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V. M. Lelevkin, I. Rafatov

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P.O. Box 94079, 1090 GB Amsterdam (NL)
Kruislaan 413, 1098 SJ Amsterdam (NL)
Telephone +31 20 592 9333
Telefax +31 20 592 4199

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ABSTRACT
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# Modeling of non-equilibrium spherical microwave discharge

Valery M. Lelevkin [1] and Ismail R. Rafatov [2,3]

*[1]Kyrgyz-Russian Slavonic University, 44, Kievskaya, 720000, Bishkek, Kyrgyzstan, krsu@krsu.edu.kg*

*[2]CWI, P.O.Box 94079, 1090 GB Amsterdam, The Netherlands, Ismail.Rafatov@cwi.nl*

*[3]American University - Central Asia, 205, Abdumomunova, 720000, Bishkek, Kyrgyzstan*

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## 1. Introduction

An interest to the study of the spherical microwave discharges arises from the various technological applications, as well as from the analogy between the spherical discharge and ball lightning, which (according to the hypothesis [1], [2]) is supported by the energy of the electromagnetic field. A spherical microwave discharge at atmosphere pressure has been studied in [3]-[6] in the framework of LTE approximation of the plasma. The present work is devoted to the modeling of a non-equilibrium spherical microwave discharge. For the construction of the model we basically follow the assumptions and methods, used in [3] and [4] for the modeling of non-equilibrium cylindrical discharges. The model is derived on the basis of MHD equations in the framework of a two-temperature partial LTE approximation for the plasma and consider two cases: the state of ionization equilibrium and deviation from this state.

## 2. Model and method of solution

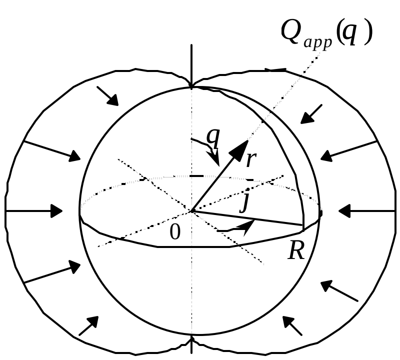


***Fig.1:*** *Scheme of spherical discharge. $R$ is the radius of discharge chamber. $r$, $j$ and $q$ are spherical coordinates.*

It is supposed that the axial symmetric electric discharge is taking place inside a spherical chamber with a dielectric wall (fig. 1) and is supported by the energy of convergent electromagnetic waves with components $\vec{E} = \vec{E}(E_r, E_q, 0)e^{iwt}$, $\vec{H} = \vec{H}(0,0,H_j)e^{iwt}$ and power $Q_{app}$. We assume that the processes are quasistationary. Maxwell's distribution for the speeds of the particles and Bolzman's distribution for the excited levels are satisfied. The plasma is quasineutral and consists of electrons and heavy particles (atoms, ions). The movement of the gas has no significant influence on the characteristics of the discharge. The energy of the electromagnetic field is absorbed basically by the electronic gas, and atoms and ions are heated up as a result of collisions with electrons.

*Note*: This work has been presented at the *Vth International Workshop on Microwave Discharges: Fundamentals and Applications*, Greifswald, Germany, July 08 -12, 2003.

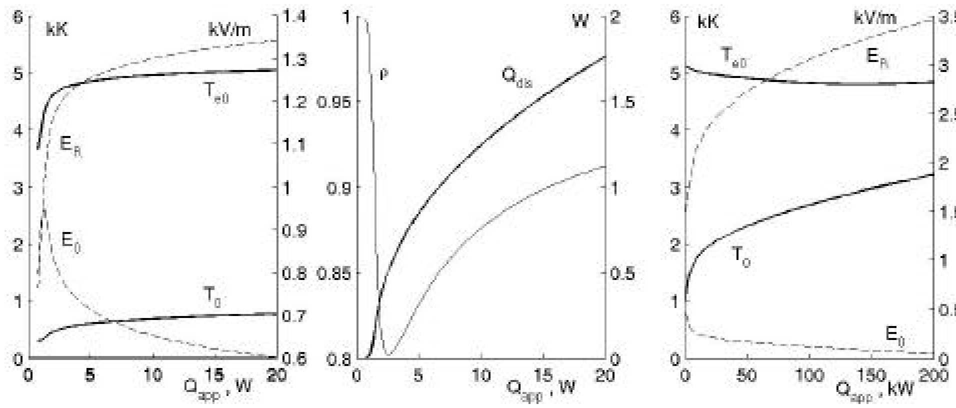


***Fig**.2: Characteristics of the discharge at the center ($T_0$, $T_{e0}$, $E_0$) and at the border ($E_R$) of the discharge chamber, reflection coefficient $\rho$ and dissipated power $Q_{dis}$ as functions of the power of the applied electromagnetic field $Q_{app}$.* $R$ =1cm, $\omega$ =15GHz (model $\dot{n}_e = 0$)

Equations, used to describe the discharge (and accordingly the problem), may conventionally be separated to three parts: electrodynamic problem, the energy balance problem, and problem of calculation of the structure of plasma.

