

Clifford Testing: Algorithms and Lower Bounds

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Abstract

We consider the problem of Clifford testing, which asks whether a black-box n -qubit unitary is a Clifford unitary or at least ε -far from every Clifford unitary. We give the first 4-query Clifford tester, which decides this problem with probability $\text{poly}(\varepsilon)$. This contrasts with the minimum of 6 copies required for the closely-related task of stabilizer testing. We show that our tester is tolerant, by adapting techniques from tolerant stabilizer testing to our setting. In doing so, we settle in the positive a conjecture of Bu, Gu and Jaffe, by proving a polynomial inverse theorem for a non-commutative Gowers 3-uniformity norm. We also consider the restricted setting of single-copy access, where we give an $O(n)$ -query Clifford tester that requires no auxiliary memory qubits or adaptivity. We complement this with a lower bound, proving that any such, potentially adaptive, single-copy algorithm needs at least $\Omega(n^{1/4})$ queries. To obtain our results, we leverage the structure of the commutant of the Clifford group, obtaining several technical statements that may be of independent interest.

CCS Concepts

• **Theory of computation** → **Quantum information theory**;
Quantum complexity theory.

Keywords

quantum information, quantum property testing, Clifford group, Gowers uniformity norm

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1 Introduction

Stabilizer testing—deciding whether an unknown state is close to a stabilizer state or far from every stabilizer state—has recently seen several remarkable advances [21, 22, 26]. While the task originates in quantum property testing [30], subsequent work has revealed deep connections to other areas of quantum information theory, mathematics and computer science. In particular, stabilizer testing is directly linked to the representation theory of the Clifford group [22], to the resource theory of magic [7, 12] and quadratic Fourier analysis [4, 6, 29]. These insights have led to steadily improving stabilizer testers, but also to surprising advances in classical algebraic property testing and algorithmic additive combinatorics [3, 8], as well as algorithms that operate in the restricted setting of single-copy access [25].

In this work, we build on all of these advances and tackle the natural dynamic analog of stabilizer testing, namely *Clifford testing*: given query access to an unknown n -qubit unitary U , determine whether it belongs to the Clifford group or is far from it. Clifford testing has structural similarities to stabilizer testing, but as a form of unitary property testing it has some extra properties that make it theoretically interesting in its own right. We will discuss some of these properties further down in the introduction, and point out informally how our work addresses these, with a more formal rundown of results given in Section 1.1. We also discuss connections to the existing literature on stabilizer testing, as well as intriguing connections to additive combinatorics.

Clifford Testing. In the context of property testing, a natural way to measure proximity to the Clifford group is in terms of the *Clifford fidelity*,

$$F_{\text{Cliff}}(U) := \max_{C \in \text{Cl}(n)} 2^{-2n} |\text{tr}(U^\dagger C)|^2, \quad (1)$$

where $\text{Cl}(n)$ denotes the n -qubit Clifford group.¹ This bears close resemblance to the stabilizer fidelity used in stabilizer testing. We say that U is ε -far from Clifford if $F_{\text{Cliff}}(U) < 1 - \varepsilon$ and ε -close otherwise. A quantum algorithm is a (one-sided) ε -Clifford tester if it accepts every Clifford unitary with probability at least $2/3$ and rejects any unitary that is ε -far from Clifford with probability at least $2/3$.

¹For discussions of suitable distance measures in the context of property testing unitary operators and, more generally, quantum channels, see Refs. [30, 33].

Inverse-Free Clifford Testing. The first Clifford testers were considered by Low [28] and Wang [35]. These testers however rely on access to the unitary U and its inverse $U^\dagger$, giving $\text{poly}(n/\varepsilon)$ - and $O(1/\varepsilon^2)$ -query ε -testers, respectively. Access to $U^\dagger$ can be achieved in circuit-based models by reversing the circuit, assuming the gate set contains or can efficiently synthesize inverses. However, in many physical or experimental settings, U represents the evolution of a device, or other process where implementing $U^\dagger$ would require reversing the system's dynamics, which may be infeasible. This motivates the question of *inverse-free Clifford testing*, which we will consider in this work.

Gross, Nezami, and Walter [22] constructed a 6-query algorithm for stabilizer testing. They noted [22, Remark 3.7], that this can be adapted to Clifford testing via the Choi isomorphism. In this work, we make this connection precise by relating stabilizer fidelity to Clifford fidelity, yielding an inverse-free 6-query Clifford tester.

