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2 3	Do modeling choices matter for the reliability of individual difference measures in conflict tasks?
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11	Abstract
12	There is a growing realization that experimental tasks that produce reliable effects in group comparisons can simultaneously provide unreliable assess- ments of individual differences. Proposed solutions to this "reliability para- dox" range from collecting more test trials to modifying the tasks and/or the way in which effects are measured from these tasks. Here we systematically compare two proposed modeling solutions in a cognitive conflict task. Using the ratio of individual variability of the conflict effect (i.e., signal) and the trial-by-trial variation in the data (i.e., noise) obtained from Bayesian hier- archical modeling, we examine whether improving statistical modeling may improve the reliability of individual differences assessment in four Stroop datasets. The proposed improvements are 1) increasing the descriptive ad- equacy of the statistical models from which conflict effects are derived, and 2) using psychologically-motivated measures from cognitive models. Our results show that modeling choices do not have a consistent effect on the signal-to-noise ratio: the proposed solutions improved reliability in only one of the four datasets. We provide analytical and simulation-based approaches to compute the signal-to-noise ratio for a range of models of varying sophis- tication and discuss their potential to aid in developing and comparing new measurement solutions to the reliability paradox.

Keywords: reliability, cognitive conflict, Bayesian hierarchical modeling

Cognitive conflict is commonly assessed using experimental paradigms by comparing conditions requiring higher versus lower levels of cognitive control. One such paradigm is the classic color-Stroop task (Stroop, 1935), in which participants indicate the color in which a color-word is printed. The Stroop effect is conventionally measured by the response time (RT) difference between trials in which the printed color and the meaning of the word are different (i.e., incongruent; e.g., "red" written in blue) and trials in which they match (i.e., congruent; e.g., "red" written in

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red). Similar cognitive conflict paradigms with a congruency manipulation are the Flanker (Eriksen & Eriksen, 1974) and the Simon task (Simon & Rudell, 1967). In all of them, the incongruent condition requires more control to inhibit automatic associations between the wrong response and irrelevant information (e.g., color). Positive conflict effects, that is the RT difference between the incongruent and the congruent conditions, provide an index of cognitive control necessary to manage the conflict (Hommel, 2011; MacLeod, 1991; Ridderinkhof, Wylie, van den Wildenberg, & et al., 2021).

Conflict effects are highly robust at the population-level – in fact, the Stroop effect is even 26 considered universal (Haaf & Rouder, 2017; MacLeod, 1991). At the individual-level, however, the 27 picture is more complicated. While almost everyone seems to show a conflict effect, true individual 28 differences are assessed with a lot of uncertainty and are masked by measurement error. This sit-29 uation has been labeled the "reliability paradox" (Hedge, Powell, & Sumner, 2018): experimental 30 control, which has the desirable effect of increasing validity, also tends to reduce individual differ-31 ences, and hence decreases reliability (Keye, Wilhelm, Oberauer, & et al., 2009; Paap & Greenberg, 32 2013; Pettigrew & Martin, 2014; Rey-Mermet, Gade, & Oberauer, 2018). 33

To better understand reliability in experimental tasks, consider typical Stroop task data from I participants with two conditions (i.e., congruent and incongruent trials), and K trials per condition. For illustrative purposes, we use the simplest model possible, the normal-normal model. Individual i's mean RT differences between the conditions follow a normal population-level distribution,

$$\overline{Y}_{i\text{inc}} - \overline{Y}_{i\text{con}} \sim \text{Normal}(\mu_{\theta}, \sigma_{\theta}^2 + \frac{2\sigma^2}{K}),$$

with mean  $\mu_{\theta}$  and variance  $\sigma_{\theta}^2 + \frac{2\sigma^2}{K}$ , where  $\mu_{\theta}$  is the population mean of the conflict effect,  $\sigma_{\theta}$  the true individual variation and  $\sigma$  the measurement error<sup>1</sup>. By "true individual variation", we mean the extent to which people differ quantitatively in their conflict effect if we had an infinite number of observations. By measurement error, we mean the confounding variability that exists because participants' RTs vary across trials, for example due to motor and perceptual processes. Using hierarchical modeling, one can estimate both  $\sigma_{\theta}$  and  $\sigma$  as well as the population mean  $\mu_{\theta}$ .

Reliability is commonly defined as the ratio between true between-subjects variance  $\sigma_{\theta}^2$  and total variance, where the latter is the sum of true between-subjects variance in the effect of interest and an error term:

$$r = \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_E^2}.$$

<sup>1</sup>In this paper, we use the terms "variability" and "variation" interchangeably to refer to both variance and standard deviation (i.e., the square root of the variance)

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In experimental tasks with two main conditions of interest, such as the Stroop task, the error term is twice the squared standard error (see Rouder & Haaf, 2019, for more details),

$$\sigma_E^2 = \frac{2\sigma^2}{K},$$

 $_{50}$  where K is the number of trials performed per condition. Therefore, the reliability term becomes

$$r = \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \frac{2\sigma^2}{K}}.$$

Rouder and Haaf (2019) pointed out that to understand the reliability paradox it is important to acknowledge that reliability is not "portable": even for otherwise identical tasks, reliability differs with K. This is due the fact that as K grows, the standard error decreases and the reliability increases. Rouder, Kumar, and Haaf (2019) proposed that the suitability of a conflict task for individual-differences research instead be evaluated in terms of the signal-to-noise ratio (SNR)

$$\gamma = \frac{\sigma_{\theta}}{\sigma},$$

where the signal is the individual variability of the conflict effect,  $\sigma_{\theta}$ , and the noise is the trial-totrial variability in RT,  $\sigma$ . Using hierarchical modeling Rouder et al. (2019) found that the ratio,  $\gamma$ , was around 1/7 across a large set of conflict tasks, leading them to conclude that for conventional conflict tasks, K > 500 (i.e., over 1000 trials per participant) is required to obtain sufficiently reliable measurement.

