REPORTRAPPORT

PNA

Probability, Networks and Algorithms

Probability, Networks and Algorithms

Mixture formulae for shot noise weighted point processes

P. Gregori; M.N.M. van Lieshout; J. Mateu.

REPORT PNA-R0310 SEPTEMBER 02, 2003

CWI is the National Research Institute for Mathematics and Computer Science. It is sponsored by the Netherlands Organization for Scientific Research (NWO).
CWI is a founding member of ERCIM, the European Research Consortium for Informatics and Mathematics.

CWI's research has a theme-oriented structure and is grouped into four clusters. Listed below are the names of the clusters and in parentheses their acronyms.

Probability, Networks and Algorithms (PNA)

Software Engineering (SEN)

Modelling, Analysis and Simulation (MAS)

Information Systems (INS)

P.O. Box 94079, 1090 GB Amsterdam (NL)
Kruislaan 413, 1098 SJ Amsterdam (NL)
Telephone +31 20 592 9333
Telefax +31 20 592 4199

ISSN 1386-3711

Mixture formulae for shot noise weighted point processes

ABSTRACT
We investigate the connections between shot noise weighted and bivariate Gibbs point processes.

2000 Mathematics Subject Classification: 60G55, 62M30.
Keywords and Phrases: Influence function; multivariate point process; potential function; shot noise weighted point processes.
Note: Work carried out under project PNA4.3 `Stochastic Geometry'.

Mixture Formulae for Shot Noise weighted Point Processes

P. Gregori[1,2], M.N.M. van Lieshout[1] and J. Mateu[2]

[1] *CWI*

P.O. Box 94079, 1090 GB Amsterdam, The Netherlands

[2] *Universitat Jaume I*

Campus Riu Sec, E-12071 Castelló de la Plana, Spain

ABSTRACT

We investigate the connections between shot noise weighted and bivariate Gibbs point processes.

2000 Mathematics Subject Classification: 60G55, 62M30.

Keywords and Phrases: Influence function, multivariate point processes, potential function, shot noise weighted point processes.

Note: Work carried out under project PNA4.3 'Stochastic Geometry'.

1. Introduction

Gibbs point processes are widely used in spatial statistics as models for finite point patterns. Such patterns may be roughly dichotomised as 'clustered' or 'regular' [6] depending on whether the presence of a point at a certain spatial location attracts or repels incidence of further points nearby. For regular patterns, the family of Markov pairwise interaction models [17, 21] is particularly appealing. Members of this class are defined by a Radon–Nikodym density with respect to a Poisson process that can be written as a product over point pairs of some interaction function, typically specified in terms of the distance between the two points. For clustered patterns, on the other hand, Neyman–Scott or Cox models [5, 19] – although not necessarily Gibbsian – are natural candidates.

In recent years, inspired by the Widom–Rowlinson model [29, 23] for liquid-vapour equilibrium, families of point process distributions were designed that can be used both for moderate clustering and regularity depending on the value of a parameter [1, 9, 10, 16, 18]. In the classic formulation, the Widom–Rowlinson mixture model [29] simply forbids molecules in different phases to penetrate each other's influence zones. The marginal distribution of a single phase is known as an attractive area-interaction model, which is a two-parameter exponential family with as sufficient statistics the number of molecules and the volume of the union of influence zones. It has interactions between clusters of arbitrarily many mutually penetrating molecules. From an intuitive point of view, it is plausible that as the intensity of both types of molecules increases, one of them tends to dominate. This phenomenon known as phase transition was proved by Ruelle [25] and further investigated in [4, 12].

Various generalisations are possible. For example, the general area-interaction model [1] allows different intensities for the two phases, spatial variability in the influence zones, and repulsion as well as attraction between molecules. A characterisation theorem for area-interaction can be found in [12] whereas limit theorems are proved in [14], its deviation from Poisson processes is studied in [26], and sampling issues are dealt with in [8, 12, 15, 28].

