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P. Dekker

Computer Science/Department of Interactive Systems

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CWI
P.O. Box 94079
1090 GB Amsterdam
The Netherlands

Scopes In Discourse

Paul Dekker

CWI, P.O. Box 94079, 1090 GB Amsterdam, The Netherlands
ILLC, Nieuwe Doelenstraat 15, 1012 CP Amsterdam, The Netherlands

Abstract

In this paper we show that the dynamic interpretation techniques of Janssen (assignment modalities), Groenendijk and Stokhof (dynamic binding), and Hendriks (flexibly scoping rules) enable a rigorous formulation of the semantics of intersentential anaphoric relationships, as well as of telescoping and periscoping phenomena in natural language.

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Introduction

In this paper we want to show that the potential of techniques of dynamic interpretation as developed by Janssen, Groenendijk and Stokhof, and Hendriks in the second half of the eighties has not yet been fully recognized. Not only do these techniques allow a rigorous formulation of the semantics of anaphoric relationships across sentential or clausal borders (including 'donkey-anaphora'), but they are also the right ones for formulating the semantics of what Craige Roberts has dubbed 'telescoping' and that of what is called 'periscoping' in this paper. The phenomenon of telescoping involves the extension of the semantic scope of operators beyond their syntactic scope; periscoping relates to the potential of a proper name, in no matter how deeply embedded a position, to serve as an antecedent for anaphoric pronouns. In discourse representation theory (DRT) these phenomena have been dealt with at the representational level of discourse representation construction. In this paper we want to show that the phenomena can also be dealt with semantically, by the application of the above-mentioned techniques.

Some things may have to be made clear from the outset. In the first place our restricted aim in this paper is not to try an in-depth analysis of the empirical phenomena at issue that can stand up against (descriptive) linguistic standards. What we are after is the selection of a limited set of phenomena, which we want to supply with a rigorous formulation within a perspicuous logical framework, idealized as such may be. In the second place, we will also try not to bother the reader with technical details in the main text. At the 'users' level these are deliberately kept as simple as possible. The essentially intensional logical underpinnings of the systems are assumed to run in the background in the main body of this paper. Only for the purpose of explaining the mere possibility of our results, the reader is sometimes allowed a snapshot of some of the systems' actions. The technical annex gives a concise specification of the required definitions and main results.

We will proceed as follows. In the first section we give a rudimentary introduction to the techniques at issue, Janssen's assignment modalities, Groenendijk and Stokhof's account of dynamic binding, and Hendriks' flexible scoping rules. In the second section these techniques are used in the formulation of a system of interpretation that deals with donkey anaphora. In section three the resulting system is extended with an account of telescoping phenomena. Section four shows how the same techniques can be used to deal with periscoping.

1. Scope and Binding

In this section we present a relatively informal description of the dynamic techniques of interpretation developed by Theo Janssen, Jeroen Groenendijk and Martin Stokhof, and Herman Hendriks. For the technical details the reader is referred to the appendix.

1.1 Assignment Modalities

Theo Janssen can be credited for the invention of a notion of *assignment modalities*. In the tenth chapter of his thesis (chapter IV of Janssen discusses the semantics of programming languages, and, more in particular, of the fairly familiar assignment statements. Assignment statements are statements declaring what the values of certain cells in a computer are in certain computer states (after, or before, certain changes of states). Variables (also called 'identifiers') are used to refer to such cells. Thus, for instance, statements of the form $d := d' + 3$ can be used as instructions to change the value of identifier d to what is the current value of identifier d' plus 3, and $d := d + 1$ as instructions to change the value of identifier d to what is the current value of d plus 1 (i.e., as instructions to increase the value of d with 1). Janssen, relying on joint work with Peter van Emde Boas, argues that for a generally adequate treatment of assignment statements some kind of an intensional analysis is called for.

In order to deal with assignment statements, Janssen employs a version of Montague's framework of *IL* (intensional logic). In Janssen's proposal, *IL*'s set of possible worlds must be understood as a set of possible (internal) machine states, so that modal operators can be understood as describing possible shifts of states. A characteristic modal operator employed by Janssen is the state switcher $\langle\alpha/d\rangle$. Such a state switcher changes the current state of evaluation into one in which identifier d has the value which α has in the present state, and which, otherwise, does not differ from the present state. Thus, a formula $\langle z/d\rangle\phi$ involves the evaluation of ϕ in a state in which the value of d is equal to what the current assignment assigns to variable z (e.g., $d+1$). Of course it must be made sure that states of the sketched kind exist, and for this reason Janssen adds three postulates ensuring that the space of possible states has the required structure. Since the information dealt with basically concerns the possible values stored in the cells which are named by identifiers, Janssen's postulates effectively make the set of states behave like the set of identifier assignments.

Of real interest to our purposes is the interaction between Janssen's state switcher and the $^\cap$ and $^\cup$ operators in Janssen's version of Montague's *IL*. Whereas *IL*'s operator $^\cap$ abstracts over possible worlds of evaluation, in Janssen's version it abstracts over possible machine states, i.e., possible identifier assignments. Likewise, the $^\cup$ operator involves application to the current state, that is, to the current identifier assignment. So, for instance, the interpretation of an expression $^\cap Ud$ is a function h from possible states to truth values, such that $h(s) = 1$ iff the value of d in s is in the extension of U in s; Furthermore, $\langle x/d\rangle Ud$ is true iff Ud is true in a state only different from the current state in that the value of d equals the current value of x, that is, iff the current value of x is in the extension of U. Interestingly, $(\lambda p\ \langle x/d\rangle{}^\cup p)(^\cap Ud)$ is true in the very same circumstances.

For a proper understanding of the sequel, it is useful to inspect the last example in a little more detail. The expression $(\lambda p\ \langle x/d\rangle{}^\cup p)(^\cap Ud)$ can be legitimately reduced as follows:

(1) $(\lambda p\ \langle x/d\rangle{}^\cup p)(^\cap Ud) = \langle x/d\rangle{}^{\cup\cap} Ud = \langle x/d\rangle Ud = Ux$

The first reduction here constitutes a licensed form of λ-conversion, since the argument $^\cap Ud$ is intensionally closed and it contains no free variables. It is important to realize here that the identifier d does not count as a 'free variable' in $^\cap Ud$. Although Janssen's postulates make identifiers behave like variables, the occurrence of d in $^\cap Ud$ is not a 'free' one, since the $^\cap$ operator abstracts over identifier assignments. The second step in (1) is an ordinary example of $^{\cup\cap}$-elimination. As a result, the formula Ud, and more in particular the identifier d, is evaluated in a state in which d is mapped on the current value of the variable x. Consequently, the original expression turns out equivalent with Ux.

In the example above the sub-expression Ud under $^\cap$ is λ-converted into the scope of a state switcher $\langle x/d\rangle$ which affects the interpretation of d. Because in this context the extension of $^\cap Ud$ is taken, it is the value of d in that context which determines the truth conditions of the whole. The really interesting bit in the example above involves this combined use of intensional application (application to the intensions of argument expressions) and extensional evaluation (putting a $^\cup$ operator in front of intensional variables that are abstracted over). The two types of constructions together function as the glue needed to smoothly transport identifiers from one context to another. Henceforth, we will follow Groenendijk and Stokhof in referring to Janssen's version of *IL* as *DIL*, short for *dynamic intensional logic*, and in using the term 'discourse markers' for Janssen's identifiers.

1.2 Dynamic Binding

Groenendijk and Stokhof have applied Janssen's techniques in the area of natural language semantics, and they have shown that his assignment modalities are a useful device in the formulation of a compositional account of inter-sentential anaphora. Anaphoric relationships between antecedent noun phrases and anaphoric pronouns are problematic for compositional accounts of natural language because the terms involved are generally understood to be co-referential, even though they are not referential. The problem consists in giving a general and separate analysis of clauses containing antecedent terms and of clauses containing anaphoric pronouns, and such that in the conjunction of the two types of clauses, antecedents and pronouns somehow get co-instantiated.

The following example serves as an illustration:

(2) A bear is walking in the forest. He hums.

Under the received analysis, the first sentence involves existential quantification over the set of individuals that are bears that walk in the forest. The problem is how to get from this to an existentially quantified set of bears who walk in the forest and who hum, which is what the two-sentence discourse amounts to. We will assume the reader is familiar with this problem, and with the kind of (representational) solution that has been given in the framework of *DRT* (cf., [8, 9]).

In [5] it is shown that the same anaphoric relationships can also be accounted for in non-representational terms. To this end Groenendijk and Stokhof extend the notion of the meaning of an existentially quantified formula to one which incorporates the formula's potential to affect the interpretation of pronouns to come. In [4] they formulate this account of donkey sentences more generally within a type-theoretical framework, that of *DIL*.

The idea of [4] is, basically, the following. Pronouns are associated with discourse markers, and possible antecedent indefinite noun phrases are associated with existentially quantified state switchers. These existentially quantified state switchers $\exists x\langle x/d\rangle\phi$ are turned into potential binders of 'free' occurrences of the discourse marker d, by a judicious use of Janssen's assignment modalities and intensional application and extensional evaluation.

To see how Groenendijk and Stokhof's notion of 'dynamic binding' works, consider again the example (1), now with an additional existential quantifier:

(3) $(\lambda p\ \exists x\langle x/d\rangle {}^\cup p)({}^\cap Wd)$

Using the same reductions as we used above, this example can be seen to be equivalent to $\exists x\langle x/d\rangle Wd$, which turns out equivalent to $\exists x Wx$. Here we see that the interpretation of the discourse marker d in $^\cap Wd$ effectively co-varies with the possible values of the variable x, which is existentially closed. In what follows we will use a handy abbreviation for such a combined modal existential quantification. For any variable x not free in ϕ:

(4) $\exists d\phi = \exists x\langle x/d\rangle\phi$

The above reduction can now be formulated as: $(\lambda p\ \exists d^{\cup}p)(^{\cap}Wd) = \exists dWd$. Now we see that what appears to act as a variable (the discourse marker d in $^{\cap}Wd$) gets bound by what appears to be a quantifier (the operator $\exists d$), which does not have the variable in its syntactic scope. This is called dynamic binding. More in general, dynamic binding can be found in any configuration of the following form, with A a variable of any type $\langle s, a\rangle$, and α an expression of type a:

(5) $(\lambda A\ \ldots\ \exists d(\ldots\ ^{\cup}A\ldots)\ \ldots)(^{\cap}\alpha)$

If d occurs in α in a configuration of this form, it eventually is evaluated within the scope of the quantifier $\exists d$, and if no other modal operators interfere, it actually gets bound by it.