Low reliability, and the impracticality of collecting that many trials, has spurred research 61 to develop more reliable ways of measuring cognitive control in general and cognitive conflict in 62 particular. One direction has been to abandon conflict measures based on RT differences, as taking 63 a difference doubles the measurement error (Draheim, Mashburn, Martin, & Engle, 2019). Instead, 64 it has been suggested that cognitive control should be assessed using accuracy-based, adaptive 65 tasks or the number of correct (in)congruent trials performed within a fixed time frame, with 66 confounding from overall speed partialed out. However, some of these approaches sacrifice the 67 validity of RT difference scores, which is evidenced by a large body of experimental literature, and 68 risk confounding from individual differences in processing speed (Kucina et al., 2023; Rouder et al., 69 2019). 70

Kucina et al. (2023) used an alternative approach to improve reliability. They used the SNR 71  $\gamma$  to compare, combine, and refine traditional conflict tasks. They also refined the RT difference 72 measure itself and took into account that RT distributions are well described by a shifted log-73 normal distribution (Heathcote & Love, 2012), improving the descriptive adequacy of the model 74 used. They reported SNRs for conflict effects in the range of 1/3 to 1/2, corresponding to clas-75 sic reliability values of 0.8 or higher, with only 100 trials. In related work, Haines et al. (2020) 76 found that modeling RT as lognormally distributed improved test-retest reliability in several con-77 flict datasets (Hedge, Powell, & Sumner, 2018), a delay-discounting task (Gawronski, Morrison, 78 Phills, & Galdi, 2017), and an Implicit Association Test (Ahn et al., 2020). More generally, Haines 79 et al. (2020) argued that simple atheoretical summaries, such as conflict effects based on mean 80 RT, are unable to validly characterize the underlying psychological processes and recommended a 81 "generative modeling approach" instead to overcome a "theory-description gap". Consistent with 82 this recommendation, Hedge, Powell, Bompas, Vivian-Griffiths, and Sumner (2018) demonstrated 83 that it can be important to simultaneously account for accuracy and RT because inconsistent re-84 lationships between accuracy-based and RT-based effects are widespread. Individual differences in 85 this speed-accuracy trade-off may potentially contribute to the unreliability of RT-based conflict 86

effects. Evidence-accumulation models can disentangle these confounding processes (Hedge, Powell, Bompas, et al., 2018).

In summary, the reliability paradox has triggered a wide range of responses in the community. The proposed solutions range from refining and reinventing traditional tasks and measures, to advocating for theoretically-motivated modeling approaches that provide psychologically meaningful measures of cognitive conflict and a more complete and descriptively more accurate account of choice RT performance. Here we investigate the degree to which different modeling choices influence the SNR.

## 95 Comparing the Reliability of Model-Based Conflict Effects

Does improving the descriptive adequacy of the model from which conflict effects are derived improve the SNR? Can a cognitive model that provides a psychologically meaningful characterization of performance detect relatively more signal in a given dataset than a purely statistical model? And to what extent does this depend on the dataset? We will use the SNR of conflict effects as measured by variants of the Stroop task to answer these questions.

Rouder and Mehrvarz (2024) suggested that the SNR can be used as an indicator for reliability that is independent of the trial size. To see the relationship between reliability and SNR, note that reliability can be re-expressed in terms of the ratio  $\gamma$ ,

$$r = \frac{\gamma^2}{\gamma^2 + \frac{2}{K}}.$$

In theory, the independence of the number of trials makes  $\gamma$  a portable indicator of reliability 104 (Rouder & Haaf, 2019). In practice, however, the SNR does not always prove to be portable. 105 Kucina et al. (2023) modeled the possibility that individual differences in conflict effect and trial-106 to-trial variability changes with K and computed the SNR as a function of the number of trials. 107 After de-trending to account for decreased mean RT with practice (Evans, Brown, Mewhort, & 108 Heathcote, 2018; Heathcote, Brown, & Mewhort, 2000), their results indicated that the between-109 subjects variability  $\sigma_{\theta}$  decreased as more trials were performed (i.e., individual differences decreased 110 with practice). Although this result raises doubts about the portability of  $\gamma$ , they showed that the 111 SNR can be useful for comparing reliability among tasks for a fixed K. Whether the SNR is portable 112 or not, however, does not affect its usefulness for our current purpose: comparing reliability among 113 models of the same task for a fixed K. 114

We investigate the effects of closing the "theory-description gap" (Haines et al., 2020) on 115 the SNR by progressively improving the descriptive adequacy and theoretical underpinning of the 116 models used to quantify the conflict effect. We examine how reliably various models can detect 117 individual differences in four Stroop datasets: we move from purely statistical models, such as 118 the normal or (shifted) lognormal distribution that provide a descriptive characterization of RT 119 distributions, to cognitive models, such as the lognormal race and the racing diffusion models that 120 simultaneously account for choice accuracy and RTs and that provide a psychologically meaningful 121 account of performance. We show how the SNR can be computed analytically for these five models 122 and outline a simulation-based approach that is more broadly applicable to any model<sup>2</sup>. 123