Even though stabilizer testing is known to require at least 6 queries [17, 22], it is not clear that (inverse-free) Clifford testing should need 6 queries. In fact, the t -fold Clifford twirl already deviates from the t -fold Haar twirl corresponding to the full unitary group at $t = 4$, which is also the reason why the Clifford group fails to be a unitary 4-design. In particular, there exists a distinguished subspace $V_{n,4} \subset ((\mathbb{C}^2)^{\otimes n})^{\otimes 4}$ that is invariant under the diagonal Clifford action $C^{\otimes 4}$ for every Clifford $C \in \text{Cl}(n)$ but that is not preserved by the corresponding fourth-moment Haar twirl of the full unitary group [36]. This suggests a natural 4-query strategy: detect whether $U^{\otimes 4}$ preserves this distinguished Clifford-invariant subspace. Our first result confirms this intuition by giving an inverse-free Clifford tester that uses only 4 entangled queries.

Let Π_4 denote the projector onto $V_{n,4}$. The test then proceeds as follows:

- (1) Prepare the state $\rho_{V_{n,4}} = \Pi_4 / \text{tr}(\Pi_4)$, i.e. the uniform mixture over $V_{n,4}$.
- (2) Apply the unitary $U^{\otimes 4}$.
- (3) Measure the POVM $\{\Pi_4, I - \Pi_4\}$ and accept iff the outcome Π_4 occurs.

In other words, the test checks whether the action of $U^{\otimes 4}$ leaves the subspace $V_{n,4}$ invariant. Every Clifford passes this test with certainty, since $C^{\otimes 4}$ preserves $V_{n,4}$ exactly. Our contribution is to show that this intuition can indeed be upgraded into a full Clifford tester: we prove that if U is far from every Clifford, then the action of $U^{\otimes 4}$ must move a detectable amount of weight outside $V_{n,4}$, leading to a nontrivial rejection probability.

It remains to show that this test can be implemented efficiently. While a naive implementation would require $4n$ qubits, the key observation is that $V_{n,4}$ admits a basis consisting of tensor products of $2n$ -qubit generalized Bell states [36]. This yields a Bell-sampling implementation of the same projector test using only $2n$ qubits at a time.

The discrepancy with the stabilizer case is technically interesting, as the stabilizer states also fail to form an exact state 4-design. However they do form an approximate additive error state 4-design (in fact even a 5-design) with exponentially small additive error [20]. Our result can be seen as showing that a similar approximate statement does not hold for the Clifford group.

Tolerant Clifford Testing. In this work we shall also be concerned with *tolerant testing*, a natural extension of the one-sided paradigm of property testing [32]. In analogy with recent works on stabilizer testing [4, 6, 29], this is more naturally expressed in terms of fidelity. For $1 > \varepsilon_1 > \varepsilon_2 \geq 0$, a quantum algorithm is an $(\varepsilon_1, \varepsilon_2)$ -*tolerant Clifford tester* if, given an n -qubit unitary U , it accepts with probability at least $2/3$ if U is $F_{\text{Cliff}}(U) \geq \varepsilon_1$ and rejects with probability at least $2/3$ if U is $F_{\text{Cliff}}(U) \leq \varepsilon_2$. Our second result shows that our 4-query Clifford tester is tolerant in this sense.

Our tolerant analysis, which extends the techniques used in tolerant stabilizer testing, has an interesting connection to recent work of Bu, Gu and Jaffe [10, 12], which introduces a non-commutative analogue of the Gowers uniformity norms from additive combinatorics. The (eight power of the) order-3 quantum uniformity norm $\|U\|_{Q^3}$ of an n -qubit unitary U is defined as the average over independent uniformly distributed Heisenberg–Weyl operators P_1, P_2, P_3 of the trace of the iterated commutator $\text{tr}[P_1, [P_2, [P_3, U]]]$. In the classical case, obtained by restricting to diagonal unitaries, the Gowers 3-uniformity norm measures how much a function oscillates after a certain “multiplicative” derivative has been applied three times in random directions. Intuition from calculus suggests that small oscillations imply that the function has some quadratic polynomial structure, and deep inverse theorems confirm this intuition [19, 34]. Bu et al. conjecture in Ref. [10] that such an inverse theorem holds for the Q^3 norm. Our tolerant analysis of our 4-query Clifford tester resolves this conjecture in the positive, showing that Clifford fidelity and the Q^3 norm are polynomially related. This result fits in a recent trend of intriguing connections between these areas and quantum information theory [3, 5, 8, 9, 11].

Single-Copy Clifford Testing. Our other results pertain to resource-restricted query models for Clifford testing, which are motivated by the practical challenges of implementing testing algorithms. In particular, we consider two key resource restrictions:

- (1) Single-copy access (or *incoherent access*, or operating *without quantum memory*).
- (2) Lack of an auxiliary system.