As we said, Groenendijk and Stokhof's notion of dynamic binding is applied in an analysis of inter-sentential anaphoric relationships. Sentences with pronouns can be understood as being in the scope of existential quantifiers in preceding sentences, precisely by means of dynamic binding. However, for dynamic binding to apply at the sentential level, it turns out that (natural language) sentences must be interpreted at some type-theoretical level of functions. So, apart from *DIL*'s assignment modalities, the analysis requires some shift of the sentential level from type t to a higher type. In the second section we will show that the independently motivated type-shift of 'value raising' can be used for this purpose. First, however, we illustrate the type of sentence interpretations that Groenendijk and Stokhof themselves employ.

In Groenendijk and Stokhof's dynamic Montague grammar (*DMG*), the dynamic binding technique is applied at the functional level of type $\langle\langle s, t\rangle, t\rangle$. Sentences of natural language are interpreted as functions in this type, which get applied to (the intensions) of discourse continuations. Two examples of such functions are the following expressions (with p a variable of type $\langle s, t\rangle$):

(6) $\lambda p\ \exists d(Bd \wedge Wd \wedge {}^{\cup}p)$

(7) $\lambda p\ Hd \wedge {}^{\cup}p$

These two expressions can be taken to specify the meanings of, e.g., *A bear is walking in the forest*, and *He hums*, respectively. The two functions can be put together by means of (intensional) functional composition, the semantic operation which is actually associated with the syntactic operation of concatenating or 'sequencing' two sentences in *DMG*. This yields, after some reductions:

(8) $\lambda p\ \exists d(Bd \wedge Wd \wedge Hd \wedge {}^{\cup}p)$

In the result the discourse marker d in Hd of example (7) turns out to be bound by the existential quantifier $\exists d$ of expression (6). Thus the whole sequence is associated with the truth conditions of there being a bear in the forest that is walking and that hums. In *DMG*, the existential quantification over walking bears in (6) is 'extended' to existential quantification over humming walking bears by the concatenation with (7). Moreover, because function composition is used, the dynamics is preserved.

1.3 Flexible Scoping

Groenendijk and Stokhof translate natural language sentences into expressions of the type $\langle\langle s, t\rangle, t\rangle$, in order to give a systematic account of the semantic dependencies that may exist between indefinite noun phrases in one sentence and pronouns in another. As has already been remarked, however, dynamic binding can be achieved with functions of any type. So, in principle, a (systematic) translation of the sentences of natural language into expressions of any (functional) type might do the job as well. In the next section we will choose a level of translation of a more involved type than Groenendijk and Stokhof use, because the shift of types needed to reach that level is a more familiar one, known as 'value raising'. As we will see, a system of type-shifting rules like that proposed in [6] gives us the tools needed for the formulation of a semantic fragment in *DIL* which deals with intersentential anaphora.

In the first chapter of his thesis, Hendriks shows that adoption of flexible type assignment leads to an elegant account of natural language scope ambiguities which arise in the presence of quantifying and coordinating expressions. The type of ambiguities dealt with can be illustrated by means of the following two examples

(9) A computer adorns every desk.

(10) Coco wants to look like an Eskimo.

Both examples can be read in two ways. On the first and least likely reading of example (9) there is a computer which adorns every desk; on the other reading, every desk is equipped with a computer, most plausibly a different computer on each desk. According to one reading of example (10) Coco doesn't mind which (type of) Eskimo to look like; the other reading entails the existence of some specific Eskimo, say Kusugak, which Coco wants to resemble. The ambiguity in both examples is usually analyzed as a ambiguity of scope.

In order to deal with scope ambiguities Hendriks elaborates upon work by, among others, Barbara Partee and Mats Rooth. He gives a formal specification of a type-shifting module which is added to a Montagovian fragment of natural language. Hendriks' system of type shifts consists of three type shifting rules, value raising, [VR], argument raising, [AR], and argument lowering, [AL]. These rules can be applied freely in the process of interpretation, and the various ways in which the rules can be applied correspond to the various possible readings that certain sentences may have. We note here that last rule [AL] is of no concern to us in the present paper, and that we will employ another rule called (generalized) division, [GD].

Hendriks' 'flexible' Montague grammar (*FMG*) is built upon a basic fragment of natural language which is simple in the sense that lexical terms are associated with the most simple 'basic' translations. Proper names are associated with basic translations of the type of individuals, and they are assumed to denote the individuals named. Ordinary (in-)transitive verbs are associated with functions which apply to individuals, too. Thus, a sentence like (11):

(11) Jordi adorns Cruyff.

is associated with a basic translation $adorn'(cruyff')(jordi')$, which is true iff the pair of individuals denoted by $jordi'$ and $cruyff'$ are in the extension of $adorn'$. Clearly, such a translation of *adorn* is not appropriate if it has to be applied to 'quantifying arguments', the translations of quantified noun phrases. In such a case, the rule of argument raising [AR] has to be applied to the translation of the transitive verb first.

Consider again example (9) in which the subject and the object are quantified noun phrases. For this reason both of the verbs' arguments have to be raised, by first argument raising, [1AR], and second argument raising, [2AR]. It so happens that the noun phrase which instantiates the argument which is raised last gets widest scope. Thus Hendriks' system obtains two readings for example (9):

(12) $([2AR][1AR](adorn'))(every\_desk')(a\_computer') =$
$a\_computer'(\lambda x\ every\_desk'(\lambda y\ adorn'(y)(x)))$

(13) $([1AR][2AR](adorn'))(every\_desk')(a\_computer') =$
$every\_desk'(\lambda y\ a\_computer'(\lambda x\ adorn'(y)(x)))$

Clearly, these are the two readings informally sketched above. This example then serves as an illustration of how the rule of argument raising makes functions with arguments of type a applicable to arguments of the type of quantifiers over objects of type a.

The rule of value raising can be used to lift expressions with values of some type a into expressions with values of the type of quantifiers over objects of type a. Typically, the values of the raised terms are the sets of properties of the values of the original terms. Now, by raising the value of a functor, its (quantifying) arguments can be given a 'view' over functors higher up in the construction tree of which they are part. In this way one can assign the noun phrase *an eskimo* in example (10) (semantic) scope over the verb *wants*, even though this verb has the noun phrase in its syntactic scope. The result of this is the second reading (15) of example (10), while the first reading (14) is obtained in the most simple way using argument raising only:

(14) $want'(([1AR](look\_like'))(an\_eskimo'))(coco') =$
$want'(\lambda x\ an\_eskimo'(\lambda y\ look\_like'(y)(x)))(coco')$

(15) $([1AR](want'))(([1AR][2VR](look\_like'))(an\_eskimo'))(coco') =$
$an\_eskimo'(\lambda y\ want'(\lambda x\ look\_like'(y)(x))(coco'))$

In both examples the first argument of (the translation of) the verb *look like* has to be raised to accommodate the quantifying noun phrase *an eskimo*. But in (15) this is done after raising the relevant second value of the verb by [VR]. The whole phrase *look_like an eskimo* is thus made into a quantifying daughter of the verb *want*, which has to be accommodated by yet another application of [AR]. As a result, the noun phrase *an eskimo* is assigned wide scope.

Although the combined application of the rules of value and argument raising suffices to make quantifying arguments climb up through application trees in all possible ways, the rules accept only a limited set of functions or operators as scope bearing expressions. For instance, the rules are not fit to deal with the flexible scope of sentential operators, which in [12] and [15] are acknowledged as scope bearing operators, too. For a generalization of the flexible typing system [1] has proposed an additional rule of division, which constitutes a generalization of the so-called Geach rule. By means of this rule of division, a function which takes arguments of type b into a type c can be made applicable to functions which take arguments from some type a into b. The divided function inherits the additional argument of type a. Neglecting aspects of intensionality, such a function χ of type $\langle b, c\rangle$ can be divided into the function $\lambda V_{\langle a,b\rangle} \lambda y_a\ \chi(V(y))$ of type $\langle\langle a, b\rangle, \langle a, c\rangle\rangle$. As a matter of fact, this division rule constitutes the unary encoding of the more well-known binary operation of (intensional) functional composition. For, schematically:

(16) $([\mathrm{GD}](\beta))(^\cap\alpha) = \lambda y\ \beta(^\cap\alpha(y)) = \beta \circ \alpha$

This concludes our informal exposition of the techniques used in the sequel of this paper. Formal definitions can be found in the appendix.

2. Donkey-Anaphora

In this section we want to show how the combined use of the techniques of Janssen, Groenendijk and Stokhof, and Hendriks enables a straightforward treatment of inter-sentential anaphoric relationships. This account is formulated in terms of a predicate logical language Σ, the semantics of which is spelled out in *DIL*. The atomic formulas of Σ and Σ's operators can be seen to be derived from *DIL*-formulas and *DIL*-operators by means of Hendriks-style type-shifting techniques. The semantic import of anaphoric relationships is accounted for using dynamic binding. The directedness of anaphoric binding is accounted for using a (derived) notion of conjunction obtained by raising the leftmost and dividing the rightmost argument of a propositional operation of conjunction. We will first present the language and its semantics, and then show how it applies to donkey anaphora.

The syntax of Σ is specified as follows (*DM* is *DIL*'s set of discourse markers):

Definition 1 (Σ Syntax) The set of well-formed Σ-formulas is the smallest set $\mathcal{WF}_\Sigma$ such that if $\phi \in \mathcal{WF}_{DIL}$, then $\uparrow\phi \in \mathcal{WF}_\Sigma$ and if Φ, $\Psi \in \mathcal{WF}_\Sigma$ and $d \in DM$, then $-\Phi$, $\mathcal{E}d\Phi$, $[\Phi\ ;\ \Psi] \in \mathcal{WF}_\Sigma$; $Ad\Phi = -\mathcal{E}d{-}\Phi$, $[\Phi \Rightarrow \Psi] = -[\Phi\ ;\ -\Psi]$, and $[\Phi \sqcup \Psi] = [-\Phi \Rightarrow \Psi]$.