## 2.1 Electrodynamic problem

We consider Maxwell's equations in the spherical coordinate system

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta H_\varphi\right) = i\omega\varepsilon_0\varepsilon_k E_r, \quad \frac{1}{r}\frac{\partial}{\partial r}\left(rH_\varphi\right) = -i\omega\varepsilon_0\varepsilon_k E_\theta,$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(rE_\theta\right) - \frac{1}{r}\frac{\partial}{\partial\theta}\left(E_r\right) = -i\mu_0\omega H_\varphi. \tag{1}$$

Here $\varepsilon_k = \varepsilon_1 - i\varepsilon_2$ is the complex dielectric permeability, $\varepsilon_1 = 1 - \sigma/\varepsilon_0\nu_e$, $\varepsilon_2 = \sigma/\varepsilon_0\omega$, $\omega$ the frequency of the electromagnetic field, $\mu_0$ and $\varepsilon_0$ the absolute magnetic and dielectric permeability's, $\nu_e$ the collision frequency, $\sigma$ the electric conductivity.

Introducing a scalar function $\chi(r,\theta) = r\sin\theta H_\varphi$ [3], we obtain from (1) the expressions for the components of the strength of the electric field $E_r$ and $E_\theta$, and an equation for $\chi$. Using a separation of variables, we have a solution in the form of the sum of modes $\chi = \sum_{n=1}^{\infty}\chi_{1n}(r)\chi_{2n}(\theta)$. Functions $\chi_{1n}$ and $\chi_{2n}$ are calculated by means of Hankel and Legendre functions $\chi_{1n} = \sqrt{z}(C_n H^{(1)}_{n+1/2}(z) + B_n H^{(2)}_{n+1/2}(z))$, $\chi_{2n} = (1-x^2)P'_n(x)$, ( $z = k_e r$, $k_e = \omega\sqrt{\varepsilon_0\mu_0}$, $x = \cos\theta$ ) outside the area of discharge, and $\chi_{1n}$ is calculated numerically inside the area. The boundary conditions, used for the solution, express the presence of an axial symmetry and a vanishing of the magnetic field in the center of the discharge. A value of the flow of Poynting vector gives us the relationship between $C_n$ and the applied power $Q_{app}$ by

$$Q_{app} = \frac{m_0 w}{k_e} p \sum_{n=1}^{\infty} \left( \left| C_n \right|^2 \frac{2}{2n+1} n(n+1) \right).$$

The coefficients of the dissipated and reflected waves are determined from the 'sewing' of the components $H_j$ and $E_q$ of strengths of electric and magnetic fields at the wall of the discharge camera $r = R$. The reflection coefficient is $r_n = | B_n |^2 / | C_n |^2$. The first mode ($n = 1$) is used in calculations.

### 2.2 Energy balance problem

The equations of energy balance for electrons and heavy particles in the vector form [3], [4] are

$$\nabla \left( \vec{v} n_e \left( \frac{5}{2} k T_e + U_I \right) \right) = \nabla \left( l_e \nabla T_e \right) + Q_E - B_{ea} \left( T_e - T \right), \tag{2}$$

$$\nabla \left( l \nabla T \right) + B_{ea} \left( T_e - T \right) = 0 , \tag{3}$$

Here $T$ is the temperature, $l$ the heat conductivity, $Q_E = \frac{1}{2} s E^2$, where $E = \sqrt{E_r E_r^* + E_q E_q^*}$ ('$*$' denotes a complex conjugate), $\vec{v} = \vec{v}_{am} + \vec{v}_t$, $\vec{v}_{am} = -D_{am} \nabla \ln n_e$, $\vec{v}_t = -D_{am} \nabla \ln T_e / 2$ are the velocities of ambipolar and thermal diffusion, $D_{am}$ a coefficient of ambipolar diffusion, $U_I$ the ionization potential, $B_{ea} = 3 d_e n_e n_e k / 2$ the coefficient of interaction between electron and heavy particles, $d_e = 2 m_e / m_a$, $m$ and $n$ are the mass and concentration of particles, $k$ the Bolzman's constant, $e, i, a$ denote electrons, ions, and atoms.

Equations (2), (3), after rewriting them in spherical coordinate system and averaging over the angle $q$, are solved with the boundary conditions $T_e'(0) = 0$, $T'(0) = 0$, $T_e'(R) = 0$, $T(R) = T_R$ with $T_R = 300 K$.

### 2.3 Problem of structure of plasma

The structure of the plasma is derived from the continuity equation for the electrons [3],

$$\nabla \left( \vec{v} n_e \right) = \dot{n}_e , \tag{4}$$

where

$$\dot{n}_e = K_I n_e n_a - K_r n_e^2 n_i . \tag{5}$$

Equations (4) and (5) are complemented by the condition of quasineutrality, the equations of state and Dalton's law $n_e \approx n_i$, $p = n_e k T_e + (n_i + n_a) k T$. Here $K_I$ and $K_r$ are ionization and recombination coefficients, and $p$ the value of pressure. Equation (4), after rewriting it in spherical coordinates and averaging over the angle $q$, is solved with boundary conditions $n_e(0) = 0$, $(D_{am} n_e(r))'_R = -n_e(R) \sqrt{k T_R / 2 p m_a}$.

