



Centrum voor Wiskunde en Informatica

**REPORTRAPPORT**

Converting the Reset

J.K. Hoogland, C.D.D. Neumann, D. Bloch

Software Engineering (SEN)

**SEN-R0108 April 30, 2001**

Report SEN-R0108  
ISSN 1386-369X

CWI  
P.O. Box 94079  
1090 GB Amsterdam  
The Netherlands

CWI is the National Research Institute for Mathematics and Computer Science. CWI is part of the Stichting Mathematisch Centrum (SMC), the Dutch foundation for promotion of mathematics and computer science and their applications.

SMC is sponsored by the Netherlands Organization for Scientific Research (NWO). CWI is a member of ERCIM, the European Research Consortium for Informatics and Mathematics.

Copyright © Stichting Mathematisch Centrum  
P.O. Box 94079, 1090 GB Amsterdam (NL)  
Kruislaan 413, 1098 SJ Amsterdam (NL)  
Telephone +31 20 592 9333  
Telefax +31 20 592 4199

# Converting the Reset

J.K. Hoogland and C.D.D. Neumann

*CWI*

*P.O. Box 94079, 1090 GB Amsterdam, The Netherlands*

*Jiri.Hoogland@cwi.nl    Dimitri.Neumann@cwi.nl*

D. Bloch

*Dresdner Kleinswort Wasserstein*

*20 Fenchurch Street, London, EC3P 3DB United Kingdom*

*Daniel.Bloch@dresdnerkw.com*

## ABSTRACT

We give a simple algorithm to incorporate the effects of resets in convertible bond prices, without having to add an extra factor to take into account the value of the reset. Furthermore we show that the effect of a notice period, and additional make-whole features, can be treated in a straightforward and simple manner. Although we present these results with the stockprice driven by geometric Brownian and a deterministic interest term structure, our results can be extended to more general cases, e.g. stochastic interest rates.

*2000 Mathematics Subject Classification:* 91B24, 60H10, 58J35, 58J70.

*Keywords and Phrases:* convertible bonds, reset feature, contingent claim pricing, partial differential equation.

## 1. INTRODUCTION

In recent years many Japanese companies have issued convertible bonds with reset-features. At specific reset-dates the conversion ratio<sup>1</sup> of the convertible bond (CB) is reset such that the conversion value is equal to the face value in cash. Typically there is one reset and the reset of the conversion price is only allowed to be downward.

The reset feature makes the contract more attractive, since it protects the holder of the bond against drops in the value of the underlying stock, but it is also more difficult to price. The usual approach in the literature [W98, N98] is to introduce an extra factor to take into account the reset-feature. The CB is then priced by solving a two- or three-dimensional PDE or using a multi-dimensional tree algorithm. This is time-consuming and error-prone.

In this article, we show that, in general, resets can be simply taken into account as jump conditions at the reset dates. This prevents the introduction of the extra factor, which greatly simplifies and improves the speed of the algorithm.

Furthermore we show that the effects of a notice period can be simply incorporated in the pricing problem by using an effective call boundary value, that follows from the remaining optionality for the holder to convert during the holding period.

Finally we show that, in a deterministic interest rate setting, the resettable convertible, without callable and puttable features, can be related to an American discrete lookback option.

The outline of this article is as follows. In Section 2 we define the convertible bond (CB) in terms of its boundary conditions etc. In Section 3 we describe the model we use to price the CB. Section 4 and section 5 shows how the notice period and resets can be incorporated in a numerical pricing scheme. In Section 6 we show that a change of numeraire provides an alternative way of understanding reset features in terms of lookbacks. Section 7 discusses implementation of the scheme and compares our

---

<sup>1</sup>The conversion ratio is the amount of stock the bond is converted into. The conversion value, or parity, is the conversion ratio times the spotprice of the stock. The conversion price is the face value, or par value, of the bond divided by the conversion ratio.

results to other results in the literature and market prices. In Section 8 we conclude and discuss possible extensions of our results to more complex settings.

## 2. DEFINITION OF THE PROBLEM

In this article we use the formulation of option pricing in terms of self-financing objects, which we call tradables. In particular we will not use cash as this is not self-financing, instead we use zero-coupon bonds. More details can be found in [HN99a].