Σ formulas are built up from the lift $\uparrow\phi$ of ordinary *DIL*-formulas ϕ using $-$ (Σ-negation), $\mathcal{E}d$ (existential Σ-quantification) and ; (Σ-conjunction). Notions of universal Σ-quantification (Ad), Σ-implication ($\Rightarrow$), and Σ-disjunction ($\sqcup$) are defined in terms of Σ-negation, existential Σ-quantification and Σ-conjunction in the usual way. The Σ-language can be made into a typed system with abstraction and application, if we adopt a suitably generalized notion of $\uparrow$ (cf., [1]). However, for the purposes of this paper it suffices to stick to the predicate logical part. In what follows, we will use capital Greek letters (Φ, Ψ, Υ) to refer to Σ-formulas, and lower case ones (ϕ, ψ, χ) to refer to *DIL*-formulas.

Σ-formulas are interpreted in *DIL*-models, and their meanings are specified using *DIL*-expressions. In what follows intensional types $\langle s, a\rangle$ are abbreviated as ${}^\star a$, and for any type a and natural number n we use a^n to indicate the type such that $a^0 = a$ and $a^{n+1} = \langle {}^\star a^n, t\rangle$. In the following definition p is a variable of type ${}^\star t$, and R a variable of type ${}^\star t^1$ $(= {}^\star\langle {}^\star t, t\rangle)$. Throughout we assume that variables which are used in the definition of the meaning of an expression Φ, do not themselves occur free in Φ.

Definition 2 (Σ Semantics) Well-formed Σ-formulas are interpreted in the type $t^2 = \langle {}^\star\langle {}^\star t, t\rangle, t\rangle$ in the following way:

$$\begin{array}{ll}
\uparrow\phi & = \lambda R\ {}^\cup R({}^\cap\phi) \\
-\Phi & = \lambda R\ {}^\cup R({}^\cap\neg\Phi({}^\cap\lambda p\ {}^\cup p)) \\
\mathcal{E}d\Phi & = \lambda R\ \exists d\Phi(R) \\
[\Phi\ ;\ \Psi] & = \lambda R\ \Phi({}^\cap\lambda p\ ({}^\cup p \wedge \Psi(R)))
\end{array}$$

The truth-conditional content $\downarrow\Phi$ of a Σ-formula Φ is $\Phi({}^\cap\lambda p\ {}^\cup p)$.

We will first comment upon the four clauses above before we discuss a couple of examples. First observe that $\uparrow\phi$ constitutes an example of value raising (the type-shifting rules referred to here are defined in the appendix):

(17) $\uparrow\phi = [1\mathrm{VR}](\phi)$

By the operation of lifting, ϕ is raised to the higher order level at which Σ's dynamics is defined, but its truth-conditional content is preserved. We can recover these truth conditions by ascribing $\uparrow\phi$ the property of being a true proposition, i.e., the property ${}^\cap\lambda p\ {}^\cup p$. An ordinary proposition has this property iff the proposition is true, and the same holds for our higher-order propositions:

(18) $\downarrow\uparrow\phi = (\lambda R\ {}^\cup R({}^\cap\phi))({}^\cap\lambda p\ {}^\cup p) = \phi$

The Σ-negation $-\Phi$ of Φ is defined as the lift of the negation of the truth-conditional content of Φ, that is:

(19) $-\Phi = \uparrow\neg\downarrow\Phi$

At the right-hand side of this (valid) equivalence we find three operations at work. The closure $\downarrow\Phi$ gives us the truth-conditional content of Φ, which is negated in $\neg\downarrow\Phi$. The result of this is raised again to type t^2 by means of the lifting operation $\uparrow$. So, although $-$ is defined at the higher-order level of dynamic types, truth-conditionally it amounts to static negation. As a consequence, the truth conditions of $-\Phi$ $(\downarrow-\Phi)$ equal the falsity conditions of Φ $(\neg\downarrow\Phi)$:

(20) $\downarrow - \Phi = \neg \downarrow \Phi$

Σ's t^2-typed existential quantification amounts to function composition. The interpretation of $\mathcal{E}d\Phi$ can be obtained by first dividing the *DIL* operation of existential quantification $\lambda p\ \exists d^{\cup} p$ to the type ${}^{\star}t^1$ and next applying it to the intension of Φ:

(21) $\mathcal{E}d\Phi = ([1\mathrm{GD}_{\star t^1}](\lambda p\ \exists d^{\cup} p))(^{\cap}\Phi)$

As we will see below, a t^2-typed existential quantifier has extended binding potential, and may bind 'free' discourse markers (pronouns) beyond its syntactic scope. However, apart from that, its truth-conditional contribution equals that of its static counterpart:

(22) $\downarrow \mathcal{E}d\Phi = \exists d \downarrow \Phi$

The Σ definition of conjunction can be derived from a propositional operation of conjunction $\lambda p \lambda q\ {}^{\cup}p \wedge {}^{\cup}q$ by dividing the second argument and raising the first:

(23) $[\Phi\ ;\Psi] = ([1\mathrm{AR}][2\mathrm{GD}_{\star t^1}](\lambda p \lambda q\ {}^{\cup}p \wedge {}^{\cup}q))(^{\cap}\Phi)(^{\cap}\Psi)$

The truth conditions of a dynamic conjunction cannot in general be stated in terms of those of its constituents and ordinary conjunction $\wedge$. Only in case the first formula is (reducible to a formula) of the form $\uparrow\phi$, such a reduction is possible:

(24) $\downarrow[\uparrow\phi\ ;\Psi] = (\phi \wedge \downarrow\Psi)$
$\downarrow[-\phi\ ;\Psi] = \downarrow[\uparrow\neg\downarrow\phi\ ;\Psi] = (\neg\downarrow\phi \wedge \downarrow\Psi)$

In the other cases (that is, in case the first conjunct of a conjunction is of the form $\mathcal{E}d\Phi$ or $[\Phi\ ;\Psi]$), the whole conjunction has to be reformulated before a proper reduction is possible. Before we can state the truth conditions of such conjunctions in terms of those of their constituent expressions and $\wedge$, we have to put them into a normal form using the following equivalences:

(25) $[\mathcal{E}d\Phi\ ;\Psi] = \mathcal{E}d[\Phi\ ;\Psi]$
$[[\Phi\ ;\Psi]\ ;\Upsilon] = [\Phi\ ;[\Psi\ ;\Upsilon]]$

From the definition of Σ-conjunction $[\Phi\ ;\Psi]$ as $\lambda R\ \Phi(^{\cap}\lambda p\ ({}^{\cup}p \wedge \Psi(R)))$ it can be seen the first conjunct Φ is assigned scope over the second Ψ. Thus, existential quantifiers in the first conjunct Φ come to range over free discourse markers in the second conjunct Ψ, and by *DIL*'s dynamics, they also bind them.

Observation (2) in the appendix tells us that the truth conditions of any Σ-formula can be determined using the equations in the observations (18), (20), (22), (24), and (25). Together, these equations show that the truth-conditional impact of the dynamic operators $-$, $\mathcal{E}d$ and ; really corresponds to that of their static counterparts $\neg$, $\exists d$ and $\wedge$, and that Σ *only* differs from an ordinary first-order system in the dynamic behaviour of the existential quantifier. End of comments.

We will show how the Σ system applies to the type of sentences which constituted (part of) the motivation for the development of *DRT*. Consider the following little discourse, followed by its 'standard' textbook translation in Σ:

(26) A bear is walking through the forest. He hums.
He is going to his favourite bee-tree.
$[[\mathcal{E}d[\uparrow Bd\ ;\uparrow Wd]\ ;\uparrow Hd]\ ;\uparrow Gdt]$

In this formula Bd, Wd, Hd, Gdt stand for the *DIL*-formulas representing that d is a bear, that d walks through the forest, that d hums, and that d goes to his favourite bee-tree t, respectively. Using the equations of (25), this formula can be rewritten as follows:

(27) $\mathcal{E}d[\uparrow Bd\,;[\uparrow Wd\,;[\uparrow Hd\,;\uparrow Gdt]]]$

This Σ-formula is in normal binding form, and its truth conditions can be determined using the reduction rules in (22), (24) and (18):

(28) $\exists d(Bd \wedge (Wd \wedge (Hd \wedge Gdt))) =$
$\exists x(Bx \wedge (Wx \wedge (Hx \wedge Gxt)))$

Thus, the little discourse (26) turns out to be true iff there is an individual that is a bear, that walks in the forest, that hums, and that goes to his favourite bee-tree t. Notice that although the three sentences in (26) are each assigned an independent interpretation, still the various occurrences of d are all co-instantiated in the interpretation of the whole.

By the definition of Ad and $\Rightarrow$ in terms of $\mathcal{E}d$, $-$ and ;, the following equivalences follow from those in (25):

(29) $[\mathcal{E}d\Phi \Rightarrow \Psi] \quad = Ad[\Phi \Rightarrow \Psi]$
$[[\Phi\,;\Psi] \Rightarrow \Upsilon] = [\Phi \Rightarrow [\Psi \Rightarrow \Upsilon]]$

We find that Σ notion of implication is a strong one (like those of *DPL*, *DMG*, and related systems). For instance, the so-called donkey sentence (30), with associated translation in Σ, has the truth conditions specified in (31):

(30) If a farmer owns a donkey he beats it.
$[\mathcal{E}d[\uparrow Fd\,;\mathcal{E}d'[\uparrow Dd'\,;\uparrow Odd']] \Rightarrow \uparrow Bdd']$

(31) $\forall x(Fx \rightarrow \forall y(Dy \rightarrow (Oxy \rightarrow Bxy)))$

For (30) to be true it is required that *every* farmer beats *every* donkey he owns. Notice that such strong reading of conditional sentences do not result from ad hoc stipulation, but, rather, from the dynamic behaviour of indefinite noun phrases (existential quantifiers) in implications, which are defined along standard standard lines in terms of $-$ and ;.

