Computational forging in the Eulerian formulation at MacNeal-Schwendler (E.D.C.) B.V.

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Abstract

This report is based on eight half-day consultancy visits of the author to MacNeal-Schwendler (E.D.C.) B.V. (MSC-EDC), in the two-month period February-March 1996. The report summarizes the main comments made on the development of computational methods for forging processes at MSC-EDC and gives suggestions for possible future work.

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1 Introduction

The common formulations in computational forging are the Lagrangian and the mixed Langrangian-Eulerian. In both formulations, the computational domain is confined to the body to be forged, which is overlaid by a grid. In case of the Lagrangian formulation, the grid remains attached to the body at *all* grid points and *all* time levels. Large deformations of the body imply great distortions of the grid and hence possible deteriorations of the spatial accuracy. A fix to this may lie in remeshing (in proper time) during the computation, which is what is done in the Lagrangian-Eulerian formulation. In case of very large deformations, remeshing may no longer help. Then, the Eulerian formulation is the only alternative. In here, the boundaries of the computational domain and the solid do not necessarily coincide. This implies an important difference with the two formulations mentioned first. In there, the boundaries of the body are *fitted* and hence crisp. In the Eulerian formulation the boundaries are *captured* and may therefore be diffused. (Of course, a diffused boundary is not necessarily less accurate; fitted boundaries will be sharp, but their location and shape may be inaccurate.) Anyhow, the Eulerian formulation is certainly most promising from the viewpoint of general applicability.

At present, computational forging in the Eulerian formulation is still *pioneering work*. This is illustrated by e.g. the fact that searches for publications on it in the open scientific literature give very little result. Given this small attention, the prospects of it in case of a successful development are promising.

The outline of the report is as follows. In Section 2, a summary is given of the major technical comments made during the consultancy period: in Section 2.1 on the computational tools developed so far and in Section 2.2 on tools that are already in development at MSC-EDC or that could be developed in future. The comments are ordered by subject, but for the remainder they are loose. The comments do *not* form a list of differences in points of view between the author and the MSC-EDC staff, or some list of shortcomings of software. (On many comments there is agreement and many tools that "should" be in the software are already in there.) In Section 3, suggestions are given for a possible research cooperation on computational forging, between MSC-EDC and the Centre for Mathematics and Computer Science (CWI).

2 Summary of technical remarks made

2.1 On existing tools

2.1.1 Discretization

- The space and time discretizations are interweaved; the numerical method is not of the method-of-lines type [10]. Understanding and controling the latter type is easier. In the method-of-lines approach, the space and time discretization are done separately: first the space discretization (a semi-discrete equation then results) and next the time discretization. Given the fact that the continuous (i.e. the non-discretized) system of forging equations is already intricate by itself, a transparent numerical approach, such as that of the method of lines, is certainly practical in case of numerical difficulties.

- The space discretization is first-order upwind and the time integration second-order centered. Von Neumann stability analysis [10] for a one-dimensional, scalar model advection problem shows that such a numerical method is labile; it has no safety margin against instabilities due to nonlinearities. (Although the differences between the discrete model equation and the discrete forging equations are very large, the lability found should not be underestimated.)

- To preserve monotonicity of the material-fraction function, which is used for the resolution of the free surfaces, a limiter function may be used which does not obey the standard higher-order accuracy requirement valid for smoothly varying solutions ($\phi(1) = 1$, [34]), because the material-fraction function varies discontinuously only. Note that when changing the limiter such that it adds more and more anti-diffusion, formally the time step that can be maximally taken for monotonicity reasons becomes smaller and smaller [13]. Moreover, as mentioned in Section 1, a more crisp solution is not necessarily more accurate.

- A fix as "glueing", to remove unstable wiggles in the solution, may be justified on theoretical grounds. (Through analysis, a decoupling of grid points has been found [1], which may explain unstable odd-even oscillations observed in practice.) Still the "glueing" should be avoided, because besides spurious high-frequency modes in the solutions, it also kills the physically correct high-frequency modes. The same holds for the Tyler viscosity [28], which is sometimes applied for stability reasons.

