \cdot 439 \cdot 449$	4	2, 8
164	$30311205 = 3 \cdot 5 \cdot 61 \cdot 157 \cdot 211$	$32587995 = 3 \cdot 5 \cdot 11 \cdot 313 \cdot 631$	4	2, 8
165	$25725545 = 5 \cdot 37 \cdot 241 \cdot 577$	$33994135 = 5 \cdot 7 \cdot 17 \cdot 19 \cdot 31 \cdot 97$	3	2, 6; 6, 9
166	$28827138 = 2 \cdot 3 \cdot 17 \cdot 409 \cdot 691$	$34233342 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 17 \cdot 2347$	7	
167	$36479275 = 5^2 7^2 97 \cdot 307$	$37548245 = 5 \cdot 7 \cdot 43 \cdot 61 \cdot 409$	3	2, 6; 6, 9
168	$34791386 = 2 \cdot 7 \cdot 43 \cdot 57793$	$38026534 = 2 \cdot 7 \cdot 13 \cdot 43^2 113$	5	
169	$35858326 = 2 \cdot 7 \cdot 29 \cdot 88321$	$38330474 = 2 \cdot 7 \cdot 13 \cdot 47 \cdot 4481$	5	
170	$34320315 = 3 \cdot 5 \cdot 181 \cdot 12641$	$38486085 = 3 \cdot 5 \cdot 11 \cdot 41 \cdot 5689$	4	2, 8
171	$36985641 = 3 \cdot 7^2 11 \cdot 89 \cdot 257$	$38708439 = 3 \cdot 7 \cdot 11 \cdot 17 \cdot 9857$	4	5, 5; 2, 8; 10, 10
172	$31331665 = 5 \cdot 19 \cdot 271 \cdot 1217$	$39585455 = 5 \cdot 7 \cdot 13 \cdot 19^2 241$	3	2, 6; 6, 9
173	$35671118 = 2 \cdot 7^2 181 \cdot 2011$	$40306882 = 2 \cdot 7 \cdot 11 \cdot 31 \cdot 8443$	5	
174	$27716559 = 3 \cdot 13 \cdot 569 \cdot 1249$	$40334385 = 3 \cdot 5 \cdot 7 \cdot 13^2 2273$	4	2, 8
175	$34791985 = 5 \cdot 23 \cdot 41 \cdot 47 \cdot 157$	$40986575 = 5^2 7 \cdot 17 \cdot 23 \cdot 599$	3	2, 6; 6, 9
176	$34634834 = 2 \cdot 13 \cdot 19 \cdot 70111$	$41083966 = 2 \cdot 7 \cdot 11 \cdot 19^2 739$	5	
177	$35817815 = 5 \cdot 11^2 73 \cdot 811$	$41164585 = 5 \cdot 7 \cdot 11 \cdot 106921$	3	2, 6; 6, 9
178	$39604726 = 2 \cdot 7 \cdot 37 \cdot 101 \cdot 757$	$42043274 = 2 \cdot 7^2 13 \cdot 61 \cdot 541$	5	
179	$40848745 = 5 \cdot 7 \cdot 491 \cdot 2377$	$42976535 = 5 \cdot 7 \cdot 23 \cdot 197 \cdot 271$	3	2, 6; 6, 9
180	$42001542 = 2 \cdot 3^2 11 \cdot 23^2 401$	$43006458 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 41 \cdot 691$	7	
181	$40217655 = 3 \cdot 5 \cdot 97 \cdot 131 \cdot 211$	$43647945 = 3 \cdot 5 \cdot 11 \cdot 113 \cdot 2341$	4	2, 8
182	$41413938 = 2 \cdot 3 \cdot 17 \cdot 23 \cdot 127 \cdot 139$	$44274126 = 2 \cdot 3 \cdot 13 \cdot 23^2 29 \cdot 37$	7	
183	$42624555 = 3 \cdot 5 \cdot 37 \cdot 76801$	$45849045 = 3 \cdot 5 \cdot 11 \cdot 241 \cdot 1153$	4	2, 8
184	$37713333 = 3 \cdot 7 \cdot 73^2 337$	$47055435 = 3 \cdot 5 \cdot 7^2 73 \cdot 877$	4	2, 8
185	$46992765 = 3 \cdot 5 \cdot 41 \cdot 43 \cdot 1777$	$48484995 = 3 \cdot 5 \cdot 13 \cdot 248641$	4	2, 8
186	$39463374 = 2 \cdot 3 \cdot 29 \cdot 337 \cdot 673$	$49047090 = 2 \cdot 3 \cdot 5 \cdot 43 \cdot 193 \cdot 197$	7	
187	$48824765 = 5 \cdot 11 \cdot 17 \cdot 79 \cdot 661$	$50016835 = 5 \cdot 11 \cdot 19 \cdot 23 \cdot 2081$	3	2, 6; 14, 7; 6, 9
188	$46697505 = 3 \cdot 5 \cdot 53 \cdot 151 \cdot 389$	$50147295 = 3 \cdot 5 \cdot 11 \cdot 313 \cdot 971$	4	2, 8
189	$39734922 = 2 \cdot 3 \cdot 73 \cdot 83 \cdot 1093$	$50525430 = 2 \cdot 3 \cdot 5 \cdot 43 \cdot 53 \cdot 739$	7	
190	$50249066 = 2 \cdot 7 \cdot 23 \cdot 113 \cdot 1381$	$51760534 = 2 \cdot 7 \cdot 23^2 29 \cdot 241$	5	
191	$49285558 = 2 \cdot 7 \cdot 29 \cdot 233 \cdot 521$	$52052042 = 2 \cdot 7 \cdot 29 \cdot 41 \cdot 53 \cdot 59$	5	
192	$50994685 = 5 \cdot 7 \cdot 71 \cdot 20521$	$52426115 = 5 \cdot 7 \cdot 31 \cdot 211 \cdot 229$	3	2, 6; 6, 9
193	$52741575 = 3^2 5^2 29 \cdot 59 \cdot 137$	$53273145 = 3 \cdot 5 \cdot 29^2 41 \cdot 103$	4	2, 8
194	$48486225 = 3 \cdot 5^2 67 \cdot 9649$	$53396655 = 3 \cdot 5 \cdot 13 \cdot 61 \cdot 67^2$	4	2, 8
195	$53050802 = 2 \cdot 7 \cdot 29 \cdot 41 \cdot 3187$	$53998798 = 2 \cdot 7 \cdot 19 \cdot 43 \cdot 4721$	5	
196	$49951165 = 5 \cdot 11 \cdot 43 \cdot 21121$	$56493635 = 5 \cdot 11 \cdot 17 \cdot 23 \cdot 37 \cdot 71$	3	2, 6; 14, 7; 6, 9
197	$54607278 = 2 \cdot 3 \cdot 11 \cdot 53 \cdot 67 \cdot 233$	$56864082 = 2 \cdot 3 \cdot 11 \cdot 17 \cdot 59 \cdot 859$	7	
198	$47579721 = 3 \cdot 7 \cdot 41 \cdot 73 \cdot 757$	$56929719 = 3 \cdot 7^2 11 \cdot 17 \cdot 19 \cdot 109$	4	5, 5; 2, 8; 10, 10
199	$55150986 = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 89 \cdot 229$	$57207414 = 2 \cdot 3 \cdot 11 \cdot 17 \cdot 67 \cdot 761$	7	
200	$56448535 = 5 \cdot 11 \cdot 13^2 6073$	$57219305 = 5 \cdot 11 \cdot 13 \cdot 79 \cdot 1013$	3	2, 6; 14, 7; 6, 9

#	a_0	a_1	k	$b, \overline{k}$
201	$51744858 = 2 \cdot 3 \cdot 11 \cdot 353 \cdot 2221$	$57656742 = 2 \cdot 3 \cdot 11^2 13 \cdot 41 \cdot 149$	7	
202	$55946975 = 5^2 7^2 109 \cdot 419$	$57815905 = 5 \cdot 7 \cdot 23 \cdot 71821$	3	2, 6; 6, 9
203	$56857785 = 3 \cdot 5 \cdot 19 \cdot 199501$	$58054215 = 3 \cdot 5 \cdot 19^2 71 \cdot 151$	4	2, 8
204	$54464242 = 2 \cdot 7 \cdot 71 \cdot 157 \cdot 349$	$59540558 = 2 \cdot 7 \cdot 11 \cdot 59 \cdot 6553$	5	
205	$56774578 = 2 \cdot 7 \cdot 31 \cdot 130817$	$60959822 = 2 \cdot 7^2 11 \cdot 193 \cdot 293$	5	
206	$56449705 = 5 \cdot 13 \cdot 23 \cdot 61 \cdot 619$	$61019735 = 5 \cdot 7 \cdot 19 \cdot 89 \cdot 1031$	3	2, 6; 6, 9
207	$59108865 = 3 \cdot 5 \cdot 83 \cdot 197 \cdot 241$	$64324095 = 3 \cdot 5 \cdot 11 \cdot 97 \cdot 4019$	4	2, 8