A convertible bond (CB) is a coupon-paying corporate bond  $B$  of maturity  $T$ , which has the embedded optionality to convert into a specified number, the conversion ratio  $k$ , of underlying stock  $S$ . The conversion is optimal, when the value  $V$  of the CB is less than the conversion value  $kS$

$$V(S, B, t) \leq kS$$

The conversion ratio may depend on time and the paths of the underlyings. When the CB is in-the-money, it behaves much like the underlying stock

$$V(S, B, t) \approx kS$$

where  $S$  has  $N_d$  dividend payments  $d_i S$  at dates  $t_{d,i}$ . When the CB is out-of-the-money it behaves like a coupon-paying corporate bond

$$V(S, B, t) \approx B = FP_T + \sum_{i:t_{c,i} \leq T} cFP_{t_{c,i}}$$

where  $P_s$  is a discount bond worth 1\$ at maturity  $s$ , the face value is denoted by  $F$ , and there are  $N_c$  coupon payments  $c$  (denoted as a fraction of the face value) at dates  $t_{c,i}$ . If the CB has not been converted, the payoff at maturity will be given by

$$V(S, B, T) = B$$

When there is mandatory conversion, this changes to  $V(S, B, T) = kS$ . A typical CB also has callable and puttable features. The callable feature allows the bond to be called by the issuer, when the CB price rises above a level  $M_C$  of cash. This limits the potential loss of the issuer. The puttable feature allows the holder to redeem an amount of cash when the CB price drops below  $M_P$  of cash. These features translate into the following bounds on the price of the convertible

$$M_P P_t \leq V(S, B, t) \leq M_C P_t$$

Here we write  $P_t = P_t(t)$  to denote that the constraint is in terms of cash at time  $t$ ,  $P_t = 1$  in units of the currency. The puttable and callable features may be time-dependent. Typically a contract is continuously callable, while the puttable feature is active only at a discrete set of times.

Also when a contract is called, the holder typically has the freedom to convert during a specified *notice* period after calling of the contract. This notice period is typically of the order of a few months. We will come back to the effects of the notice period in Section 4.

Furthermore the conversion ratio may be reset at  $N_r$  prespecified dates  $t_{r,i}$  according to specific rules. The reset feature is in general considered a difficult problem to deal with since it introduces a path-dependancy in the pricing of the contract. It turns out however that the path-dependancy is of the soft sort, similar to the case of barrier- and lookback- contracts. This means that we can move the path-dependancy to the jump conditions, keeping the pricing relatively simple.

## 3. MODELLING CONVERTIBLE BONDS

The modelling of a CB is a relatively complex issue, due to its sensitivity to interest rates, credit risk, and stock volatility. A proper model should, of course, include all these features. The more

realistic case with a stochastic interest-rate model and credit risk will be discussed in another article. In the present article we will keep things simple, since we want to focus on how to deal with resets in the pricing problem. Also we will consider a complete market with the usual assumptions; we are allowed to short stocks, have no transaction costs etc. When there is a deterministic relation between discount bonds with different maturities we have assuming, for simplicity, continuous compounding and constant rate  $r$ ,

$$P_s(t) = e^{r(T-s)} P_T(t)$$

This implies that we can rewrite every occurrence of a discount bond with a maturity different from  $T$  into one maturing at  $T$  times some deterministic time-dependent factor. This simplifies the discussion considerably. Cashflows at any time  $s$  can be converted easily into discount bonds maturing at time  $T$  using the above relation. For example, the coupon bond at time  $t$  becomes

$$B = FP_T + \sum_{i:t \leq t_{c,i} \leq T} cF e^{r(T-t_{c,i})} P_T$$

Here and in the following we will use the shorthand notation  $P \equiv P_T$ . Since the contract defines the exchange between corporate bonds and stock, credit risk is involved with every occurrence of a corporate discount bond. The rate  $r$  should therefore be understood as being the risky rate, including a credit spread, for that particular corporate discount bond. This is clearly a rather simple approach to credit risk since it does, for example, not incorporate the correlation between a drop in the stockprice and an increase in the credit spread. Still by formulating the pricing problem in terms of the underlying instruments it should provide a reasonable first order approximation to incorporating credit risk. We will come back to the credit risk issue in more detail in the second article. The stockprice is modelled by a geometric Brownian motion:

$$\frac{dS}{S} = \sigma dW + \dots$$

w.r.t. the discount bond-price  $P$  maturing at  $T$  and  $\sigma$  denotes the deterministic volatility function<sup>2</sup> of the stock. The  $W$  is Brownian motion (under the forward- $T$  measure) and the dots denote irrelevant drift terms.