To conclude the introductory part of this paper, let us sum up what we have done. We have formulated the semantics of a small system Σ which deals with inter-sentential anaphora drawing from Janssen's assignment modalities in *DIL*, employing Groenendijk and Stokhof's technique of dynamic binding, and using Hendriks-style type-shifting techniques. In *DIL* an existential quantifier can be defined as a *modal* operator $\lambda p\ \exists d^{\cup}p$, which can figure as a dynamic binder in the sense of Groenendijk and Stokhof. Employing Hendriks' techniques of lifting, *DIL*-formulas can next lifted to a level of interpretation where the dynamics of binding is accounted for in a systematic way. Appropriate higher-order notions of negation, existential quantification and conjunction also can be derived from their lower-order counterparts, using the rules of [VR], [AR], [GD] and $\downarrow$ (that is: application to the identity function).

3. Telescoping

In the system of Σ, like in that of *DPL* and *DMG*, there are three basic operators two of which are properly called dynamic. The dynamics of Σ's existential quantifier is clearly brought out in the general equivalence of $[\mathcal{E}d\Phi\,;\Psi]$ and $\mathcal{E}d[\Phi\,;\Psi]$. Σ's notion of conjunction can be called dynamic, since conjunctions inherit the dynamics of their conjuncts. This leaves us with negation as the only non-dynamic (basic) operator of Σ. Thus the question pops up what a system of interpretation would or should look like in which all three operators are dynamic. As we will see in this section, this question is not only interesting from a purely theoretical perspective, but of practical interest, too.

Groenendijk and Stokhof have already a notion of negation as complementation, under which dynamic effects of negated formulas are preserved. A notion of negation as complementation of course satisfies the law of double negation, something which, in Groenendijk and Stokhof's *DMG*, means that the double negation of a formula has the same dynamics as the formula itself. Interestingly, for this phenomenon motivation has been found in natural language. Consider:

(32) It is not the case that Archie doesn't own a car.
It is a 2CV and it's parked in front of the house.

The first sentence of (32) can be conceived of as the double negation of the sentence *Archie owns a car*, and not only do these two sentences have the same truth conditions, but they also have the same dynamic impact. For both *Archie owns a car* and *It is not the case that Archie doesn't own a car* present a 'live' antecedent for the pronouns in the second sentence *It is a 2CV and it's parked in front of the house*. Obviously, this effect can only be properly accounted for if the dynamic impact of a formula is somehow preserved under negation. (That is, as long as the expressions of our language are associated with singular objects and operations. For an alternative, cf., the double-barrelled semantics of [10].)

Groenendijk and Stokhof observe that in a system in which all three basic operations are dynamic, also the derived notions of universal quantification, implication and disjunction are dynamic. As we will see later on in this section natural language motivation has been found for a dynamic conception of these operations, too. So, it seems, there is good motivation for a notion of dynamic negation. Yet there is a substantial problem with Groenendijk and Stokhof's notion of negation. If a formula Φ in a conjunction $[\sim\Phi\,;\Psi]$ is able to affect the context of Ψ, the last formula must somehow be evaluated in the scope of $\sim$. But in such a case, evidently, we should not want the truth-conditional content of Ψ to contribute to the content of what is negated with $\sim\Phi$. Still, as Groenendijk and Stokhof themselves observe, this is precisely what happens in their system.

In [1] I have shown that a dynamic notion of negation which has the wanted properties and lacks the unwanted ones, cannot be defined as long as we are dealing with the dynamics of sentences of natural language in the type t^1 used in *DMG*. For a proper definition of such a notion we need to resort to the higher type t^2 employed in Σ. In the remainder of this section I will present this higher-order definition of dynamic negation, and show how it applies to the examples which are and those which are not dealt with in Groenendijk and Stokhof's system.

We now turn to this Σ system extended with a dynamic notion of negation. We will distinguish the dynamic negation $\sim$ from the static one $-$ used above, and the system which is obtained from Σ by substituting $\sim$ for $-$ is referred to as Σ^τ. Its syntax is given as follows:

Definition 3 (Σ^τ Syntax) The set of well-formed Σ^τ-formulas $\mathcal{WF}_{\Sigma^\tau}$ is that of $\mathcal{WF}_\Sigma$ with $-$ replaced by $\sim$; $\mathcal{A}d\Phi = \sim\mathcal{E}d\sim\Phi$, $[\Phi \gg \Psi] = \sim[\Phi\,;\sim\Psi]$, and $[\Phi \amalg \Psi] = [\sim\Phi \gg \Psi]$.

In order to see what our dynamic negation must achieve, we have to focus on conjunctions of the form $[\sim\Phi\,;\Psi]$. In order for Φ in such a conjunction to dynamically affect the evaluation of Ψ, Ψ must somehow be evaluated in the scope of Φ. Furthermore, for the dynamic negation of Φ to involve a genuine negation of Φ's contents, the definition of $\sim$ must somehow bring Φ in the scope of an ordinary negation $\neg$. We have seen above that a Σ-conjunction $[\Phi\ ;\ \Psi]$ involves ascribing Φ the property of propositions ${}^\cap\lambda p\ ({}^\cup p \wedge \Psi(R))$, which is that of being true in conjunction with $\Psi(R)$. So, for a conjunction $[\sim\Phi\ ;\ \Psi]$ we find that Φ must be *denied* some property of propositions, which is somehow derived from the property of being true in conjunction with $\Psi(R)$. In order to determine *which* property Φ in such a case is denied to have, we can take a clue from the classical case.

What property of propositions is the proposition ϕ denied to have when its classical negation is conjoined with a proposition ψ? Put formally, for which term Q of type ${}^\star t^1$ do we find that $(\neg\phi \wedge \psi) = \neg({}^\cup Q)({}^\cap\phi)$? This equation can be solved by equating Q with the *dual* ${}^\cap\lambda p \neg(\neg{}^\cup p \wedge \psi)$ of the property of being true in conjunction with ψ, since:

(33) $(\neg\phi \wedge \psi) = \neg({}^\cup({}^\cap\lambda p \neg(\neg{}^\cup p \wedge \psi)))({}^\cap\phi)$

To this type t equivalence corresponds the following type t^2 one:

(34) $[\sim\Phi\ ;\ \Psi] = \lambda R\ \neg\Phi({}^\cap\lambda p \neg(\neg{}^\cup p \wedge \Psi(R)))$

The last, wanted, equivalence now can be validated by defining $\sim\Phi$ as that set of properties of propositions the *duals* of which are *not* in Φ:

Definition 4 (Σ^τ Semantics) $\sim\Phi = \lambda R\ \neg\Phi({}^\cap\lambda p\ \neg{}^\cup R({}^\cap\neg{}^\cup p))$

This definition of the negation $\sim\Phi$ of Φ involves a negation $\neg\Phi(\ldots)$ of Φ, which preserves a landing site R in $(\ldots)$ for further expansions in the scope of Φ. However, upon the present definition, such expansions do not strengthen the proposition that is dynamically negated, but they weaken it. Within the negative scope of $\neg\Phi$, these expansions occur under an additional negation, i.e., as $\neg{}^\cup R(\ldots)$. Thus, (upward entailing) monotonicity properties of this argument are preserved. A third negation $\neg{}^\cup p$ finally preserves the (downward entailing) monotonicity properties of the embedded propositional argument p of R, which is supplied inside Φ, and which contributes to the contents of this negated formula. Thus, the expansion variable R finds itself strictly enclosed by two negations, and, consequently, the contents of the negated formula and of further expansions are treated separately.

Now we have defined a notion of dynamic negation we may ask ourselves whether it makes much sense. The following observations are intended to show that it does, indeed. First, observe that if R is trivial (that is, if it is the identity function ${}^\cap\lambda p\ {}^\cup p$), the two embedded negations in definition (4) collapse. In other words, the content of $\sim\Phi$ equals the negation of the content of Φ:

(35) $\downarrow\sim\Phi = \neg\Phi({}^\cap\lambda p\ \neg\neg{}^\cup p) = \neg\downarrow\Phi$

Second, observe that $\sim$ behaves as a classical negation when applied to Σ^τ's atomic formulas, the lifts of *DIL*-formulas:

(36) $\sim\uparrow\phi = \lambda R\ \neg\neg{}^\cup R({}^\cap\neg\phi) = \uparrow\neg\phi$

The dynamic negation of the mere lift of a *DIL*-formula ϕ to the level of dynamic types equals the lift of the static negation of ϕ. Since atomic Σ^τ-formulas have no dynamic impact, their dynamic negation amounts to negation of truth-conditional content only. Our notion of dynamic negation also obeys the law of double negation:

(37) $\sim\sim\Phi = \lambda R\ \neg\neg\Phi({}^\cap\lambda p\ \neg\neg{}^\cup R({}^\cap\neg\neg{}^\cup p)) = \Phi$

From this equation we may conclude that Σ^{τ} can deal with example (32) above.

The present observations show that the Σ^{τ} notion of negation is dynamic (it satisfies double negation) and that it behaves as a proper negation at the truth-conditional level. It also interacts in a classical way with the other operators:

(38) $\sim\mathcal{E}d\Phi = \mathcal{A}d\sim\Phi \qquad \sim[\Phi\,;\Psi] = [\Phi \gg \sim\Psi] \qquad \sim[\Phi \amalg \Psi] = [\sim\Phi\,;\sim\Psi]$
$\sim\mathcal{A}d\Phi = \mathcal{E}d\sim\Phi \qquad \sim[\Phi \gg \Psi] = [\Phi\,;\sim\Psi]$

Since our basic operators $\sim$, $\mathcal{E}d$, and ; are all dynamic now, and since $\mathcal{A}$, $\gg$ and $\amalg$ are defined in terms of these three, $\mathcal{A}$, $\gg$ and $\amalg$ are dynamic, too. The following equations bring this to light:

(39) $[\mathcal{A}d\Phi\,;\Psi] = \mathcal{A}d[\Phi\,;\Psi]$
$[[\Phi \gg \Psi]\,;\Upsilon] = [\Phi \gg [\Psi\,;\Upsilon]]$
$[[\Phi \amalg \Psi]\,;\Upsilon] = [\Phi \amalg [\Psi\,;\Upsilon]]$

These equations clearly display the 'telescoping' impact of our negation. If the first conjunct of a conjunction is formed with $\mathcal{A}d$, $\gg$, or $\amalg$, this operator takes scope over the second conjunct.

The equations in (39) correspond to a kind of telescoping which has been observed in natural language. The following examples have been discussed in the work of Gareth Evans, Lauri Karttunen, Barbara Partee, Craige Roberts, and Peter Sells, to name a few.