The first restriction, single-copy access, has already received significant attention in quantum learning theory and property testing [1, 2, 13–16, 18, 23]. For state-related tasks, single-copy algorithms only process one copy of the state at a time, in contrast to multi-copy algorithms that can act jointly on several copies. For tasks involving unitaries or channels, single-copy algorithms are those that keep no quantum memory between queries: each round consists of preparing an input, applying the channel once, and measuring the entire output system. The restriction to single-copy access is motivated by the technological difficulty of maintaining a coherent quantum memory or performing joint multi-copy operations. However, single-copy algorithms can exhibit dramatically increased sample complexities, often even exponentially, compared to the multi-copy setting.

The second restriction, lack of an auxiliary system, arises more specifically in the context of learning and testing unitaries or channels. With access to an auxiliary register, an algorithm can prepare entangled inputs, send only part of the state through the channel, and then measure the entire joint system. This entanglement can

provide a significant advantage. Following the nomenclature laid out in Ref. [33], we refer to algorithms without such an auxiliary system as *auxiliary-free* (ancilla-free), and to those that make use of it as *auxiliary-assisted* (ancilla-assisted).

Here, we investigate Clifford testing in these resource-restricted query models. To construct single-copy Clifford testing algorithms, our starting point is the work [25] which gives a single-copy stabilizer testing algorithm using $O(n/\epsilon^2)$ copies of the unknown state. In the auxiliary-assisted setting, by preparing copies of the Choi state and feeding them one at a time into this algorithm, one can obtain an auxiliary-assisted single-copy algorithm that inherits the complexity of the scheme in Ref. [25]. However, this tester is not auxiliary-free (as we need memory for the Choi states). We also give an auxiliary-free single-copy ϵ -Clifford tester (which is substantially more difficult to derive) that uses $O(n/\epsilon^3)$ queries and time $O(n^3/\epsilon)$. Finally, we prove that any auxiliary-free tester requires $\Omega(n^{1/4})$ queries, even when the tester is allowed to make *adaptive* queries (which can make a qualitative difference in some scenarios [33]).

Table 1: Summary of our results: Upper and lower bounds on the query complexity for inverse-free Clifford testing and comparison to sample complexity of stabilizer testing.

	Multi-copy	Single-copy	
		Aux.-free	Aux.-assisted
Clifford testing	$t = 4$	$\Omega(n^{1/4}) \leq t \leq O(n)$	Open
Stabilizer testing	$t = 6$ [22]	$\Omega(n^{1/2}) \leq t \leq O(n)$ [25]	

1.1 Results

Below, we summarize our results in detail. The first two results give performance guarantees for Clifford testing algorithms.

THEOREM 1.1 (ONE-SIDED 4-QUERY CLIFFORD TESTER). *There exists a quantum algorithm that, given an n -qubit unitary U , makes 4 queries to U and for any $\epsilon > 0$, has the following completeness and soundness guarantees:*

- It accepts if U is a Clifford unitary.
- It rejects with probability $\min(\frac{1}{4}, \frac{\epsilon}{2})$ if $F_{\text{Cliff}}(U) \leq 1 - \epsilon$.

THEOREM 1.2 (TWO-SIDED 4-QUERY CLIFFORD TESTER). *There exists a quantum algorithm that, given an n -qubit unitary U , makes 4 queries to U and for any $\epsilon > 0$, has the following completeness and soundness guarantees:*

- It accepts with probability $\text{poly}(\epsilon)$ if $F_{\text{Cliff}}(U) \geq \epsilon$.
- It rejects with probability $1 - \text{poly}(\epsilon)$ if $F_{\text{Cliff}}(U) \leq \epsilon$.

By repeating these testers $\text{poly}(\epsilon)$ times, we obtain constant-query testers with perfect (resp. constant) completeness and soundness. In proving the existence of a tolerant Clifford tester we also settle a conjecture due to [10], pertaining to a non-commutative generalization of the Gowers uniformity norms.

THEOREM 1.3 (INVERSE THEOREM FOR THE Q^3 NORM). *For any n -qubit unitary U , we have that*

$$F_{\text{Cliff}}(U) \geq \text{poly}(\|U\|_{Q^3}). \quad (2)$$

Finally, we prove upper and lower bounds for Clifford testing in the single-copy access model.