Related models where volume is replaced by other functionals from convex geometry such as the Euler-Poincaré characteristic are introduced in [16].

In contrast to distance based pairwise interaction functions, the Widom–Rowlinson style models described above focus on the influence of points or molecules on their environment. The same principle underlies the shot noise weighted point process models of [18]. Briefly, given a non-negative influence function, the sum of influences over all points in a pattern is filtered by means of an interaction potential to yield the sufficient statistic of an exponential family. It is the object of this paper to relate bivariate pairwise interaction and shot noise weighted point process models, thus generalising the mixture formulae for the Widom–Rowlinson model. Although we focus on probabilistic aspects, it should be noted that statistical inference for finite point processes defined by a parametric density is moderately well-developed and relies heavily on Monte Carlo methods [11]. A review on parameter estimation and goodness of fit testing can be found in [9], for non-parametric inference on the influence or interaction function, see e.g. [3, 7, 13, 2].

The plan of this paper is as follows. In Section 2, we review pairwise interaction and shot noise weighted point processes and fix notation. Sections 3 and 4 present the main results on the marginal distributions of multi-type pairwise interaction and bivariate shot noise weighted point processes, which are summarised in Section 5.

2. Finite point processes

Let W be a compact set in $\mathbb{R}^d$ of positive volume $|W| > 0$. In this paper, we shall consider point processes X on W defined by their density $p(\cdot)$ with respect to a unit rate Poisson process on W. Realisations of X are finite point configurations $\mathbf{x} = \{x_1, \ldots, x_n\} \subseteq W$, where $n = n(\mathbf{x}) \in \mathbb{N}_0$ denotes the cardinality of $\mathbf{x}$. Note that almost surely all points are different.

2.1 Pairwise interaction point processes

A widely used class of models for random configurations in which the points tend to repel each other is that of the 'pairwise interaction point processes' (see e.g. [20, 27] or the monographs [17, 21] and the references therein). Such models have a density of the form

$$p(\mathbf{x}) = \alpha\, \beta^{n(\mathbf{x})} \prod_{1 \le i < j \le n(\mathbf{x})} \varphi(x_i, x_j) \tag{2.1}$$

for some symmetric Borel measurable interaction function φ with values in $[0, 1]$. The intensity parameter β is strictly positive, and $\alpha \in (0, \infty)$ is the normalising constant (i.e. $\alpha^{-1} = \mathbb{E}_1 \left[\beta^{n(X)} \prod_{i<j} \varphi(X_i, X_j) \right]$ where $\mathbb{E}_1$ denotes the expectation with respect to the reference unit rate Poisson process).

Example 1. *The 'Strauss' or 'soft core' model has interaction function [27]*

$$\varphi(x, y) = \begin{cases} \delta & \text{if } ||x - y|| \le r \\ 1 & \text{otherwise} \end{cases} \tag{2.2}$$

for some $\delta \in (0, 1)$ and $r > 0$. Here, and in the sequel, we use the notation $||x - y||$ for the Euclidean distance between x and y. For $\delta = 1$, (2.1) defines a Poisson process with rate β. If $\delta = 0$, (2.2) is the so-called 'hard core' interaction function; it enforces that almost surely all points are separated by a distance larger than r.

2.2 Shot noise weighted point processes

'Shot noise weighted point processes' [18] have a density that is defined in terms of functionals of shot noise. More precisely, let $\kappa : W \times W \to [0,\infty)$ be a Borel function and f a real-valued Borel function with $f(0) = 0$. Then set

$$p(\mathbf{x}) = \alpha\, \beta^{n(\mathbf{x})} \exp\left[-\log\gamma \int_W f\left(\sum_{x\in\mathbf{x}} \kappa(a,x) \right) da \right]. \tag{2.3}$$