The main argument for more complex statistical models of the conflict effect is that descriptive adequacy may increase the SNR (Haines et al., 2020). A better fit means more accurate measurement of individual differences and the noise term, the components of the SNR. Additionally, cognitive models may further increase the SNR. By adding psychologically meaningful parameters, some variability that is attributed to noise in statistical models may be explained by cognitive

<sup>&</sup>lt;sup>2</sup>Any model with unbiased (e.g., maximum likelihood) estimators

models. Therefore, cognitive modeling may increase the SNR by decreasing its denominator, the
 noise term, which is critical for reliable assessment of individual differences in Stroop effects.

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#### Method

### <sup>132</sup> From the Normal Model to the Racing Diffusion Model

The five models employed are illustrated in Figure 1. The first three are purely RT-based 133 Bayesian hierarchical models that were also used by Haines et al. (2020). They are shown in in-134 creasing order of descriptive adequacy: a normal model, which makes the unrealistic assumption 135 that RTs are normally distributed and assigns probability to negative RTs, followed by two models 136 that account for the characteristic right skew of empirical RT distribution by assuming a lognormal 137 parametrization, one with and one without an estimated shift parameter. The shift parameter en-138 sures that the lower bound of the RT distribution is shifted away from zero, excluding unrealistically 139 fast RTs. 140

We then use two psychologically-grounded evidence-accumulation models, the lognormal race 141 model (LNR; Heathcote & Love, 2012) and the racing diffusion model (RDM; Tillman, Van Zandt, 142 & Logan, 2020), both providing a comprehensive account of performance by simultaneously ac-143 counting for response choices and the corresponding RTs (see also Matzke, Logan, & Heathcote, 144 2020). Racing evidence-accumulation models have a long history in psychology because they pro-145 vide a principled vet flexible approach to describe and explain performance in a broad range of 146 tasks (Heathcote & Matzke, 2022). Both the LNR and the RDM assume that decisions are made 147 by a process of accumulating evidence until a threshold amount is reached, which then triggers a 148 response. In particular, the models assume an independent race between a set of evidence accumu-149 lators, corresponding to the different response options. In terms of the color-Stroop task, this means 150 separate racers for each response option (e.g., left button for "green" and right button for "red") 151 that race against each other. The racer that wins determines the response. Individuals may start 152 accumulating evidence at different start points and/or set different thresholds to vary the trade-off 153 between the speed of their responses and the accuracy of their choices (Donkin & Brown, 2018). 154 For example, a smaller distance from the starting point of evidence accumulation to the response 155 threshold leads to faster but less accurate responses. Conversely, larger distances lead to slower 156 but more accurate responses. Similar to the shifted lognormal model, the cognitive models also 157 feature a "non-decision time" parameter that accounts for time spent to encode evidence from the 158 choice stimulus and to produce a response. This parameter shifts the finishing time distributions 159 and ensures that the lower bound of the RT distributions is greater than zero. 160

The LNR (Heathcote & Love, 2012) was chosen because it forms a natural extension of the 161 RT-only models by assuming a lognormal parameterization but also accounting for choice accuracy. 162 The model assumes that the rate of evidence accumulation and the distance between the start 163 point and threshold follow independent lognormal distributions. As a result, the time for a single 164 accumulator to reach its threshold (i.e., the finishing time) also follows a lognormal distribution. 165 When accuracy is perfect, the observed RT distribution follows a shifted lognormal distribution, 166 consistent with the use of this distribution by Haines et al. (2020). However, when performance is 167 not perfect, as is typically the case for Stroop data (see Table 1 for average error rates in the four 168 data sets we examine), the only RT-based shifted lognormal model is mis-specified. Here we use 169 the LNR model to assess whether the SNR improves when accuracy is also taken into account. 170

A limitation of the LNR model is that the rate of evidence accumulation and the distance from starting point to threshold are not separately identified because they combine linearly. As a result, the conflict effect of the LNR confounds individual differences in response cuation and the effects of congruency on the rate of evidence accumulation. The RDM (Tillman et al., 2020) addresses this challenge to validity by allowing us to separately estimate evidence-accumulation rates and response thresholds.

The RDM (Tillman et al., 2020) assumes that each of the accumulators is a Wiener diffusion 177 process with an evidence-accumulation rate v, a starting point of 0, and a response threshold  $\beta$ . 178 Trial-to-trial variability in behavior is caused by stochastic accumulation, that is, during accu-179 mulation, the evidence-accumulation rate changes from moment to moment due to the addition 180 of fractional Gaussian noise. The finishing time distribution of each accumulator is an inverse 181 Gaussian distribution (i.e., Wald distribution; Wald, 1947). Here we use the RDM to investigate 182 whether using psychologically-grounded measures of cognitive conflict as captured in the evidence-183 accumulation rate parameter improves the SNR. 184