2.1.2 Numerical experiments

- The validity of the exact solutions for test cases (such as the "flat billet") should be critically reviewed all the time. (Are they correct for all time and all geometrical properties of the bodies, are they unique, ... ?)

- Computations to investigate the accuracy behavior of a numerical method should be preferably be done on a family of at least three grids (say with mesh sizes h, $\frac{1}{2}h$, $\frac{1}{4}h$) and – if possible – on each grid with at least three different time steps as well (say τ, $\frac{1}{2}\tau$, $\frac{1}{4}\tau$).

2.2 On tools in development and possible future tools

2.2.1 Continuous equations

- In one, two and three dimensions, the forging equations may be written as a system of three, seven and thirteen coupled, first-order partial differential equations, respectively. Denoting the dimension by d, we have one continuity equation, d momentum equations and d^2 transport equations for the components of the stress deviation tensor [4]. (The stress deviation tensor is not taken symmetric.) The number of unknowns exceeds the number of differential equations by one. The system can be balanced by an algebraic equation relating density and pressure for the specific solid considered (the equation of state).

- To allow for discontinuous solutions, the system should not be solved in differential form, but in integral form instead. For that, the system should first be written, as far as possible, in divergence form. (Because the system is not completely based on conservation laws, a complete divergence form is not possible.)

2.2.2 Space discretization

- When developing an improved computational method for forging, for reasons of transparency, as mentioned, the method-of-lines approach should be followed.

- To respect the conservation laws underlying the system of forging equations, the space discretization should be a finite-volume method [10]. The finite-volume method should preferably not be of cell-vertex type but of cell-center type instead. I.e., in each finite volume, all solution components are supposed to lie in the center of that finite volume. (A cell-vertex method may lead to an unbalance between the number of grid unknowns and finite-volume equations, because the numbers of vertices and finite volumes may not be the same.)

- The derivation of a Roe-type approximate Riemann solver [29] for the system of forging equations written as a hyperbolic system with source terms [35], should be continued. An approximate Riemann solver obeys a great deal of the physics modeled by the continuous system of equations. In many applications it guarantees a good solution quality. The complexity of an approximate Riemann solver is compensated by the simplicity it yields with respect to e.g. a physically correct boundary-condition formulation. In the case of a simple density-pressure relation, besides a Roe-type approximate Riemann solver, an Osher-type one [27] might be constructed as well. The advantages of Osher's over Roe's approximate Riemann solver, are mathematically even better defined boundary conditions [26] and continuously differentiable fluxes [8], which latter may be a necessity in case an implicit time stepping method is applied. Analogously to the testing of approximate Riemann solvers for the equations of gas dynamics on the standard shock-tube problem, newly derived approximate Riemann solvers for the forging equations should also be tested on a well-defined test case. (This test case still has to be prescribed.)

- The evaluation of the source terms should be consistent with that of the convection terms [5, 15, 30].

- Multi-D upwinding (by either rotated state interpolation [16, 18], rotated flux computation [2, 22, 32], or even multi-D wave decomposition [3, 12, 31]), should not yet be applied to the full system of equations. Multi-D upwind techniques are developing at a rapid pace, but they are not yet well-proven for the present full system of equations. An exception might be made for the advection equation for the material-fraction function. That equation is not coupled to the system of equations, in the sense that the material-fraction function does not appear in any of the other equations. However, note that most existing multi-D upwind schemes have been specifically derived for *steady* problems. For a three-dimensional, unsteady, hyperbolic equation as the advection equation for the material-fraction function, a new multi-D upwind scheme may be derived.