For each $a, b \in W$, the term $\kappa(a,b)$ measures the influence of a point b felt at the location a, and the function f (called the 'potential function') acts as a filter on the total influence $\sum_{x\in\mathbf{x}} \kappa(a,x)$ of the configuration $\mathbf{x}$ at a. The model parameters are $\beta > 0$ and $\gamma > 0$. As in Section 2.1, we write α for the normalising constant. In order to ensure that $\alpha \in (0,\infty)$, some conditions need to be imposed. For example the Ruelle criterion [24]

$$\left| f\left(\sum_{x\in\mathbf{x}} \kappa(a,x) \right) \right| \leq C\, n(\mathbf{x}) \tag{2.4}$$

for some $C > 0$ is sufficient for integrability of (2.3). Further constraints serve to avoid the ambiguity in scale between f and γ. For instance, under the assumption that $f(\cdot)$ is absolutely integrable, one might require that f integrates to unity [18].

The main influence function considered in [18] is the coverage function

$$\kappa(a,x) = \mathbf{1}\{a \in B(x,r)\} = \mathbf{1}\{||a-x|| \leq r\} \tag{2.5}$$

where $B(x,r)$ is the ball of radius $r > 0$ centered at x. Clearly, the choice of alternatives is huge. In many situations in practice, it is natural to assume that the influence function is isotropic and decreasing in the distance between its arguments. In such a case, $\kappa(a,x) = \kappa(||a-x||)$ with slight abuse of notation. Examples include, for $r, \sigma > 0$,

$$\kappa(t) = \begin{cases} \exp\left[-(t/\sigma)^2\right] \mathbf{1}\{t \leq 2\sigma\} & \textbf{bell} \\ 1/\left(1+(t/\sigma)^2\right)^2 \mathbf{1}\{t \leq 3\sigma\} & \textbf{Cauchy} \\ \left(1-(t/\sigma)^2\right)^2 \mathbf{1}\{t \leq \sigma\} & \textbf{quadratic} \end{cases}$$

which all have compact support and are normalised so that $\kappa(0) = 1$. As in [1, 16, 18], inhomogeneity in the interaction range may be modelled by taking $\kappa(a,x) = \mathbf{1}\{a \in Z(x)\}$, where $Z(x)$ is a compact set related to x, whereas replacing the constant β by a function $\beta(a)$, $a \in W$, results in first order spatial heterogeneity.

In contrast to pairwise interaction models, shot noise weighted point processes can exhibit a wide range of interaction structures.

Example 2. *The 'area-interaction' point process [1] is the special case of (2.3) with the coverage function for κ, and potential function $f(t) = \mathbf{1}\{t \geq 1\}$. Its density can be written as*

$$p(\mathbf{x}) = \alpha\, \beta^{n(\mathbf{x})}\, \gamma^{-|U_r(\mathbf{x})|} \tag{2.6}$$

where $U_r(\mathbf{x}) = \cup_{x\in\mathbf{x}} B(x,r) \cap W$ is the union of balls in W with radius $r > 0$ centered at the points x of the configuration $\mathbf{x}$. Note that for $\gamma > 1$, realisations tend to be clustered to cover a minimum of space, whereas for $\gamma < 1$, repulsive configurations are more likely to occur. If $\gamma = 1$, (2.6) reduces to the density of a Poisson process with rate β.

3. Bivariate mixture models

The area-interaction model (2.6) is intimately connected with the Widom–Rowlinson two-type mixture model [29]. Indeed, consider two independent Poisson processes, say X and Y, and impose the condition that the shortest distance $d(X, Y)$ between a point in X and one in Y is larger than r. Then the joint density with respect to the product measure of two independent unit rate Poisson processes is given by

$$p(\mathbf{x}, \mathbf{y}) = \alpha\, \beta_1^{n(\mathbf{x})}\, \beta_2^{n(\mathbf{y})}\, \mathbf{1}\{d(\mathbf{x}, \mathbf{y}) > r\} \tag{3.1}$$

where $\beta_1, \beta_2 > 0$ are the intensities of the component Poisson processes. It can be shown that the marginal distribution of X is that of an area-interaction point process (2.6) with interaction parameter $\gamma = e^{\beta_2}$ [29, 12].