208	$59261006 = 2 \cdot 7 \cdot 89 \cdot 199 \cdot 239$	$65146354 = 2 \cdot 7 \cdot 13 \cdot 29 \cdot 12343$	5	
209	$62569353 = 3 \cdot 7 \cdot 11 \cdot 439 \cdot 617$	$66938487 = 3 \cdot 7 \cdot 11 \cdot 23 \cdot 43 \cdot 293$	4	5, 5; 2, 8; 10, 10
210	$65585289 = 3 \cdot 7 \cdot 11 \cdot 97 \cdot 2927$	$69244791 = 3 \cdot 7^2 11^2 17 \cdot 229$	4	5, 5; 2, 8; 10, 10
211	$67838246 = 2 \cdot 7^2 19 \cdot 36433$	$69874714 = 2 \cdot 7 \cdot 13 \cdot 379 \cdot 1013$	5	
212	$67854955 = 5 \cdot 7^2 419 \cdot 661$	$71188565 = 5 \cdot 7 \cdot 23 \cdot 191 \cdot 463$	3	2, 6; 6, 9
213	$64660323 = 3 \cdot 7 \cdot 13 \cdot 433 \cdot 547$	$71201949 = 3 \cdot 7^2 13 \cdot 19 \cdot 37 \cdot 53$	4	5, 5; 2, 8; 10, 10; 110, 11
214	$61531118 = 2 \cdot 11 \cdot 23 \cdot 277 \cdot 439$	$71445682 = 2 \cdot 7 \cdot 11 \cdot 23^2 877$	5	
215	$66870573 = 3 \cdot 7 \cdot 11 \cdot 337 \cdot 859$	$71507667 = 3 \cdot 7 \cdot 11 \cdot 23 \cdot 43 \cdot 313$	4	5, 5; 2, 8; 10, 10
216	$61129835 = 5 \cdot 13^2 73 \cdot 991$	$72306325 = 5^2 7 \cdot 13 \cdot 37 \cdot 859$	3	2, 6; 6, 9
217	$67654002 = 2 \cdot 3 \cdot 13 \cdot 59 \cdot 61 \cdot 241$	$72659598 = 2 \cdot 3 \cdot 11 \cdot 17 \cdot 31 \cdot 2089$	7	
218	$69964615 = 5 \cdot 7 \cdot 881 \cdot 2269$	$73735865 = 5 \cdot 7 \cdot 19 \cdot 110881$	3	2, 6; 6, 9
219	$75449154 = 2 \cdot 3 \cdot 11 \cdot 23^2 2161$	$77565246 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 37 \cdot 1381$	7	
220	$72587697 = 3 \cdot 7 \cdot 13^2 113 \cdot 181$	$78370383 = 3 \cdot 7 \cdot 13 \cdot 19 \cdot 29 \cdot 521$	4	5, 5; 2, 8; 10, 10; 110, 11
221	$69750694 = 2 \cdot 13^2 17 \cdot 61 \cdot 199$	$78511706 = 2 \cdot 7 \cdot 13 \cdot 37 \cdot 89 \cdot 131$	5	
222	$63443037 = 3 \cdot 7 \cdot 67^2 673$	$79192995 = 3 \cdot 5 \cdot 7 \cdot 67 \cdot 11257$	4	2, 8
223	$80115135 = 3 \cdot 5 \cdot 17^2 18481$	$80734785 = 3 \cdot 5 \cdot 17 \cdot 137 \cdot 2311$	4	2, 8;
224	$67382601 = 3 \cdot 11^2 13 \cdot 109 \cdot 131$	$80879799 = 3 \cdot 7 \cdot 11 \cdot 13 \cdot 23 \cdot 1171$	4	5, 5; 2, 8; 10, 10
225	$61601478 = 2 \cdot 3 \cdot 10266913$	$82135290 = 2 \cdot 3 \cdot 5 \cdot 19 \cdot 103 \cdot 1399$	7	
226	$77133406 = 2 \cdot 7 \cdot 71 \cdot 73 \cdot 1063$	$83440994 = 2 \cdot 7 \cdot 13 \cdot 37 \cdot 12391$	5	
227	$78807218 = 2 \cdot 7 \cdot 43 \cdot 109 \cdot 1201$	$84488782 = 2 \cdot 7 \cdot 19 \cdot 41 \cdot 61 \cdot 127$	5	
228	$79441934 = 2 \cdot 11 \cdot 13 \cdot 89 \cdot 3121$	$85294066 = 2 \cdot 11 \cdot 13 \cdot 17 \cdot 53 \cdot 331$	5	
229	$75136558 = 2 \cdot 7 \cdot 449 \cdot 11953$	$85498322 = 2 \cdot 7 \cdot 13 \cdot 29 \cdot 97 \cdot 167$	5	
230	$75928255 = 5 \cdot 13 \cdot 37 \cdot 131 \cdot 241$	$85812545 = 5 \cdot 7 \cdot 13 \cdot 151 \cdot 1249$	3	2, 6; 6, 9
231	$65415894 = 2 \cdot 3 \cdot 457 \cdot 23857$	$86880810 = 2 \cdot 3 \cdot 5 \cdot 29 \cdot 37 \cdot 2699$	7	
232	$81802045 = 5 \cdot 13 \cdot 17 \cdot 181 \cdot 409$	$87403715 = 5 \cdot 7 \cdot 17^2 8641$	3	2, 6; 6, 9
233	$81025165 = 5 \cdot 13 \cdot 31 \cdot 79 \cdot 509$	$90150515 = 5 \cdot 7 \cdot 13^2 15241$	3	2, 6; 6, 9
234	$85363558 = 2 \cdot 7 \cdot 41 \cdot 127 \cdot 1171$	$91540442 = 2 \cdot 7 \cdot 19 \cdot 37 \cdot 71 \cdot 131$	5	
235	$83863005 = 3 \cdot 5 \cdot 211 \cdot 26497$	$94190115 = 3 \cdot 5 \cdot 17 \cdot 29 \cdot 47 \cdot 271$	4	2, 8
236	$90348545 = 5 \cdot 7 \cdot 1117 \cdot 2311$	$95264575 = 5^2 7^2 19 \cdot 4093$	3	2, 6; 6, 9
237	$95426338 = 2 \cdot 7 \cdot 17 \cdot 547 \cdot 733$	$96416222 = 2 \cdot 7^2 19 \cdot 53 \cdot 977$	5	
238	$94704915 = 3 \cdot 5 \cdot 23 \cdot 277 \cdot 991$	$97656045 = 3 \cdot 5 \cdot 23^2 31 \cdot 397$	4	2, 8
239	$93217335 = 3 \cdot 5 \cdot 43^2 3361$	$100963785 = 3 \cdot 5 \cdot 13 \cdot 43 \cdot 12041$	4	2, 8
240	$85972434 = 2 \cdot 3 \cdot 17 \cdot 113 \cdot 7459$	$101133870 = 2 \cdot 3 \cdot 5 \cdot 113 \cdot 29833$	7	
241	$88711494 = 2 \cdot 3 \cdot 19 \cdot 43 \cdot 18097$	$102816570 = 2 \cdot 3 \cdot 5 \cdot 523 \cdot 6553$	7	
242	$99237045 = 3 \cdot 5 \cdot 31 \cdot 101 \cdot 2113$	$103514955 = 3 \cdot 5 \cdot 17 \cdot 41 \cdot 9901$	4	2, 8
243	$101509415 = 5 \cdot 7 \cdot 83^2 421$	$104304025 = 5^2 7 \cdot 43 \cdot 83 \cdot 167$	3	2, 6; 6, 9
244	$102586862 = 2 \cdot 7 \cdot 29^2 8713$	$109637458 = 2 \cdot 7 \cdot 23 \cdot 29 \cdot 59 \cdot 199$	5	
245	$105370055 = 5 \cdot 7 \cdot 181 \cdot 16633$	$110180665 = 5 \cdot 7^2 31 \cdot 89 \cdot 163$	3	2, 6; 6, 9
246	$95921045 = 5 \cdot 11 \cdot 1123 \cdot 1553$	$113040235 = 5 \cdot 7 \cdot 11 \cdot 89 \cdot 3299$	3	2, 6; 6, 9
247	$112768922 = 2 \cdot 7 \cdot 17 \cdot 199 \cdot 2381$	$113426278 = 2 \cdot 7^2 17 \cdot 103 \cdot 661$	5	
248	$113483238 = 2 \cdot 3 \cdot 11^2 19^2 433$	$114042522 = 2 \cdot 3 \cdot 11 \cdot 19 \cdot 199 \cdot 457$	7	
249	$101461262 = 2 \cdot 7^2 311 \cdot 3329$	$115191538 = 2 \cdot 7 \cdot 11 \cdot 29 \cdot 25793$	5	
250	$116940775 = 5^2 7 \cdot 83^2 97$	$118274585 = 5 \cdot 11 \cdot 13 \cdot 83 \cdot 1993$	3	2, 6; 6, 9

#	a_0	a_1	k	b, $\overline{k}$
251	$102117235 = 5 \cdot 11 \cdot 97 \cdot 19141$	$118375565 = 5 \cdot 7 \cdot 11 \cdot 349 \cdot 881$	3	2, 6; 6, 9