Due to the deterministic interest rate term structure, the only two relevant variables in the problem become  $S$  and  $P$ . The value of the CB at time  $t$  is therefore denoted by  $V(S, P, t)$  and it satisfies the following PDE (see Ref. [HN99a] for details)

$$\left( \partial_t + \frac{1}{2} \sigma^2 S^2 \partial_S^2 \right) V(S, P, t) = 0 \tag{3.1}$$

which is the symmetric version of the Black-Scholes equation. They both lead to the same answer, but the above equation is much more convenient to use both in deriving analytic results as well as in solving pricing equations numerically. The convertible price should satisfy the following constraints

$$V(S, P, t) \geq kS$$

and

$$M_P e^{r(T-t)} P \leq V(S, P, t) \leq M_C e^{r(T-t)} P$$

Note that these constraints may be time-dependant, e.g. CB's are typically call-protected during the first years. The boundary conditions are as follows. At maturity we have the payoff

$$V(S, P, T) = (F + cF)P$$

---

<sup>2</sup>It is of course easy to introduce a local volatility function:  $\sigma(t) \rightarrow \sigma(S, P, t)$ . We will not deal with this case in the present work.

When there is a mandatory conversion at maturity this changes accordingly to

$$V(S, P, T) = kS$$

For  $S \rightarrow \infty$  we have the condition

$$V(S, P, t) \rightarrow kS$$

For  $S \rightarrow 0$  we have the condition

$$V(S, P, t) = FP + \sum_{i:t \leq t_{c,i} \leq T} cF e^{r(T-t_{c,i})} P$$

At the coupon-payment dates  $t_{c,i}$  we have the jump-conditions

$$V(S, P, t_{c,i}^-) = V(S, P, t_{c,i}) + cF e^{r(T-t_{c,i})} P$$

These are trivial to implement. At the dividend-payment dates  $t_{d,i}$  we have the jump-conditions

$$V(S, P, t_{d,i}^-) = V(S(1 + d_i), P, t_{d,i})$$

The jump condition at ex-dividend dates can be removed by using a different variable instead of  $S$  as we will show now. At a dividend payment date  $t_{d,i}$  we have

$$S(t_{d,i}) = \frac{S(t_{d,i}^-)}{1 + d_i}$$

Now introduce a new variable  $\tilde{S}$ , which is proportional to the self-financing portfolio, hence a tradable, consisting of the stock together with its dividends. In the case of known stock dividends  $d_i S$ , this new tradable  $\tilde{S}$  is just proportional to  $S$  itself, where the factor of proportionality,  $D \leq 1$ , jumps at dividend payments. The variable is normalized such that it coincides with  $S$  at maturity.

$$\tilde{S}(t) \equiv S(t) \prod_{i:t \leq t_{d,i} \leq T} (1 + d_i)^{-1} \equiv S(t) D(t)$$

Note that  $\tilde{S}$  just follows geometric Brownian motion without jumps. In terms of  $\tilde{S}$ , we do not need to include jump conditions for the dividends in the PDE at all. The effect of dividends<sup>3</sup> moves entirely to the boundary conditions. In the present case this means that we have to change the conversion condition to

$$V(S, P, t) \equiv \tilde{V}(\tilde{S}, P, t) \leq \frac{k\tilde{S}}{D(t)}$$

Here the tilde is used to indicate the price as a function of the tradable  $\tilde{S}$ . Since  $D(t)$  is an increasing function of time, it increases the incentive to early convert the CB. In the case of discrete dividends, the optimal conversion will be just before an ex-dividend date, since the conversion value decreases at that moment. The usefulness of this parametrization extends clearly beyond the present case. Similar considerations allows us to treat cash-dividends in a consistent manner [HN00a].

---

<sup>3</sup>The case of continuous dividends can be treated in a similar manner, but we do not have to care about any jump-conditions. In the continuous case we have  $D(t) = e^{-q(T-t)}$ .

## 4. THE NOTICE PERIOD EFFECT

The notice period  $\tau_n$  is normally not taken into account in the numerical evaluation. At first sight it looks like a complex boundary condition. But a closer look reveals that it can be dealt with in a straightforward way. The CB may be called when

$$V(S, P, t) \geq M_C e^{r(T-t)} P \quad (4.1)$$

When the value of the CB is above this boundary, the issuer is allowed to call the CB. In the case of calling, the holder is still allowed to convert during the notice period. So the holder has in fact another CB with value  $V'$ , a lifetime equal to  $\tau_n$ , conversion ratio  $k$ , and face value  $M_C \exp(r(T-t_n))P$ , where  $t_n$  is the time of calling. But there are no more callable or puttable features to take care of. This fact allows one to compute the value  $V'$  (semi-)analytically.