The Widom–Rowlinson mixture model (3.1) is an example of a bivariate pairwise cross interaction point process. Such a process $Z = (X, Y)$ has a density of the type [17, Ch. 2]

$$p(\mathbf{x}, \mathbf{y}) = \alpha\, \beta_1^{n(\mathbf{x})}\, \beta_2^{n(\mathbf{y})} \prod_{\substack{1 \le i \le n(\mathbf{x}) \\ 1 \le j \le n(\mathbf{y})}} \varphi(x_i, y_j) \tag{3.2}$$

with respect to the product measure of unit rate Poisson processes on W. Here φ is a $[0, 1]$-valued Borel measurable interaction function that describes the cross interaction between the two types of points. For the Widom–Rowlinson mixture model, $\varphi(x, y) = \mathbf{1}\{||x - y|| > r\}$.

Theorem 1. *Suppose $Z = (X, Y)$ is a bivariate point process with density (3.2) for some β_1, $\beta_2 > 0$, and (jointly) measurable, $[0, 1]$-valued interaction function φ. Then the marginal distribution of X is a shot noise weighted point process with intensity parameter β_1, interaction parameter e^{β_2}, potential function $f(t) = 1 - e^{-t}$ $(t \ge 0)$, and influence function $\kappa(a, x) = -\log \varphi(x, a)$ $(a, x \in W)$ under the conventions $\log 0 = -\infty$ and $e^{-\infty} = 0$.*

Proof: By integration over the second component,

$$p(\mathbf{x}) \propto \beta_1^{n(\mathbf{x})}\, \mathbb{E}_{\beta_2} \left[\prod_{x \in \mathbf{x}, y \in Y} \varphi(x, y) \right].$$

Since by assumption $0 \le \varphi(\cdot, \cdot) \le 1$, the expectation with respect to a Poisson process with intensity β_2 on the right hand side of the above expression is the generating functional $G(h)$ evaluated for $h(y) = \prod_{x \in \mathbf{x}} \varphi(x, y)$. Using the fact that for a Poisson process $\log G(h) = -\beta_2 \int_W (1 - h(a)) da$ (see e.g. [5, p. 225]), one obtains

$$\begin{aligned} p(\mathbf{x}) &\propto \beta_1^{n(\mathbf{x})} \exp \left[-\beta_2 |W| + \beta_2 \int_W \prod_{x \in \mathbf{x}} \varphi(x, a)\, da \right] \\ &= \beta_1^{n(\mathbf{x})} \exp \left[-\log \gamma \int_W \left(1 - \prod_{x \in \mathbf{x}} \varphi(x, a) \right) da \right] \end{aligned}$$

if $\gamma := e^{\beta_2}$. Since $\prod_{x \in \mathbf{x}} \varphi(x, a) = \exp \left[\sum_{x \in \mathbf{x}} \log \varphi(x, a) \right]$ under the usual convention for zeroes of φ, the claim follows. □

Clearly, the marginal potential function is increasing. Hence, as the interaction parameter $\gamma > 1$, the marginal density of X favours configurations that exert little influence. In other words, the repulsion between points of different type leads to clustered components. The strength of the influence depends logarithmically on the cross-interaction function. In particular, if φ is isotropic and increasing in the distance between its arguments, then κ is a decreasing function of the distance. Regarding the range of influence, if $\varphi(x,a) \equiv 1$ whenever $||x - a|| > r$, the marginal distributions of the components are Markov at range $2r$, that is, the Papangelou conditional intensity

$$\lambda(\xi; \mathbf{x}) := \frac{p(\mathbf{x} \cup \{\xi\})}{p(\mathbf{x})} = \beta_1 \exp\left[-\beta_2 \int_W \prod_{x \in \mathbf{x}} \varphi(x,a)(1 - \varphi(\xi,a))\, da\right]$$