(40) Every chess-set comes with a spare pawn. It is taped to the top of the box.

(41) If a client comes in, you treat him politely. You first offer him a cup of coffee, and then ask for the reason of his visit.

(42) Either there is no bathroom in this house, or it is in a funny place. In any case it is not on the ground floor.

The main operator in the first sentences of these examples must be understood dynamically, because it takes scopes over the second sentence with which the first is conjoined. So, for instance, example (40) must be understood as saying that every chess-set comes with a spare pawn which is taped to the top of the box, and this is the reading obtained in Σ^{τ}. (The first sentence of example chess-set would be translated as $\mathcal{A}d[\uparrow Cd \gg \mathcal{E}d'[\uparrow Sd'\,;\uparrow Cdd']]$, and thus invokes both the first and the second equation of (39).) In example (41) the second sentence must be understood as being dependent upon the *if*-clause of the first. The example is, thus, taken to mean that if a client comes in, then you (i) treat him politely, (ii) offer him a cup of coffee, and (iii) ask for the reason of his visit. With our strong implication, this amounts to requiring you to give this polite treatment to *every* customer. Example (42), finally, can be reformulated as (43), which, in its turn, is Σ^{τ}-equivalent to (44):

(43) Either there is no bathroom in this house, or it is in a funny place and not on the ground floor.

(44) If there is a bathroom in this house, it is in a funny place and not on the ground floor.

The reading of example (42) obtained in Σ^{τ} is that if there are any bathrooms in the house at all, then there are none on the ground floor, but then they (all) are located in a funny place. It may be worthwhile to notice that the plausible reading (44) of the puzzling bathroom disjunction (42) is obtained by employing a dynamic notion of negation in the analysis of a disjunction $[\Phi \amalg \Psi]$ as the dual of the conjunction (that is, as $[\sim\Phi \gg \Psi] = \sim[\sim\Phi\,;\sim\Psi]$). Thus, we can agree with [16, p. 243] that a viable analysis of this example requires an independent exploration of the semantics (and pragmatics) of disjunctive assertions. If anything is found on such an exploration, it must at least be that the (classical) disjunction is the dual of the (classical) conjunction.

Sofar we have sketched how Σ^τ deals with examples which are also properly dealt with using Groenendijk and Stokhof's dynamic negation. Let us now turn to some examples which fall beyond the scope of Groenendijk and Stokhof's system, examples involving only a single negation:

(45) The salesman doesn't leave a client waiting. He sends him up to me as soon as he has determined the reason of his visit.

(46) No computer leaves this building with a Zonnebloem-chip. It is removed beforehand.

On Groenendijk and Stokhof's account of dynamic negation $[\sim\Phi\,;\Psi]$ equals $\sim[\Phi\,;\Psi]$. So, using their notion, example (45) is taken to mean that the salesman does not both leave a client waiting and send him up to me after determining the reason of his visit. This non-reading can be satisfied if lots of clients are left waiting, as long as they are *not* send up to me when the reasons of their visits have been determined. Upon our account, *not a client* is equivalent to *every client not*. Thus, example (45) turns out to be satisfied, correctly, iff, for every client c, (i) the salesman does not leave c waiting, and (ii) the salesman sends c up to me as soon as he has determined the reason of c's visit.

In a similar way the first sentence of example (46) can be seen to be fully equivalent with the sentence *Every computer does not leave the building with a Zonnebloem-chip*. The whole example is taken to mean that every computer has all of its Zonnebloem-chips removed before it leaves the building, apparently, the most obvious reading of the example. A nice pair of examples, finally, is the following:

(47) Either there is a bathroom downstairs, or it is upstairs.

(48) If there is no bathroom downstairs, then it is upstairs.

The two examples are equivalent in Σ^τ, as they are upon their most intuitive interpretation. Both correctly entail the existence of a bathroom which is either downstairs or upstairs. (We note that the examples would also entail the existence of a bathroom upon Groenendijk and Stokhof's account, although that account would wrongly require it to be a bathroom which is downstairs and not upstairs.)

The examples above provide empirical motivation for our dynamic interpretation of $\sim$, $\mathcal{A}d$, $\gg$, and $\amalg$. Not only does our notion of dynamic negation do well with respect to the examples (40)–(44) which Groenendijk and Stokhof's system can handle, but also with the other examples (45)–(48), all of this in a uniform manner. By a suitable adjustment of Groenendijk and Stokhof's notion of dynamic negation, severe counterexamples to their 'insertion'-style treatment of telescoping can be dealt with properly and systematically. (For an alternative, representational, treatment of telescoping by accommodation the reader is referred to [14, 13].)

Obviously, the examples which we have discussed here are in a certain sense special. The most standard interpretation of the respective operators is evidently the more static one which we presented within the Σ framework. For this reason it may be useful to point out that the more static Σ operations can be derived from the corresponding Σ^τ operations using the closure operator:

(49)
$$\begin{aligned} -\Phi &= \uparrow\downarrow\sim\Phi \\ Ad\Phi &= \uparrow\downarrow\mathcal{A}d\Phi \\ [\Phi \Rightarrow \Psi] &= \uparrow\downarrow[\Phi \gg \Psi] \end{aligned}$$

Both the really dynamic as well as the more standard interpretation of the operators can be seen to be present in Σ^τ.

We may also observe that the Σ^τ operations $\sim$, $\mathcal{A}d$, $\gg$, and $\amalg$ are themselves derivable from corresponding *DIL* operations with (variants of) Hendriks-style type-shifting rules, which employ a notion of a 'generalized dual'. (Cf. [1, pp. 98ff] for the required definitions and motivation.) That is, we find that:

(50) $\sim\Phi \quad = ([\text{1dGD}_{\star t^1}](\lambda p\ \neg{}^{\cup}p))({}^{\cap}\Phi)$
$\mathcal{A}d\Phi \quad = ([\text{1GD}_{\star t^1}](\lambda p\ \forall d^{\cup}p))({}^{\cap}\Phi)$
$[\Phi \gg \Psi] = ([\text{1dAR}][\text{2GD}_{\star t^1}](\lambda p\lambda q\ {}^{\cup}p \rightarrow {}^{\cup}q))({}^{\cap}\Phi)({}^{\cap}\Psi)$
$[\Phi \amalg \Psi] \ = ([\text{1AR}][\text{2GD}_{\star t^1}](\lambda p\lambda q\ {}^{\cup}p \vee {}^{\cup}q))({}^{\cap}\Phi)({}^{\cap}\Psi)$

where a 'd' in the label [dAR] or [dGD] of a type shift indicates that a dual is taken of the raised or inherited argument. The results of the present section can thus be seen to be obtainable by adding suitably adapted type shifts to a framework in which assignment modalities can be used to model dynamic binding.

To conclude this section we have to point at two types of predictions which Σ^{τ} makes which do not seem to be motivated by linguistic facts. For, as a kind of counterpart of the donkey corollaries displayed under (29), we also find the following ones:

(51) $[\mathcal{A}d\Phi \gg \Psi] = \mathcal{E}d[\Phi \gg \Psi]$
$[[\Phi \gg \Psi] \gg \Upsilon] = [\Phi\ ;[\Psi \gg \Upsilon]]$

In [2, pp. 275–6] it has already been observed that such equivalences follow from extended binding phenomena. They fall out as necessary consequences of (i) the adoption of a (motivated) notion of dynamic negation, and (ii) the use of the standard definitions of universal quantification and implication in terms of existential quantification, conjunction and negation. The conclusion may have to be that these standard definitions eventually have to be given up.

For instance, compare our example (48) above with example (52):

((48)) If there is no bathroom downstairs, then it is upstairs.

(52) If every bathroom is not downstairs, then it is upstairs.

Upon the (fairly standard) equivalence of $\sim\mathcal{E}d[\Phi\ ;\Psi]$ and $\mathcal{A}d[\Phi \gg \sim\Psi]$, these two sentences are predicted to be fully equivalent. However, apparently they are not. Example (48) seems to entail the existence of a bathroom which the pronoun *it* refers to, and which, if it is not downstairs, is upstairs. Contrariwise, example (52) doesn't seem to entail anything about the existence of bathrooms. As a matter of fact, it appears to be impossible to establish any anaphoric relationship in example (52) at all. Although we have found a negation which figures as the driving force behind the extended binding phenomena discussed in this chapter, it appears that the dynamics of the derived operators of universal quantification, implication and disjunction does not so rigidly follow from that of Σ^{τ}'s negation as these standard equivalence schemes suggest. It remains to be seen, of course, how Σ^{τ}'s positive results can be preserved when these equivalence schemes are adjusted.

4. Periscoping

In standard versions of *DRT* as presented in [8, 9] proper names receive a special treatment. Like indefinite noun phrases they may serve as antecedents for subsequent pronouns, but unlike indefinites they *always* allow anaphora, from no matter what position in the structure in which they occur. If proper names occur embedded under a number of operators, and if, consequently, the minimal clause in which they occur is analyzed relative to some sub-*DRS* of the main *DRS* which represents the discourse under analysis, they still make a semantic contribution to the main level of discourse representation. For instance, consider the following conditional sentence in which the proper names *Cees* and *Gert* occur in the antecedent and the consequent, respectively:

(53) If Cees bakes an apple-pie, Gert eats it.

When the *DRS* construction algorithm applies to an example like this in a main *DRS* K, it will involve the introduction of a condition $K' \Rightarrow K''$ in K, and the antecedent *if Cees bakes an apple-pie* will be further analyzed in the sub-*DRS* K', and the consequent *Gert eats it* in the sub-*DRS* K''. Still, unlike the indefinite *an apple-pie*, which triggers the introduction of a discourse referent in its local *DRS* K', the two proper names induce the introduction of two discourse referents in the main *DRS* K. The reason is that, no matter from how deeply embedded a position, proper names may serve as antecedents for pronouns in other positions. Although such proper names actually occur below the main discourse level, they are able to 'see' pronouns at the surface, and take them in their scope, as it were. That's why we will call this phenomenon 'periscoping' here.

In [11] it is observed that compositional semantic systems like *DMG*, or other compositional reformulations of *DRT*, have difficulties with such a non-local treatment of proper names. For, as easy as it may seem to write the semantic contribution of proper names on the top of your metaphorical sheet that constitutes the current discourse representation, just as impossible it may appear to be to model the semantics of this when one is engaged in the local computation of some subformula. Apparently, such local computation is what compositional interpretation seems to amount to.