Figure 1. Model overview. The plots show exemplary shapes of the corresponding response and finishing time distributions as assumed by the respective models. The lightgrey vertical line indicates response time (RT) = 0 on the x-axis. The **normal model** crosses this line, demonstrating that the normal model also allows for negative RTs. The lognormal model accounts for the fact that RTs can only be positive and that their distribution is typically skewed. The shifted lognormal model additionally accounts for the fact that RT distributions are shifted away from zero, whereby  $\psi$  indicates the shift parameter. The lognormal race model (LNR) additionally accounts for choice errors. The model conceptualizes decision-making as an independent race between evidence accumulators, one for each response option, until a threshold is reached which then triggers a response. Depicted are the finishing time distributions of the matching and mismatching accumulators (i.e., stimulus and response (mis)match). The dashed lines show observed distributions of the respective racers (i.e., those that won the race), scaled by their winning proportions.  $\psi$  indicates the non-decision time parameter. The racing diffusion model (RDM) separately estimates the evidence-accumulation rate and the threshold parameters. The yellow paths depict an exemplary race between the matching and mismatching accumulators. The matching accumulator (darker yellow) reaches the threshold  $\beta$  first, resulting in a correct response on that trial.  $\nu_1$  and  $\nu_2$  refer to the matching and matching evidence-accumulation rate parameters.

We deliberately chose to explore the performance of relatively standard evidenceaccumulation models as opposed to more complex models specifically developed for conflict tasks. Although conflict models such as the spotlight diffusion model (White, Ratcliff, & Starns, 2011) align more closely with the cognitive processes assumed to underlie performance in these tasks, they are limited in terms of tractability: the models grapple with pronounced parameter trade-offs and hierarchical estimation is not straightforward. As a result, despite their theoretical merits, they are less suitable as measurement models to examine individual differences in conflict tasks.

The full Bayesian hierarchical model specification can be found in the supplementary mate-192 rials. The three purely RT-based models, the normal, the lognormal, and the shifted lognormal 193 models, are parameterized such that the (log) means are decomposed into an intercept  $\alpha_i$  and a 194 conflict or congruency effect  $\theta_i$  for individual *i*. We allow the (log) standard deviation to vary 195 across individuals,  $\sigma_i$ , but not across conditions (i.e., congruent vs. incongruent). For the shifted 196 lognormal model, we also estimate individual shift parameters  $\psi_i$  which remain the same across 197 conditions. The LNR is parameterized such that the log mean of the accumulator that matches 198 the stimulus is decomposed into an intercept  $\alpha_{1i}$  and a conflict effect  $\theta_i$ . The corresponding log 199 standard deviation,  $\sigma_{1i}$ , is allowed to vary across individuals but again not across conditions. Due 200 to high accuracy rates in our analyzed datasets (see Table 1), we fix the log mean and log standard 201 deviation of the accumulator that mismatches the stimulus across individuals and estimate an in-202 tercept only and no conflict effect (i.e., we estimate  $\alpha_2$  and  $\sigma_2$ ). Just like in the shifted lognormal 203 model, we also estimate a non-decision time parameter for each individual,  $\psi_i$ . Lastly, for the RDM. 204 we decompose the matching evidence-accumulation rate into an intercept  $\alpha_{1i}$  and a conflict effect 205  $\theta_i$ . Note that we only model a conflict effect on the rate because we expect that the cognitive con-206 flict will impact the evidence accumulation rate at which participants respond to the (in)congruent 207 stimuli. In contrast, the individual response thresholds are set at the beginning of a trial and we 208 do not expect stimulus encoding or motor control processed to differ across conditions. Similar to 209 the LNR specification, we estimate an intercept only and do not allow for individual differences in 210 the mismatching evidence-accumulation rate,  $\alpha_2$  (see Lüken, Heathcote, Haaf, & Matzke, 2023). 211 Furthermore, we estimate individual response thresholds,  $\beta_i$ , and non-decision time parameters,  $\psi_i$ , 212 which were fixed across congruency conditions and accumulators. 213

### 214 Estimating Signal-to-Noise Ratios

All models have a designated parameter reflecting the numerator of the SNR,  $\sigma_{\theta}$  (i.e., individual differences in the conflict effect). However, this is not the case for its denominator. To determine the trial-by-trial variation, we estimate the standard error of the conflict effect. Note that in the case of the normal model, the standard error is simply  $\frac{2\sigma^2}{K}$  and that  $\sigma$  can be directly estimated using hierarchical modeling and interpreted on the real line (see Rouder & Haaf, 2019; Rouder et al., 2019).

For the normal model, we could use the formula provided by Rouder and Mehrvarz (2024) 221 to obtain a point estimate of  $\gamma$  from RT data. However, we prefer to use Bayesian hierarchical 222 modeling (Lee, 2011) for all models for several reasons. First, hierarchical models partition the 223 variability in data into between and within-participant components. Second, Bayesian estimation 224 naturally provides measures of uncertainty in  $\gamma$  estimates, providing a basis for inference about 225 the SNR. Third, Bayesian hierarchical models are well suited for estimating the parameters of 226 evidence-accumulation models such as the LNR and RDM, as well as the shift parameter of the 227 lognormal distribution when it is estimated (e.g., Heathcote et al., 2019; Stevenson et al., 2024). 228

Hierarchical modeling typically directly provides an estimate of the between-subjects variability of interest,  $\sigma_{\theta}$  for all models. The models just need to have parameters reflecting the difference