The conversion, after calling, depends completely on the holder of the contract. Since the typical period is between one and four months, there will be at most one coupon and one dividend payment during the notice period. If there are no coupon- and dividend payments during the notice period, there is no reason to convert early by the holder. The CB is just valued as an European call plus bond. In the case of one dividend payment, the only optimal early conversion is just before the ex-dividend date. In that case the CB is valued by a bivariate integral. Again this can also be incorporated without too much work. Finally coupon payments are trivial to deal with. So in all cases one is able to compute  $V'$ . Since  $V'$  has time value it will be above the call boundary value.

The issuer is allowed to call when Eq. (4.1) holds. Whether it is optimal to call depends on the value of  $V$  and  $V'$ . When called the contract will be worth  $V'$  to the holder. So for the issuer calling will depend on

$$V(S, P, t_-) = \min(V(S, P, t), V'(S, P, M_C, t))$$

When  $V < V'$  on the RHS, there is clearly no reason to call. On the other hand, when  $V > V'$  on the LHS it is optimal to call. The effect of the notice period translates to an effective call boundary value  $V'$ ; in a numerical scheme we have to replace the condition of calling, Eq. (4.1), by

$$V(S, P, t) \leq V'(S, P, M_C, t)$$

Note that when  $\tau_n \rightarrow 0$ , the time value of  $V'$  goes to zero too and the call boundary value will coincide with the original call value.

Often CB's contain a make-whole feature, which compensates the holder by paying an extra amount of cash or increasing the conversion ratio, when the contract is called. Clearly such features can be simply incorporated by adjusting  $V'$  appropriately.

By taking into account the notice period one adds extra optionality for the holder of the CB. Hence the contract will become slightly more expensive. The numerical effect is however small for typical notice periods and we will neglect it in the numerical computations of Section 7.

## 5. RESETTING CONVERSION RATIOS

When the stockprice drops it becomes less attractive to convert the CB. Here enters the reset, which allows the conversion price to be refixed in order to get parity around par during the lifetime of the contract. The reset increases the value of the contract for the holder as it improves the probability of conversion even with falling stock prices. For the issuer resets are also attractive because the higher price allows them to lower the coupon rate. This also makes it understandable why Japanese corporations were especially interested in such contracts. In the deteriorating Japanese market resets were added to CB's to increase their attractiveness.

For example, a CB with a downward reset protects the holder from large drops in value of the underlying stock by resetting the conversion price  $F\$/k_i$  downward such that the conversion value  $k_i S$  is at-the-money with the face value  $F\$ = F P_{t_{r,i}}(t_{r,i})$  at prespecified dates  $t_{r,i}$  ( $i = 1, \dots, n \equiv N_r$ )

during the lifetime of the contract. Since a downward reset of the conversion price increases the conversion ratio and hence increases the dilution, the reset is often floored and capped by multiples of previous conversion prices. In practice many of the contracts have only one reset, a few have two or even three resets.

Since resets, in general, introduce path-dependance in the contract, the usual approach to price resettable CB's is to introduce an extra degree of freedom to take care of that fact. In many cases found in practice this turns out to be an unnecessary complication. Instead the resets can be treated as advanced jump-conditions at the reset-dates. This makes the pricing of resettable convertibles not more complicated than other types of convertibles.

In Ref. [ML99] two types of resets are considered. The first case is the step-down reset, which is simply a convertible with a deterministic time-dependant conversion price. Clearly, this does not require any additional trickery to price above standard CB's. So we will not discuss this type of contract any further.

In the second case the conversion price is reset in order to get parity around par<sup>4</sup>, but capped and floored by multiples of previous values of the conversion price. In this article we will focus on the reset rule, given in Eq. (5.1). In terms of the conversion ratio, it boils down to the following:

$$k_i = \max \left( \alpha k_{i-1}, \min \left( \beta k_{i-1}, \frac{FP_{t_r,i}(t_{r,i})}{S(t_{r,i})} \right) \right) \quad (5.1)$$

where the conversion ratio is capped and floored by multiples of the *previous* conversion ratio  $k_{i-1}$ , with  $\alpha \leq 1$  and  $\beta \geq 1$  typically.