is a function of ξ and those points in $\mathbf{x}$ that are within range $2r$ of ξ. More generally, define the influence zone of a point $u \in W$ to be the subset $Z(u) = \{a \in W : \varphi(u,a) < 1\}$ of W. Then, it is easily seen that $\lambda(\xi;\mathbf{x})$ depends only on those $x \in \mathbf{x}$ for which $Z(x) \cap Z(\xi) \neq \emptyset$ (cf. [18, Thm. 3.3]). Moreover, the conditional intensity at a fixed point $\xi \in W$ is increasing with respect to set inclusion in its second argument, confirming the clustered nature of the component distribution.

Example 3. *The analytic expression of the integrals involved in Theorem 1 is tedious even in simple cases. For example, the marginal distribution of the first component of a bivariate Strauss density (3.2) with interaction function given by (2.2) for some* $\delta \in (0,1)$ *and* $r > 0$ *has density* $p(\mathbf{x}) \propto \beta_1^{n(\mathbf{x})} \exp\left[\log\gamma \int_W \delta^{n(\{x \in \mathbf{x}:\, ||x-a|| \leq r\})}\, da\right]$, *where* $\gamma = e^{\beta_2}$, *which can be rewritten as*

$$p(\mathbf{x}) \propto \beta_1^{n(\mathbf{x})} \exp\left[\log\gamma \left(|W \setminus U_r(\mathbf{x})| + \sum_{k=1}^{n(\mathbf{x})} \delta^k |U_r^k(\mathbf{x})|\right)\right]$$

with

$$U_r^k(\mathbf{x}) = \bigcup_{\substack{1 \leq i_1 < \cdots \\ \cdots < i_k \leq n(\mathbf{x})}} \bigcap_{j=1}^{k} B(x_{i_j}, r)$$

the union of pieces of k*-overlapping balls in* W*. Note that for the hard core choice* $\delta = 0$*, one returns to the Widom–Rowlinson set-up.*

4. Shot noise mixture models

In this section, we consider bivariate mixture models in which points of the first component exert an influence on points of the second component. More specifically, assume that f is a real-valued, measurable function with $f(0) = 0$, and $\kappa : W \times W \to [0, \infty)$ a Borel measurable influence function such that (2.4) holds. Set

$$p(\mathbf{x}, \mathbf{y}) = \alpha\, \beta_1^{n(\mathbf{x})}\, \beta_2^{n(\mathbf{y})} \exp\left[-\log\gamma \sum_{y \in \mathbf{y}} f\left(\sum_{x \in \mathbf{x}} \kappa(y,x)\right)\right] \tag{4.1}$$

where as before $\alpha \in (0,\infty)$ is the normalising constant, β_1 and $\beta_2 > 0$ are intensity parameters, and $\gamma > 0$ is the cross-interaction parameter. The sum $\sum_{x\in\mathbf{x}} \kappa(y,x)$ expresses the total influence of the pattern $\mathbf{x}$ on $y \in \mathbf{y}$.

Theorem 2. *Suppose $Z = (X,Y)$ is a bivariate point process with density (4.1) for some $\beta_1, \beta_2 > 0$, $\gamma > 0$, potential function f, and (jointly) measurable, non-negative influence function κ. Then the marginal distribution of X is a shot noise weighted point process with intensity parameter β_1, interaction parameter e^{β_2}, potential function $1-\gamma^{-f(t)}$ $(t \geq 0)$, and influence function κ.*

Proof: Consider a finite configuration $\mathbf{x}$ in W. Without loss of generality (rescaling $\log\gamma$ by $C\,n(\mathbf{x})$ otherwise), $|f(\sum_{x\in\mathbf{x}} \kappa(y,x))| \leq 1$ uniformly in $y \in W$. If we write $\delta = \max\{\gamma, 1/\gamma\} \geq 1$, it follows by integration over the second component that

$$p(\mathbf{x}) \propto \beta_1^{n(\mathbf{x})} \mathbb{E}_{\beta_2\delta}\left[\prod_{y\in Y} \delta^{-1}\gamma^{-f(\sum_{x\in\mathbf{x}}\kappa(y,x))}\right].$$