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between the congruent and incongruent conditions (i.e.,  $\theta_i$ ). For all but the RDM, the individ-231 ual noise parameters are also directly provided (i.e.,  $\sigma_i$  for the normal and the three lognormal 232 models):  $\sigma_i$ , the shape parameter of the lognormal distribution and standard deviation of the log-233 RTs, directly reflects the individual trial-by-trial variation. Moreover, within models,  $\sigma_{\theta}$  and  $\sigma_i$ 234 are on the same scale (i.e., on the real line in the normal model and on the log scale in the three 235 lognormal models). Therefore, the units cancel out, making the ratios comparable to each other. 236 Since we assume individual noise parameters, we need to average estimates of  $\sigma_i$  across individuals, 237 i = 1, ..., I: 238

$$\gamma = \frac{\hat{\sigma}_{\theta}}{\sqrt{\frac{1}{I}\sum_{i=1}^{I}\hat{\sigma}_{i}^{2}}} \tag{1}$$

Finding an expression for the noise term  $\sigma_i$  in the RDM is more complicated. Contrary to the 239 other models, trial-by-trial variation is not explicitly represented as a model parameter. Besides the 240 hierarchical structure that we place on top of the model parameters, our specification of the RDM 241 has one inherent source of variability: within-trial variation of the drift rate, which is described 242 by a Wiener diffusion process (Tillman et al., 2020). However, this parameter reflects variation 243 within trials and is conventionally set to 1 to make the model identifiable. To get an estimate of 244 between-trial variation or measurement error, the standard error of the conflict effect needs to be 245 determined. Let  $\hat{\nu}_{i_{con}}$  and  $\hat{\nu}_{i_{inc}}$  be individual *i*'s matching evidence-accumulation rate estimates in 246 the congruent and incongruent condition such that  $\Delta_{\hat{\nu}_i} = \hat{\nu}_{i_{con}} - \hat{\nu}_{i_{inc}}$  represents the conflict effect. 247 To get an expression for the noise, we need to compute the variance  $V(\Delta_{\hat{\nu}_i})$ . An expression for this 248 variance can be derived as follows: 240

$$\begin{split} \mathbf{V}(\hat{\nu}_{i_{con}} - \hat{\nu}_{i_{inc}}) &= \mathbf{V}(\hat{\nu}_{i_{con}}) + \mathbf{V}(\hat{\nu}_{i_{inc}}) - 2\mathbf{Cov}(\hat{\nu}_{i_{con}}, \hat{\nu}_{i_{inc}}) \\ \mathbf{V}(\hat{v}) &= (\mathbf{E}(\hat{v}))^2 - \mathbf{E}(\hat{v}^2) \\ \mathbf{E}(\hat{v}) &= \sqrt{K} \frac{\Gamma(K - 3/2)}{\Gamma(K - 1)} \left(\frac{1}{K\beta} + v\right) \\ \mathbf{E}(\hat{v}^2) &= \frac{K}{K - 2} \left(v^2 + \frac{3}{K^2\beta^2} + \frac{3v}{K\beta}\right) \\ \mathbf{E}(\hat{v}_1 \hat{v}_2) &= \frac{K}{K - 2} \left(\frac{1}{K\beta} + \nu_1\right) \left(\frac{1}{K\beta} + \nu_2\right) \\ \mathbf{Cov}(\hat{v}_1, \hat{v}_2) &= \mathbf{E}(\hat{v}_1 \hat{v}_2) - \mathbf{E}(\hat{v}_1)\mathbf{E}(\hat{v}_2) \end{split}$$

Note that  $\Gamma$  is the gamma function. The proof is provided in the supplementary materials. This effect variance reflects the standard error  $\frac{\sqrt{2}\sigma_i}{\sqrt{K_i}}$ , which can be transformed to get an estimate of the individual noise terms  $\sigma_i$ :  $\sqrt{\frac{V(\Delta_{\nu_i})K_i}{2}}$  where  $K_i$  is the number of trials per condition that individual *i* performed. Averaging across individuals, the SNR for the RDM is:

$$\gamma = \frac{\hat{\sigma}_{\theta}}{\sqrt{\frac{1}{I}\sum_{i=1}^{I}\frac{V(\Delta_{\dot{\nu}_{i}})K_{i}}{2}}}.$$
(2)

In order to capture uncertainty in  $\gamma$  estimates, we calculate the ratio for all available posterior samples of the individual RDM parameters.

The standard error of the effect of interest within a model can also be computed using a simulation-based approach. The details are provided in the supplementary materials. The advantage of the simulation-based approach is that it is more broadly applicable than an analytical expression which is specific to the particular model. However, the simulation-based approach Table 1

Descriptive statistics (mean RT and accuracy), number of participants (I), number of trials per congruency condition (K), and task type.

	RT (SD)	Accuracy	Ι	K	Task type
Enkavi et al. (2019)	.73 (.21)	.96	522	48	Color Stroop
Pratte et al. $(2010)$	.77(.37)	.96	38	164	Color Stroop
Rey-Mermet et al. $(2018)$	.59(.15)	.96	128	95	Number Stroop
Von Bastian et al. $(2016)$	.75 $(.25)$	.97	121	48	Number Stroop

requires an unbiased maximum-likelihood estimator for the quantity of interest, which may not always be available. For the RDM, this quantity is the evidence-accumulation rate, which has such a maximum-likelihood estimator (i.e., the natural estimator; see supplementary materials).<sup>3</sup>

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#### Results

We applied the five models to four datasets: number-Stroop tasks by (Rey-Mermet et al., 2018, younger age group) and by Von Bastian, Souza, and Gade (2016), color-Stroop tasks by Pratte, Rouder, Morey, and Feng (2010) and by Enkavi et al. (2019). We only analyzed the (in)congruent conditions and retrieved all datasets from the preprocessed database made available by Haaf, Hoffstadt, and Lesche (2024).