252	$93312471 = 3 \cdot 7 \cdot 2017 \cdot 2203$	$119770665 = 3 \cdot 5 \cdot 7 \cdot 37 \cdot 30829$	4	2, 8
253	$119731215 = 3 \cdot 5 \cdot 23^2 79 \cdot 191$	$120234225 = 3 \cdot 5^2 23 \cdot 47 \cdot 1483$	4	2, 8
254	$103685421 = 3 \cdot 7^2 31 \cdot 61 \cdot 373$	$121300179 = 3 \cdot 7 \cdot 11 \cdot 13 \cdot 31 \cdot 1303$	4	5, 5; 2, 8; 10, 10
255	$121663115 = 5 \cdot 7 \cdot 71 \cdot 173 \cdot 283$	$122797045 = 5 \cdot 7 \cdot 31 \cdot 113177$	3	2, 6; 6, 9
256	$117929245 = 5 \cdot 7 \cdot 757 \cdot 4451$	$124293155 = 5 \cdot 7^2 19 \cdot 26701$	3	2, 6; 6, 9
257	$125367158 = 2 \cdot 7 \cdot 23 \cdot 139 \cdot 2801$	$129656842 = 2 \cdot 7^2 23^2 41 \cdot 61$	5	
258	$124467014 = 2 \cdot 7 \cdot 29 \cdot 113 \cdot 2713$	$130677946 = 2 \cdot 7 \cdot 17 \cdot 43 \cdot 113^2$	5	
259	$131996242 = 2 \cdot 7 \cdot 37^2 71 \cdot 97$	$136534958 = 2 \cdot 7 \cdot 29 \cdot 37 \cdot 61 \cdot 149$	5	
260	$123165555 = 3 \cdot 5 \cdot 2221 \cdot 3697$	$139398285 = 3 \cdot 5 \cdot 13 \cdot 23 \cdot 31081$	4	2, 8
261	$135981685 = 5 \cdot 7 \cdot 71 \cdot 54721$	$139807115 = 5 \cdot 7 \cdot 29 \cdot 181 \cdot 761$	3	2, 6; 6, 9
262	$129066555 = 3 \cdot 5 \cdot 73 \cdot 311 \cdot 379$	$140916165 = 3 \cdot 5 \cdot 13 \cdot 37 \cdot 19531$	4	2, 8
263	$109831701 = 3 \cdot 7 \cdot 2017 \cdot 2593$	$140990955 = 3 \cdot 5 \cdot 7 \cdot 97 \cdot 109 \cdot 127$	4	2, 8
264	$133558978 = 2 \cdot 7 \cdot 29 \cdot 313 \cdot 1051$	$141625022 = 2 \cdot 7 \cdot 11 \cdot 491 \cdot 1873$	5	
265	$138043955 = 5 \cdot 7 \cdot 313 \cdot 12601$	$145002445 = 5 \cdot 7 \cdot 37 \cdot 41 \cdot 2731$	3	2, 6; 6, 9
266	$137450894 = 2 \cdot 7 \cdot 29 \cdot 233 \cdot 1453$	$145514866 = 2 \cdot 7 \cdot 17 \cdot 29^2 727$	5	
267	$109181958 = 2 \cdot 3 \cdot 18196993$	$145575930 = 2 \cdot 3 \cdot 5 \cdot 17 \cdot 397 \cdot 719$	7	
268	$144358795 = 5 \cdot 7 \cdot 97 \cdot 101 \cdot 421$	$145945205 = 5 \cdot 7 \cdot 37 \cdot 251 \cdot 449$	3	2, 6; 6, 9
269	$137064578 = 2 \cdot 7 \cdot 31 \cdot 313 \cdot 1009$	$145981822 = 2 \cdot 7 \cdot 17 \cdot 53 \cdot 71 \cdot 163$	5	
270	$128270121 = 3 \cdot 7 \cdot 19 \cdot 151 \cdot 2129$	$147518679 = 3 \cdot 7 \cdot 11 \cdot 19^2 29 \cdot 61$	4	5, 5; 2, 8; 10, 10
271	$146519925 = 3 \cdot 5^2 19 \cdot 229 \cdot 449$	$147654795 = 3 \cdot 5 \cdot 19 \cdot 71 \cdot 7297$	4	2, 8
272	$129124798 = 2 \cdot 11 \cdot 19 \cdot 541 \cdot 571$	$147895202 = 2 \cdot 7 \cdot 13 \cdot 19^2 2251$	5	
273	$139006077 = 3 \cdot 7 \cdot 29 \cdot 31 \cdot 37 \cdot 199$	$148394883 = 3 \cdot 7^2 13 \cdot 19 \cdot 61 \cdot 67$	4	5, 5; 2, 8; 10, 10; 110, 11
274	$137514435 = 3 \cdot 5 \cdot 61 \cdot 137 \cdot 1097$	$148673085 = 3 \cdot 5 \cdot 11 \cdot 137 \cdot 6577$	4	2, 8
275	$136875705 = 3 \cdot 5 \cdot 101 \cdot 167 \cdot 541$	$149972295 = 3 \cdot 5 \cdot 11 \cdot 73 \cdot 12451$	4	2, 8
276	$145119254 = 2 \cdot 7 \cdot 37 \cdot 41 \cdot 6833$	$150023146 = 2 \cdot 7 \cdot 13 \cdot 193 \cdot 4271$	5	
277	$141220982 = 2 \cdot 7 \cdot 73 \cdot 138181$	$157247818 = 2 \cdot 7 \cdot 13 \cdot 43 \cdot 71 \cdot 283$	5	
278	$150178042 = 2 \cdot 7^2 37 \cdot 83 \cdot 499$	$158542118 = 2 \cdot 7 \cdot 19 \cdot 43 \cdot 83 \cdot 167$	5	
279	$152100074 = 2 \cdot 7 \cdot 31 \cdot 97 \cdot 3613$	$159976726 = 2 \cdot 7 \cdot 13 \cdot 73 \cdot 12041$	5	
280	$161003465 = 5 \cdot 7^2 37 \cdot 17761$	$161233975 = 5^2 7 \cdot 37^2 673$	3	2, 6; 6, 9
281	$147419115 = 3 \cdot 5 \cdot 1861 \cdot 5281$	$166846485 = 3 \cdot 5 \cdot 13 \cdot 23 \cdot 37201$	4	2, 8
282	$159011335 = 5 \cdot 7 \cdot 701 \cdot 6481$	$167580665 = 5 \cdot 7 \cdot 19 \cdot 252001$	3	2, 6; 6, 9
283	$165189626 = 2 \cdot 7 \cdot 29 \cdot 251 \cdot 1621$	$175010374 = 2 \cdot 7 \cdot 11 \cdot 631 \cdot 1801$	5	
284	$144510586 = 2 \cdot 11 \cdot 61 \cdot 257 \cdot 419$	$176513414 = 2 \cdot 7 \cdot 11 \cdot 17 \cdot 191 \cdot 353$	5	
285	$168318038 = 2 \cdot 7^2 41 \cdot 163 \cdot 257$	$180046762 = 2 \cdot 7 \cdot 17 \cdot 43 \cdot 73 \cdot 241$	5	
286	$139795835 = 5 \cdot 61 \cdot 67 \cdot 6841$	$185240965 = 5 \cdot 7 \cdot 13 \cdot 23 \cdot 31 \cdot 571$	3	2, 6; 6, 9
287	$184755305 = 5 \cdot 7 \cdot 43 \cdot 122761$	$186470935 = 5 \cdot 7 \cdot 37 \cdot 311 \cdot 463$	3	2, 6; 6, 9
288	$164895945 = 3 \cdot 5 \cdot 1753 \cdot 6271$	$186625335 = 3 \cdot 5 \cdot 13 \cdot 23 \cdot 41611$	4	2, 8
289	$174548685 = 3 \cdot 5 \cdot 41 \cdot 463 \cdot 613$	$187363635 = 3 \cdot 5 \cdot 23 \cdot 29 \cdot 61 \cdot 307$	4	2, 8
290	$181324335 = 3 \cdot 5 \cdot 43 \cdot 73 \cdot 3851$	$191232465 = 3 \cdot 5 \cdot 23 \cdot 37 \cdot 71 \cdot 211$	4	2, 8
291	$185803135 = 5 \cdot 7 \cdot 101 \cdot 52561$	$192628865 = 5 \cdot 11 \cdot 13 \cdot 41 \cdot 6571$	3	2, 6; 6, 9
292	$192607195 = 5 \cdot 11^2 17 \cdot 61 \cdot 307$	$195156005 = 5 \cdot 11 \cdot 17 \cdot 31 \cdot 6733$	3	2, 6; 14, 7; 6, 9
293	$193057165 = 5 \cdot 7 \cdot 71 \cdot 77689$	$198490355 = 5 \cdot 7 \cdot 29 \cdot 167 \cdot 1171$	3	2, 6; 6, 9