THEOREM 1.4 (EFFICIENT AUXILIARY-FREE, SINGLE-COPY CLIFFORD TESTER). *There exists an auxiliary-free single-copy ϵ -Clifford tester that uses $\tilde{O}(n/\epsilon^3)$ queries and time $\tilde{O}(n^3/\epsilon^3)$.*

THEOREM 1.5 (LOWER BOUND FOR AUXILIARY-FREE, SINGLE-COPY CLIFFORD TESTERS). *Any auxiliary-free single-copy algorithm for Clifford testing requires at least $\Omega(n^{1/4})$ queries.*

This bound holds also against *adaptive* algorithms which may choose input states and measurements for subsequent rounds based on measurement outcomes from previous round. Interestingly, we find that our proof technique for the lower bound does not straightforwardly extend to the auxiliary-assisted setting.

1.2 Technical Overview

From Stabilizer Testing to Clifford Testing. To connect Clifford testing to stabilizer testing, we need to understand the relation between Clifford fidelity and stabilizer fidelity. Via the Choi–Jamiołkowski isomorphism, every unitary U corresponds to its Choi state $|U\rangle\rangle$. We then observe that, since every Clifford Choi state is a stabilizer state,

$$F_{\text{Cliff}}(U) \leq \max_{S \in \text{Stab}(2n)} |\langle S|U\rangle\rangle|^2 = F_{\text{Stab}}(|U\rangle\rangle). \quad (3)$$

However, this one-sided inequality alone is insufficient to reduce Clifford testing to stabilizer testing.

Our first technical contribution resolves this by proving that the two fidelities are in fact polynomially equivalent,

$$F_{\text{Stab}}(|U\rangle\rangle)^6 \leq F_{\text{Cliff}}(U) \leq F_{\text{Stab}}(|U\rangle\rangle), \quad (4)$$

and that they coincide whenever $F_{\text{Stab}}(|U\rangle\rangle) > 1/2$. This sandwich inequality establishes a precise quantitative link between stabilizer and Clifford fidelity, thereby allowing Clifford testing to be reduced to stabilizer testing even in the tolerant sense. Importantly, the same inequality underlies the proof of [Theorem 1.2](#), yielding performance guarantees for the novel 4-query tester.

Expected Stabilizer Fidelity and the Auxiliary-Free Tester. The auxiliary-free algorithm is based on the following simple procedure, repeated $O(1/\epsilon)$ times:

- (1) Sample a uniformly random n -qubit stabilizer state $|S\rangle$, and apply the unknown unitary U to prepare $U|S\rangle$.
- (2) Prepare $O(n/\epsilon^2)$ independent copies of $U|S\rangle$, and feed them into the single-copy stabilizer tester of Ref. [25].

Thus, the overall query complexity is $O(n/\epsilon^3)$.

Intuitively, the idea is the following: since $|S\rangle$ is drawn uniformly at random, there should be a good chance that any non-Cliffordness in U translates into non-stabilizerness of $U|S\rangle$. Our main technical contribution in this section is to show that this intuition is correct. Specifically, we show that if U has Clifford fidelity $1 - \epsilon$, then for a random stabilizer input $|S\rangle$, the output state $U|S\rangle$ has stabilizer fidelity $1 - \Omega(\epsilon)$ with probability $\Omega(\epsilon)$. Thus, by repeating the above procedure $O(1/\epsilon)$ times, we obtain a constant probability of encountering an output state in the parameter regime detected by the single-copy stabilizer tester.

As the key ingredient, we prove the following strong sandwich inequality relating Clifford fidelity to expected stabilizer fidelity:

$$\begin{aligned} F_{\text{Cliff}}(U) &\leq \mathbb{E}_{|S\rangle \in \text{Stab}(n)} [F_{\text{Stab}}(U|S)] \\ &\leq \left[\frac{1}{8} F_{\text{Stab}}(|U\rangle\langle U|) + \frac{7}{8} + O(2^{-n}) \right]^{1/4}, \end{aligned} \quad (5)$$

where $F_{\text{Stab}}(|U\rangle\langle U|) = F_{\text{Cliff}}(U)$ whenever $F_{\text{Cliff}}(U) > 1/2$. We believe this sandwich inequality is of independent interest.

Clifford Group Forms an Approximate Unitary Design for PPT Operators. To prove our lower bound on auxiliary-free single-copy Clifford testers in [Theorem 1.5](#), we analyze the ability of such testers to distinguish the t -fold Haar and Clifford twirls. Our key technical contribution here is a new structural statement about the Clifford group.