- As far as the calculation of the left and right cell-face states for the approximate Riemann solver is concerned, here higher-order accuracy and monotonicity should still simply be obtained by the well-proven MUSCL approach [11]. I.e., left and right cell-face states should be calculated through higher-order accurate, monotone, piecewise polynomial interpolation of cell-center states. (A specifically tailored limiter may be quickly constructed and it may be quickly investigated which state variables behave most smoothly and are thus best suited for interpolation.) It is expected that this separate topic of calculating left and right cell-face states will take little effort here.

- The forging problems are uniformly stiff; over the entire computational domain they have a very small ratio of flow speed over speed of sound (very low "Mach"). This stiffness may give rise to poor accuracy properties of the discrete system [17]. For steady problems, a remedy may be found in proper preconditioning of the continuous flow equations, for unsteady equations such a fix is

not available yet. A "brute-force" fix as solution-adaptive local grid refinement (next item) might be helpful as long as no fundamental remedy has been found.

- In resolving free surfaces and coupling surfaces, for simplicity, one may first refrain from subcell resolution (see next item), i.e. first take the surfaces coincident with the cell faces (the Godunov approach [6]). If wanted, an improved resolution at relatively low increase of computational costs can then be obtained by solution-adaptive local grid refinement. Software that can be used for this purpose has already been transferred from CWI to MSC-EDC (see [7, 9] for descriptions of that software). Given the availability of an approximate Riemann solver, the wave speeds of all convection phenomena are known and hence a time-accurate, catch-all grid-refinement criterion can be defined.

- If solution-adaptive local grid refinement solely cannot meet the accuracy and efficiency requirements set by industries, then the computational method may also make use (again) of subcell resolution. (In general terms, subcell resolution is that one allows the solution to have structure in a cell, instead of allowing it to be piecewise constant only.) For this, the system of equations should be extended (again) with a transport equation for a passive scalar function, which marks the free surface, the material-fraction function. (The function may be defined to be one inside the body and zero outside. Then, the location of the free surface may be defined by the value $\frac{1}{2}$ of the material-fraction function.) In many application areas other than computational forging (e.g. computational aerodynamics), subcell resolution is no well-proven technology and research on it is rare, probably simply because local grid refinement does a satisfactory job in these areas. At SMC-EDC, a probably unique experience exists on subcell resolution.

- If the system of equations is not valid across free surfaces and coupling surfaces, then discretizing across these surfaces should be avoided and both types of surfaces should be treated as boundaries. At these boundaries, the grid may have sliced-off cells, which may be merged (called blended at MSC-EDC) with non-cut neigboring cells [19, 20, 21]. (At these boundaries, the difference in solution quality for a Roe-type and an Osher-type approximate Riemann solver might become significant.)

2.2.3 Time integration

- Given the numerical properties of the once chosen space discretization and given the accuracy and efficiency requirements set by industries, a suitable off-the-shelf time integrator can be chosen from the vast collection of time integrators available today. (A satisfactory time integrator is expected to be available for *any* space discretization, also for space discretizations with e.g. very small cells due to strong local grid refinement or due to non-blended, sliced-off cells.) It is expected that no development work needs to be done here.

3 Suggestions for future cooperation

The existing computational methods for forging in the Eulerian formulation at MSC-EDC are not yet sufficiently well-developed for industrial applications. Fundamental research, mainly numerical mathematics research, still needs to be done. A possibility for MSC-EDC is to contract out such research to an institute as CWI (see Appendix for some general information on CWI). A research project can be formulated, which is partially funded from a government program and which is still primarily directed towards the long-term company interests of MSC-EDC.