294	$189052017 = 3 \cdot 7 \cdot 11 \cdot 631 \cdot 1297$	$202858383 = 3 \cdot 7^2 13 \cdot 19 \cdot 37 \cdot 151$	4	5, 5; 2, 8; 10, 10
295	$203983425 = 3^2 5^2 17^2 3137$	$205452735 = 3 \cdot 5 \cdot 17 \cdot 137 \cdot 5881$	4	2, 8
296	$205570455 = 3 \cdot 5 \cdot 31 \cdot 59^2 127$	$208354665 = 3 \cdot 5 \cdot 19 \cdot 59 \cdot 12391$	4	2, 8
297	$197366818 = 2 \cdot 11 \cdot 23 \cdot 43 \cdot 47 \cdot 193$	$210671582 = 2 \cdot 11 \cdot 17 \cdot 19 \cdot 23 \cdot 1289$	5	
298	$186787455 = 3 \cdot 5 \cdot 1297 \cdot 9601$	$211343745 = 3 \cdot 5 \cdot 17 \cdot 19 \cdot 181 \cdot 241$	4	2, 8
299	$209406705 = 3 \cdot 5 \cdot 31^2 73 \cdot 199$	$214851855 = 3 \cdot 5 \cdot 23 \cdot 31 \cdot 20089$	4	2, 8
300	$162960681 = 3 \cdot 13 \cdot 23 \cdot 139 \cdot 1307$	$217680855 = 3 \cdot 5 \cdot 7 \cdot 23^2 3919$	4	2, 8

#	a_0	a_1	k	$b, \overline{k}$
301	$211726405 = 5 \cdot 11^2 19 \cdot 113 \cdot 163$	$219375035 = 5 \cdot 11 \cdot 23 \cdot 37 \cdot 43 \cdot 109$	3	2, 6; 14, 7; 6, 9
302	$210279335 = 5 \cdot 7^2 331 \cdot 2593$	$220822105 = 5 \cdot 7 \cdot 37 \cdot 41 \cdot 4159$	3	2, 6; 6, 9
303	$222649935 = 3 \cdot 5 \cdot 17^2 51361$	$224387505 = 3^2 5 \cdot 17 \cdot 137 \cdot 2141$	4	2, 8
304	$221525031 = 3 \cdot 7^2 13^2 37 \cdot 241$	$231349209 = 3 \cdot 7 \cdot 13 \cdot 17 \cdot 79 \cdot 631$	4	5, 5; 2, 8; 10, 10; 110, 11
305	$223395305 = 5 \cdot 7 \cdot 1051 \cdot 6073$	$235647895 = 5 \cdot 7 \cdot 31 \cdot 47 \cdot 4621$	3	2, 6; 6, 9
306	$233031694 = 2 \cdot 7 \cdot 19 \cdot 197 \cdot 4447$	$237532946 = 2 \cdot 7 \cdot 19^2 43 \cdot 1093$	5	
307	$229011198 = 2 \cdot 3 \cdot 13 \cdot 31 \cdot 53 \cdot 1787$	$239063682 = 2 \cdot 3 \cdot 11 \cdot 13^2 21433$	7	
308	$196913134 = 2 \cdot 11 \cdot 67 \cdot 103 \cdot 1297$	$239320466 = 2 \cdot 7 \cdot 11 \cdot 19 \cdot 89 \cdot 919$	5	
309	$241040535 = 3 \cdot 5 \cdot 17 \cdot 397 \cdot 2381$	$241509225 = 3 \cdot 5^2 17 \cdot 307 \cdot 617$	4	2, 8
310	$226743946 = 2 \cdot 11 \cdot 13 \cdot 67 \cdot 11833$	$241803254 = 2 \cdot 7 \cdot 11^2 349 \cdot 409$	5	
311	$230554714 = 2 \cdot 13 \cdot 17 \cdot 23 \cdot 22679$	$248404646 = 2 \cdot 7 \cdot 17 \cdot 23^2 1973$	5	
312	$234659282 = 2 \cdot 11 \cdot 13 \cdot 239 \cdot 3433$	$255430318 = 2 \cdot 11 \cdot 13^2 23 \cdot 29 \cdot 103$	5	
313	$246331035 = 3^2 5 \cdot 23 \cdot 238001$	$256324965 = 3 \cdot 5 \cdot 13 \cdot 251 \cdot 5237$	4	2, 8
314	$218844162 = 2 \cdot 3^2 17 \cdot 113 \cdot 6329$	$257426430 = 2 \cdot 3 \cdot 5 \cdot 113 \cdot 75937$	7	
315	$239358182 = 2 \cdot 7 \cdot 71 \cdot 113 \cdot 2131$	$261617818 = 2 \cdot 7 \cdot 11 \cdot 71^2 337$	5	
316	$260474585 = 5 \cdot 7 \cdot 73 \cdot 97 \cdot 1051$	$262072615 = 5 \cdot 7 \cdot 41 \cdot 181 \cdot 1009$	3	2, 6; 6, 9
317	$258440835 = 3 \cdot 5 \cdot 41 \cdot 61 \cdot 83^2$	$264259965 = 3 \cdot 5 \cdot 31 \cdot 41 \cdot 83 \cdot 167$	4	2, 8
318	$251777735 = 5 \cdot 11 \cdot 17 \cdot 269281$	$265239865 = 5 \cdot 11^2 17^2 37 \cdot 41$	3	2, 6; 14, 7; 6, 9
319	$243841143 = 3 \cdot 7 \cdot 13^2 127 \cdot 541$	$265642377 = 3 \cdot 7^2 11 \cdot 13 \cdot 12637$	4	5, 5; 2, 8; 10, 10
320	$264872342 = 2 \cdot 7^2 17 \cdot 173 \cdot 919$	$265658218 = 2 \cdot 7 \cdot 17 \cdot 103 \cdot 10837$	5	
321	$216363021 = 3 \cdot 7 \cdot 73 \cdot 113 \cdot 1249$	$266702835 = 3 \cdot 5 \cdot 7^2 109 \cdot 3329$	4	2, 8
322	$242265062 = 2 \cdot 13 \cdot 17 \cdot 31 \cdot 17681$	$266918938 = 2 \cdot 11 \cdot 13^2 17 \cdot 41 \cdot 103$	5	
323	$263410329 = 3 \cdot 7^2 13^2 23 \cdot 461$	$267039591 = 3 \cdot 7 \cdot 13 \cdot 23 \cdot 71 \cdot 599$	4	5, 5; 2, 8; 10, 10; 110, 11
324	$273236138 = 2 \cdot 7 \cdot 17 \cdot 211 \cdot 5441$	$275115862 = 2 \cdot 7 \cdot 17^2 97 \cdot 701$	5	
325	$236381405 = 5 \cdot 13 \cdot 61 \cdot 59617$	$278700835 = 5 \cdot 7 \cdot 19 \cdot 37 \cdot 47 \cdot 241$	3	2, 6; 6, 9
326	$279243958 = 2 \cdot 7 \cdot 29^2 37 \cdot 641$	$282010442 = 2 \cdot 7 \cdot 29 \cdot 59 \cdot 61 \cdot 193$	5	
327	$269194646 = 2 \cdot 7 \cdot 29 \cdot 151 \cdot 4391$	$283945354 = 2 \cdot 7 \cdot 11 \cdot 1843801$	5	
328	$274908765 = 3 \cdot 5 \cdot 23 \cdot 577 \cdot 1381$	$284686755 = 3 \cdot 5 \cdot 23 \cdot 47 \cdot 97 \cdot 181$	4	2, 8
329	$273815738 = 2 \cdot 7 \cdot 29 \cdot 79 \cdot 8537$	$285462982 = 2 \cdot 7 \cdot 23 \cdot 43 \cdot 53 \cdot 389$	5	
330	$273501445 = 5 \cdot 7 \cdot 967 \cdot 8081$	$288478715 = 5 \cdot 7 \cdot 31 \cdot 47 \cdot 5657$	3	2, 6; 6, 9
331	$284495162 = 2 \cdot 7 \cdot 29^2 73 \cdot 331$	$294298438 = 2 \cdot 7 \cdot 29 \cdot 31 \cdot 67 \cdot 349$	5	
332	$295029878 = 2 \cdot 11 \cdot 13 \cdot 23 \cdot 44851$	$296990122 = 2 \cdot 11 \cdot 13^2 23^2 151$	5	
333	$287520495 = 3 \cdot 5 \cdot 41 \cdot 53 \cdot 8821$	$299538705 = 3 \cdot 5 \cdot 19 \cdot 71 \cdot 113 \cdot 131$	4	2, 8
334	$295407606 = 2 \cdot 3 \cdot 13^2 17 \cdot 17137$	$303392778 = 2 \cdot 3 \cdot 13 \cdot 17^2 43 \cdot 313$	7	
335	$232247046 = 2 \cdot 3 \cdot 449 \cdot 86209$	$308449530 = 2 \cdot 3^2 5 \cdot 17 \cdot 449^2$	7	
336	$296739422 = 2 \cdot 7 \cdot 23 \cdot 953 \cdot 967$	$310217698 = 2 \cdot 7 \cdot 23 \cdot 29 \cdot 139 \cdot 239$	5	