The reset rule, most often found in practice, is as follows:

$$k_i = \max \left( \alpha k_{i-1}, \min \left( \beta k_0, \frac{FP_{t_r,i}(t_{r,i})}{S(t_{r,i})} \right) \right) \quad (5.2)$$

Here the conversion ratio is capped by a multiple of the previous conversion ratio  $k_{i-1}$  from above and floored by a multiple of the initial conversion ratio  $k_0$  from below. Contracts with the reset rule Eq. (5.2) can be valued using our method, when there is *only* one reset. In that case, Eqs. (5.1) and (5.2) are identical. With deterministic interest rates, we then get:

$$k_1 = \max \left( \alpha k_0, \min \left( \beta k_0, \frac{Fe^{r(T-t_r)}P(t_r)}{S(t_r)} \right) \right)$$

Note that the reset rule of Eq. (5.1) is more expensive as the size of the window of opportunity at a given reset time  $t_{r,i}$ ,  $[\alpha k_{i-1}S(t_{r,i}), \beta k_{i-1}S(t_{r,i})]$ , remains constant over time, while in the case of Eq. (5.2) this window-size may shrink over time, thus losing some value.

In the market, one approximates the price of contracts, with resets according Eq. (5.2), as if only the first reset is present assuming that the stock price at the first reset date will have dropped so much as to kill all remaining resets. When this is not the case the game is repeated but now with the second reset date as the only reset. Such an approximation will clearly underestimate the price of the contract, since it neglects the value of the remaining resets. On the other hand when one would use the reset rule Eq. (5.1) as a proxy for Eq. (5.2), the price is overestimated, since the size of the window of opportunity remains constant over time. A better proxy might be to take an average of both cases, where both contributions are weighted with their respective probabilities of occurring. In a deteriorating market however, the first proxy may not be such a bad proxy after all.

From section 2 it is clear, that the only way  $S$  enters the boundary conditions is through the payoff condition at maturity and the reset conditions. The boundary conditions for callable and puttable features remain the same. Now what happens to the reset conditions? If we can rewrite a reset

---

<sup>4</sup>In practice the reset depends on the arithmetic average of the closing stockprices over a specified period before and at the reset date. We will just take the stockprice at the reset-date.

condition in terms of the conversion value, then we will be able to put all the effects of the resets in jump conditions of the following form

$$v(y, t_-) = v(\max(\alpha y, \min(\beta y, \gamma)), t)$$

and we do not have to add any extra degree of freedom to solve the problem. This turns out to be the case, with multiple resets, for the reset rule of Eq. (5.1) and, with only one reset<sup>5</sup>, for Eq. (5.1).

As an example, first consider the simplest case with one reset at time  $t_1$ . There are no callable or puttable features, no coupons and dividends etc. At maturity the value of the CB is simply

$$V(x, 1, T) = \max(k_1 x, F)$$

where we pulled out the numeraire  $P$  to simplify the equation, and introduced  $x \equiv S/P$ . Note that this is just a function of the conversion value  $y_1 \equiv k_1 x$  only. So the value of the contract at  $t_1$  is

$$f(y_1) = \int \max(y_1 \phi(z - \sigma \sqrt{T - t_1}), F \phi(z)) dz$$

of course still a function of  $y_1$ . Here  $\phi$  denotes the standard normal pdf. The derivation of the above, very useful, formula can be found in Ref. [HN99a]. The reset condition of  $k_1$  at  $t_1$  is given by

$$\begin{aligned} y_1 &= \max(\alpha k_0 x, \min(\beta k_0 x, F_1)) \\ &= \max(\alpha y_0, \min(\beta y_0, F_1)) \end{aligned}$$

where  $y_0 \equiv k_0 x$  and  $F_1 \equiv F \exp(r(T - t_1))$ . At time  $t_1$ ,  $y_1$  is a function of  $y_0$  only. The payoff at time  $t_1$  in terms of variable  $y_0$  can thus be written as

$$f(\max(\alpha y_0, \min(\beta y_0, F)))$$

This is used as initial value for the PDE in terms of variable  $y_0$  until the present time  $t$ . In Fig. 1 the adjusted payoff at time  $t_1$  is shown for the case  $\alpha = 0.95$  and  $\beta = 1.5$ . In the case where there