Clearly, then, a treatment of periscoping may require a general shift in the semantic architecture, but, still, this shift need not be too revolutionary. The aim of this section is to show that a sophisticated, but relatively simple application of Hendriks' type shifting rules suffices to send the contribution of all proper names to the main level, from no matter how deeply embedded a position. This account of periscoping is perfectly general. It can be implemented just as easily in an ordinary static system, as in the dynamic fragment of section two or in that of section three. We also want to note here that the application of our periscoping technique is not restricted to the interpretation of proper names. As is easily seen it is also useful for a treatment of specific indefinites and, for instance, interjections. These phenomena, too, involve global effects of local constructions, which is precisely what we model by means of periscoping.

Our treatment of periscoping is first formulated for a small first order system Σ^π. Its syntax is specified as follows:

Definition 5 (Σ^π Syntax) The set of well-formed Σ^π-formulas is the smallest set $\mathcal{WF}_{\Sigma^\pi}$ such that if $\phi \in \mathcal{WF}_{DIL}$, then $\uparrow\phi \in \mathcal{WF}_{\Sigma^\pi}$ and if $\Phi, \Psi \in \mathcal{WF}_{\Sigma^\pi}$, $d \in DM$ and $\text{PN} \in PN$, then $\text{NOT}\Phi$, $\text{SM}d\Phi$, $[\Phi \text{ AND } \Psi]$, $\text{PN}d\Phi \in \mathcal{WF}_{\Sigma^\pi}$.

The Σ^π-language is also constructed from the lift $\uparrow\phi$ of ordinary *DIL*-formulas, but now we use NOT (Σ^π-negation), SMd (existential Σ^π-quantification) and AND (Σ^π-conjunction). An additional clause deals with proper names, which are associated with discourse markers and behave like quantifiers. (Universal quantification, implication and disjunction can be dealt with in the usual manner.)

Σ^π-formulas, like Σ-formulas, are interpreted in *DIL*-models, and their meanings are specified using *DIL*-expressions, too.

Definition 6 (Σ^π Semantics) Well-formed Σ^π-formulas are interpreted in type ${}^\star t^2 = \langle {}^\star\langle {}^\star t, t\rangle, t\rangle$ in the following way:

$$\begin{array}{ll}
\uparrow\phi & = \lambda R\ {}^\cup R({}^\cap\phi) \\
\text{NOT}\Phi & = \lambda R\ \Phi({}^\cap\lambda p\ {}^\cup R({}^\cap\neg{}^\cup p)) \\
\text{SM}d\Phi & = \lambda R\ \Phi({}^\cap\lambda p\ {}^\cup R({}^\cap\exists d{}^\cup p)) \\
[\Phi \text{ AND } \Psi] & = \lambda R\ \Phi({}^\cap\lambda p\ \Psi({}^\cap\lambda q\ {}^\cup R({}^\cap({}^\cup p \wedge {}^\cup q)))) \\
\text{PN}d\Phi & = \lambda R\ \exists d(d = \text{PN} \wedge \Phi(R))
\end{array}$$

The truth-conditional content $\downarrow\Phi$ of a Σ-formula Φ is defined by $\Phi({}^\cap\lambda p\ {}^\cup p)$

As may appear from this definition, the operators NOT, SM and AND all involve a local application of the operators $\neg$, $\exists$ and $\wedge$, respectively, within the scope of the formulas which they are applied to. They serve to construct the (propositional) contents of Σ^π-formulas which are always the arguments of the functions R abstracted over. The only really global effects come in with the fifth clause of Σ^π's semantics, which deals with proper names. Proper names are always assigned wide scope.

It should be noticed that Σ^π-formulas and operators can be derived from *DIL*-formulas and operators by means of value and argument raising:

(54)
$$\begin{array}{ll} \uparrow\phi & = [1\text{VR}](\phi) \\ \text{NOT}\Phi & = ([1\text{AR}][2\text{VR}](\lambda p\ \neg{}^\cup p))({}^\cap\Phi) \\ \text{SM}d\Phi & = ([1\text{AR}][2\text{VR}](\lambda p\ \exists d{}^\cup p))({}^\cap\Phi) \\ [\Phi\ \text{AND}\ \Psi] & = ([1\text{AR}][2\text{AR}][3\text{VR}](\lambda p\lambda q\ {}^\cup p\wedge{}^\cup q))({}^\cap\Phi)({}^\cap\Psi) \end{array}$$

Of our original 0-, 1- and 2-place operators, first the values are raised from t^0 to t^2, and next the propositional arguments (if any) are raised to this higher type. As shown in [6], the various raisings of values and arguments cancel out if no other scope bearing operators interfere, since:

(55) $[1\text{AR}](\beta)({}^\cap[1VR](\alpha)) = \beta({}^\cap\alpha)$

The import of this equivalence for Σ^π is that the derived operators retain their ordinary truth-conditional effects:

(56)
$$\begin{array}{ll} \text{NOT}\uparrow\phi & = \uparrow\neg\phi \\ \text{SM}d\uparrow\phi & = \uparrow\exists d\phi \\ [\uparrow\phi\ \text{AND}\uparrow\psi] & = \uparrow[\phi\wedge\psi] \end{array}$$

These equivalences also show that all Σ^π-formulas without proper names can be rewritten as the respective lifts of the corresponding *DIL*-formulas.

The semantic behaviour of proper names in Σ will become visible in the following equivalences:

(57)
$$\begin{array}{ll} \text{NOTPN}d\Phi & = \text{PN}d\text{NOT}\Phi \\ \text{SM}d'\text{PN}d\Phi & = \text{PN}d\text{SM}d'\Phi \\ [\text{PN}d\Phi\ \text{AND}\ \Psi] & = \text{PN}d[\Phi\ \text{AND}\ \Psi] \\ [\uparrow\phi\ \text{AND PN}d\Psi] & = \text{PN}d[\uparrow\phi\ \text{AND}\ \Psi] \end{array}$$

Here we see that proper names as a matter of fact 'look through', or 'float up through' other operators the scope of which they find themselves in. The equivalences in (57) and (56) can be used to reduce any Σ^π-formula Φ to an instance of the following Σ^π normal form:

(58) $\text{PN}_1 d_1 \ldots \text{PN}_n d_n \uparrow\phi$

The truth conditions of any Σ^π-formula Φ can be computed next by applying the down operator $\downarrow$ to its Σ^π normal form, and using the following equivalences:

(59)
$$\begin{array}{ll} \downarrow\uparrow\phi & = \phi \\ \downarrow\text{PN}d\Phi & = \exists d(d = \text{PN}\wedge\downarrow\Phi) \end{array}$$

Thus, the equivalences in (56)–(59) serve to show that proper names deliver their semantic contribution at the main sentence level in Σ^π. By way of conclusion we see that a purely semantic and compositional account of periscoping is possible, and also that it can be obtained by a judicious use of an existing system of type shifts.

The Σ^π account of periscoping employs the ordinary propositions and propositional operators of *DIL*, Hendriks' rules of value raising and argument raising, and a quantifier treatment of proper names, and that's all. We now want to show that this account is sufficiently general to be implemented in different semantic systems, in a similar type-theoretical framework. For instead of building our treatment of periscoping on top of a static fragment, like we did above, we might as well build it on top of the dynamic system Σ or, for that matter, Σ^τ. To substantiate this claim, we show how the periscoping account can be implemented in Σ^τ.

We first revise the syntax of Σ^π in the following way:

Definition 7 ($\Sigma^{\tau\pi}$ Syntax)
The set of $\Sigma^{\tau\pi}$-formulas $\mathcal{WF}_{\Sigma^{\tau\pi}}$ is $\mathcal{WF}_{\Sigma^\pi}$ with $\uparrow$ replaced by $\Uparrow$

In $\Sigma^{\tau\pi}$ $\Uparrow$ must be understood as being short for $\uparrow\uparrow$, the leftmost $\uparrow$ of which is the periscoping lift, and the rightmost one the telescoping lift. The semantics clearly displays this double lifting, since formulas are raised to the type $t^4 = (t^2)^2$. In the following definition of the semantics of $\Sigma^{\tau\pi}$, $\mathcal{R}$ is a variable of type ${}^\star t^3$, and P and Q are variables of type ${}^\star t^2$, the type of intensions of Σ-formulas:

Definition 8 ($\Sigma^{\tau\pi}$ Semantics) $\Sigma^{\tau\pi}$-formulas are interpreted in the type t^4 in the following way:

$$
\begin{aligned}
\Uparrow\phi &= \lambda\mathcal{R}\ {}^\cup\mathcal{R}({}^\cap{\uparrow}\phi)\\
\text{NOT}\Phi &= \lambda\mathcal{R}\ \Phi({}^\cap\lambda P\ {}^\cup\mathcal{R}({}^\cap{\sim}{}^\cup P))\\
\text{SM}d\Phi &= \lambda\mathcal{R}\ \Phi({}^\cap\lambda P\ {}^\cup\mathcal{R}({}^\cap\mathcal{E}d{}^\cup P))\\
[\Phi \text{ AND } \Psi] &= \lambda\mathcal{R}\ \Phi({}^\cap\lambda P\ \Psi({}^\cap\lambda Q\ {}^\cup\mathcal{R}({}^\cap[{}^\cup P\,;{}^\cup Q])))\\
\text{PN}d\Phi &= \lambda\mathcal{R}\ \exists d(d = \text{PN} \wedge \Phi(\mathcal{R}))
\end{aligned}
$$

The truth-conditional content $\Downarrow\Phi$ of a Σ-formula Φ is defined by $\Phi({}^\cap\lambda P\ {\downarrow}{}^\cup P)$

The double lift in $\Uparrow\phi$ is obvious. The definitions of NOT, SMd and $\wedge$ are similar to the original ones, except for (i) the higher type used, and (ii) the use of $\sim$, $\mathcal{E}d$ and ; in stead of $\neg$, $\exists d$ and $\wedge$. Notice that the $\Sigma^{\tau\pi}$ operators are derived from corresponding Σ^τ operators in precisely the same way as the Σ^τ operators were derived from corresponding *DIL* operators:

(60)
$$
\begin{aligned}
\Uparrow\phi &= [1\text{VR}]({\uparrow}\phi)\\
\text{NOT}\Phi &= ([1\text{AR}][2\text{VR}](\lambda P\ {\sim}{}^\cup P))({}^\cap\Phi)\\
\text{SM}d\Phi &= ([1\text{AR}][2\text{VR}](\lambda P\ \mathcal{E}d{}^\cup P))({}^\cap\Phi)\\
[\Phi \text{ AND } \Psi] &= ([1\text{AR}][2\text{AR}][3\text{VR}](\lambda P\lambda Q\ [{}^\cup P\,;{}^\cup Q]))({}^\cap\Phi)({}^\cap\Psi)
\end{aligned}
$$

Again, the respective values are raised first, and the arguments (if any) next. The operators also remain associated with their natural counterparts, this time the dynamic ones:

(61)
$$
\begin{aligned}
\text{NOT}{\uparrow}\Phi &= {\uparrow}{\sim}\Phi\\
\text{SM}d{\uparrow}\Phi &= {\uparrow}\mathcal{E}d\Phi\\
[{\uparrow}\Phi \text{ AND } {\uparrow}\Psi] &= {\uparrow}[\Phi\,;\Psi]
\end{aligned}
$$

Proper names again float up through other operators:

(62)
$$\begin{aligned}
\text{NOTPN}d\Phi &= \text{PN}d\text{NOT}\Phi\\
\text{SM}d'\text{PN}d\Phi &= \text{PN}d\text{SM}d'\Phi\\
[\text{PN}d\Phi \text{ AND } \Psi] &= \text{PN}d[\Phi \text{ AND } \Psi]\\
[\uparrow\phi \text{ AND PN}d\Psi] &= \text{PN}d[\uparrow\phi \text{ AND } \Psi]
\end{aligned}$$

The closure operator $\Downarrow$ finally reduces $\Sigma^{\tau\pi}$-formulas from the higher-order type t^4 to the propositional level of type t^0, and, again, such a down operator defines the truth-conditional contents of our higher order formulas. The truth conditions of any $\Sigma^{\tau\pi}$-formula Φ can be obtained by first putting it in $\Sigma^{\tau\pi}$ normal form. The $\Sigma^{\tau\pi}$ normal form entirely corresponds to the Σ^{π} normal form, with the difference that it has a dynamic instead of a static 'propositional core'. After closing this $\Sigma^{\tau\pi}$ normal form, the following equivalences can be used to compute its truth conditions:

(63)
$$\begin{aligned}
\Downarrow\uparrow\Phi &= \downarrow\Phi\\
\Downarrow\text{PN}d\Phi &= \exists d(d = \text{PN} \wedge \Downarrow\Phi)
\end{aligned}$$

We see that, indeed, the account of periscoping formulated for a 'static' first order fragment can be just as well applied to the dynamic Σ^{τ} fragment, the result of which is a system which combines the results of section 3 and 4.

To conclude this section, it is worthwhile to point out that the periscoping technique is also very useful for a treatment of specific indefinites. Consider the indefinite *a certain woman* ... in the following example:

(64) Didi thought that he could not convince a certain woman I met in Rochester last year.

Upon its specific interpretation, this indefinite is understood to have wide scope, and the sentence can be taken to mean that there is a certain woman whom I met in Rochester last year, and whom Didi thought he could not convince. More in general we find that specific indefinites, in *DRT*-terms, "always establish their discourse referents in the universe of the main DRS and thus are not properly within the scope of any other NP" ([9, p. 290]). This is exactly what we can account for by means of periscoping.

Specific indefinites have also been analyzed as referential expressions (as in [3]). When conceived of as a referential expression, a specific indefinite has its denotation determined by its descriptive contents and the context of utterance, including the psychological state of the speaker. The problem with this analysis for a hearer-oriented semantics is that the interpreter may fail to know whom the speaker has in mind, without this disallowing him a proper understanding of the speaker's utterance. For a proper understanding of example (64), the hearer need not (be able to) identify the specific woman the speaker has in mind. It is sufficient that the sentence entails the existence of a woman whom the speaker met last year in Rochester, and about whom Didi has the belief reported. Notice that this is precisely what a periscoping account of specific indefinites would give us.

5. Conclusion

In this paper we have given a treatment of telescoping and periscoping by applying techniques which previously have been used to deal with assignment statements, donkey anaphora and scope ambiguities. We have defined a small toy fragment Σ^{τ}, in which all sentential operators are given a dynamic interpretation, and a fragment Σ^{π} in which proper names are assigned unrestricted wide scope. The two types of dynamics have been combined in the system referred to as $\Sigma^{\tau\pi}$. We thus have shown that a fully semantic account of the phenomena is possible. With regard to, in particular, periscoping, this might not have appeared to be very likely in the first place.

Our treatment of telescoping has been found appropriate for several hard examples. Still, one may of course object that our treatment of telescoping is too rigid. Apparently, the dynamic readings of

$\sim$, $\mathcal{A}d$, $\gg$ and Π are less usual than the static readings. So, whereas we find that the kind of donkey dynamics dealt with in *DRT* and *DPL* is pervasive on the one hand, on the other hand we see that the extended dynamics deriving from a dynamic negation constitutes the exception, rather than the rule.

As has also been argued in [1, p. 110], such observations can be taken to motivate the use of a telescoping module on top of a system of interpretation which itself already deals with the basic dynamics of natural language. Such a distinction between the basic dynamics of ordinary anaphora, and the extended dynamics involved in telescoping can be further motivated. For although the particular examples discussed in section 3 can be dealt with in terms of (extended) scopes, pronominal anaphors cannot in general be dealt with by stretching the scopes of their antecedents (notably, plural anaphora, witness, e.g., the work of Evans). As a conclusion we therefore opt for the use of a telescoping module like that of Σ^{τ} to deal with exceptional scopings, on top of a system which independently deals with inter-sentential anaphora more in general.

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6. Technical Annex

Definition 9 (Dynamic Intensional Logic) The system of dynamic intensional logic differs from intensional logic in three respects: (i) there is a distinguished set of discourse markers DM among the constants CON_e of type e; (ii) instead of IL's modal and temporal operators, DIL has state switchers $\langle\alpha/d\rangle$; (iii) three postulates make states behave like discourse marker assignments.

(DIL Types) The set of types T_{DIL} is the smallest set such that: $e, t \in T_{DIL}$ and if $a, b \in T_{DIL}$, then $\langle s, a\rangle$ and $\langle a, b\rangle \in T_{DIL}$.

(DIL Syntax) The set of well-formed DIL-expressions is built up from sets of constants CON_a and variables VAR_a of type a, by means of functional application, λ-abstraction, identity, the usual connectives and quantifiers, the intension operator $^\cap$ and the extension operator $^\cup$, and state switchers $\langle\alpha/d\rangle$ (for $d \in DM$ and α of type e). A formula $\exists d\phi$ is short for $\exists x\langle x/d\rangle\phi$, for an x not free in ϕ.

(DIL Models and Domains) A model for DIL is a triple $M = \langle S, D, F\rangle$ with S and D non-empty sets of states and individuals, respectively, and F an interpretation function for the constants of DIL. The domain D_a of a type a is defined by $D_e = D$; $D_t = \{0,1\}$; $D_{\langle a,b\rangle} = D_b^{D_a}$; $D_{\langle s,a\rangle} = D_a^S$.

(DIL Postulates)
(Rigidness) $F(c)(s) = F(c)(s')$ for all $c \in (CON \setminus DM)$;
(Distinctness) $s = s'$ if $\forall d \in DM$: $F(d)(s) = F(d)(s')$;
(Update) For all $s \in S$, $d \in DM$, $z \in D_e$ there is an $s' \in S$ such that
$\forall d \in DM$: $F(d')(s') = z$ if $d' = d$, and $F(d')(s') = F(d')(s)$ otherwise.

The postulates ensure that for any state s, discourse marker d and object z, there is a unique state s', such that in s' the value of all constants except d is the same as in s and such that the value of d in s' is z. This state is referred to as $s[d/z]$ in the interpretation of the state switcher.

(DIL Semantics)
$[\![t]\!]_{M,s,g} = F(t)(s)$, if $t \in CON$, $[\![t]\!]_{M,s,g} = g(t)$, if $t \in VAR$;
$[\![\beta(\alpha)]\!]_{M,s,g} = [\![\beta]\!]_{M,s,g}([\![\alpha]\!]_{M,s,g})$
$[\![\lambda x_a\beta_b]\!]_{M,s,g} = h \in {D_b}^{D_a}$ such that $h(z) = [\![\beta]\!]_{M,s,g[x/z]}$ for any $z \in D_a$;
$[\![\alpha = \beta]\!]_{M,s,g} = 1$ iff $[\![\alpha]\!]_{M,s,g} = [\![\beta]\!]_{M,s,g}$;
$[\![\neg\phi]\!]_{M,s,g} = 1$ iff $[\![\phi]\!]_{M,s,g} = 0$
$[\![\exists x_a\phi]\!]_{M,s,g} = 1$ iff there is a $z \in D_a$ such that $[\![\phi]\!]_{M,s,g[x/z]} = 1$
$[\![\phi \wedge \psi]\!]_{M,s,g} = 1$ iff $[\![\phi]\!]_{M,s,g} = [\![\psi]\!]_{M,s,g} = 1$;
$[\![^\cap\alpha_a]\!]_{M,s,g} = h \in {D_a}^I$ such that $h(s') = [\![\alpha]\!]_{M,s',g}$ for any $s' \in I$
$[\![^\cup\alpha]\!]_{M,s,g} = [\![\alpha]\!]_{M,s,g}(s)$;
$[\![\langle\alpha/d\rangle\phi]\!]_{M,s,g} = [\![\phi]\!]_{M,s[d/[\![\alpha]\!]_{M,s,g}],g}$

An occurrence of a discourse marker d in ϕ is free iff it is not in the scope of an intension operator $^\cap$ or of a state quantifier $\exists d$ in ϕ. An occurrence of $^\cup$ in ϕ is called free iff it is not in the scope of an intension operator $^\cap$ in ϕ. A DIL-expression α is intensionally closed, notation: $\alpha \in ICE$, iff α contains no free occurrence of a discourse marker d or of $^\cup$.