337	$296777602 = 2 \cdot 11 \cdot 17 \cdot 23 \cdot 34501$	$310422398 = 2 \cdot 11 \cdot 13 \cdot 23 \cdot 41 \cdot 1151$	5	
338	$294657605 = 5 \cdot 11 \cdot 19 \cdot 457 \cdot 617$	$312077755 = 5 \cdot 11^2 17 \cdot 19 \cdot 1597$	3	2, 6; 14, 7; 6, 9
339	$310205922 = 2 \cdot 3 \cdot 13^2 23 \cdot 47 \cdot 283$	$313072734 = 2 \cdot 3 \cdot 13 \cdot 23 \cdot 47^2 79$	7	
340	$315241355 = 5 \cdot 11 \cdot 13 \cdot 353 \cdot 1249$	$317344885 = 5 \cdot 11^2 13 \cdot 157 \cdot 257$	3	2, 6; 14, 7; 6, 9
341	$316460586 = 2 \cdot 3 \cdot 13 \cdot 29 \cdot 31 \cdot 4513$	$320272854 = 2 \cdot 3 \cdot 11 \cdot 19 \cdot 97 \cdot 2633$	7	
342	$310970145 = 3 \cdot 5 \cdot 31 \cdot 73 \cdot 9161$	$322169055 = 3 \cdot 5 \cdot 13 \cdot 601 \cdot 2749$	4	2, 8
343	$247913913 = 3 \cdot 13 \cdot 31 \cdot 53^2 73$	$323570247 = 3 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 53 \cdot 107$	4	5, 5; 2, 8; 10, 10
344	$314482542 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 207169$	$323594898 = 2 \cdot 3 \cdot 11^2 17 \cdot 157 \cdot 167$	7	
345	$326133206 = 2 \cdot 7 \cdot 31 \cdot 61 \cdot 97 \cdot 127$	$327050794 = 2 \cdot 7^2 17 \cdot 109 \cdot 1801$	5	
346	$321452945 = 5 \cdot 11 \cdot 23 \cdot 59^2 73$	$329000815 = 5 \cdot 11^2 13 \cdot 59 \cdot 709$	3	2, 6; 14, 7; 6, 9
347	$312369575 = 5^2 7 \cdot 953 \cdot 1873$	$329202265 = 5 \cdot 7 \cdot 19 \cdot 495041$	3	2, 6; 6, 9
348	$267942974 = 2 \cdot 13 \cdot 37 \cdot 223 \cdot 1249$	$330497986 = 2 \cdot 7 \cdot 13 \cdot 17 \cdot 37 \cdot 2887$	5	
349	$296243285 = 5 \cdot 13 \cdot 31 \cdot 79 \cdot 1861$	$330502315 = 5 \cdot 11 \cdot 13^2 31^2 37$	3	2, 6; 14, 7; 6, 9
350	$293452275 = 3 \cdot 5^2 1873 \cdot 2089$	$331945485 = 3 \cdot 5 \cdot 17 \cdot 19 \cdot 131 \cdot 523$	4	2, 8

#	a_0	a_1	k	$b, \overline{k}$
351	$290732845 = 5 \cdot 13 \cdot 41 \cdot 127 \cdot 859$	$331969235 = 5 \cdot 13^2 19 \cdot 23 \cdot 29 \cdot 31$	3	2, 6; 14, 7; 6, 9
352	$314300998 = 2 \cdot 11 \cdot 17 \cdot 41 \cdot 103 \cdot 199$	$331971002 = 2 \cdot 11^2 17 \cdot 19 \cdot 31 \cdot 137$	5	
353	$238876673 = 7 \cdot 17 \cdot 103 \cdot 19489$	$333602815 = 5 \cdot 7 \cdot 13 \cdot 17^2 43 \cdot 59$	3	2, 6; 6, 9
354	$311979382 = 2 \cdot 11^2 13 \cdot 131 \cdot 757$	$336668618 = 2 \cdot 11 \cdot 13^2 23 \cdot 31 \cdot 127$	5	
355	$276099295 = 5 \cdot 17 \cdot 67 \cdot 48481$	$338239265 = 5 \cdot 7 \cdot 13 \cdot 23 \cdot 32321$	3	2, 6; 6, 9
356	$331327194 = 2 \cdot 3 \cdot 11 \cdot 31 \cdot 67 \cdot 2417$	$338388006 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 41 \cdot 5437$	7	
357	$284186318 = 2 \cdot 13 \cdot 53 \cdot 271 \cdot 761$	$356037682 = 2 \cdot 7 \cdot 11 \cdot 13 \cdot 177841$	5	
358	$355293285 = 3 \cdot 5 \cdot 17 \cdot 487 \cdot 2861$	$356366235 = 3 \cdot 5 \cdot 31 \cdot 79 \cdot 89 \cdot 109$	4	2, 8
359	$359276015 = 5 \cdot 7 \cdot 127 \cdot 131 \cdot 617$	$367209745 = 5 \cdot 7 \cdot 29 \cdot 331 \cdot 1093$	3	2, 6; 6, 9
360	$349183245 = 3 \cdot 5 \cdot 37 \cdot 281 \cdot 2239$	$372706035 = 3 \cdot 5 \cdot 13 \cdot 61 \cdot 31333$	4	2, 8
361	$369130542 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 241 \cdot 1009$	$375983058 = 2 \cdot 3 \cdot 11^2 19 \cdot 97 \cdot 281$	7	
362	$298855734 = 2 \cdot 3 \cdot 37 \cdot 433 \cdot 3109$	$377842890 = 2 \cdot 3 \cdot 5 \cdot 37 \cdot 73 \cdot 4663$	7	
363	$372521555 = 5 \cdot 7 \cdot 73 \cdot 211 \cdot 691$	$378640045 = 5 \cdot 7 \cdot 31 \cdot 461 \cdot 757$	3	2, 6; 6, 9
364	$368241302 = 2 \cdot 11 \cdot 13 \cdot 59 \cdot 139 \cdot 157$	$380933098 = 2 \cdot 7 \cdot 13^2 233 \cdot 691$	5	
365	$325119674 = 2 \cdot 11 \cdot 23 \cdot 642529$	$381661126 = 2 \cdot 7 \cdot 11 \cdot 23 \cdot 277 \cdot 389$	5	
366	$357163095 = 3 \cdot 5 \cdot 41 \cdot 271 \cdot 2143$	$383112105 = 3 \cdot 5 \cdot 19 \cdot 31 \cdot 103 \cdot 421$	4	2, 8
367	$357544054 = 2 \cdot 7 \cdot 43 \cdot 257 \cdot 2311$	$387569546 = 2 \cdot 7 \cdot 13 \cdot 71 \cdot 89 \cdot 337$	5	
368	$346109698 = 2 \cdot 11 \cdot 17 \cdot 443 \cdot 2089$	$392207102 = 2 \cdot 7 \cdot 17^2 31 \cdot 53 \cdot 59$	5	
369	$330386874 = 2 \cdot 3 \cdot 17 \cdot 281 \cdot 11527$	$392523846 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 17^2 1583$	7	
370	$377200902 = 2 \cdot 3 \cdot 13^2 41 \cdot 43 \cdot 211$	$393314298 = 2 \cdot 3 \cdot 13 \cdot 29 \cdot 31 \cdot 71 \cdot 79$	7	
371	$361640715 = 3 \cdot 5 \cdot 181 \cdot 133201$	$405591285 = 3 \cdot 5 \cdot 11 \cdot 73 \cdot 151 \cdot 223$	4	2, 8
372	$321240102 = 2 \cdot 3 \cdot 53 \cdot 433 \cdot 2333$	$412164570 = 2 \cdot 3 \cdot 5 \cdot 53^2 67 \cdot 73$	7	
373	$393140022 = 2 \cdot 3 \cdot 11 \cdot 37 \cdot 199 \cdot 809$	$413179338 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 397 \cdot 1213$	7	
374	$278619987 = 3 \cdot 17^2 97 \cdot 3313$	$413243565 = 3 \cdot 5 \cdot 7 \cdot 17 \cdot 37 \cdot 6257$	4	2, 8
375	$411739419 = 3 \cdot 7 \cdot 13 \cdot 29 \cdot 131 \cdot 397$	$418530021 = 3 \cdot 7^2 13^2 17 \cdot 991$	4	5, 5; 2, 8; 10, 10; 110, 11
376	$384985785 = 3 \cdot 5 \cdot 71 \cdot 193 \cdot 1873$	$420123975 = 3 \cdot 5^2 17 \cdot 43 \cdot 79 \cdot 97$	4	2, 8