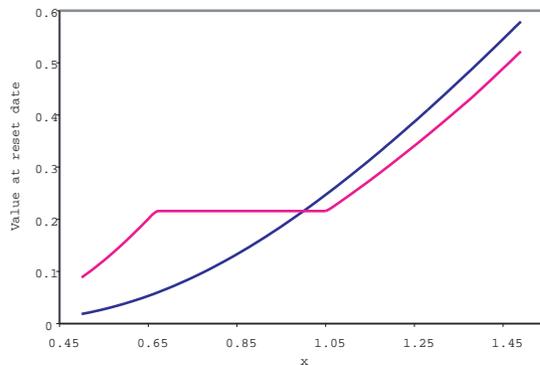


Figure 1: The effect of the jump condition at the reset date  $t_1$ .

are coupons, dividends, callable and puttable features we can use the same trick, but then we have to solve the PDE for  $y_1$  of course numerically from  $T$  back to  $t_1$ .

<sup>5</sup>For Eq. (5.2) with multiple resets Monte Carlo experiments show that our method serves as a good proxy. We compared the European versions of resettable convertibles with the first reset condition, a cap  $\alpha = 1$ , floor  $\beta = 0.8$ , and volatility  $\sigma = 50\%$  gives a difference in price of order 1%. Since the second condition is more risky, it will have the lowest price of the two. The resets are not really sensitive to callable and puttable features, so the difference in prices should also hold for the more general case.

Now consider the case of multiple resets with the reset condition specified by

$$k_i x = \max(\alpha k_{i-1} x, \min(\beta k_{i-1} x, F_i))$$

where  $F_i \equiv F \exp(r(T - t_i))$ . Since the contract only depends on the products  $y_i \equiv k_i x$ , we can proceed as above. Starting at maturity with the payoff

$$v(y_n, T) = \max(y_n, F)$$

we solve the PDE backward, in terms of variable  $y_n$ , in time until the last reset date  $t_{r,n}$ . At that time the value of the contract is given by  $v(y_n, t_{r,n})$ . Now we use the definition of the conversion ratio reset-rule of  $k_n$  at  $t_{r,n}$  to rewrite  $y_n$  at time  $t_{r,n}$  as a function of the new variable  $y_{n-1} \equiv k_{n-1} x$ .

$$\begin{aligned} y_n &= \max(\alpha k_{n-1}, \min(\beta k_{n-1}, \frac{F_n}{x}))x \\ &= \max(\alpha y_{n-1}, \min(\beta y_{n-1}, F_n)) \end{aligned}$$

So the value of the contract at time  $t_{r,n}$  can now be expressed as

$$v(\max(\alpha y_{n-1}, \min(\beta y_{n-1}, F)), t_{r,n})$$

This in turn is a function of  $y_{n-1}$  only. The PDE in terms of  $y_{n-1}$  is identical to the one in terms of  $y_n$  and it is solved backward in time until time  $t_{r,n-1}$ . The payoff at time  $t_{r,n-1}$  in terms of variable  $y_{n-1}$  can again be rewritten in terms of  $y_{n-2}$ . Clearly the procedure can be repeated ad infinitum. Also note that the reset rule may be time-dependent.

## 6. RESETTABLE CB'S ARE LOOKBACKS IN DISGUISE

In this section we show that for a particular choice of the reset there is a close relation between convertibles and lookbacks. This provides another understanding of why the resets only introduce soft path-dependancy. To this end we drop the ceiling ( $\beta \rightarrow \infty$ ) on the conversion ratio. Working out the recursion relation with given  $k_0$ , we get for  $i = 1 \dots n$ :

$$k_i = \max(k_0, \max_{j=1 \dots i} e^{r(T-t_{r,j})} \frac{FP(t_{r,j})}{S(t_{r,j})})$$

As we saw earlier the only combination of relevance is the following expression, defined for  $t_{r,i} \leq t \leq t_{r,i+1}$ :

$$k_i S(t) = \max(k_0 S(t), \max_{j=1 \dots i} e^{r(T-t_{r,j})} \frac{FP(t_{r,j})}{S(t_{r,j})} S(t))$$

Using the formulation of Ref. [HN99b] we can relate this to the tradables  $X_s(t)$  defined through

$$X_s(t) = \begin{cases} P(t) & t < s \\ \frac{P(s)}{S(s)} S(t) & t \geq s \end{cases}$$