Research can be carried out in CWI's group Industrial Mathematics. In this group, fundamental and applied research is done in the field of scientific computing for industrial applications. Recent applications include computational fluid dynamics, semi-conductor device simulation and parameter identification in chemical kinetics. Last ten years, the major part of the research concerned the development of accurate and efficient numerical methods for computational fluid dynamics, in particular for aircraft aerodynamics. Beside in research, the group was active in scientific and industrial consultancy, and in transferring software (developed in the group) to institutes and industries in- and outside

the Netherlands. Research on computational fluid dynamics led to three doctoral degrees [14, 23, 33], research on semi-conductor-device simulation to two [24, 25]. The group has the following scientific staff: prof.dr. P.W. Hemker (group leader and professor of Industrial Mathematics at the University of Amsterdam), prof.dr.ir. P. Wesseling (advisor of the group and professor of Numerical Mathematics at the Delft University of Technology) and the author (senior researcher). Prof. Hemker can be active as supervising professor for doctoral degrees to be obtained in future research for and by SMC-EDC.

References

[1] P. BRAND, Internal note on stability in case of rigid plastic, MacNeal-Schwendler (E.D.C.) B.V., Gouda, 1996.

[2] S.F. DAVIS, A rotationally biased upwind difference scheme for the Euler equations, *Journal of Computational Physics*, **56**, 65-92, 1984.

[3] H. DECONINCK AND B. KOREN, EDS., Euler and Navier-Stokes Solvers Using Multi-Dimensional Upwind Schemes and Multigrid Acceleration, *Notes on Numerical Fluid Mechanics*, **XX**, Vieweg, Braunschweig (in preparation).

[4] Y.C. FUNG, *Foundations of Solid Mechanics*, Prentice-Hall, Englewood Cliffs, NJ, 1965.

[5] P. GLAISTER, Flux difference splitting techniques for the Euler equations in non-Cartesian geometry, *Numerical Analysis Report 8/85*, Department of Mathematics, University of Reading, 1985.

[6] S.K. GODUNOV, Finite difference method for numerical computation of discontinuous solutions of the equations of fluid dynamics (Cornell Aeronautical Lab. Transl. from the Russian), *Matematicheskii Sbornik* **47**, 271-306, 1959.

[7] P.W. HEMKER, H.T.M. VAN DER MAAREL AND C.T.H. EVERAARS, BASIS: a data structure for adaptive multigrid computations, *Report NM-R9014*, CWI, Amsterdam, 1990.

[8] P.W. HEMKER AND S.P. SPEKREIJSE, Multiple grid and Osher's scheme for the efficient solution of the steady Euler equations, *Applied Numerical Mathematics*, **2**, 475-493, 1986.

[9] P.W. HEMKER AND P.M. DE ZEEUW, BASIS3: a data structure for 3-dimensional sparse grids, in Euler and Navier-Stokes Solvers Using Multi-Dimensional Upwind Schemes and Multigrid Acceleration (H. DECONINCK AND B. KOREN, EDS.), *Notes on Numerical Fluid Mechanics*, **XX**, Vieweg, Braunschweig (in preparation).

[10] CH. HIRSCH, *Numerical Computation of Internal and External Flows, Fundamentals of Numerical Discretization*, **1**, Wiley, Chichester, 1988.

[11] CH. HIRSCH, *Numerical Computation of Internal and External Flows, Computational Methods for Invsicid and Viscous Flows*, **2**, Wiley, Chichester, 1990.

[12] CH. HIRSCH, C. LACOR AND H. DECONINCK, Convection algorithms based on a diagonalization procedure for the multidimensional Euler equations, *AIAA paper 87-1163*, 1987.

[13] W. HUNDSDORFER, B. KOREN, M. VAN LOON AND J.G. VERWER, A positive finite-difference advection scheme, *Journal of Computational Physics*, **117**, 35-46, 1995.

[14] B. KOREN, *Multigrid and Defect Correction for the Steady Navier-Stokes Equations, Application to Aerodynamics*, doctoral thesis, Delft University of Technology, 1989.

[15] B. KOREN, A robust upwind discretization method for advection, diffusion and source terms, in Numerical Methods for Advection-Diffusion Problems (C.B. VREUGDENHIL AND B. KOREN, EDS.), *Notes on Numerical Fluid Mechanics*, **45**, 117-138, Vieweg, Braunschweig, 1993.