Before estimation, we removed the first three trials of every block, neutral trials, and RTs 269 <= 0.25 seconds. Two studies (Enkavi et al., 2019; Rey-Mermet et al., 2018), used response 270 windows to ensure fast responding. This approach lead to somewhat bimodal RT distributions 271 and convergence problems for some models. We therefore removed very slow responses, specifically 272 responses that were at the upper limit of the response window (Rey-Mermet et al., 2018: RT 273 >= 1.9 s; Enkavi et al., 2019: RT > 1.5 s). Pratte et al. (2010) did not have a response 274 window, but the very long RTs caused some convergence problems, so we excluded them (RT 275 >= 3 s). We also excluded participants with fewer than 50% remaining trials. Overall, only 276 few trials and participants were excluded (von Bastian et al., 2016: 0.9%, Rey-Mermet et al., 277 2018: 5% of the observations including all observations from one participant due to having 278 less than 50% trials after data cleaning; Pratte et al., 2010: 5%; Enkavi et al., 2019: 4% of the 279 observations including all observations from one participant due to having less than 50% trials after 280 data cleaning). For the fitting of the (shifted log)normal models, we also removed all incorrect trials. 281 282

We estimated all models using stan(Stan Development Team, 2023)<sup>4</sup> using weakly informative priors based on prior predictive checks and known plausible ranges of RTs in seconds<sup>5</sup>. The descriptive adequacy of the normal model was consistently worse than for the other four models

<sup>5</sup>See the supplementary materials for the prior specification, estimation results, and the detailed posterior predictive assessment of the absolute descriptive adequacy of the models (i.e., the match between the observed response time distributions and the posterior predictions).

<sup>&</sup>lt;sup>3</sup>There exists also a straightforward method-of-moments estimator for the evidence-accumulation rate, but this is not a maximum-likelihood estimator and is biased (see the supplementary materials).

<sup>&</sup>lt;sup>4</sup>We used the following R packages: R (Version 4.2.1; R Core Team, 2022) and the R-packages *DBI* (Version 1.1.3; R Special Interest Group on Databases (R-SIG-DB), Wickham, & Müller, 2022), *dplyr* (Version 1.1.3; Wickham, François, Henry, Müller, & Vaughan, 2023), *extraDistr* (Version 1.9.1; Wolodzko, 2020), *ggplot2* (Version 3.4.3; Wickham, 2016), *papaja* (Version 0.1.1; Aust & Barth, 2022), *patchwork* (Version 1.1.3; Pedersen, 2023), *RSQLite* (Version 2.3.1; Müller, Wickham, James, & Falcon, 2023), *rstan* (Version 2.21.5; Stan Development Team, 2022), *scales* (Version 1.2.1; Wickham & Seidel, 2022), *StanHeaders* (Version 2.21.0.7; Stan Development Team, 2020), *stringr* (Version 1.5.0; Wickham, 2022), *tidybayes* (Version 3.0.2; Kay, 2022), *tidyr* (Version 1.3.0; Wickham, Vaughan, & Girlich, 2023), and *tinylabels* (Version 0.2.3; Barth, 2022).

because the normal model cannot account for the characteristic skewness of RT data (see Figure 286 2). The shifted lognormal, the LNR, and the RDM showed good descriptive adequacy, however the 287 RDM and the LNR both had some difficulties capturing slow errors in the incongruent condition, 288 particularly for the dataset by (Rey-Mermet et al., 2018, see the Appendix and supplementary 289 materials). To quantify descriptive adequacy, we computed the root mean squared error (RMSE) 290 for the correct RTs and in the case of the LNR and RDM, additionally for the proportion of correct 291 responses<sup>6</sup>. We simulated 500 datasets from the posterior predictive distributions and computed 292 the RMSE for each of the simulated datasets and several quantiles (see Table 2 and 3 in the Ap-293 pendix). For most datasets, RMSE steadily decreased for the first three models and the RMSE for 294 295 the LNR tended to be smaller than for the RDM.



*Figure 2*. Observed (black) and predicted (grey) correct response time distributions collapsed across participants and conditions for the dataset by Von Bastian et al. (2016). The grey lines represent 500 samples from the posterior predictive distribution. The lognormal race and the racing diffusion model distributions also contain the incorrect responses.

We then computed the SNRs by plugging the parameter posterior samples into Equations (1) and (2) (see Figure 3). Note that the simulation-based approach leads to approximately the same result<sup>7</sup>. Within datasets, the credible intervals overlap substantially, suggesting that the choice of the model does not substantially influence the ratio in the analyzed datasets. The only exception is the Von Bastian et al. (2016) dataset, where the posterior median increases from below  $\frac{1}{10}$  for the normal model to around  $\frac{1}{5}$  for the RDM. Notably, the SNRs of the dataset by Enkavi et al. (2019) are considerably higher than those of the other datasets.

<sup>&</sup>lt;sup>6</sup>We did not use model selection criteria such as the deviance information criterion (DIC) due to the fact that the models are not comparable as they were fitted to different data (i.e., the (shifted log)normal models were fitted to correct responses only).

<sup>&</sup>lt;sup>7</sup>see the supplementary materials for the comparisons



Figure 3. Posterior medians and 95% credible interval of the signal-to-noise ratios  $\gamma$  as computed by the analytical method.