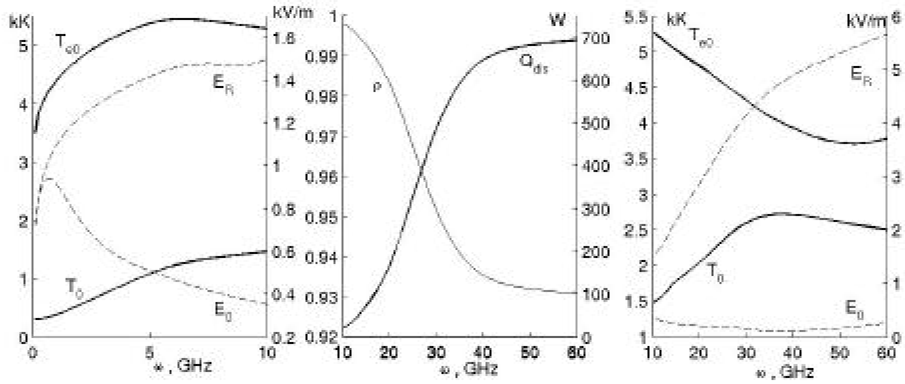


***Fig**.3: Characteristics of the discharge at the center ($T_0$, $T_{e0}$, $E_0$) and at the border ($E_R$) of the discharge chamber, reflection coefficient $r$ and dissipated power $Q_{dis}$ as functions of the frequency $w$ of the applied electromagnetic field.* $R$ =1cm, $Q_{app}$ =10 kW (model $\dot{n}_e = 0$)

The plasma coefficients are determined as functions of $T_e$, $T$, $p$ and concentrations of particles [3].

## 2.4 Ionization equilibrium model

If we assume that the processes of ionization in the plasma are compensated by processes of recombination ($\dot{n}_e = 0$), the problem is simplified essentially. In this case, we do not need the equation of continuity (4), the left term of the (2) vanishes, and (5) turns to the Saha equation [3].

## 3. Results

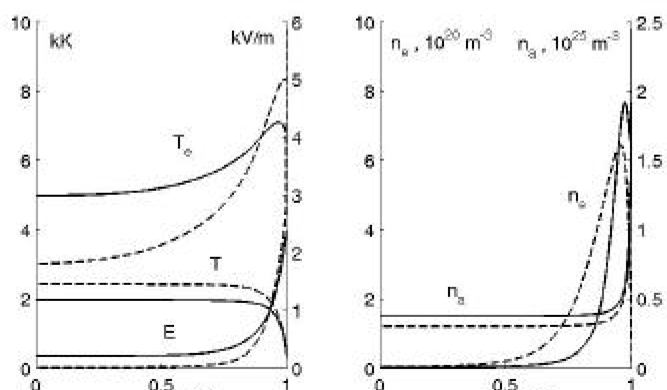


***Fig**.4: Profiles of temperatures $T$ and $T_e$, concentrations $n_e$ and $n_a$, and $E$ in the case of $\dot{n}_e = 0$ (solid lines) and $\dot{n}_e \neq 0$ (dotted line). $R = 1cm$, $w$ =15GHz, $Q_{app}$ =20kW.*

Calculations show that the characteristics of the spherical discharge are close to the results, presented in [3] and [4] for cylindrical discharges.

The plasma of a spherical microwave discharge in argon at atmospheric pressure is essentially non-equilibrium. At $R$=1cm and $w$ =15GHz, and sufficiently small power $Q_{app}$ essentially constant radial distributions of $T$ and $T_e$ are realized, and the temperature of heavy particles is close to $T_R = 300K$, the plasma is almost transparent for electromagnetic waves (fig. 2). With the increase the $Q_{app}$, the profiles of $T_e$, $n_e$, and the distribution of Joule's heat transfer get strongly expressed peaks at the wall of the chamber. The values $E$ and $n_e$, and the difference between $T_e$ at $T$ decreases in the axis region and increases at the peripheries (fig.2). The area of dissipation of the applied power shifts to the periphery of the discharge. Like in the case of a cylindrical microwave discharge [3], the area of "hot" electrons, which intensively dissipate the energy of the applied electromagnetic field and shield its penetration in the axis zone, is realized at the periphery. With the increase of $w$ or $R$, the dissipation of the electric field in the plasma grows, and the characteristics of the discharge change qualitatively resemble with the increase of $Q_{app}$ (fig. 3). Comparing the models, corresponding to $\dot{n}_e = 0$ and $\dot{n}_e \neq 0$, show that in the case of deviation of the state of ionization equilibrium, the density of electrons $n_e$, strength of electric field $E$ and a difference between $T_e$ and $T$ are smaller near the center of discharge, and bigger near the wall of chamber (fig.4).

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