377	$407417055 = 3 \cdot 5 \cdot 23 \cdot 691 \cdot 1709$	$422261025 = 3 \cdot 5^2 23^2 29 \cdot 367$	4	2, 8
378	$419461365 = 3 \cdot 5 \cdot 29^2 41 \cdot 811$	$422420235 = 3 \cdot 5 \cdot 29 \cdot 59 \cdot 109 \cdot 151$	4	2, 8
379	$418010142 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 509 \cdot 541$	$426895458 = 2 \cdot 3 \cdot 11 \cdot 19 \cdot 67 \cdot 5081$	7	
380	$336829942 = 2 \cdot 17 \cdot 31 \cdot 313 \cdot 1021$	$426946058 = 2 \cdot 7 \cdot 11 \cdot 17^2 53 \cdot 181$	5	
381	$365313918 = 2 \cdot 3 \cdot 17^2 457 \cdot 461$	$433452162 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 17 \cdot 29717$	7	
382	$433676075 = 5^2 7 \cdot 37 \cdot 66977$	$434332885 = 5 \cdot 7 \cdot 41 \cdot 313 \cdot 967$	3	2, 6; 6, 9
383	$399820545 = 3^2 5 \cdot 83 \cdot 167 \cdot 641$	$436500735 = 3 \cdot 5 \cdot 11 \cdot 83 \cdot 31873$	4	2, 8
384	$400978851 = 3 \cdot 7 \cdot 13 \cdot 281 \cdot 5227$	$441870429 = 3 \cdot 7 \cdot 11 \cdot 13 \cdot 269 \cdot 547$	4	5, 5; 2, 8; 10, 10
385	$430545434 = 2 \cdot 11 \cdot 17 \cdot 19 \cdot 60589$	$441921766 = 2 \cdot 11^2 13 \cdot 17 \cdot 8263$	5	
386	$439668515 = 5 \cdot 11 \cdot 13 \cdot 307 \cdot 2003$	$442492765 = 5 \cdot 11^2 13 \cdot 127 \cdot 443$	3	2, 6; 14, 7; 6, 9
387	$401124705 = 3 \cdot 5 \cdot 151 \cdot 409 \cdot 433$	$444904095 = 3 \cdot 5 \cdot 19 \cdot 31 \cdot 37 \cdot 1361$	4	2, 8
388	$427076405 = 5 \cdot 7^2 991 \cdot 1759$	$450095275 = 5^2 7 \cdot 19 \cdot 135367$	3	2, 6; 6, 9
389	$402547082 = 2 \cdot 7 \cdot 163 \cdot 176401$	$454756918 = 2 \cdot 7 \cdot 11 \cdot 31 \cdot 95257$	5	
390	$385145882 = 2 \cdot 11 \cdot 41 \cdot 67 \cdot 6373$	$455958118 = 2 \cdot 7 \cdot 11 \cdot 23 \cdot 109 \cdot 1181$	5	
391	$404339955 = 3 \cdot 5 \cdot 2917 \cdot 9241$	$457862925 = 3 \cdot 5^2 13 \cdot 43 \cdot 67 \cdot 163$	4	2, 8
392	$442033025 = 5^2 7 \cdot 127 \cdot 19889$	$460086655 = 5 \cdot 7 \cdot 31 \cdot 67 \cdot 6329$	3	2, 6; 6, 9
393	$317331807 = 3 \cdot 13^2 389 \cdot 1609$	$461300385 = 3 \cdot 5 \cdot 7 \cdot 13 \cdot 337949$	4	2, 8
394	$462097706 = 2 \cdot 7 \cdot 17^2 181 \cdot 631$	$463246294 = 2 \cdot 7^2 17 \cdot 109 \cdot 2551$	5	
395	$379043302 = 2 \cdot 11 \cdot 73 \cdot 236017$	$470614298 = 2 \cdot 7 \cdot 11 \cdot 17 \cdot 67 \cdot 2683$	5	
396	$440366914 = 2 \cdot 13 \cdot 19 \cdot 29 \cdot 59 \cdot 521$	$471671486 = 2 \cdot 11 \cdot 13 \cdot 29^2 37 \cdot 53$	5	
397	$429051795 = 3 \cdot 5 \cdot 109 \cdot 397 \cdot 661$	$474208365 = 3 \cdot 5 \cdot 19 \cdot 23 \cdot 73 \cdot 991$	4	2, 8
398	$470259202 = 2 \cdot 7 \cdot 17 \cdot 409 \cdot 4831$	$475647998 = 2 \cdot 7^2 17 \cdot 73 \cdot 3911$	5	
399	$495745815 = 3 \cdot 5 \cdot 19 \cdot 199 \cdot 8741$	$501033705 = 3 \cdot 5 \cdot 19^2 67 \cdot 1381$	4	2, 8
400	$483104825 = 5^2 7 \cdot 127 \cdot 21737$	$502840135 = 5 \cdot 7 \cdot 29 \cdot 79 \cdot 6271$	3	2, 6; 6, 9

#	a_0	a_1	k	$b, \overline{k}$
401	$426182214 = 2 \cdot 3 \cdot 17 \cdot 193 \cdot 21649$	$504854970 = 2 \cdot 3 \cdot 5 \cdot 83 \cdot 202753$	7	
402	$468651705 = 3 \cdot 5 \cdot 53 \cdot 139 \cdot 4241$	$504987975 = 3^2 5^2 17 \cdot 47 \cdot 53^2$	4	2, 8
403	$487791015 = 3 \cdot 5 \cdot 23 \cdot 461 \cdot 3067$	$505102425 = 3 \cdot 5^2 23^2 29 \cdot 439$	4	2, 8
404	$506000362 = 2 \cdot 7^2 19 \cdot 331 \cdot 821$	$516867638 = 2 \cdot 7 \cdot 23 \cdot 29 \cdot 55351$	5	
405	$499936675 = 5^2 7 \cdot 109 \cdot 26209$	$519030365 = 5 \cdot 7 \cdot 31 \cdot 73 \cdot 6553$	3	2, 6; 6, 9
406	$498769766 = 2 \cdot 11 \cdot 17 \cdot 23^2 2521$	$521326234 = 2 \cdot 11 \cdot 13 \cdot 23 \cdot 41 \cdot 1933$	5	
407	$505783685 = 5 \cdot 11 \cdot 23 \cdot 47^2 181$	$521598715 = 5 \cdot 11 \cdot 23 \cdot 31 \cdot 47 \cdot 283$	3	2, 6; 14, 7; 6, 9
408	$508223086 = 2 \cdot 7 \cdot 29 \cdot 61 \cdot 20521$	$525984914 = 2 \cdot 7^2 13 \cdot 181 \cdot 2281$	5	
409	$480141222 = 2 \cdot 3 \cdot 11 \cdot 83 \cdot 87649$	$526057818 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 83^2 89$	7	
410	$511488845 = 5 \cdot 7 \cdot 131 \cdot 281 \cdot 397$	$526347955 = 5 \cdot 7^2 41 \cdot 61 \cdot 859$	3	2, 6; 6, 9
411	$462816158 = 2 \cdot 7 \cdot 937 \cdot 35281$	$527846242 = 2 \cdot 7 \cdot 11 \cdot 43 \cdot 79 \cdot 1009$	5	
412	$516413595 = 3 \cdot 5 \cdot 23 \cdot 241 \cdot 6211$	$532828005 = 3 \cdot 5 \cdot 23 \cdot 41 \cdot 139 \cdot 271$	4	2, 8
413	$482327566 = 2 \cdot 7 \cdot 71 \cdot 631 \cdot 769$	$533736434 = 2 \cdot 7 \cdot 11 \cdot 73 \cdot 197 \cdot 241$	5	
414	$503657098 = 2 \cdot 7 \cdot 37 \cdot 263 \cdot 3697$	$542163062 = 2 \cdot 7 \cdot 19 \cdot 29 \cdot 67 \cdot 1049$	5	
415	$519169525 = 5^2 7 \cdot 757 \cdot 3919$	$547153355 = 5 \cdot 7^2 19 \cdot 117541$	3	2, 6; 6, 9
416	$504750946 = 2 \cdot 7 \cdot 97 \cdot 491 \cdot 757$	$562116254 = 2 \cdot 7 \cdot 17 \cdot 19 \cdot 197 \cdot 631$	5	
417	$524969438 = 2 \cdot 7^2 31 \cdot 172801$	$563670562 = 2 \cdot 7 \cdot 13 \cdot 71 \cdot 181 \cdot 241$	5	
418	$541172842 = 2 \cdot 7 \cdot 23 \cdot 281 \cdot 5981$	$563931158 = 2 \cdot 7 \cdot 23 \cdot 29 \cdot 131 \cdot 461$	5	