*Figure 4*. Posterior medians and 95% credible intervals of the numerator (i.e., signal) and denominator (i.e., noise) of the signal-to-noise ratios.

Looking at the components of the SNRs separately (Figure 4), the ratio differences across 303 datasets and within models seem to be driven by both the extent of individual differences in the 304 Stroop effect (i.e., signal) and by trial-by-trial variation (i.e., noise). Note that the credible interval 305 (CrI) of the noise term are narrower than those of the signal term because the former is estimated 306 from all observations. In contrast, the CrI of  $\sigma_{\theta}$  differ in width across datasets as more participants 307 leads to more precise estimation (see Table 1). Across models, the order of the terms stays largely 308 the same – with two exceptions. (1) Moving from the normal model to the RDM for the data by 309 Von Bastian et al. (2016), we gain relatively more signal, though note that the scales of the signal 310 and the noise terms are not directly comparable. Within the components of the other datasets, 311

there does not appear to be a dominating signal or noise term across models, which is not surprising given that the CrI of the SNRs are overlapping.

314

#### Discussion

Experimental tasks that produce robust group-level differences can simultaneously produce 315 unreliable individual differences. Proposed solutions to this "reliability paradox" (Hedge, Powell, 316 & Sumner, 2018) include collecting more trials per participant, modifying the cognitive tasks, 317 using more descriptively adequate models, and using psychologically-motivated measures derived 318 from cognitive models to quantify individual differences in the construct of interest. Here we 319 focused on the last two solutions, and built on the work by Rouder and Mehrvarz (2024) and 320 Rouder et al. (2019) by using signal-to-noise ratios (SNRs) as a tool for comparing models of 321 varying sophistication. The SNR is a useful measure of reliability quantifying how well individual 322 differences can be detected relative to measurement error. We provided analytical solutions to 323 compute the ratio for five models from the output of Bayesian hierarchical modeling: the normal, 324 lognormal, shifted lognormal, lognormal race (LNR), and racing diffusion models (RDM). Moreover, 325 we provided a general algorithm that - whenever an unbiased estimator of the quantity of interest 326 is available - can be used to compute the SNR for any model. We then applied the five models to 327 four Stroop datasets. 328

Our analysis showed that modeling choices do not have a consistent effect on the SNR: the 329 proposed solutions improved reliability in only one of the four datasets, and we found no consistent 330 ordering or pattern as to which model yielded the highest SNR across datasets. Notably, improved 331 descriptive accuracy did not correspond to a higher SNR. The normal model is clearly unable 332 to accommodate the slow tail of typical RT distributions, yet the corresponding SNRs are not 333 consistently worse than those of the RDM or lognormal models that can account for the skewness 334 in observed RTs. These differences did not appear to be explained by an effect of trial number or 335 sample size. Perhaps a more systematic assessment of more datasets might help identify dataset 336 or task characteristics that are predictive of higher SNRs for one model over another. As of now, 337 the preferred model can only be established after fitting all models using hierarchical modeling and 338 computing the SNRs. 339

In our analyses, we only looked at a subset of potential models. For example, other relevant 340 models are the diffusion decision model (Ratcliff & McKoon, 2008; Ratcliff & Smith, 2004) or 341 the linear ballistic accumulator model (Heathcote & Love, 2012). However, our approach can be 342 extended to these models. In both cases the trial-by-trial variation is not directly reflected in 343 a model parameter. Yet, there are two ways of computing the measurement error term of the 344 SNR: (1) one can try to derive it analytically by computing the variance of the effect of interest 345 or standard error, as we have done for the RDM. (2) one can use a simulation-based approach if 346 unbiased maximum likelihood estimators are available. 347

The focus of this paper is the reliability of measures. However, the validity of a measure 348 (i.e., its ability to measure the intended construct) is also important. Cognitive model parameters 349 have a clear psychological interpretation and so intrinsically support valid inferences as long as the 350 model accurately represents the data and the process that generates it. So, even if a particular 351 model parameter might not lead to a higher SNR, it might still be a better measure. For instance, 352 suppose we analyze a dataset in which speed-accuracy trade-offs are present. Because the LNR or 353 RDM can account for such trade-offs by combining information about accuracy and speed they are 354 likely to provide more valid measures. However, estimating a model providing a valid psychological 355 account may also require particular qualities in the data and the design from which it comes. For 356 example, Lüken et al. (2023) showed that low error rates compromised the quality of estimates of 357

the parameters of the diffusion decision model and the linear ballistic accumulator. It is possible that the low error rates in the datasets we analyzed here may have have had the same effect on the LNR and RDM, and that is the reason why they did not consistently perform better than the rest.

Clearly, both reliability and validity need to be taken into account whenever researchers attempt to answer substantive questions about the nature of individual differences in cognitive control. Descriptive adequacy is also important, as a model that clearly misfits the data is unlikely to be valid. However, good fit alone does not ensure validity, the model must also provide a sensible account of psychological processes that could plausibly generate the data, preferably one that is backed up by converging evidence from prior literature (i.e., the same type of model has provided accurate and coherent accounts of data from related tasks and manipulations).

In sum, the SNR can be used as a tool to identify the statistical or cognitive model that is best suited to examine individual differences in conflict data. We have provided analytical and simulation-based approaches to compute the SNR for a range of models of varying sophistication and showed that models that provide a better, and potentially more valid, description do not necessarily consistently improve the reliability with which individual differences in cognitive conflict are measured. Hence, we recommend that our methodology be deployed on a case-by-case basis to assess the effects of model choices on reliability.