If we set the initial conversion ratio at-the-money,  $k_0 \equiv F e^{r(T-t_0)} P(t_0) / S(t_0)$ , we get

$$k_i S(t) = \max_{j=0 \dots i} e^{r(T-t_{r,j})} F X_{t_{r,j}}(t) \tag{6.1}$$

Thus the value of  $k_i S(t)$  at times  $t \geq t_{r,i}$  is equal to the value of the weighted maximum of a set of tradables  $X_{t_{r,j}}(t)$ . In fact, since we have the freedom to switch numeraires, we can exchange  $S$  and  $P$  and with tradable objects  $Y_s(t)$  defined through

$$Y_s(t) = \begin{cases} S(t) & t < s \\ \frac{S(s)}{P(s)} P(t) & t \geq s \end{cases}$$

we can relate the expression with  $k_i S$  to

$$\hat{k}_i P(t) = \max_{j=0\dots i} e^{r(T-t_{r,j})} F Y_{t_{r,j}}(t)$$

The  $Y_s$  are tradables, that transport the value of  $S$  at time  $s$  to a later time. So the term  $\hat{k}_i$  keeps track of a weighted maximum of the stock price. Now the governing PDE remains unchanged. This can be simply understood from the homogeneity of the price as a function of the tradables. Thus an appropriate change of variables links a convertible bond with an American lookback.

## 7. RESULTS

We tested the model using various contracts traded on the Japanese markets. To this end we solved the one-dimensional PDE numerically. In Ref. [HN00b] an alternative mixed finite-difference (FD) scheme, dubbed ‘tradable scheme’ is proposed to solve the PDE in Eq. (3.1). It has attractive features compared to the conventional Crank-Nicholson (CN) scheme and we discuss it shortly here. The idea of ‘tradable schemes’ is as follows. We assume that we are able to solve analytically a given PDE for a given set of simple boundary conditions. This solution  $R$  is then used to construct a FD-scheme such that the scheme solves  $R$  exactly at the grid points. In contrast to the usual schemes such as CN these schemes behave very nicely, when there are boundary layer problems, e.g. asian options. Furthermore it can be formulated in a very compact manner. More details can be found in the article mentioned above.

To compute the price of the convertible, using the tradable scheme, we need a tradable for which we can compute an exact solution. In the case of deterministic interest rate we can use a power-tradable

$$R_\alpha(S, P, \Sigma) = \left( \frac{S}{e^{-\frac{1}{2}\alpha\Sigma^2} P} \right)^\alpha e^{-\frac{1}{2}\alpha\Sigma^2} P$$

where  $\Sigma \equiv \sigma\sqrt{T-t}$  in the case of constant volatility. We use  $R_2$  to fit the scheme to.

Our results compare favorable with results from commercial packages for the same parameter settings. Clearly our approach is much faster and more accurate, than the usual algorithms, due to the reduction of the statespace.

In Figures 2, 3, and 5 we have plotted the price, the delta, and gamma for three CB’s differing in the number of resets. The effect of the resets becomes more pronounced as the number of resets increases. The resets make the convertible less sensitive to changes of the conversion-value in the region where the contract is allowed to reset.

In Figure 4 the absolute difference of the delta of the two contracts with resets w.r.t. the one without resets is given. The impact is quite dramatic. A similar effect can be seen for the gamma’s and hence also for the theta’s and vega’s. Here one sees in fact that the gamma may become negative.

When one compares the results with the actual prices in the market, there is a clear discrepancy. For all contracts we have considered the model gives a too high price. The implied vol of the contract is much lower than the historical vol over say the last month or so. Typical values are 40% and 55% respectively. In a way this signals that one does not price the embedded option correctly, too cheap. So this might provide an explanation for the interest of hedge funds in resettable convertibles.

## 8. CONCLUSIONS

We have shown that resets, when using the right coordinates, do not introduce any extra factor in the PDE used to price the convertible bond. This significantly reduces the complexity of the problem. In fact one can show that convertibles are related to lookbacks via a change of numeraire. A simple algorithm is presented to take into account the effects of the notice period. All results are given in a setting with stockprices driven by geometric Brownian motion and deterministic interest rates, but they carry over to more complex and realistic situation with stochastic interest rates and credit risk too. This will be discussed in a follow-up article.

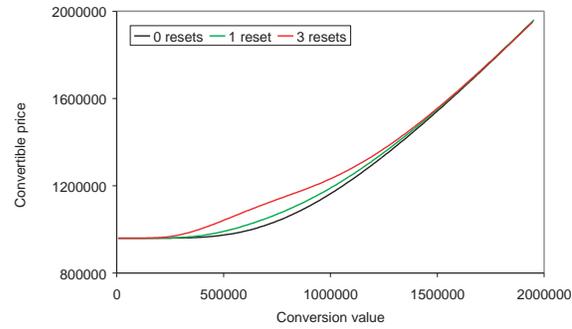


Figure 2: The price of a CB with 0,1, and 3 resets as a function of the conversion value.