[16] B. KOREN AND P.W. HEMKER, Multi-D upwinding and multigridding for steady Euler flow computations, in Proceedings of the Ninth GAMM Conference on Numerical Methods in Fluid Mechanics, Lausanne, 1991 (J.B. VOS, A. RIZZI AND I.L. RYHMING, EDS.), *Notes on Numerical Fluid Mechanics*, **35**, 89-98, Vieweg, Braunschweig, 1992.

[17] B. KOREN AND B. VAN LEER, Analysis of preconditioning and multigrid for Euler flows with low-subsonic regions, *Advances in Computational Mathematics*, **4**, 127-144, 1995.

[18] B. KOREN AND H.T.M. VAN DER MAAREL, Monotone higher-order accurate, multi-dimensional upwinding, in Proceedings of the 13th International Conference on Numerical Methods in Fluid Dynamics, Rome, 1992 (M. NAPOLITANO AND F. SABETTA, EDS.), *Lecture Notes in Physics*, **414**, 110-114, Springer, Berlin, 1993.

[19] R.J. LEVEQUE, A large time step generalization of Godunov's method for systems of conservation laws, *SIAM Journal on Numerical Analysis*, **22**, 1051-1073, 1985.

[20] R.J. LEVEQUE, High resolution finite volume methods on arbitrary grids via wave propagation, *Journal of Computational Physics*, **78**, 36-63, 1988.

[21] R.J. LEVEQUE, Cartesian grid methods for flow in irregular regions, in *Numerical Methods for Fluid Dynamics*, **III** (K.W. MORTON AND M.J. BAINES, EDS.), 375-382, Clarendon, Oxford, 1988.

[22] D.W. LEVY, K.G. POWELL AND B. VAN LEER, An implementation of a grid-independent upwind scheme for the Euler equations, *AIAA paper 89-1931*, 1989.

[23] H.T.M. VAN DER MAAREL, *A Local Grid Refinement Method for the Euler Equations*, doctoral thesis, University of Amsterdam, 1993.

[24] J. MOLENAAR, *Multigrid Methods for Semiconductor Device Simulation*, doctoral thesis, University of Amsterdam, 1992.

[25] R.R.P. VAN NOOYEN, *Some Aspects of Mixed Finite Element Methods for Semiconductor Simulation*, doctoral thesis, University of Amsterdam, 1992.

[26] S. OSHER AND S. CHAKRAVARTHY, Upwind schemes and boundary conditions with applications to Euler equations in general geometries, *Journal of Computational Physics*, **50**, 447-481, 1983.

[27] S. OSHER AND F. SOLOMON, Upwind difference schemes for hyperbolic systems of conservation laws, *Mathematics of Computation*, **38**, 339-374, 1982.

[28] P.J. ROACHE, *Computational Fluid Dynamics*, Hermosa, Albuquerque, NM, 1976.

[29] P.L. ROE, Approximate Riemann solvers, parameter vectors, and difference schemes, *Journal of Computational Physics*, **43**, 357-372, 1981.

[30] P.L. ROE, Characteristic-based schemes for the Euler equations, *Annual Review of Fluid Mechanics*, **18**, 337-365, 1986.

[31] P.L. ROE, Discrete models for the numerical analysis of time-dependent multidimensional gas dynamics, *Journal of Computational Physics*, **63**, 458-476, 1986.

[32] C.L. RUMSEY, B. VAN LEER AND P.L. ROE, A grid-independent approximate Riemann solver with applications to the Euler and Navier-Stokes equations, *AIAA Paper 91-0239*, 1991.

[33] S.P. SPEKREIJSE, *Multigrid Solution of the Steady Euler Equations*, doctoral thesis, Delft University of Technology, 1987.