Observation 1 For α, β, and x of appropriate types
$^{\cup\cap}\alpha = \alpha$;
$(\lambda x\beta)(\alpha) = [\alpha/x]\beta$ if the free variables in α are free for x in β and $\alpha \in ICE$;
$\exists d\phi = \exists x[x/d]\phi$ if x and $^\cup$ do not occur freely in ϕ, and x is free for d in ϕ.

Definition 10 (Dynamic Montague grammar) The first order part of the language of Groenendijk and Stokhof's dynamic Montague grammar is constructed from the lift of atomic *DIL*-formulas, using $-$, $\mathcal{E}d$ and ;. *DMG*-formulas are interpreted in the type $t^1 = \langle {}^\star t, t\rangle$ in the following way (here p is of type ${}^\star t$; $\top$ abbreviates $x = x$):

$$\begin{array}{llll} \uparrow\phi & = \lambda p\ (\phi \wedge {}^\cup p) & \mathcal{E}d\Phi & = \lambda p\ \exists d(\Phi(p)) \\ -\Phi & = \lambda p\ (\neg\Phi({}^\cap\top) \wedge {}^\cup p) & [\Phi\,;\Psi] & = \lambda p\ \Phi({}^\cap\Psi(p)) \end{array}$$

The truth-conditional content $\downarrow\Phi$ of a *DMG*-formula Φ is defined by $\Phi({}^\cap\top)$.

In the following definition $\beta : b$ means that β is an expression of type b and $\vec{x} : \vec{a}$ that the variables in the sequence $\vec{x}$ have the types $\vec{a}$, respectively.

Definition 11 (Value Raising, Argument Raising, Division)

for $\vec{a}$ a sequence of types $a_1 \ldots a_n$, let $\langle \vec{a}, b\rangle = \langle a_1, \ldots \langle a_n, b\rangle \ldots\rangle$ and
for $\vec{x}$ a sequence of variables $x_1 \ldots x_n$ of types $a_1 \ldots a_n$, respectively, let
$\lambda\vec{x}\phi = \lambda x_1 \ldots \lambda x_n \phi$ (of type $\langle \vec{a}, b\rangle$, if ϕ is of type b) and
$\psi(\vec{x}) = \psi(x_1) \ldots (x_n)$ (of type b, if ψ is of type $\langle \vec{a}, b\rangle$)

then for $\phi : \langle \vec{a}, b\rangle$, $\psi : \langle \vec{a}, \langle b, \langle \vec{c}, t\rangle\rangle\rangle$, and $\chi : \langle \vec{a}, \langle {}^\star t, t\rangle\rangle$:

$$\begin{array}{lll} [n\mathrm{VR}](\phi) & = \lambda\vec{x}\lambda V\ {}^\cup V({}^\cap\phi(\vec{x})) & \text{of type } \langle \vec{a}, b^2\rangle \\ [n\mathrm{AR}](\psi) & = \lambda\vec{x}\lambda Y\lambda\vec{z}\ {}^\cup Y({}^\cap\lambda y\ \psi(\vec{x})(y)(\vec{z})) & \text{of type } \langle \vec{a}, \langle {}^\star b^2, \langle \vec{c}, t\rangle\rangle\rangle \\ [n\mathrm{GD}_b](\chi) & = \lambda\vec{x}\lambda V\lambda y\ \chi(\vec{x})({}^\cap({}^\cup V(y))) & \text{of type } \langle \vec{a}, \langle {}^\star\langle b, t\rangle, \langle b, t\rangle\rangle\rangle \end{array}$$

where $\vec{a} = a_1, \ldots, a_{n-1}$, $\vec{x} = x_1, \ldots, x_{n-1}$, $\vec{c} = c_1, \ldots, c_m$, $\vec{z} = z_1, \ldots, z_m$, $\vec{x} : \vec{a}$, $\vec{z} : \vec{c}$, $V : {}^\star b^1$, $Y : {}^\star b^2$, and $y : b$.

Observation 2 (Σ to *DIL*) The equivalences in (65) suffice to reduce every closed Σ-formula to a truth-conditionally equivalent *DIL*-formula.

$$(65)\quad \begin{array}{llll} \downarrow\uparrow\phi & = \phi & \downarrow[\uparrow\phi\,;\Psi] & = (\downarrow\uparrow\phi \wedge \downarrow\Psi) \\ \downarrow\sim\Phi & = \neg\downarrow\Phi & \downarrow[-\Phi\,;\Psi] & = (\downarrow-\Phi \wedge \downarrow\Psi) \\ \downarrow\mathcal{E}d\Phi & = \exists d\downarrow\Phi & \downarrow[\mathcal{E}d\Phi\,;\Psi] & = \downarrow\mathcal{E}d[\Phi\,;\Psi] \\ & & \downarrow[[\Phi\,;\Psi]\,;\Upsilon] & = \downarrow[\Phi\,;[\Psi\,;\Upsilon]] \end{array}$$

Proof: by induction on the dynamic complexity $dc(\Phi)$ of Φ, where:

$$(66)\quad \begin{array}{llllll} dc(\phi) & = 0 & dc(\uparrow\phi) & = dc(\phi) + 1 & cd(\uparrow\phi) & = 0 \\ dc(\neg\phi) & = dc(\phi) & dc(-\Phi) & = dc(\Phi) + 1 & cd(-\Phi) & = 0 \\ dc(\exists d\phi) & = dc(\phi) & dc(\mathcal{E}d\Phi) & = dc(\Phi) + 1 & cd(\mathcal{E}d\Phi) & = cd(\Phi) + 1 \\ dc(\phi \wedge \psi) & = dc(\phi) + dc(\psi) & & & cd(\Phi\,;\Psi) & = cd(\Psi) + 1 \\ dc(\downarrow\Phi) & = dc(\Phi) & dc(\Phi\,;\Psi) & = cd(\Phi) + dc(\Phi) + dc(\Psi) + 1 & & \end{array}$$

$dc(\Phi)$ equals the number of steps needed to reduce a closed Σ-formula $\downarrow\Phi$ to a truth-conditionally equivalent *DIL*-formula using the equivalences in (65). ($cd(\Phi)$ is an auxiliary notion used in defining $dc(\Phi\,;\Psi)$.) If $dc(\phi) = 0$, then ϕ is a *DIL*-formula. If $dc(\Phi) > 0$, then one of the equivalences in (65) is of the form $\downarrow\Phi = \Phi'$, where $dc(\Phi') = dc(\Phi) - 1$. End of proof.

Observation 3 (Σ to *DMG*) Σ is truth-conditionally equivalent to *DMG*

Proof: The equivalences in (65) are valid in both *DMG* and Σ. End of proof.

Observation 4 (Telescoping Σ^τ) The equivalences in (67) suffice to reduce every closed Σ^τ-formula to a truth-conditionally equivalent *DIL*-formula.

(67) (a)

$$\begin{aligned} \downarrow\uparrow\phi &= \phi \\ \downarrow\sim\Phi &= \neg\downarrow\phi \\ \downarrow\mathcal{E}d\Phi &= \exists d\downarrow\Phi \\ \downarrow\mathcal{A}d\Phi &= \forall d\downarrow\Phi \\ \downarrow[\uparrow\phi\,;\Psi] &= (\phi\wedge\downarrow\Psi) \\ \downarrow[\uparrow\phi\gg\Psi] &= (\phi\rightarrow\downarrow\Psi) \end{aligned}$$

(b)

$$\begin{aligned} \sim\uparrow\phi &= \uparrow\neg\Phi \\ \sim\sim\Phi &= \Phi \\ \sim\mathcal{E}d\Phi &= \mathcal{A}d\sim\Phi \\ \sim\mathcal{A}d\Phi &= \mathcal{E}d\sim\Phi \\ \sim[\Phi\,;\Psi] &= [\Phi\gg\sim\Psi] \\ \sim[\Phi\gg\Psi] &= [\Phi\,;\sim\Psi] \end{aligned}$$

(c)

$$\begin{aligned} [\mathcal{E}d\Phi\,;\Psi] &= \mathcal{E}d[\Phi\,;\Psi] \\ [\mathcal{A}d\Phi\,;\Psi] &= \mathcal{A}d[\Phi\,;\Psi] \\ [[\Phi\,;\Psi]\,;\Upsilon] &= [\Phi\,;[\Psi\,;\Upsilon]] \\ [[\Phi\gg\Psi]\,;\Upsilon] &= [\Phi\gg[\Psi\,;\Upsilon]] \end{aligned}$$

(d)

$$\begin{aligned} [\mathcal{E}d\Phi\gg\Psi] &= \mathcal{A}d[\Phi\gg\Psi] \\ [\mathcal{A}d\Phi\gg\Psi] &= \mathcal{E}d[\Phi\gg\Psi] \\ [[\Phi\,;\Psi]\gg\Upsilon] &= [\Phi\gg[\Psi\gg\Upsilon]] \\ [[\Phi\gg\Psi]\gg\Upsilon] &= [\Phi\,;[\Psi\gg\Upsilon]] \end{aligned}$$

Proof: similar to that of observation (2). Sketch: The equivalences under (b) can be used to eliminate *all* occurrences of $\sim$ from a Σ^τ-formula Φ; those under (c) and (d) can next be used to obtain the normal binding form Φ' of Φ, an expression in which all subformulas of the form $[\Phi\,;\Psi]$ and $[\Phi\gg\Psi]$ have a left-hand side constituent $\Phi=\uparrow\phi$; the closure $\downarrow\Phi'$ of such a formula Φ' can be turned into a *DIL*-expression using the equivalences under (a). End of sketch.

Observation 5 (Periscoping Σ^π) The equivalences in (56) and (57) can be used to reduce any Σ^π-formula Φ to a formula of the form $\text{PN}_1 d_1 \ldots \text{PN}_n d_n \uparrow\phi$

Proof: by induction on the number n of proper names in Φ. For $n=0$ a straightforward induction on the complexity of Φ shows that a Σ^π-formula Φ without proper names can be turned into an equivalent formula of the form $\uparrow\phi$ using the rules of (56). For $n>0$ the left-most proper name can be scoped out using (57) and (56) to produce a formula of the form $\text{PN}_i d_i \Phi'$, and the induction hypothesis applies to Φ'. End of proof.