Note that $0 \leq \delta^{-1}\gamma^{-f(\sum_{x\in\mathbf{x}}\kappa(y,x))} \leq 1$, hence the expectation on the right hand side is the generating functional $G(h)$ for $h(y) = \delta^{-1}\gamma^{-f(\sum_{x\in\mathbf{x}}\kappa(y,x))}$ of a Poisson process with intensity $\beta_2\,\delta$. Hence

$$p(\mathbf{x}) \propto \beta_1^{n(\mathbf{x})} \exp\left[-\beta_2\delta|W| + \beta_2\int_W \gamma^{-f(\sum_{x\in\mathbf{x}}\kappa(a,x))} da\right].$$

We conclude that X is distributed as a shot noise weighted point process with the desired parameters, potential and influence functions. □

The marginal potential function $f_X(t) = 1-\gamma^{-f(t)}$ depends on the cross potential and the interaction parameter; its derivative is given by $f'_X(t) = \gamma^{-f(t)} f'(t) \log\gamma$. Thus, if f is increasing (respectively decreasing), f_X is increasing (decreasing) if $\log\gamma$ is positive and decreasing (increasing) otherwise. Thus, the strength and type of interaction depend on f and γ. To determine the range of interaction, define the influence zone $Z_\kappa(u) = \{a \in W : \kappa(a,u) > 0\}$ of u as that subset of W on which u exerts a non-vanishing influence. Then, by arguments similar to those employed in Section 3, it can be shown that the conditional intensity depends only on those $x \in \mathbf{x}$ for which $Z_\kappa(x) \cap Z_\kappa(\xi) \neq \emptyset$. In particular, if $\kappa(a,x) = 0$ whenever $||a-x|| > r$, then X is Markov at range $2r$.

Example 4. *For $f(t) = \mathbf{1}\{t \geq 1\}$ and κ given by (2.5), the coverage function, (4.1) reduces to*

$$p(\mathbf{x},\mathbf{y}) = \alpha\beta_1^{n(\mathbf{x})}\,\beta_2^{n(\mathbf{y})} \exp\left[-\log\gamma\, n(\mathbf{y} \cap U_r(\mathbf{x}))\right]$$

which has marginal density

$$\begin{aligned} p(\mathbf{x}) &\propto \beta_1^{n(\mathbf{x})} \exp\left[\beta_2 \int_W \left(\gamma^{-\mathbf{1}\{a\in U_r(\mathbf{x})\}} - 1\right) da\right] \\ &= \beta_1^{n(\mathbf{x})} \exp\left[\beta_2 \int_W \frac{1-\gamma}{\gamma}\mathbf{1}\{a \in U_r(\mathbf{x})\}\right] = \beta_1^{n(\mathbf{x})} \exp\left[\beta_2 \frac{1-\gamma}{\gamma}|U_r(\mathbf{x})|\right] \end{aligned}$$

for the first component, that is, an area-interaction point process (2.6) with interaction parameter $\exp[\beta_2(1 - 1/\gamma)]$. *Note that for* $\gamma > 1$, *since there is repulsion between points of different types, the points in the first component tend to cluster in the holes left by the second one. Similarly, if* $\gamma < 1$, *the attraction in the bivariate density results in repulsion in the marginal distribution of* X. *For* $\gamma = 1$, $p(\cdot, \cdot)$ *defines the distribution of a random vector consisting of two independent Poisson processes, the marginal distributions of which are also Poisson.*

Theorem 2 remains valid if the total influence of the configuration felt at $a \in W$ is computed in other ways than summation. For instance, let

$$\kappa_p(y, \mathbf{x}) = \begin{cases} (\sum_{x \in \mathbf{x}} \kappa(y, x)^p)^{1/p} & p \in \mathbb{N} \\ \max_{x \in \mathbf{x}} \kappa(y, x) & p = \infty \end{cases}$$

with $p = 1$ corresponding to the classic choice. Only for $p = \infty$, however, a genuinely new family of models is obtained, as only then the influence and potential functions cannot be simply transformed to obtain a shot noise weighted density.