419	$497940755 = 5 \cdot 13^2 31 \cdot 19009$	$569548525 = 5^2 7 \cdot 13 \cdot 79 \cdot 3169$	3	2, 6; 6, 9
420	$522761395 = 5 \cdot 13 \cdot 29^2 73 \cdot 131$	$571684685 = 5 \cdot 11 \cdot 13 \cdot 29 \cdot 79 \cdot 349$	3	2, 6; 14, 7; 6, 9
421	$459330594 = 2 \cdot 3 \cdot 29 \cdot 367 \cdot 7193$	$572520030 = 2 \cdot 3 \cdot 5 \cdot 29 \cdot 658069$	7	
422	$561934135 = 5 \cdot 7 \cdot 61 \cdot 263201$	$575089865 = 5 \cdot 7 \cdot 29 \cdot 241 \cdot 2351$	3	2, 6; 6, 9
423	$397629453 = 3 \cdot 13^2 367 \cdot 2137$	$578026995 = 3 \cdot 5 \cdot 7 \cdot 13 \cdot 423463$	4	2, 8
424	$583521638 = 2 \cdot 7 \cdot 23 \cdot 47 \cdot 38557$	$587038522 = 2 \cdot 7^2 23 \cdot 37 \cdot 7039$	5	
425	$592818655 = 5 \cdot 11 \cdot 13 \cdot 157 \cdot 5281$	$593280545 = 5 \cdot 11^2 13 \cdot 241 \cdot 313$	3	2, 6; 14, 7; 6, 9
426	$590684295 = 3 \cdot 5 \cdot 17 \cdot 1429 \cdot 1621$	$593756025 = 3 \cdot 5^2 17 \cdot 163 \cdot 2857$	4	2, 8
427	$590075134 = 2 \cdot 11^2 17 \cdot 19 \cdot 7549$	$605528066 = 2 \cdot 11 \cdot 13 \cdot 17 \cdot 124543$	5	
428	$586556925 = 3 \cdot 5^2 23 \cdot 337 \cdot 1009$	$605624835 = 3 \cdot 5 \cdot 29 \cdot 31 \cdot 97 \cdot 463$	4	2, 8
429	$544697646 = 2 \cdot 3 \cdot 17 \cdot 37 \cdot 101 \cdot 1429$	$606841554 = 2 \cdot 3 \cdot 11 \cdot 17 \cdot 31 \cdot 73 \cdot 239$	7	
430	$520648154 = 2 \cdot 13 \cdot 17 \cdot 443 \cdot 2659$	$607194406 = 2 \cdot 7 \cdot 13 \cdot 17 \cdot 443^2$	5	
431	$599291511 = 3 \cdot 7^2 13 \cdot 53 \cdot 61 \cdot 97$	$608373129 = 3 \cdot 7 \cdot 13^2 37 \cdot 41 \cdot 113$	4	5, 5; 2, 8; 10, 10; 110, 11
432	$529986545 = 5 \cdot 11 \cdot 89 \cdot 108271$	$613344655 = 5 \cdot 7 \cdot 11 \cdot 331 \cdot 4813$	3	2, 6; 6, 9
433	$483976014 = 2 \cdot 3 \cdot 47 \cdot 163 \cdot 10529$	$614389170 = 2 \cdot 3 \cdot 5 \cdot 47^2 73 \cdot 127$	7	
434	$497086638 = 2 \cdot 3 \cdot 37^2 73 \cdot 829$	$614632530 = 2 \cdot 3 \cdot 5 \cdot 37 \cdot 277 \cdot 1999$	7	
435	$584270178 = 2 \cdot 3 \cdot 17 \cdot 19 \cdot 103 \cdot 2927$	$619088286 = 2 \cdot 3 \cdot 17^2 19^2 23 \cdot 43$	7	
436	$555852462 = 2 \cdot 3 \cdot 11^2 613 \cdot 1249$	$620362578 = 2 \cdot 3 \cdot 11 \cdot 19 \cdot 23 \cdot 137 \cdot 157$	7	
437	$553836598 = 2 \cdot 7 \cdot 211 \cdot 313 \cdot 599$	$621592202 = 2 \cdot 7 \cdot 11 \cdot 47 \cdot 157 \cdot 547$	5	
438	$552984014 = 2 \cdot 11 \cdot 31 \cdot 83 \cdot 9769$	$648479986 = 2 \cdot 7 \cdot 11 \cdot 23 \cdot 223 \cdot 821$	5	
439	$597763785 = 3 \cdot 5 \cdot 73 \cdot 113 \cdot 4831$	$648608055 = 3 \cdot 5 \cdot 17 \cdot 29 \cdot 139 \cdot 631$	4	2, 8
440	$627313674 = 2 \cdot 3 \cdot 13 \cdot 29^2 73 \cdot 131$	$649540086 = 2 \cdot 3 \cdot 13 \cdot 29 \cdot 31 \cdot 59 \cdot 157$	7	
441	$627486222 = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 67 \cdot 3461$	$651329778 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 31 \cdot 13841$	7	
442	$551924737 = 7 \cdot 13 \cdot 17 \cdot 43 \cdot 8297$	$652256255 = 5 \cdot 7 \cdot 17 \cdot 29 \cdot 103 \cdot 367$	3	2, 6; 6, 9
443	$579713275 = 5^2 19 \cdot 23 \cdot 47 \cdot 1129$	$653145605 = 5 \cdot 7 \cdot 23 \cdot 47 \cdot 61 \cdot 283$	3	2, 6; 6, 9
444	$641430685 = 5 \cdot 7 \cdot 71 \cdot 359 \cdot 719$	$654071075 = 5^2 7 \cdot 29 \cdot 359^2$	3	2, 6; 6, 9
445	$652655925 = 3^2 5^2 17^2 10037$	$657644235 = 3 \cdot 5 \cdot 17 \cdot 131 \cdot 19687$	4	2, 8
446	$662328058 = 2 \cdot 7 \cdot 17 \cdot 433 \cdot 6427$	$670167302 = 2 \cdot 7 \cdot 17^2 73 \cdot 2269$	5	
447	$666854115 = 3 \cdot 5 \cdot 19 \cdot 331 \cdot 7069$	$676631325 = 3 \cdot 5^2 19^2 67 \cdot 373$	4	2, 8
448	$664545486 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 317 \cdot 1381$	$678580914 = 2 \cdot 3 \cdot 11 \cdot 23 \cdot 41 \cdot 10903$	7	
449	$644240345 = 5 \cdot 7 \cdot 661 \cdot 27847$	$679001575 = 5^2 7^2 19 \cdot 29173$	3	2, 6; 6, 9
450	$552690622 = 2 \cdot 11 \cdot 61 \cdot 411841$	$682829378 = 2 \cdot 7 \cdot 11^2 17 \cdot 131 \cdot 181$	5	

#	a_0	a_1	k	$b, \overline{k}$
451	$562154061 = 3 \cdot 7 \cdot 37 \cdot 723493$	$688040115 = 3 \cdot 5 \cdot 7^2 163 \cdot 5743$	4	2, 8
452	$590543178 = 2 \cdot 3 \cdot 17^2 97 \cdot 3511$	$692600502 = 2 \cdot 3 \cdot 11 \cdot 17 \cdot 19 \cdot 53 \cdot 613$	7	
453	$686155715 = 5 \cdot 7 \cdot 71^2 3889$	$705126205 = 5 \cdot 7 \cdot 37 \cdot 71 \cdot 7669$	3	2, 6; 6, 9
454	$706509285 = 3 \cdot 5 \cdot 23 \cdot 71 \cdot 28843$	$714824475 = 3 \cdot 5^2 23^2 43 \cdot 419$	4	2, 8
455	$679455942 = 2 \cdot 3 \cdot 11 \cdot 61 \cdot 71 \cdot 2377$	$717632058 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 181 \cdot 4621$	7	
456	$670855857 = 3 \cdot 7 \cdot 19 \cdot 43 \cdot 61 \cdot 641$	$722603343 = 3 \cdot 7^2 11 \cdot 17 \cdot 97 \cdot 271$	4	5, 5; 2, 8; 10, 10
457	$676389142 = 2 \cdot 11 \cdot 13 \cdot 89 \cdot 26573$	$726612458 = 2 \cdot 11 \cdot 13^2 23 \cdot 29 \cdot 293$	5	
458	$657146622 = 2 \cdot 3 \cdot 11 \cdot 601 \cdot 16567$	$734397378 = 2 \cdot 3 \cdot 11 \cdot 13 \cdot 31 \cdot 27611$	7	
459	$577065741 = 3 \cdot 7 \cdot 233 \cdot 117937$	$736269555 = 3 \cdot 5 \cdot 7 \cdot 59 \cdot 157 \cdot 757$	4	2, 8