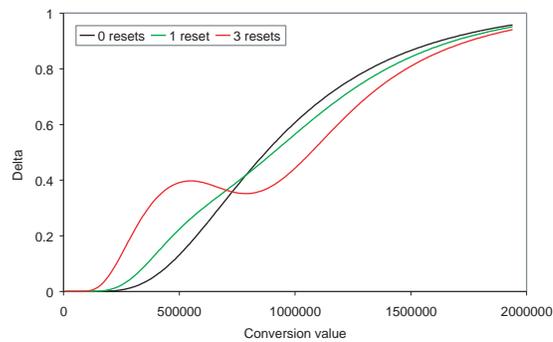


Figure 3: The delta of a CB with 0,1, and 3 resets as a function of the conversion value.

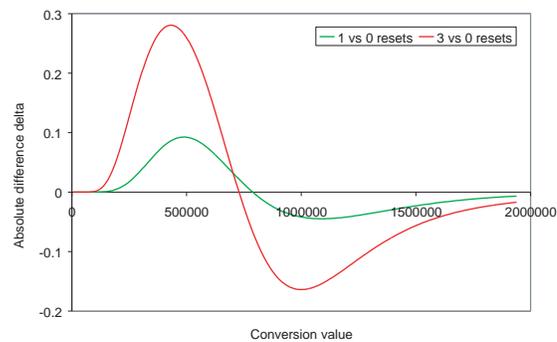


Figure 4: The absolute difference of the delta of a CB with 1 and 3 resets w.r.t. one without reset as a function of the conversion value.

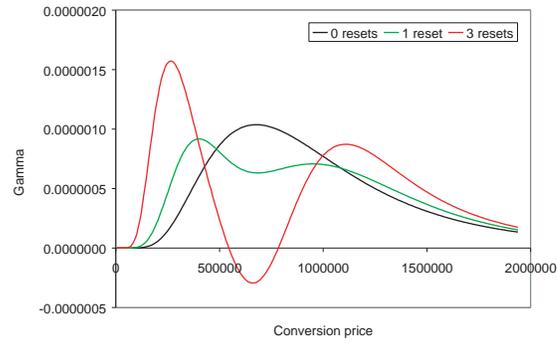


Figure 5: The gamma of a CB with 0,1, and 3 resets as a function of the conversion value.

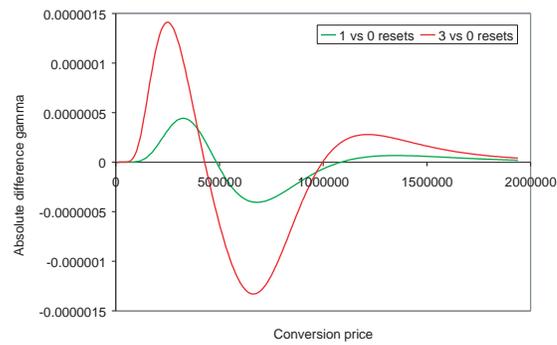


Figure 6: The absolute difference of the gamma of a CB with 1 and 3 resets w.r.t. one without resets as a function of the conversion value.

## References

- [HN99a] J.K. Hoogland and C.D.D. Neumann, *Local scale invariance and contingent claim pricing II: path-dependent contingent claims*, IJTAF, **4**, (2001), 1, 1–21.
- [HN99b] J.K. Hoogland and C.D.D. Neumann, *Local scale invariance and contingent claim pricing II: path-dependent contingent claims*, IJTAF, **4**, (2001), 1, 23–43.
- [HN00a] J.K. Hoogland and C.D.D. Neumann, *Asians and cash dividends: exploiting symmetries in pricing theory*, Technical Report MAS-R0019, CWI, (2000).
- [HN00b] J.K. Hoogland and C.D.D. Neumann, *Tradable schemes*, Technical Report MAS-R0024, CWI, (2000).
- [ML99] Merrill Lynch, *Japanese Refixable Convertibles*, Global Securities Research & Economics Group/Global Convertibles Research Group, Merrill Lynch, (1999) .
- [N98] I. Nelken, *Japanese reset convertible bonds*, <http://www.supercc.com/papers/japan.htm>, (1998).
- [W98] P. Wilmott, *Derivatives*, Wiley, (1998).