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### Appendix

<sup>500</sup> Below, we present plots of the model fits (observed and predicted cumulative distribution <sup>501</sup> functions) and tables displaying the RMSE for all models and datasets. Additional figures assessing <sup>502</sup> the fits can be found in the supplementary materials.

### 503 Posterior predictive checks



Figure 5. Observed and predicted cumulative distribution functions (CDFs) for Enkavi et al. (2019). Observed = pink, blue = predicted, RDM = racing diffusion model, LNR = lognormal race model, SLN = shifted lognormal model, LN = lognormal model, N = normal model. The predictions are based on 500 sampled datasets from the posterior predictive distribution and plotted is the median including 95% credible interval. The points show the 10%, 30%, 50%, 70%, and 90% quantiles averaged across participants, separately for the two congruency conditions. Note that for the RDM and the LNR, both the (defective) CDFs of correct and incorrect responses are shown, whereas for the other models, only the CDFs of correct responses are depicted.

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Figure 6. Observed and predicted cumulative distribution functions (CDFs) for Rey-Mermet et al. (2018). Observed = pink, blue = predicted, RDM = racing diffusion model, LNR = lognormal race model, SLN = shifted lognormal model, LN = lognormal model, N = normal model. The predictions are based on 500 sampled datasets from the posterior predictive distribution and plotted is the median including 95% credible interval. The points show the 10%, 30%, 50%, 70%, and 90% quantiles averaged across participants, separately for the two congruency conditions. Note that for the RDM and the LNR, both the (defective) CDFs of correct and incorrect responses are shown, whereas for the other models, only the CDFs of correct responses are depicted.



Figure 7. Observed and predicted cumulative distribution functions (CDFs) for Pratte et al. (2010). Observed = pink, blue = predicted, RDM = racing diffusion model, LNR = lognormal race model, SLN = shifted lognormal model, LN = lognormal model, N = normal model. The predictions are based on 500 sampled datasets from the posterior predictive distribution and plotted is the median including 95% credible interval. The points show the 10%, 30%, 50%, 70%, and 90% quantiles averaged across participants, separately for the two congruency conditions. Note that for the RDM and the LNR, both the (defective) CDFs of correct and incorrect responses are shown, whereas for the other models, only the CDFs of correct responses are depicted.



Figure 8. Observed and predicted cumulative distribution functions (CDFs) for Von Bastian et al. (2016). Observed = pink, blue = predicted, RDM = racing diffusion model, LNR = lognormal race model, SLN = shifted lognormal model, LN = lognormal model, N = normal model. The predictions are based on 500 sampled datasets from the posterior predictive distribution and plotted is the median including 95% credible interval. The points show the 10%, 30%, 50%, 70%, and 90% quantiles averaged across participants, separately for the two congruency conditions. Note that for the RDM and the LNR, both the (defective) CDFs of correct and incorrect responses are shown, whereas for the other models, only the CDFs of correct responses are depicted.

# 504 Goodness of fit (RMSE)

## Table 2 $\,$

Median and 95 % credible interval of the root mean squared error based on the 1%, 10%, 30%, 50%, 70%, 90%, and 99% quantiles of the correct response times and 500 samples from the posterior predictive distribution.

	Enkavi et al. $(2019)$	Pratte et al. $(2010)$	Rey-Mermet et al. $(2018)$	Von Bastian et al. (2016)
Ν	$0.054 \ [0.051, \ 0.057]$	$0.168 \ [0.152, \ 0.184]$	$0.069 \ [0.064, \ 0.073]$	$0.126 \ [0.116, \ 0.136]$
LN	$0.011 \ [0.009, \ 0.013]$	$0.073 \ [0.045, \ 0.100]$	$0.054 \ [0.047, \ 0.059]$	$0.086 \ [0.071, \ 0.100]$
$\operatorname{SLN}$	$0.019 \ [0.013, \ 0.024]$	$0.016 \ [0.006, \ 0.049]$	$0.027 \ [0.018, \ 0.035]$	$0.033 \; [0.013,  0.051]$
LNR	$0.016 \ [0.011, \ 0.021]$	$0.018 \ [0.007, \ 0.048]$	$0.031 \ [0.023, \ 0.038]$	$0.031 \ [0.023, \ 0.038]$
RDM	$0.025 \ [0.018, \ 0.031]$	$0.049 \ [0.018, \ 0.077]$	$0.027 \ [0.019, \ 0.034]$	$0.027 \ [0.019, \ 0.034]$

# Table 3

Median and 95 % credible interval of the root mean squared error computed on the proportion of correct responses using 500 samples from the posterior predictive distribution.

	Enkavi et al. $(2019)$	Pratte et al. $(2010)$	Rey-Mermet et al. $(2018)$	Von Bastian et al. $(2016)$
LNR	$0.001 \ [0.000, \ 0.003]$	$0.002 \ [0.000, \ 0.005]$	$0.001 \ [0.000, \ 0.004]$	$0.002 \ [0.000, \ 0.005]$
RDM	$0.001 \ [0.000, \ 0.003]$	$0.011 \ [0.006, \ 0.016]$	$0.001 \ [0.000, \ 0.004]$	$0.006 \ [0.002, \ 0.010]$