Note that for κ_∞, the Ruelle condition (2.4) is satisfied if either f is bounded or κ is bounded in $W \times W$ by some $s > 0$ and f is bounded on $[0, s]$. Alternatively, one may assume that $|f(\cdot)|$ is dominated by a linear function, and that the $\sup_{x \in W} \int_W \kappa(a, x)\, da$ is finite.

Some models, for instance (2.6), may be written in both L_1 and L_∞ terms. Indeed,

$$|U_r(\mathbf{x})| = \int_W f_1(\kappa_1(a, \mathbf{x}))\, da = \int_W f_2(\kappa_\infty(a, \mathbf{x}))\, da$$

where, of course, the underlying κ is the coverage function, $f_1(t) = \mathbf{1}\{t \geq 1\}$ as before, and $f_2(t) = t$ for $t \geq 0$. The family of models with potential f_2, the supremum norm and general influence function κ may be dubbed *generalised area-interaction*. For $\gamma > 1$, such models are attractive, for $\gamma < 1$ repulsive (in terms of a conditional intensity that is increasing or decreasing with respect to set inclusion).

Remark 1. *Suppose the influence function* κ *is binary, i.e. takes values in* $\{0, 1\}$. *Then the model based on* κ_∞ *may still be rewritten in terms of a shot noise weighted point process with the modified potential function* $\tilde{f}(t) := f(1)\mathbf{1}\{t \geq 1\}$. *The reverse statement is not true, hence classical shot noise weighting is more flexible than* κ_∞ *weighting for binary influence functions.*

5. Summary

In this paper, we presented links between two popular classes of models in stochastic geometry, namely those whose density is defined in terms of repulsion between pairs of points, and those whose density is defined in terms of geometric characteristics such as set coverage. We proved that the components of any bivariate pairwise cross interaction point process form a shot noise weighted point process, thus extending results for the well-known Widom–Rowlinson penetrable spheres model. We also showed that the first component of a bivariate shot noise weighted point process, with influence from points of the first component on those of the second one, is distributed as a shot noise weighted point process with the same influence function but a different potential. We noted that the latter theorem remains true for a wider class of models.

Acknowledgements
This research was carried out during two visits by the first author to CWI in 2002 and 2003. He is grateful to Fundació Caixa Castelló-Bancaixa (grant 02G014.13/1) and Ministerio de Ciencia y Tecnología (grant BFM2001-3286) who supported the first visit, and to CWI for its hospitality and financial support.

References

1. A.J. Baddeley and M.N.M. van Lieshout. Area-interaction point processes. *Annals of the Institute of Statistical Mathematics*, 47:601–619, 1995.
2. K.K. Berthelsen and J. Møller. Perfect simulation and inference for spatial point processes. Technical report, R-01-2009, Department of Mathematical Sciences, Aalborg University, 2001.
3. J.M. Billiot and M. Goulard. An estimation method of the pair potential function for Gibbs point processes on spheres. *Scandinavian Journal of Statistics*, 28:185–203, 2001.
4. J.T. Chayes, L. Chayes, and R. Kotecky. The analysis of the Widom–Rowlinson model by stochastic geometric methods. *Communications in Mathematical Physics*, 172:551–569, 1995.
5. D.J. Daley and D. Vere-Jones. *An introduction to the theory of point processes*. Springer Verlag, New York, 1988.
6. P.J. Diggle. *Statistical analysis of spatial point patterns*. Academic Press, London, 1983.
7. P.J. Diggle, D.J. Gates and A. Stibbard. A non-parametric estimator for pairwise-interaction point processes. *Biometrika*, 74:763–770, 1987.
8. P.A. Ferrari, R. Fernández and N.L. Garcia. Perfect simulation for interacting point processes, loss networks and Ising models. *Stochastic Processes and their Applications*, 102:63–88, 2002.
9. C.J. Geyer. Likelihood inference for spatial point processes. In: *Stochastic geometry, likelihood, and computation*, O. Barndorff-Nielsen, W.S. Kendall and M.N.M. van Lieshout (Eds.) CRC Press/Chapman and Hall, Boca Raton, 1999.
10. P. Grabarnik and A. Särkkä. Interacting neighbour point processes: some models for clustering. *Journal of Statistical Computation and Simulation*, 68:103–125, 2001.
11. P.J. Green, N.L. Hjort and S. Richardson. *Highly structured stochastic systems*. Oxford University Press, 2003.