460	$646105638 = 2 \cdot 3 \cdot 17 \cdot 139 \cdot 199 \cdot 229$	$749385690 = 2 \cdot 3 \cdot 5 \cdot 419 \cdot 59617$	7	
461	$735765261 = 3 \cdot 7 \cdot 11 \cdot 71 \cdot 113 \cdot 397$	$754461939 = 3 \cdot 7 \cdot 11 \cdot 23 \cdot 211 \cdot 673$	4	5, 5; 2, 8; 10, 10
462	$645639115 = 5 \cdot 11 \cdot 397 \cdot 29569$	$759432245 = 5 \cdot 11 \cdot 13 \cdot 17 \cdot 43 \cdot 1453$	3	2, 6; 14, 7; 6, 9
463	$747024369 = 3 \cdot 7 \cdot 13 \cdot 29 \cdot 157 \cdot 601$	$762556431 = 3 \cdot 7 \cdot 13 \cdot 19 \cdot 113 \cdot 1301$	4	5, 5; 2, 8; 10, 10; 110, 11
464	$644619666 = 2 \cdot 3 \cdot 23 \cdot 83 \cdot 167 \cdot 337$	$764058990 = 2 \cdot 3 \cdot 5 \cdot 83^2 3697$	7	
465	$714596145 = 3 \cdot 5 \cdot 43 \cdot 181 \cdot 6121$	$765954255 = 3 \cdot 5 \cdot 11 \cdot 307 \cdot 15121$	4	2, 8
466	$754502126 = 2 \cdot 7 \cdot 17^2 186481$	$767174674 = 2 \cdot 7^2 17 \cdot 61 \cdot 7549$	5	
467	$720686078 = 2 \cdot 7 \cdot 31 \cdot 1049 \cdot 1583$	$771456322 = 2 \cdot 7 \cdot 13 \cdot 71 \cdot 227 \cdot 263$	5	
468	$768747182 = 2 \cdot 7^2 19 \cdot 181 \cdot 2281$	$782564818 = 2 \cdot 7 \cdot 19 \cdot 73 \cdot 191 \cdot 211$	5	
469	$752977435 = 5 \cdot 7 \cdot 281 \cdot 76561$	$790472165 = 5 \cdot 7^2 23 \cdot 151 \cdot 929$	3	2, 6; 6, 9
470	$692843578 = 2 \cdot 7 \cdot 4091 \cdot 12097$	$791335622 = 2 \cdot 7 \cdot 11 \cdot 43 \cdot 73 \cdot 1637$	5	
471	$684006115 = 5 \cdot 13 \cdot 61 \cdot 167 \cdot 1033$	$796129565 = 5 \cdot 7 \cdot 11 \cdot 173 \cdot 11953$	3	2, 6; 6, 9
472	$793152794 = 2 \cdot 7 \cdot 31 \cdot 37 \cdot 49393$	$807148006 = 2 \cdot 7 \cdot 19 \cdot 73 \cdot 197 \cdot 211$	5	
473	$786383871 = 3 \cdot 7 \cdot 13^2 43 \cdot 5153$	$833899521 = 3 \cdot 7 \cdot 13 \cdot 17 \cdot 47 \cdot 3823$	4	5, 5; 2, 8; 10, 10; 110, 11
474	$691838022 = 2 \cdot 3 \cdot 23 \cdot 103 \cdot 48673$	$837241530 = 2 \cdot 3 \cdot 5 \cdot 53 \cdot 613 \cdot 859$	7	
475	$755152785 = 3^2 5 \cdot 173 \cdot 97001$	$846511215 = 3 \cdot 5 \cdot 11 \cdot 41 \cdot 125131$	4	2, 8
476	$808093923 = 3 \cdot 7 \cdot 13 \cdot 79 \cdot 89 \cdot 421$	$852444957 = 3 \cdot 7 \cdot 13^2 17 \cdot 71 \cdot 199$	4	5, 5; 2, 8; 10, 10; 110, 11
477	$814697499 = 3 \cdot 7 \cdot 11 \cdot 229 \cdot 15401$	$870678501 = 3 \cdot 7^2 11 \cdot 23 \cdot 41 \cdot 571$	4	5, 5; 2, 8; 10, 10
478	$800624482 = 2 \cdot 7 \cdot 43 \cdot 1329941$	$875099918 = 2 \cdot 7^2 11 \cdot 59 \cdot 13759$	5	
479	$788662574 = 2 \cdot 7 \cdot 97 \cdot 271 \cdot 2143$	$876956626 = 2 \cdot 7^2 13 \cdot 41 \cdot 103 \cdot 163$	5	
480	$780394678 = 2 \cdot 7^2 1361 \cdot 5851$	$890365322 = 2 \cdot 7 \cdot 11 \cdot 31 \cdot 421 \cdot 443$	5	
481	$810964785 = 3 \cdot 5 \cdot 211 \cdot 257 \cdot 997$	$902473935 = 3 \cdot 5 \cdot 11 \cdot 97 \cdot 113 \cdot 499$	4	2, 8
482	$878758286 = 2 \cdot 7 \cdot 23 \cdot 113 \cdot 24151$	$906409714 = 2 \cdot 7 \cdot 23 \cdot 41 \cdot 71 \cdot 967$	5	
483	$860558226 = 2 \cdot 3 \cdot 11 \cdot 43 \cdot 353 \cdot 859$	$915295854 = 2 \cdot 3 \cdot 11 \cdot 19 \cdot 29 \cdot 25169$	7	
484	$913115105 = 5 \cdot 7 \cdot 43 \cdot 606721$	$921606175 = 5^2 7 \cdot 37 \cdot 317 \cdot 449$	3	2, 6; 6, 9
485	$842263665 = 3 \cdot 5 \cdot 83 \cdot 167 \cdot 4051$	$921851535 = 3 \cdot 5 \cdot 11 \cdot 83^2 811$	4	2, 8
486	$756791562 = 2 \cdot 3 \cdot 37 \cdot 137 \cdot 149 \cdot 167$	$927197430 = 2 \cdot 3 \cdot 5 \cdot 37 \cdot 835313$	7	
487	$917435015 = 5 \cdot 7 \cdot 101 \cdot 109 \cdot 2381$	$933252985 = 5 \cdot 7 \cdot 31 \cdot 281 \cdot 3061$	3	2, 6; 6, 9
488	$908289865 = 5 \cdot 7 \cdot 71 \cdot 365509$	$933870455 = 5 \cdot 7 \cdot 67 \cdot 71^2 79$	3	2, 6; 6, 9
489	$904577715 = 3^2 5 \cdot 29 \cdot 101 \cdot 6863$	$939927885 = 3 \cdot 5 \cdot 13 \cdot 293 \cdot 16451$	4	2, 8
490	$629115190 = 2 \cdot 5 \cdot 11 \cdot 89 \cdot 179 \cdot 359$	$941044170 = 2 \cdot 3 \cdot 5 \cdot 11 \cdot 89 \cdot 179^2$	7	
491	$863098509 = 3 \cdot 7 \cdot 13 \cdot 193 \cdot 16381$	$948398451 = 3 \cdot 7 \cdot 11 \cdot 13 \cdot 313 \cdot 1009$	4	5, 5; 2, 8; 10, 10
492	$910247865 = 3 \cdot 5 \cdot 43 \cdot 137 \cdot 10301$	$972427335 = 3 \cdot 5 \cdot 11 \cdot 619 \cdot 9521$	4	2, 8
493	$933132954 = 2 \cdot 3 \cdot 13 \cdot 23 \cdot 157 \cdot 3313$	$976486758 = 2 \cdot 3 \cdot 13 \cdot 23 \cdot 37 \cdot 47 \cdot 313$	7	
494	$947180605 = 5 \cdot 7 \cdot 127 \cdot 409 \cdot 521$	$977534915 = 5 \cdot 7 \cdot 41 \cdot 53 \cdot 12853$	3	2, 6; 6, 9
495	$897725654 = 2 \cdot 7 \cdot 61 \cdot 137 \cdot 7673$	$980379946 = 2 \cdot 7^2 13 \cdot 41 \cdot 137^2$	5	
496	$927660657 = 3 \cdot 7 \cdot 11^2 181 \cdot 2017$	$988345743 = 3 \cdot 7 \cdot 11 \cdot 19 \cdot 67 \cdot 3361$	4	5, 5; 2, 8; 10, 10
497	$958575045 = 3 \cdot 5 \cdot 61 \cdot 73 \cdot 113 \cdot 127$	$992267835 = 3 \cdot 5 \cdot 13 \cdot 673 \cdot 7561$	4	2, 8
498	$984292595 = 5 \cdot 11 \cdot 13 \cdot 401 \cdot 3433$	$992539405 = 5 \cdot 11^2 13 \cdot 97 \cdot 1301$	3	2, 6; 14, 7; 6, 9
499	$934904386 = 2 \cdot 13 \cdot 19 \cdot 31 \cdot 41 \cdot 1489$	$993543614 = 2 \cdot 7 \cdot 17 \cdot 31 \cdot 311 \cdot 433$	5	