[34] P.K. SWEBY, High resolution schemes using flux limiters for hyperbolic conservation laws, *SIAM Journal on Numerical Analysis*, **21**, 995-1011, 1984.

[35] A.C.J. VENIS, Internal notes on the derivation of a Roe-type approximate Riemann solver for forging equations, MacNeal-Schwendler (E.D.C.) B.V., Gouda, 1996.

Appendix

CWI - National Research Centre for Mathematics and Computer Science

Mission

CWI is the research institute of the Foundation Mathematical Centre (SMC), a non-profit organization located in Amsterdam. The Foundation's mission – already laid down in its 1946 founding charter – is to foster frontier research in mathematics and computer science, as well as to transfer new knowledge in these fields to society in general and trade and industry in particular.

Research

Research at CWI focuses on fundamental aspects, both pure scientific and application-oriented. At the same time, much attention is paid to the applicability of results of fundamental research. Strategic considerations and applicability play an important role in the selection of new projects. In several projects there is cooperation with institutes for applied research like the Netherlands Organization for Applied Scientific Research (TNO), other technological institutes (ECN, NLR, WL, MARIN, ...) and research laboratories in industry.

Academic links

CWI cherishes its wide range of links with the academic research community. A considerable number of young mathematicians and computer scientists have followed work on a doctoral thesis at CWI by winning a post in the academic or industrial sectors. Several senior CWI researchers hold part-time professorships at Dutch universities. In their turn, universities are strongly represented in commissions advising on CWI's research policy. In addition, the SMC Foundation, of which CWI forms part, manages some eight national mathematical working parties at Dutch universities.

Knowledge transfer

Results of research carried out at CWI are published, presented at conferences and transferred in advanced short courses. Knowledge transfer also takes place through the exchange of scientific staff with universities, research institutes or industry. Contract research is considered an important form of knowledge transfer, in which interaction with society generates new research areas.

Research programs

CWI has a staff of about 180 fte, 50 of whom are seconded. Annually some 200 scientists spend some time at the institute. CWI's research program focuses on carefully selected topics, of which the following are of special interest.

Mathematics:

- Computer assisted mathematics (computer algebra, specific knowledge bases, systems for symbolic computation, systems for finding information),
- Biomathematics and Non-linear dynamics (models for infection, structured populations),
- HPCN (computational fluid dynamics, ordinary differential equations, differential-algebraic equations),

- Research at the interface between optimal control and computer science, combinatorics, queueing and control theoretical aspects of computer and communication networks,
- Computer intensive methods in stochastics (statistical analysis and modeling of digital images and spatial data).

Computer science:

- Multiple computing agents (theoretical foundations, autonomous learning systems, programming methods),
- Formal methods in software technology (programming environments, user interfaces, compiler generation for parallel machines),
- Data mining ((semi-)automatic searching for interrelations in large databases by applying integral database algorithms, learning systems, statistics and HPCN),
- Multimedia (integration of computer graphics, image reconstruction and pattern recognition),
- HPCN (distributed programming environments, effective architectures for parallel database management systems).

International relations

CWI has extensive international contacts - formal and informal - and provides hospitality to many visitors from abroad. CWI participates in many European Union research projects and is a founding member of ERCIM, the European Research Consortium for Informatics and Mathematics. ERCIM aims to promote research in information technology and applied mathematics on the European level, and to intensify contacts between research organizations and industry. Its current activities focus on active cooperation between researchers through joint workshops and conferences and jointly organized advanced courses.

Finances

A major part (about 70%) of SMC's funding comes from the Netherlands Organization for Scientific Research (NWO). The remaining 30% is obtained through national research programs, international programs (ESPRIT, RACE, BRITE, ...) and contract research commissioned by industry.

Infrastructure

CWI has the disposal of an advanced computer network which is connected to national and international computer networks and has access to the national Cray supercomputer facility, as well as a massively parallel Parsytec computer. Furthermore, the institute possesses a first-rate library of international importance.