THEOREM 1.6 (CLIFFORD GROUP IS AN APPROXIMATE $t = o(n^{1/4})$ -DESIGN FOR PPT OPERATORS). *Let $\Phi_H^{(t)} = \mathbb{E}_{U \sim \mu_H} [U^{\otimes t}(\cdot)U^{\dagger, \otimes t}]$ be the t -fold Haar twirling channel and $\Phi_C^{(t)} = \mathbb{E}_{C \sim \text{Cl}(n)} [C^{\otimes t}(\cdot)C^{\dagger, \otimes t}]$ be the t -fold Clifford twirling channel. Then,*

$$\max_{\rho, M \in \text{PPT}} \left| \text{tr}(M \Phi_H^{(t)}(\rho)) - \text{tr}(M \Phi_C^{(t)}(\rho)) \right| \leq 2^{-n+O(t^4)}. \quad (6)$$

Here, PPT (positive partial transpose) denotes the set of operators that remain positive semidefinite under all partial transpositions across the t copies; in particular, this set includes all product and separable operators.

This result can be viewed as showing that the Clifford group forms an approximate unitary design when the distinguishability metric is restricted to PPT operators—a relaxation of the usual diamond norm. Apart from our main application in the single-copy lower bound, this result also finds an application in the recent work [\[27\]](#), which establishes an unconditional quantum–classical separation in memory usage. In their argument, a key step is to show that an anti-concentration-type quantity, $\mathbb{E}_{C \sim \text{Cl}(n)} |\langle \psi | C | 0^n \rangle|^{2t}$, is well-approximated by its Haar-averaged counterpart.

To show [Theorem 1.6](#), we analyze the commutant of the t -fold tensor action of the Clifford group, or Clifford commutant for short, defined as

$$\{A \in \mathcal{L}((\mathbb{C}^2)^{\otimes n})^{\otimes t} \mid [A, C^{\otimes t}] = 0, \forall C \in \text{Cl}(n)\}, \quad (7)$$

i.e., the space of operators that commute with all $C^{\otimes t}$ for $C \in \text{Cl}(n)$, which is precisely the subspace onto which the t -fold Clifford twirl projects. This commutant was characterized in the seminal work [\[22\]](#) in terms of a set of generators characterized by a class of self-dual binary error correction codes. Since we restrict the distinguishability metric to PPT operators, it is essential to understand the behavior of these commutant generators under partial transposition. In previous work [\[25\]](#), it was shown that every nontrivial generator of the Clifford commutant admits a non-unitary partial transpose. Here, we continue this line of study and complement it by showing that each generator also admits a unitary partial transpose², which can be found efficiently in the number of copies t . We believe this technical result to be of independent interest.

²A similar unitarity result was recently obtained in Ref. [\[7\]](#) using a completely different approach.

Our proof leverages the characterization of the Clifford commutant generators in terms of self-dual binary codes from [\[22\]](#) and establishes a connection to matroid theory: by viewing the generator matrices of these codes as matroids, we can apply matroid intersection results, most notably Rado’s theorem [\[31\]](#), to establish the existence of the desired partial transpose.

1.3 Outlook and Open Problems

Looking ahead, several open problems emerge naturally from our work. We outline these below:

Higher-Order Inverse Theorems. In the tolerant testing result of this paper, we mainly focus on the inverse theorem for the Q^3 norm. A natural future direction is to investigate inverse theorems for Q^k norms with $k \geq 4$.

Matching Upper and Lower Bounds. Currently, our query complexity upper and lower bounds for auxiliary-free single-copy Clifford testing are not matching (c.f. [Table 1](#)). Similar to the case of single-copy stabilizer testing [\[25\]](#), we expect that the lower bound may be tightened, possibly by refining the PPT relaxation or leveraging additional structure of the Clifford commutant.

Lower Bound for Auxiliary-Assisted Single-Copy Clifford Testing. Our proof strategy for lower bounding the query complexity of single-copy Clifford testing does not seem to extend to the auxiliary-assisted setting. It remains open whether the PPT relaxation technique we use, can be adapted to this regime, or whether new analytical tools are required to capture the query complexity in this access model.

Characterizing the Role of Inverse Access for Single-Copy Algorithms. While our focus here has been on the inverse-free access model, it remains of interest to understand how access to inverse queries, i.e. to $U^\dagger$, can help with single-copy Clifford testing (the multi-copy complexity has already been settled by [\[35\]](#)). Determining whether inverse access can asymptotically reduce the single-copy query complexity would clarify the operational difference between forward-only and reversible testing models.

Author Contributions

MH and JH conceived of the project and derived the main results. MH wrote the bulk of the manuscript, with input from JH, JB, PvD, and ZB contributed [Theorem 1.2](#) and [Theorem 1.3](#). All authors participated in finalizing the manuscript.

Full Version. The full version of this manuscript, including all proofs, can be found at [\[24\]](#).

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