12. O. Häggström, M.N.M. van Lieshout and J. Møller. Characterization results and Markov chain Monte Carlo algorithms including exact simulation for some point processes. *Bernoulli*, 5:641–659, 1999.
13. J. Heikkinen and A. Penttinen. Bayesian smoothing in the estimation of the pair potential function of Gibbs point processes. *Bernoulli*, 5:1119–1136, 1999.
14. S.R. Jammalamadaka and M.D. Penrose. Poisson limits for pairwise and area interaction point processes. *Advances in Applied Probability*, 32:75–85, 2000.
15. W.S. Kendall. Perfect simulation for the area-interaction point process. In: *Proceedings of the Symposium on Probability towards the year 2000*, L. Accardi and C. Heyde (Eds.) Springer-Verlag, Berlin, 1998.
16. W.S. Kendall, M.N.M. van Lieshout and A.J. Baddeley. Quermass-interaction processes: conditions for stability. *Advances in Applied Probability (SGSA)*, 31:315–342, 1999.
17. M.N.M. van Lieshout. *Markov point processes and their applications*. Imperial College Press/World Scientific Publishing, London/Singapore, 2000.
18. M.N.M. van Lieshout and I.S. Molchanov. Shot noise weighted processes: a new family of spatial point processes. *Communications in Statistics: Stochastic Models*, 14:715–734, 1998.
19. J. Møller and R.P. Waagepetersen. Statistical inference for Cox processes. In: *Spatial cluster modelling*, D. Denison and A.B. Lawson (Eds.) Chapman and Hall/CRC, Boca Raton, 2002.
20. Y. Ogata and M. Tanemura. Estimation for interaction potentials of spatial point patterns through the maximum likelihood procedure. *Annals of the Institute of Statistical Mathematics*, 33:315–338, 1981.
21. B.D. Ripley. *Statistical inference for spatial processes*. Cambridge University Press, Cambridge, 1988.
22. B.D. Ripley and F.P. Kelly. Markov point processes. *Journal of the London Mathematical Society*, 15:188–192, 1977.
23. J.S. Rowlinson. Penetrable sphere models of liquid-vapor equilibrium. *Advances in Chemical Physics*, 41:1–57, 1980.
24. D. Ruelle. *Statistical mechanics*. John Wiley and Sons, New York, 1969.
25. D. Ruelle. Existence of a phase transition in a continuous classical system. *Physical Review Letters*, 27:1040–1041, 1971.
26. A. Särkkä and A.J. Baddeley. Parameter estimation and identifiability for the Widom–Rowlinson process. Manuscript, 2003.
27. D.J. Strauss. A model for clustering. *Biometrika*, 63:467–475, 1975.
28. E. Thőnnes. Perfect simulation of some point processes for the impatient user. *Advances in Applied Probability (SGSA)*, 31:69–87, 1999.
29. B. Widom and J.S. Rowlinson. A new model for the study of liquid-vapor phase transition. *Journal of Chemical Physics*, 52:1670–1684, 1970.